MECA482 - Furuta Pendulum Model

This is the model from Wikipedia. The correspodning publication is "Cazzolato, B.S and Prime, Z (2011) "On the Dynamics of the Furuta Pendulum", Journal of Control Science and Engineering, Volume 2011 (2011), Article ID 528341, 8 pages." However the first model has been not used to implement the feedback controller. Infact, this model has been developed initially to be run in Simulink.

```
> restart;
> with(plots):
  with(CodeGeneration):
  with(VectorCalculus):
  with(LinearAlgebra):
> merge := proc(x,y)
     [op(x), op(y)]
  end proc;
```

$$merge := \mathbf{proc}(x, y)\ \ [op(x), op(y)]\ \mathbf{end\ proc} \tag{1.1}$$

```
> linearize := proc(eqs, lin_point)
  local var, vardot, f, var_sub_eqs,var_sub,lin_point_sub,f_sub,
  J,deltax,f_subx0,f_lin_sub,var_sub_eqs_inv,f_lin,
  eqs_space_state;

      #Getting rid of time dependency
      var := [seq(lhs(lin_point[i]),i=1..numelems(lin_point))];
      vardot := diff(var,t);
      f:= map(rhs,map(op,solve(eqs,vardot)));
      var_sub_eqs := [seq(var[i]=cat(x,__,i), i = 1..numelems
  (var))];
      var_sub := [seq(rhs(var_sub_eqs[i]), i=1..numelems
  (var_sub_eqs))];
      lin_point_sub := subs(var_sub_eqs, lin_point);
      f_sub := subs(var_sub_eqs, f);
      #Linearizing
      J := Matrix(subs(lin_point_sub,Jacobian(f_sub,var_sub)));
      deltax:= Transpose(Matrix([seq(var_sub[i] - rhs
  (lin_point_sub[i]), i=1..numelems(var_sub))]));
      f_subx0 := Transpose(Matrix(subs(lin_point_sub,(f_sub))));
      f_lin_sub := (f_subx0 + J.deltax);
      #Putting back time dependency
      var_sub_eqs_inv := [seq(rhs(var_sub_eqs[i]) = lhs
  (var_sub_eqs[i]), i =1..numelems(var_sub))];
      f_lin := subs(var_sub_eqs_inv,f_lin_sub);
      eqs_space_state := [seq(vardot[i] = f_lin[i,1],i=1..
  numelems(var))];

  end proc;
```

$linearize := \mathbf{proc}(eqs, lin\_point)$ **(1.2)**

$\quad \mathbf{local}\ var, vardot, f, var\_sub\_eqs, var\_sub, lin\_point\_sub, f\_sub, J, deltax, f\_subx0,$
$\quad f\_lin\_sub, var\_sub\_eqs\_inv, f\_lin, eqs\_space\_state;$

$\quad var := [seq(lhs(lin\_point[i]), i = 1 ..numelems(lin\_point))];$

$\quad vardot := VectorCalculus{:}\text{-}diff(var, t);$

$\quad f := map(rhs, map(op, solve(eqs, vardot)));$

$\quad var\_sub\_eqs := [seq(var[i] = cat(x, \_\_, i), i = 1 ..numelems(var))];$

```
var_sub := [seq(rhs(var_sub_eqs[i]), i = 1 ..numelems(var_sub_eqs))];
lin_point_sub := subs(var_sub_eqs, lin_point);
f_sub := subs(var_sub_eqs, f);
J := Matrix(subs(lin_point_sub, VectorCalculus:-Jacobian(f_sub, var_sub)));
deltax := LinearAlgebra:-Transpose(Matrix([seq(VectorCalculus:-`+`(var_sub[i],
VectorCalculus:-`-`(rhs(lin_point_sub[i]))), i = 1 ..numelems(var_sub))]));
f_subx0 := LinearAlgebra:-Transpose(Matrix(subs(lin_point_sub, f_sub)));
f_lin_sub := VectorCalculus:-`+`(f_subx0, VectorCalculus:-`.`(J, deltax));
var_sub_eqs_inv := [seq(rhs(var_sub_eqs[i]) = lhs(var_sub_eqs[i]), i = 1
 ..numelems(var_sub))];
f_lin := subs(var_sub_eqs_inv, f_lin_sub);
eqs_space_state := [seq(vardot[i] = f_lin[i, 1], i = 1 ..numelems(var))]
end proc
```

Equations of Motion with and without parameters

```
> eq1 := diff(theta1(t), t, t)*(J__1zz+m__1*(l__1)^2+(m__2)*
(L__1)^2+(J__2yy+m__2*(l__2)^2)*sin(theta2(t))^2+J__2xx*cos
(theta2(t))^2)+diff(theta2(t), t, t)*m__2*L__1*l__2*cos
(theta2(t))-m__2*L__1*l__2*sin(theta2(t))*diff(theta2(t),t)
^2+diff(theta1(t), t)*diff(theta2(t), t)*sin(2*theta2(t))*
(m__2*l__2^2+J__2yy-J__2xx) + b__1*diff(theta1(t),t) =
eta__g*k__g*eta__m*k__t*((V__m - k__g*k__m*diff(theta1(t), t)
)/r__m)
```

$$eq1 := \left(\frac{d^2}{dt^2}\,\theta 1(t)\right)\left(m_2 L_1^2 + m_1 l_1^2 + J_{1zz} + \left(m_2 l_2^2 + J_{2yy}\right)\sin(\theta 2(t))^2 + J_{2xx}\cos(\theta 2(t))^2\right) + \left(\frac{d^2}{dt^2}\,\theta 2(t)\right) m_2 L_1 l_2 \cos(\theta 2(t)) - m_2 L_1 l_2 \sin(\theta 2(t))\left(\frac{d}{dt}\,\theta 2(t)\right)^2 + \left(\frac{d}{dt}\,\theta 1(t)\right)\left(\frac{d}{dt}\,\theta 2(t)\right)\sin(2\,\theta 2(t))\left(m_2 l_2^2 - J_{2xx} + J_{2yy}\right) + b_1\left(\frac{d}{dt}\,\theta 1(t)\right) = \frac{\eta_g k_g \eta_m k_t\left(V_m - k_g k_m\left(\frac{d}{dt}\,\theta 1(t)\right)\right)}{r_m} \tag{1.1.1}$$

```
> eq2 := diff(theta1(t), t, t)*m__2*L__1*l__2*cos(theta2(t))+
diff(theta2(t), t, t)*(m__2*l__2^2+J__2zz)+1/2*diff(theta1
(t), t)^2*sin(2*theta2(t))*(-m__2*l__2^2-J__2yy+J__2xx)+b__2*
diff(theta2(t), t)+g*m__2*l__2*sin(theta2(t)) = tau__2
```

