# LAB 3

VATTIKUTI SAI ANVITH – IMT2018528

2 September 2020

## CONTENTS



## 1 THEORY

- **Amplitude Spectrum:**
  It gives the variation of amplitude of the signal at each frequency. Fourier Transform is applied on the given signal to convert the signal from time-domain into frequency-domain.

- **Phase Spectrum:**
  It gives the variation of phase at each frequency. Fourier Transform is applied on the given signal to convert the signal from time-domain into frequency-domain.

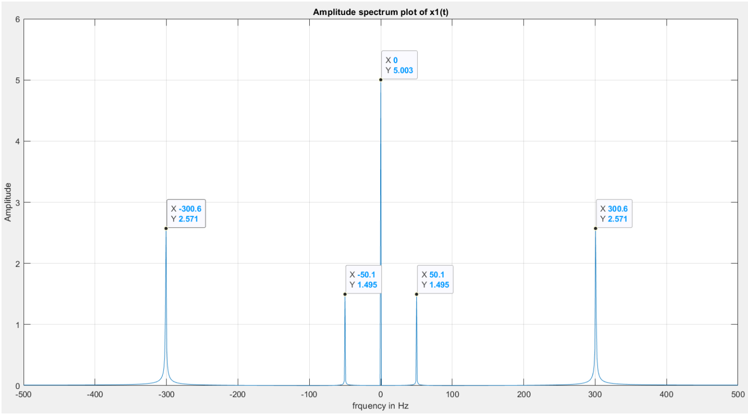
- **White Gaussian Noise:**
  If each sample has a normal distribution with zero mean, then that signal is said to be a Gaussian White noise.

# 2 QUESTION 1

1. Consider a signal $x_1(t) = 5 + 3\cos\left(2\pi 50t + \frac{\pi}{8}\right) + 6\cos\left(2\pi 300t + \frac{\pi}{2}\right)$
   a) Plot the amplitude spectrum of the signal $x_1(t)$.
   b) Plot the phase spectrum of the signal $x_1(t)$.

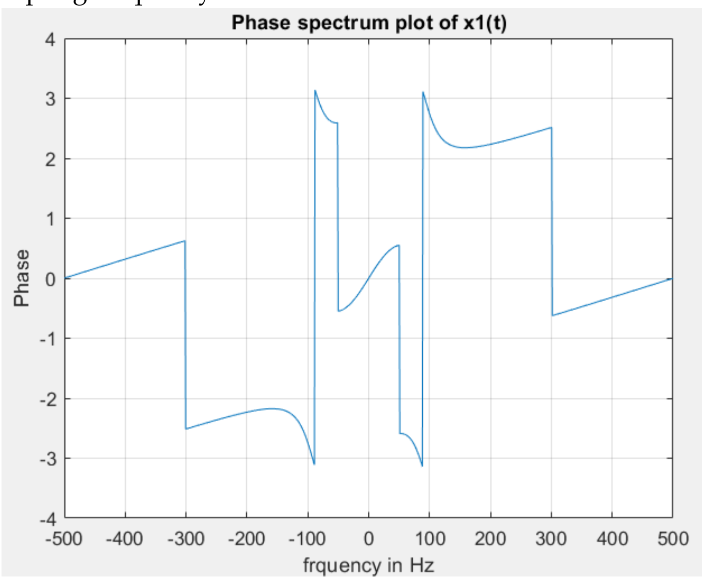
## 2.1 Plotting the amplitude spectrum of the signal x1(t)

Sampling frequency of 1000Hz is used.




## 2.2 Plotting the phase spectrum of the signal x1(t)

Sampling frequency of 1000Hz is used.




## 2.3 Observations

### 2.3.1 *Section A*

After plotting the amplitude spectrum of x1(t), 5 peaks were observed at 0Hz, ±50.1Hz and ±300.6Hz frequencies with amplitude of 5, 1.5 and 2.57 respectively. These peaks correspond to the 0Hz(DC) and 50Hz, 300Hz frequency terms in the summation. The amplitudes correspond nearly to half times the factor of cosine terms. It can also be noted that when I decrease the sampling frequency, the peak values are maintained. Also we could note the peaks get broader near 0 Amplitude. The frequency range on plot also decreases.

#### 2.3.2 *Section B*

After plotting the phase spectrum of x1(t), It can also be noted that when I decrease the sampling frequency, the frequency range on plot decreases. Unlike the amplitude spectrum, here I observed that both the frequency and phase angle at peaks is increased.