$$eq2 := \left(\frac{d^2}{dt^2}\,\theta 1(t)\right) m_2 L_1 l_2 \cos(\theta 2(t)) + \left(\frac{d^2}{dt^2}\,\theta 2(t)\right)\left(m_2 l_2^2 + J_{2zz}\right) + \frac{\left(\frac{d}{dt}\,\theta 1(t)\right)^2 \sin(2\,\theta 2(t))\left(-m_2 l_2^2 + J_{2xx} - J_{2yy}\right)}{2} + b_2\left(\frac{d}{dt}\,\theta 2(t)\right) + g\, m_2 l_2 \sin(\theta 2(t)) = \tau_2 \tag{1.1.2}$$

```
> data_mechanical := [J__1zz = 0.0023, m__1 = 0, l__1 = 0.215,
  m__2 = 0.2, L__1 = 0.215, J__2yy = 0.0023, l__2 = 0.1675,
  J__2xx = 0, J__2zz = 0.0023, b__1 = 0, Vm = 0, g=9.81, tau__2
  = 0, b__2 = 0]
```

$$data\_mechanical := \left[J_{1zz}=0.0023, m_1=0, l_1=0.215, m_2=0.2, L_1=0.215, J_{2yy}=0.0023, l_2=0.1675, J_{2xx}=0, J_{2zz}=0.0023, b_1=0, Vm=0, g=9.81, \tau_2=0, b_2=0\right] \quad \textbf{(1.1.3)}$$

```
> data_electrical := [eta__g = 0.85, eta__m = 0.87, k__g = 70,
  k__m = 0.0076 , k__t=0.0076, r__m=2.6, V__m = 10];
```

$$data\_electrical := \left[\eta_g=0.85, \eta_m=0.87, k_g=70, k_m=0.0076, k_t=0.0076, r_m=2.6, V_m=10\right] \quad \textbf{(1.1.4)}$$

```
> data := merge(data_mechanical, data_electrical);
```

$$data := \left[J_{1zz}=0.0023, m_1=0, l_1=0.215, m_2=0.2, L_1=0.215, J_{2yy}=0.0023, l_2=0.1675, J_{2xx}=0, J_{2zz}=0.0023, b_1=0, Vm=0, g=9.81, \tau_2=0, b_2=0, \eta_g=0.85, \eta_m=0.87, k_g=70, k_m=0.0076, k_t=0.0076, r_m=2.6, V_m=10\right] \quad \textbf{(1.1.5)}$$

```
> eqs := [eq1 ,eq2];
```

$$eqs := \left[\left(\frac{d^2}{dt^2}\theta1(t)\right)\left(m_2 L_1^2 + m_1 l_1^2 + J_{1zz} + \left(m_2 l_2^2 + J_{2yy}\right)\sin(\theta2(t))^2 + J_{2xx}\cos(\theta2(t))^2\right) + \left(\frac{d^2}{dt^2}\theta2(t)\right) m_2 L_1 l_2 \cos(\theta2(t)) - m_2 L_1 l_2 \sin(\theta2(t))\left(\frac{d}{dt}\theta2(t)\right)^2 + \left(\frac{d}{dt}\theta1(t)\right)\left(\frac{d}{dt}\theta2(t)\right)\sin(2\,\theta2(t))\left(m_2 l_2^2 - J_{2xx} + J_{2yy}\right) + b_1\left(\frac{d}{dt}\theta1(t)\right) = \frac{\eta_g k_g \eta_m k_t\left(V_m - k_g k_m\left(\frac{d}{dt}\theta1(t)\right)\right)}{r_m}, \left(\frac{d^2}{dt^2}\theta1(t)\right) m_2 L_1 l_2 \cos(\theta2(t)) + \left(\frac{d^2}{dt^2}\theta2(t)\right)\left(m_2 l_2^2 + J_{2zz}\right) + \frac{\left(\frac{d}{dt}\theta1(t)\right)^2 \sin(2\,\theta2(t))\left(-m_2 l_2^2 + J_{2xx} - J_{2yy}\right)}{2} + b_2\left(\frac{d}{dt}\theta2(t)\right) + g\, m_2 l_2 \sin(\theta2(t)) = \tau_2\right] \quad \textbf{(1.1.6)}$$

Simulation

Initial condition --> Vertical Arm at Pi/2

```
> ics1 := [diff(theta1(t), t) = 0, diff(theta2(t), t) = 0,
  theta1(t) = 0, theta2(t) = Pi/2]
```

$$ics1 := \left[\frac{d}{dt}\,\theta 1(t) = 0,\ \frac{d}{dt}\,\theta 2(t) = 0,\ \theta 1(t) = 0,\ \theta 2(t) = \frac{\pi}{2}\right] \qquad \textbf{(1.2.1.1)}$$

```
> ics10 := subs(t=0,convert(ics1, D))
```

$$ics10 := \left[\mathrm{D}(\theta 1)(0) = 0,\ \mathrm{D}(\theta 2)(0) = 0,\ \theta 1(0) = 0,\ \theta 2(0) = \frac{\pi}{2}\right] \qquad \textbf{(1.2.1.2)}$$