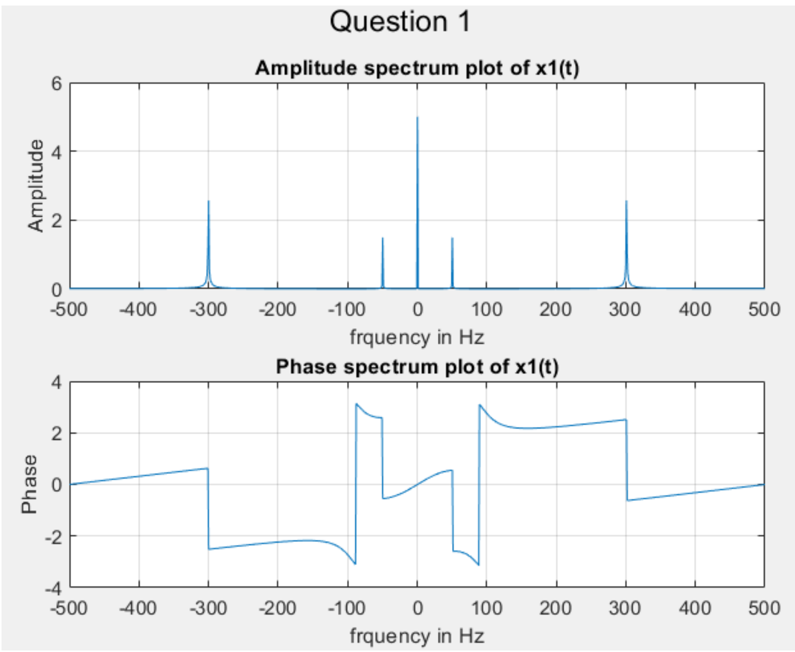
### 2.4 Code

```
close all; clc;
sgtitle('Question 1')

fs = 1000;
t = linspace(0,1,fs-1);

a = 5;
b = 3*cos((2*pi*50*t)+(pi/8));
c = 6*cos((2*pi*300*t)+(pi/2));
x1 = a+b+c;
dft_x1 = fftshift(fft(x1))/length(fft(x1));
freq = linspace(-fs/2,fs/2,fs-1);
%% Amplitude spectrum plot
subplot(2,1,1);
plot(freq, abs(dft_x1)); grid ON
xlabel('frquency in Hz'); ylabel('Amplitude');
title("Amplitude spectrum plot of x1(t)")
%% Phase spectrum plot
subplot(2,1,2);
plot(freq,(angle(dft_x1))); grid ON
xlabel('frquency in Hz'); ylabel('Phase');
title("Phase spectrum plot of x1(t)")
```

### 2.5 Output

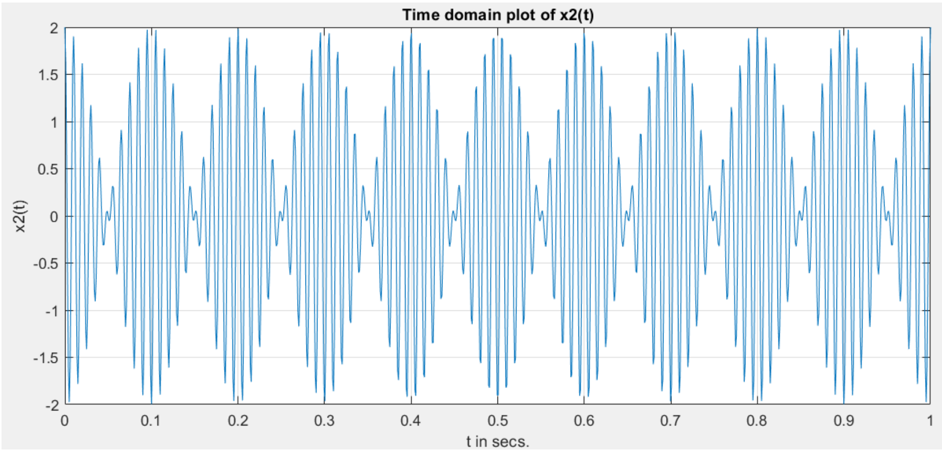


# 3 QUESTION 2

2. Consider a signal $x_2(t) = 2\cos(2\pi 5t) * \cos(2\pi 100t)$.
   a) Plot the time domain representation of the signal $x_2$(t).
   b) Plot the amplitude spectrum of the signal $x_2$(t).

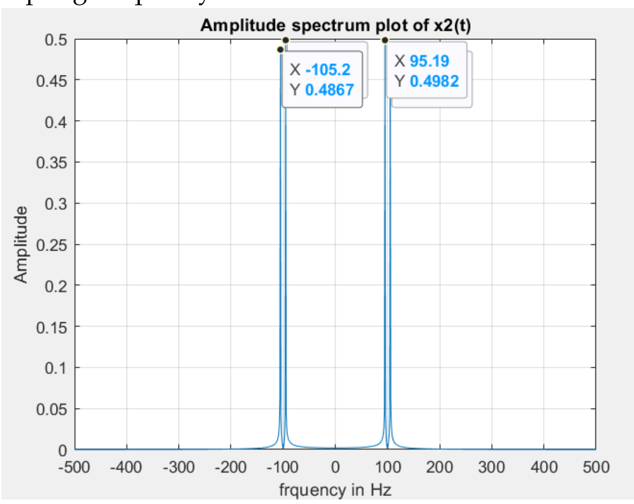
## 3.1 Plotting the time domain representation of x2(t)

Sampling frequency of 1000Hz is used.




## 3.2 Plotting the Amplitude spectrum of x2(t)

Sampling frequency of 1000Hz is used.




## 3.3 Observations

### 3.3.1 *Section A*

From the time domain plot of x2(t) I see that the output multiplication of the 2 signals has a time period which appears to be 0.2 seconds. The first signal in multiplication has time period of 0.2 and seconds one has time period of 0.01 seconds. Now changing the given function into sum of 2 cosines, we get to find that the actual period is LCM(1/105, 1/95) seconds. This is almost 71607.5 seconds.

### 3.3.2 *Section B*

After plotting the amplitude spectrum of x2(t), 4 peaks were observed at ±95.19Hz and ±105.2Hz frequencies with amplitude of 0.498 and 0.486 respectively. We get these peaks because when written as sum of 2 cosines, we get the same corresponding frequencies. The amplitudes correspond to half of the amplitudes of the cosines. It can also be noted that when I decrease the sampling frequency, the peak values are maintained. However, the peaks get broader near 0 Amplitude. The frequency range on plot also decreases.

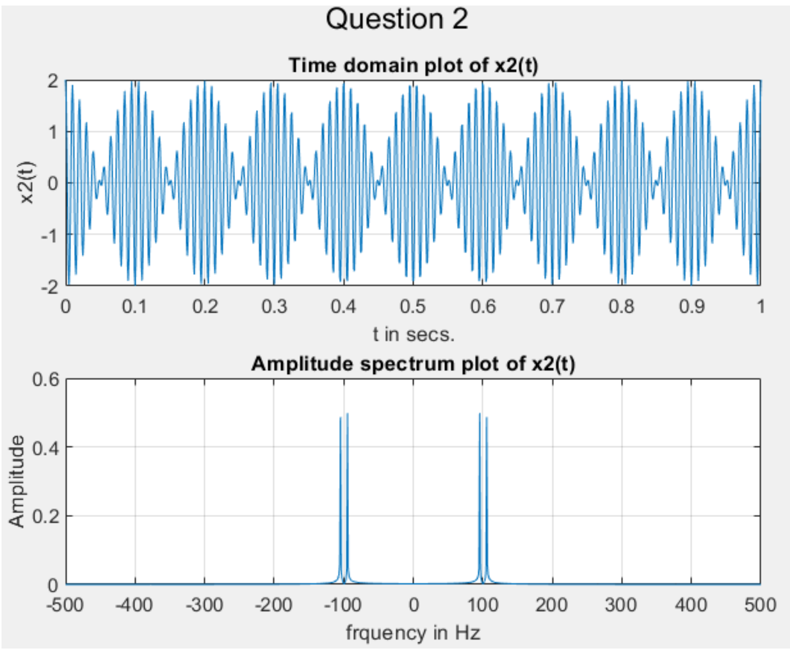
## 3.4 Code

```
close all; clc;
sgtitle('Question 2')

fs = 1000;
t = linspace(0,1,fs-1);

a = 2*cos(2*pi*5*t);
b = cos(2*pi*100*t);
x2 = a.*b;
dft_x2 = fftshift(fft(x2))/length(fft(x2));
freq = linspace(-fs/2,fs/2,fs-1);
%% Time domain plot
subplot(2,1,1);
plot(t, x2); grid ON
xlabel("t in secs."); ylabel("x2(t)");
title("Time domain plot of x2(t)")
%% Amplitude spectrum plot
subplot(2,1,2);
plot(freq,abs(dft_x2)); grid ON
xlabel('frquency in Hz');ylabel('Amplitude');
title("Amplitude spectrum plot of x2(t)")
```