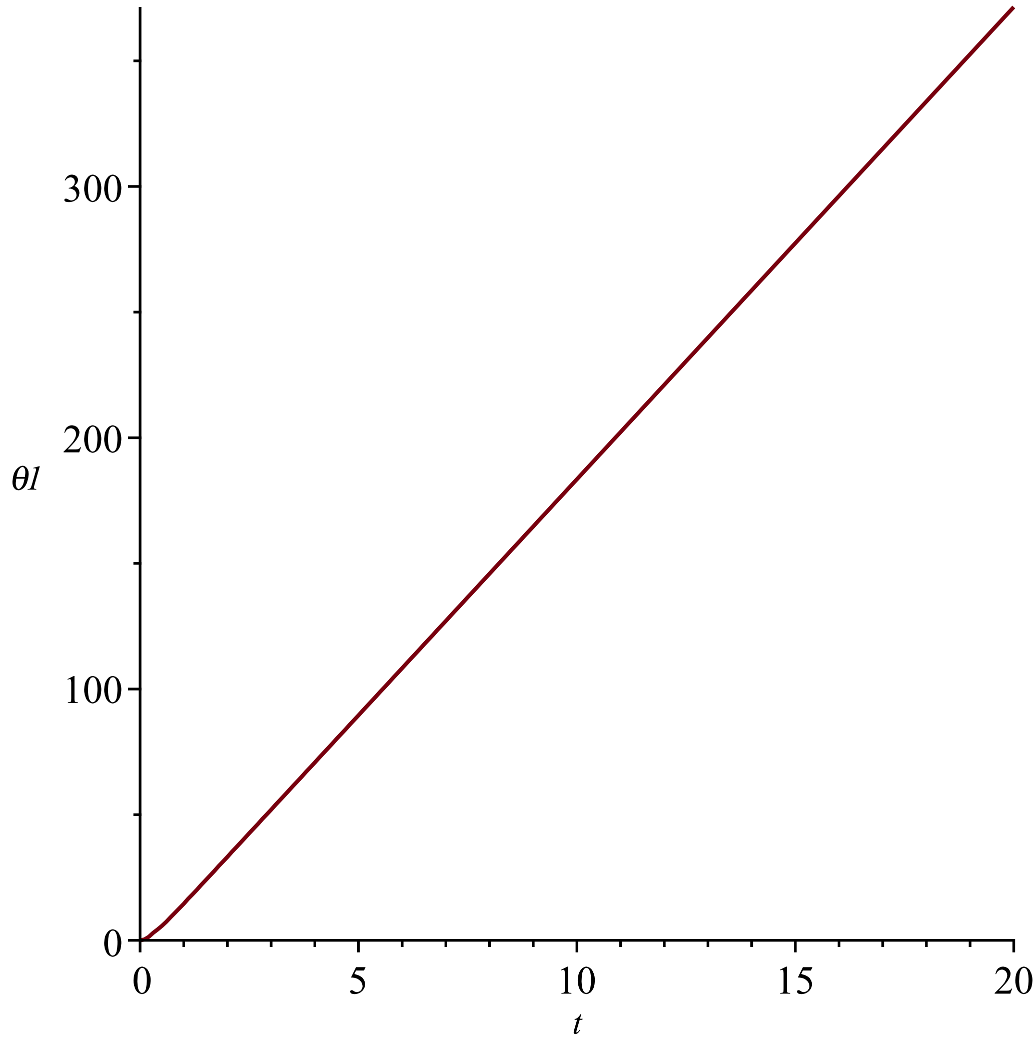
```
> ODEs1 := merge(subs(data, eqs), ics10);
```

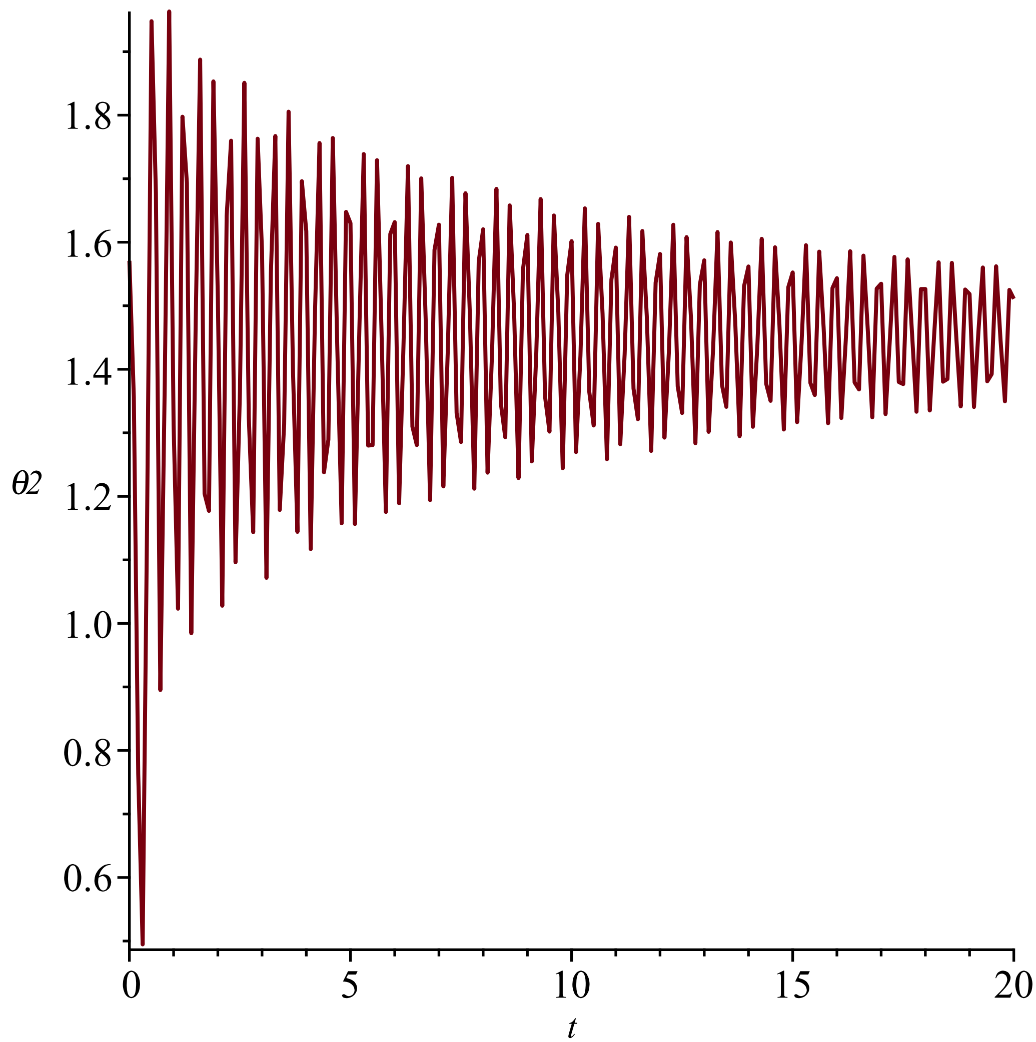
$$ODEs1 := \left[\left(\frac{d^2}{dt^2}\,\theta 1(t)\right)\left(0.0115450 + 0.007911250\sin(\theta 2(t))^2\right) + 0.00720250\left(\frac{d^2}{dt^2}\,\theta 2(t)\right)\cos(\theta 2(t)) - 0.00720250\sin(\theta 2(t))\left(\frac{d}{dt}\,\theta 2(t)\right)^2 + 0.007911250\left(\frac{d}{dt}\,\theta 1(t)\right)\left(\frac{d}{dt}\,\theta 2(t)\right)\sin(2\,\theta 2(t)) = 1.513130769 - 0.08049855691\left(\frac{d}{dt}\,\theta 1(t)\right),\ 0.00720250\left(\frac{d^2}{dt^2}\,\theta 1(t)\right)\cos(\theta 2(t)) + 0.007911250\left(\frac{d^2}{dt^2}\,\theta 2(t)\right) - 0.003955625000\left(\frac{d}{dt}\,\theta 1(t)\right)^2\sin(2\,\theta 2(t)) + 0.3286350\sin(\theta 2(t)) = 0,\ \mathrm{D}(\theta 1)(0) = 0,\ \mathrm{D}(\theta 2)(0) = 0,\ \theta 1(0) = 0,\ \theta 2(0) = \frac{\pi}{2}\right] \qquad \textbf{(1.2.1.3)}$$

```
> ode_sol1 := dsolve(ODEs1, numeric);
```

$ode\_sol1 :=$ **proc**(x_rkf45) ... **end proc** **(1.2.1.4)**

```
> odeplot(ode_sol1, [t, theta1(t)], t=0..20);
  odeplot(ode_sol1, [t, theta2(t)], t=0..20);
```


300
200
100
0
θ1
0
5
10
15
20
t



Reducing to first order system of ODEs and Exporting model to Matlab(to simulate it in Simulink)

```
> add_diff := [diff(theta1(t), t) = theta1dot(t), diff(theta2(t), t) = theta2dot(t)];
```

$$add\_diff := \left[\frac{d}{dt}\, \theta 1(t) = theta1dot(t),\ \frac{d}{dt}\, \theta 2(t) = theta2dot(t) \right] \tag{1.3.1}$$

```
> eqs_first_order := merge(subs(add_diff, eqs), add_diff);
```

$$eqs\_first\_order := \left[\left(\frac{d}{dt}\, theta1dot(t) \right) \left(m_2 L_1^2 + m_1 l_1^2 + J_{1zz} + \left(l_2^2 m_2 + J_{2yy} \right) \sin\left(\theta 2(t)\right)^2 + J_{2xx} \cos\left(\theta 2(t)\right)^2 \right) + \left(\frac{d}{dt}\, theta2dot(t) \right) m_2 L_1 l_2 \cos\left(\theta 2(t)\right) - m_2 L_1 l_2 \sin\left(\theta 2(t)\right) theta2dot(t)^2 \right. \tag{1.3.2}$$

$$+ theta1dot(t)\, theta2dot(t)\, \sin(2\, \theta2(t))\, \left(l_2^2\, m_2 - J_{2xx} + J_{2yy}\right) + b_1\, theta1dot(t)$$

$$= \frac{\eta_g\, k_g\, \eta_m\, k_t\, (V_m - k_g\, k_m\, theta1dot(t))}{r_m}, \left(\frac{d}{dt}\, theta1dot(t)\right) m_2\, L_1\, l_2 \cos(\theta2(t))$$

$$+ \left(\frac{d}{dt}\, theta2dot(t)\right) \left(l_2^2\, m_2 + J_{2zz}\right)$$

$$+ \frac{theta1dot(t)^2 \sin(2\, \theta2(t))\, \left(-l_2^2\, m_2 + J_{2xx} - J_{2yy}\right)}{2} + b_2\, theta2dot(t)$$

$$+ g\, m_2\, l_2 \sin(\theta2(t)) = \tau_2, \frac{d}{dt}\, \theta1(t) = theta1dot(t), \frac{d}{dt}\, \theta2(t) = theta2dot(t)\Bigg]$$

```
> var := [diff(theta1(t), t), diff(theta2(t), t), diff
  (theta1dot(t), t), diff(theta2dot(t), t)];
```

$$var := \left[\frac{d}{dt}\, \theta1(t), \frac{d}{dt}\, \theta2(t), \frac{d}{dt}\, theta1dot(t), \frac{d}{dt}\, theta2dot(t)\right] \quad \textbf{(1.3.3)}$$

```
> var_sub := [theta1(t) = theta1, theta2(t) = theta2, theta1dot
  (t) = theta1dot, theta2dot(t) = theta2dot]
```

$$var\_sub := [\theta1(t) = \theta1, \theta2(t) = \theta2, theta1dot(t) = theta1dot, theta2dot(t) = theta2dot] \quad \textbf{(1.3.4)}$$

```
> solve(eqs_first_order, var)[1]:
  map(rhs, %):
  F := simplify(subs(var_sub, %));
```