## 3.5 Output

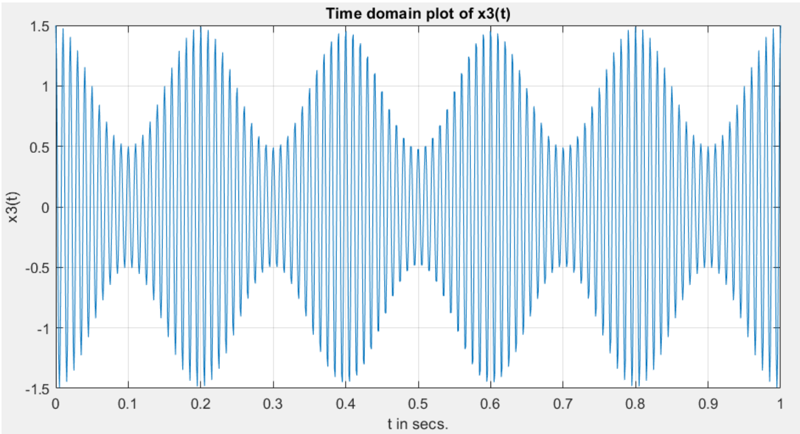


# 4 QUESTION 3

3. Consider a signal $x_3(t) = (1 + 0.5\cos(2\pi 5t)) * \cos(2\pi 100t)$.
   a) Plot the time domain representation of the signal x3(t).
   b) Plot the amplitude spectrum of the signal x3(t).
   c) Compare the results obtained in 2(b) and 3(b). Write your observations.
   d) Corrupt the signal x3(t) with white gaussian noise signal having zero mean and variance=9. Consider the length of noise signal same as the length of signal x3(t).
   e) Plot the corrupted signal in time domain.

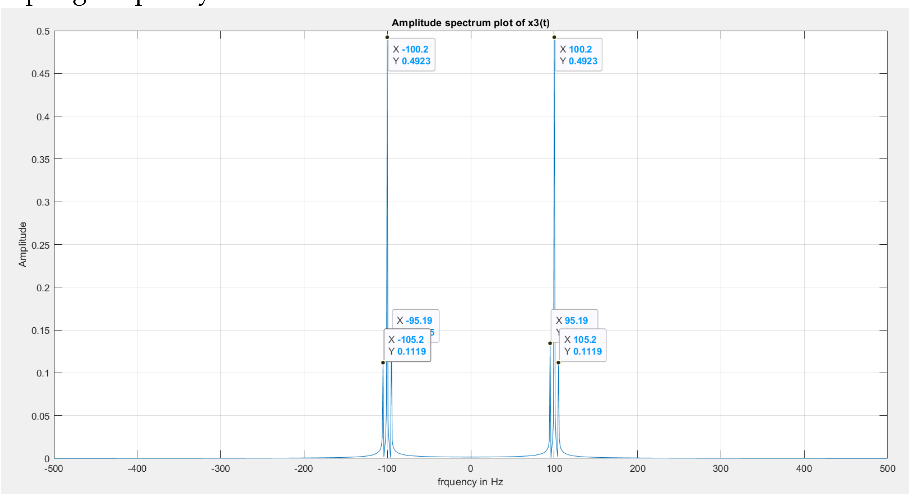
## 4.1 Plotting the time domain representation of the signal x3(t)

Sampling frequency of 1000Hz is used.




## 4.2 Plotting the amplitude spectrum of the signal x3(t)

Sampling frequency of 1000Hz is used.