$$F := \Big[theta1dot, theta2dot, \big(l_2\, m_2\, r_m\, L_1\, theta1dot^2 \sin(\theta2)\, \big(l_2^2\, m_2 - J_{2xx} \quad \textbf{(1.3.5)}$$

$$+ J_{2yy}\big) \cos(\theta2)^2 - \Big(\Big(l_2^2\, \big(-2\, l_2^2\, theta1dot\, theta2dot + L_1\, g\big)\, m_2^2$$

$$+ 2\, theta2dot\, theta1dot\, l_2^2\, (J_{2xx} - J_{2yy} - J_{2zz})\, m_2 + 2\, theta2dot\, theta1dot\, J_{2zz}\, (J_{2xx}$$

$$- J_{2yy})\Big) \sin(\theta2) - l_2\, m_2\, L_1\, \big(-theta2dot\, b_2 + \tau_2\big)\Big)\, r_m \cos(\theta2) - \big(l_2^2\, m_2$$

$$+ J_{2zz}\big)\, \big(l_2\, m_2\, r_m\, L_1\, theta2dot^2 \sin(\theta2) - r_m\, theta1dot\, b_1$$

$$- \eta_g\, \eta_m\, k_g\, k_t\, (k_g\, k_m\, theta1dot - V_m)\big)\big) \Big/ \Big(r_m\, \Big(\Big(\big(L_1^2\, l_2^2 + l_2^4\big)\, m_2^2 - l_2^2\, (J_{2xx} - J_{2yy}$$

$$-J_{2zz}\big)\, m_2 - J_{2zz}\, \big(J_{2xx} - J_{2yy}\big)\big)\, \cos(\theta 2)^2 - \big(l_2^2\, m_2 + J_{2zz}\big)\, \big(\big(L_1^2 + l_2^2\big)\, m_2 + m_1\, l_1^2$$

$$+ J_{1zz} + J_{2yy}\big)\big)\big), \Big(r_m\, \mathit{theta1dot}^2 \sin(\theta 2)\, \big(l_2^2\, m_2 - J_{2xx} + J_{2yy}\big)^2 \cos(\theta 2)^3$$

$$- \big(l_2\, m_2\, \big(2\, L_1\, \mathit{theta1dot}\ \mathit{theta2dot} + g\big) \sin(\theta 2) + \mathit{theta2dot}\, b_2 - \tau_2\big)\, \big(l_2^2\, m_2 - J_{2xx}$$

$$+ J_{2yy}\big)\, r_m \cos(\theta 2)^2 + \Big(-r_m\, \Big(l_2^2\, \big(L_1^2\, \mathit{theta1dot}^2 - L_1^2\, \mathit{theta2dot}^2$$

$$+ l_2^2\, \mathit{theta1dot}^2\big)\, m_2^2 + \Big(\big(m_1\, l_1^2 + J_{1zz} - J_{2xx} + 2\, J_{2yy}\big)\, l_2^2 - L_1^2\, \big(J_{2xx}$$

$$- J_{2yy}\big)\Big)\, \mathit{theta1dot}^2\, m_2 - \mathit{theta1dot}^2\, \big(J_{2xx} - J_{2yy}\big)\, \big(m_1\, l_1^2 + J_{1zz} + J_{2yy}\big)\Big) \sin(\theta 2)$$

$$- m_2\, l_2\, \big(r_m\, \mathit{theta1dot}\, b_1 + \eta_g\, \eta_m\, k_g\, k_t\, \big(k_g\, k_m\, \mathit{theta1dot} - V_m\big)\big)\, L_1\Big) \cos(\theta 2)$$

$$+ r_m\, \big(g\, l_2\, m_2 \sin(\theta 2) + \mathit{theta2dot}\, b_2 - \tau_2\big)\, \big(\big(L_1^2 + l_2^2\big)\, m_2 + m_1\, l_1^2 + J_{1zz} + J_{2yy}\big)\Big)$$

$$\Big/ \Big(r_m\, \Big(\big(\big(L_1^2\, l_2^2 + l_2^4\big)\, m_2^2 - l_2^2\, \big(J_{2xx} - J_{2yy} - J_{2zz}\big)\, m_2 - J_{2zz}\, \big(J_{2xx}$$

$$- J_{2yy}\big)\big) \cos(\theta 2)^2 - \big(l_2^2\, m_2 + J_{2zz}\big)\, \big(\big(L_1^2 + l_2^2\big)\, m_2 + m_1\, l_1^2 + J_{1zz} + J_{2yy}\big)\Big)\Big)\Big]$$

```
> Matlab(F[1], resultname="dtheta1_dt");
dtheta1_dt = theta1dot;
> Matlab(F[2], resultname="dtheta2_dt");
dtheta2_dt = theta2dot;
> Matlab(F[3], resultname="d2theta1_dt2");
d2theta1_dt2 = (l__2 * m__2 * r__m * L__1 * theta1dot ^ 2 *
sin(theta2) * (l__2 ^ 2 * m__2 - J__2xx + J__2yy) * cos
(theta2) ^ 2 - ((l__2 ^ 2 * (-0.2e1 * l__2 ^ 2 * theta1dot *
theta2dot + L__1 * g) * m__2 ^ 2 + 0.2e1 * theta2dot *
theta1dot * l__2 ^ 2 * (J__2xx - J__2yy - J__2zz) * m__2 +
0.2e1 * theta2dot * theta1dot * J__2zz * (J__2xx - J__2yy))
* sin(theta2) - l__2 * m__2 * L__1 * (-theta2dot * b__2 +
tau__2)) * r__m * cos(theta2) - (l__2 ^ 2 * m__2 + J__2zz) *
(l__2 * m__2 * r__m * L__1 * theta2dot ^ 2 * sin(theta2) -
r__m * theta1dot * b__1 - eta__g * eta__m * k__g * k__t *
(k__g * k__m * theta1dot - V__m))) / r__m / (((L__1 ^ 2 *
l__2 ^ 2 + l__2 ^ 4) * m__2 ^ 2 - l__2 ^ 2 * (J__2xx -
J__2yy - J__2zz) * m__2 - J__2zz * (J__2xx - J__2yy)) * cos
(theta2) ^ 2 - (l__2 ^ 2 * m__2 + J__2zz) * ((L__1 ^ 2 +
l__2 ^ 2) * m__2 + m__1 * l__1 ^ 2 + J__1zz + J__2yy));
> Matlab(F[4], resultname="d2theta2_dt2");
d2theta2_dt2 = (r__m * theta1dot ^ 2 * sin(theta2) * (l__2 ^
2 * m__2 - J__2xx + J__2yy) ^ 2 * cos(theta2) ^ 3 - (l__2 *
m__2 * (0.2e1 * L__1 * theta1dot * theta2dot + g) * sin
(theta2) + theta2dot * b__2 - tau__2) * (l__2 ^ 2 * m__2 -
J__2xx + J__2yy) * r__m * cos(theta2) ^ 2 + (-r__m * (l__2 ^
2 * (L__1 ^ 2 * theta1dot ^ 2 - L__1 ^ 2 * theta2dot ^ 2 +
l__2 ^ 2 * theta1dot ^ 2) * m__2 ^ 2 + ((m__1 * l__1 ^ 2 +
J__1zz - J__2xx + 0.2e1 * J__2yy) * l__2 ^ 2 - L__1 ^ 2 *
(J__2xx - J__2yy)) * theta1dot ^ 2 * m__2 - theta1dot ^ 2 *
(J__2xx - J__2yy) * (m__1 * l__1 ^ 2 + J__1zz + J__2yy)) *
```

```
sin(theta2) - m__2 * l__2 * (r__m * theta1dot * b__1 +
eta__g * eta__m * k__g * k__t * (k__g * k__m * theta1dot -
V__m)) * L__1) * cos(theta2) + r__m * (g * l__2 * m__2 * sin
(theta2) + theta2dot * b__2 - tau__2) * ((L__1 ^ 2 + l__2 ^
2) * m__2 + m__1 * l__1 ^ 2 + J__1zz + J__2yy)) / r__m / ((
(L__1 ^ 2 * l__2 ^ 2 + l__2 ^ 4) * m__2 ^ 2 - l__2 ^ 2 *
(J__2xx - J__2yy - J__2zz) * m__2 - J__2zz * (J__2xx -
J__2yy)) * cos(theta2) ^ 2 - (l__2 ^ 2 * m__2 + J__2zz) * (
(L__1 ^ 2 + l__2 ^ 2) * m__2 + m__1 * l__1 ^ 2 + J__1zz +
J__2yy));
```