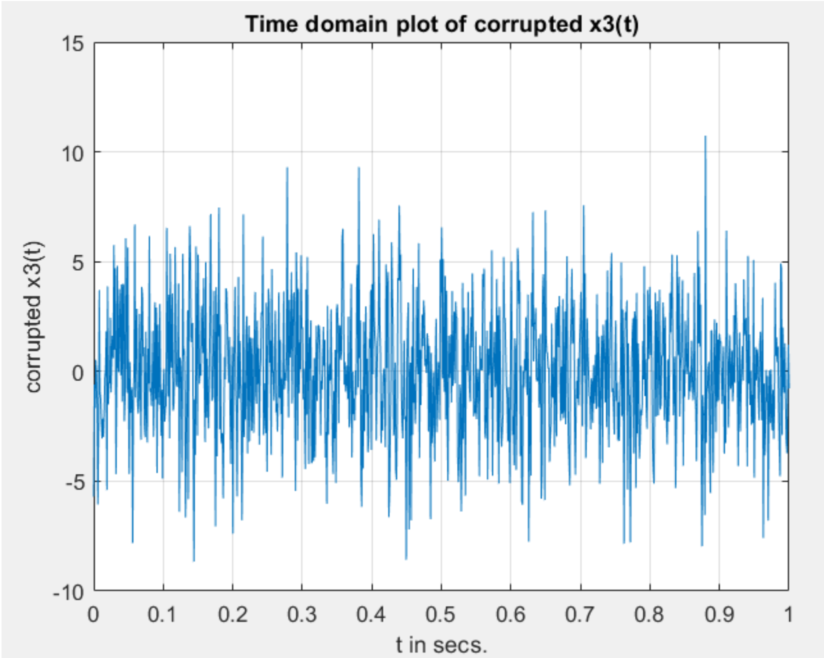



## 4.3 Comparing the results obtained in 2(b) and 3(b)

In 2(b) we had 4 peaks at ±95.19Hz and ±105.2Hz frequencies. Whereas in 3(b) we had 6 peaks at ±95.19Hz, ±105.2Hz and ±100.2Hz. Observations are recorded below in observations section.

### 4.4 Plotting the time domain representation of the corrupted signal x3(t)

Sampling frequency of 1000Hz is used.




### 4.5 Observations

#### 4.5.1 *Section A*

This signal is a combination of signal x2(from previous question) with the amplitude multiplied to 0.5 instead of 2 and the other signal being cos(200*pi*t) multiplied with 1. This signal has time period of LCM(71607.5,0.1) seconds, which makes the time period of the signal x3 again 71607.5 seconds.

#### 4.5.2 *Section B*

After plotting the amplitude spectrum of x3(t), 6 peaks were observed at ±95.19Hz, ±105.2Hz and ±100.2Hz frequencies with amplitude of 0.1345, 0.1119 and 0.4923 respectively. We get these peaks because when written as sum of cosines, we get the same corresponding frequencies(95, 105 and 100Hz). The amplitudes correspond to half of the amplitudes of the cosines.It can also be noted that when I decrease the sampling frequency, the peak values are maintained. However, the peaks get broader near 0 Amplitude. The frequency range on plot also decreases.

#### 4.5.3 *Section C*

I've observed in these 2 plots that they both share 4 common peaks at ±95.19Hz and ±105.2Hz frequencies. It's because when represented in sum of cosines, the cosine terms with frequencies 95Hz and 105Hz are common in both cases. The extra peak in 3(b) is due to the presence of cosine term with 100Hz frequency in the summation.

#### 4.5.4 *Section D, E*

Code used to add white gaussian noise having zero mean and variance=9:

```
var = 9;
w = sqrt(var).*randn(1, size(t,2));
c = x3 + w;
```

Here w is the white gaussian noise being added to the signal x3. We could see the original signal x3, being disturbed due the the white noise.

### 4.6 Code

```
close all; clc;
sgtitle('Question 3')

fs = 1000;
t = linspace(0,1,fs-1);

a = 1 + (0.5*cos(2*pi*5*t));
b = cos(2*pi*100*t);
x3 = a.*b;
dft_x3 = fftshift(fft(x3))/length(fft(x3));
freq = linspace(-fs/2,fs/2,fs-1);

var = 9;
w = sqrt(var).*randn(1, size(t,2));
c = x3 + w;
%% Time domain plot
subplot(3,1,1);
plot(t, x3); grid ON
xlabel("t in secs."); ylabel("x3(t)");
title("Time domain plot of x3(t)")
%% Amplitude spectrum plot
subplot(3,1,2);
plot(freq,abs(dft_x3)); grid ON
xlabel('frquency in Hz');ylabel('Amplitude');
title("Amplitude spectrum plot of x3(t)")
%% Time domain plot of corrupted signal
subplot(3,1,3);
plot(t,c); grid ON
xlabel("t in secs."); ylabel("corrupted x3(t)");
title("Time domain plot of corrupted x3(t)")
```

### 4.7 Output