Jacobian DF/Dx

```
> DF_DX := simplify(Jacobian(F, [theta1, theta2, theta1dot,
  theta2dot] )):
> Matlab(DF_DX, resultname="Jacobian");
Jacobian = [0 0 1 0; 0 0 0 1; 0 (theta1dot ^ 2 * ((L__1 ^
2 * l__2 ^ 2 + l__2 ^ 4) * m__2 ^ 2 - l__2 ^ 2 * (J__2xx -
J__2yy - J__2zz) * m__2 - J__2zz * (J__2xx - J__2yy)) *
m__2 * l__2 * (l__2 ^ 2 * m__2 - J__2xx + J__2yy) * r__m *
L__1 * cos(theta2) ^ 5 - 0.3e1 * (l__2 ^ 2 * m__2 +
J__2zz) * m__2 * l__2 * ((L__1 ^ 2 + l__2 ^ 2) * l__2 ^ 2
* (theta1dot ^ 2 - theta2dot ^ 2 / 0.3e1) * m__2 ^ 2 + ((
(m__1 * l__1 ^ 2 + J__1zz - J__2xx + 0.2e1 * J__2yy) *
theta1dot ^ 2 + theta2dot ^ 2 * (J__2xx - J__2yy - J__2zz)
/ 0.3e1) * l__2 ^ 2 - L__1 ^ 2 * theta1dot ^ 2 * (J__2xx -
J__2yy)) * m__2 - ((m__1 * l__1 ^ 2 + J__1zz + J__2yy) *
theta1dot ^ 2 - J__2zz * theta2dot ^ 2 / 0.3e1) * (J__2xx
- J__2yy)) * r__m * L__1 * cos(theta2) ^ 3 + (((L__1 ^ 2 *
l__2 ^ 2 + l__2 ^ 4) * m__2 ^ 2 - l__2 ^ 2 * (J__2xx -
J__2yy - J__2zz) * m__2 - J__2zz * (J__2xx - J__2yy)) *
m__2 * l__2 * (-theta2dot * b__2 + tau__2) * L__1 * sin
(theta2) + (l__2 ^ 2 * (-0.2e1 * l__2 ^ 2 * theta1dot *
theta2dot + L__1 * g) * m__2 ^ 2 + 0.2e1 * theta2dot *
theta1dot * l__2 ^ 2 * (J__2xx - J__2yy - J__2zz) * m__2 +
0.2e1 * theta2dot * theta1dot * J__2zz * (J__2xx - J__2yy)
) * ((L__1 ^ 2 * l__2 ^ 2 + l__2 ^ 4) * m__2 ^ 2 + ((0.2e1
* m__1 * l__1 ^ 2 + 0.2e1 * J__1zz + J__2xx + J__2yy +
J__2zz) * l__2 ^ 2 + 0.2e1 * J__2zz * L__1 ^ 2) * m__2 +
0.2e1 * J__2zz * (m__1 * l__1 ^ 2 + J__1zz + J__2xx /
0.2e1 + J__2yy / 0.2e1))) * r__m * cos(theta2) ^ 2 + 0.2e1
* (l__2 ^ 2 * m__2 + J__2zz) * (((L__1 ^ 2 * l__2 ^ 2 +
l__2 ^ 4) * m__2 ^ 2 - l__2 ^ 2 * (J__2xx - J__2yy -
J__2zz) * m__2 - J__2zz * (J__2xx - J__2yy)) * (r__m *
theta1dot * b__1 + eta__g * eta__m * k__g * k__t * (k__g *
k__m * theta1dot - V__m)) * sin(theta2) + m__2 * l__2 * (
(L__1 ^ 2 + l__2 ^ 2) * (theta1dot ^ 2 - theta2dot ^ 2 /
0.2e1) * l__2 ^ 2 * m__2 ^ 2 + (((m__1 * l__1 ^ 2 + J__1zz
- J__2xx + 0.2e1 * J__2yy) * theta1dot ^ 2 + theta2dot ^ 2
* (m__1 * l__1 ^ 2 + J__1zz + 0.2e1 * J__2xx - J__2yy -
J__2zz) / 0.2e1) * l__2 ^ 2 - ((J__2xx - J__2yy) *
theta1dot ^ 2 - J__2zz * theta2dot ^ 2 / 0.2e1) * L__1 ^
2) * m__2 - theta1dot ^ 2 * (J__2xx - J__2yy) * (m__1 *
l__1 ^ 2 + J__1zz + J__2yy) + J__2zz * theta2dot ^ 2 *
(m__1 * l__1 ^ 2 + J__1zz + 0.2e1 * J__2xx - J__2yy) /
0.2e1) * r__m * L__1) * cos(theta2) - (l__2 ^ 2 * m__2 +
J__2zz) * (-l__2 * m__2 * L__1 * (-theta2dot * b__2 +
tau__2) * sin(theta2) + l__2 ^ 2 * (-0.2e1 * l__2 ^ 2 *
```

```
theta1dot * theta2dot + L__1 * g) * m__2 ^ 2 + 0.2e1 *
theta2dot * theta1dot * l__2 ^ 2 * (J__2xx - J__2yy -
J__2zz) * m__2 + 0.2e1 * theta2dot * theta1dot * J__2zz *
(J__2xx - J__2yy)) * r__m * ((L__1 ^ 2 + l__2 ^ 2) * m__2
+ m__1 * l__1 ^ 2 + J__1zz + J__2yy)) / r__m / (((L__1 ^ 2
* l__2 ^ 2 + l__2 ^ 4) * m__2 ^ 2 - l__2 ^ 2 * (J__2xx -
J__2yy - J__2zz) * m__2 - J__2zz * (J__2xx - J__2yy)) *
cos(theta2) ^ 2 - (l__2 ^ 2 * m__2 + J__2zz) * ((L__1 ^ 2
+ l__2 ^ 2) * m__2 + m__1 * l__1 ^ 2 + J__1zz + J__2yy)) ^
2 (0.2e1 * L__1 * r__m * theta1dot * sin(theta2) * l__2 *
m__2 * (-l__2 ^ 2 * m__2 + J__2xx - J__2yy) * cos(theta2)
^ 2 + 0.2e1 * r__m * theta2dot * sin(theta2) * (l__2 ^ 2 *
m__2 + J__2zz) * (-l__2 ^ 2 * m__2 + J__2xx - J__2yy) *
cos(theta2) - (l__2 ^ 2 * m__2 + J__2zz) * (k__t * k__m *
k__g ^ 2 * eta__m * eta__g + b__1 * r__m)) / r__m / (((-
L__1 ^ 2 * l__2 ^ 2 - l__2 ^ 4) * m__2 ^ 2 + l__2 ^ 2 *
(J__2xx - J__2yy - J__2zz) * m__2 + J__2zz * (J__2xx -
J__2yy)) * cos(theta2) ^ 2 + (l__2 ^ 2 * m__2 + J__2zz) *
((L__1 ^ 2 + l__2 ^ 2) * m__2 + m__1 * l__1 ^ 2 + J__1zz +
J__2yy)) ((-0.2e1 * theta1dot * (l__2 ^ 2 * m__2 - J__2xx
+ J__2yy) * (l__2 ^ 2 * m__2 + J__2zz) * sin(theta2) +
l__2 * m__2 * L__1 * b__2) * cos(theta2) + 0.2e1 * L__1 *
theta2dot * sin(theta2) * l__2 * m__2 * (l__2 ^ 2 * m__2 +
J__2zz)) / (((-L__1 ^ 2 * l__2 ^ 2 - l__2 ^ 4) * m__2 ^ 2
- l__2 ^ 2 * (-J__2xx + J__2yy + J__2zz) * m__2 + J__2zz *
(J__2xx - J__2yy)) * cos(theta2) ^ 2 + (l__2 ^ 2 * m__2 +
J__2zz) * ((L__1 ^ 2 + l__2 ^ 2) * m__2 + m__1 * l__1 ^ 2
+ J__1zz + J__2yy)); 0 (0.2e1 * theta1dot ^ 2 * ((L__1 ^ 2
* l__2 ^ 2 + l__2 ^ 4) * m__2 ^ 2 - l__2 ^ 2 * (J__2xx -
J__2yy - J__2zz) * m__2 - J__2zz * (J__2xx - J__2yy)) *
(l__2 ^ 2 * m__2 - J__2xx + J__2yy) ^ 2 * r__m * cos
(theta2) ^ 6 - (0.2e1 * L__1 * theta1dot * theta2dot + g)
* ((L__1 ^ 2 * l__2 ^ 2 + l__2 ^ 4) * m__2 ^ 2 - l__2 ^ 2
* (J__2xx - J__2yy - J__2zz) * m__2 - J__2zz * (J__2xx -
J__2yy)) * m__2 * l__2 * (l__2 ^ 2 * m__2 - J__2xx +
J__2yy) * r__m * cos(theta2) ^ 5 - 0.5e1 * theta1dot ^ 2 *
((L__1 ^ 2 * l__2 ^ 2 + l__2 ^ 4) * m__2 ^ 2 + ((0.4e1 /
0.5e1 * m__1 * l__1 ^ 2 + 0.4e1 / 0.5e1 * J__1zz - J__2xx
/ 0.5e1 + J__2yy + J__2zz) * l__2 ^ 2 + 0.4e1 / 0.5e1 *
J__2zz * L__1 ^ 2) * m__2 + 0.4e1 / 0.5e1 * J__2zz * (m__1
* l__1 ^ 2 + J__1zz - J__2xx / 0.4e1 + 0.5e1 / 0.4e1 *
J__2yy)) * (l__2 ^ 2 * m__2 - J__2xx + J__2yy) ^ 2 * r__m
* cos(theta2) ^ 4 + 0.2e1 * m__2 * l__2 * r__m * ((L__1 ^
2 + l__2 ^ 2) * m__2 + m__1 * l__1 ^ 2 + J__1zz + J__2yy)
* (l__2 ^ 2 * ((0.3e1 * L__1 * theta1dot * theta2dot + g)
* l__2 ^ 2 - g * L__1 ^ 2 / 0.2e1) * m__2 ^ 2 - l__2 ^ 2 *
(J__2xx - J__2yy - J__2zz) * (0.3e1 * L__1 * theta1dot *
theta2dot + g) * m__2 - J__2zz * (J__2xx - J__2yy) *
(0.3e1 * L__1 * theta1dot * theta2dot + g)) * cos(theta2)
^ 3 + (-((L__1 ^ 2 * l__2 ^ 2 + l__2 ^ 4) * m__2 ^ 2 -
l__2 ^ 2 * (J__2xx - J__2yy - J__2zz) * m__2 - J__2zz *
(J__2xx - J__2yy)) * m__2 * l__2 * (r__m * theta1dot *
b__1 + eta__g * eta__m * k__g * k__t * (k__g * k__m *
theta1dot - V__m)) * L__1 * sin(theta2) + 0.4e1 * ((L__1 ^
2 + l__2 ^ 2) * (l__2 ^ 2 * theta1dot ^ 2 + L__1 ^ 2 *
theta1dot ^ 2 / 0.4e1 - L__1 ^ 2 * theta2dot ^ 2 / 0.4e1)
* l__2 ^ 4 * m__2 ^ 4 + 0.3e1 / 0.2e1 * l__2 ^ 2 *
```

```
(theta1dot ^ 2 * (m__1 * l__1 ^ 2 + J__1zz - J__2xx +
0.2e1 * J__2yy + 0.2e1 / 0.3e1 * J__2zz) * l__2 ^ 4 + (
(m__1 * l__1 ^ 2 + J__1zz - 0.7e1 / 0.3e1 * J__2xx +
0.10e2 / 0.3e1 * J__2yy + 0.2e1 * J__2zz) * theta1dot ^ 2
- 0.2e1 / 0.3e1 * (m__1 * l__1 ^ 2 + J__1zz + J__2xx /
0.2e1 + J__2yy / 0.2e1 + J__2zz / 0.2e1) * theta2dot ^ 2)
* L__1 ^ 2 * l__2 ^ 2 / 0.2e1 - ((J__2xx - J__2yy - 0.2e1
* J__2zz) * theta1dot ^ 2 + 0.2e1 * J__2zz * theta2dot ^
2) * L__1 ^ 4 / 0.6e1) * m__2 ^ 3 + (theta1dot ^ 2 * (
(0.3e1 * m__1 * l__1 ^ 2 + 0.3e1 * J__1zz - 0.3e1 * J__2xx
+ 0.6e1 * J__2yy) * J__2zz + 0.6e1 * J__2yy ^ 2 + (0.6e1 *
m__1 * l__1 ^ 2 + 0.6e1 * J__1zz - 0.6e1 * J__2xx) *
J__2yy + J__2xx ^ 2 + (-0.4e1 * m__1 * l__1 ^ 2 - 0.4e1 *
J__1zz) * J__2xx + (m__1 * l__1 ^ 2 + J__1zz) ^ 2) * l__2
^ 4 / 0.2e1 - 0.3e1 / 0.4e1 * (((-0.4e1 / 0.3e1 * m__1 *
l__1 ^ 2 - 0.4e1 / 0.3e1 * J__1zz + 0.8e1 / 0.3e1 * J__2xx
- 0.4e1 * J__2yy) * J__2zz + (m__1 * l__1 ^ 2 + J__1zz -
0.2e1 / 0.3e1 * J__2xx + 0.5e1 / 0.3e1 * J__2yy) * (J__2xx
- J__2yy)) * theta1dot ^ 2 + 0.2e1 / 0.3e1 * J__2zz *
(m__1 * l__1 ^ 2 + J__1zz + J__2xx / 0.2e1 + J__2yy /
0.2e1) * theta2dot ^ 2) * L__1 ^ 2 * l__2 ^ 2 - J__2zz *
L__1 ^ 4 * theta1dot ^ 2 * (J__2xx - J__2yy) / 0.2e1) *
m__2 ^ 2 - theta1dot ^ 2 * (((-0.6e1 * J__2yy ^ 2 +
(-0.6e1 * m__1 * l__1 ^ 2 - 0.6e1 * J__1zz + 0.6e1 *
J__2xx) * J__2yy - J__2xx ^ 2 + (0.4e1 * m__1 * l__1 ^ 2 +
0.4e1 * J__1zz) * J__2xx - (m__1 * l__1 ^ 2 + J__1zz) ^ 2)
* J__2zz + (J__2xx - J__2yy) * (m__1 * l__1 ^ 2 + J__1zz +
J__2yy) * (m__1 * l__1 ^ 2 + J__1zz - J__2xx + 0.2e1 *
J__2yy)) * l__2 ^ 2 + 0.2e1 * J__2zz * (m__1 * l__1 ^ 2 +
J__1zz - J__2xx / 0.2e1 + 0.3e1 / 0.2e1 * J__2yy) * L__1 ^
2 * (J__2xx - J__2yy)) * m__2 / 0.2e1 - J__2zz * theta1dot
^ 2 * (J__2xx - J__2yy) * (m__1 * l__1 ^ 2 + J__1zz +
J__2yy) * (m__1 * l__1 ^ 2 + J__1zz - J__2xx + 0.2e1 *
J__2yy) / 0.2e1) * r__m) * cos(theta2) ^ 2 - m__2 * l__2 *
(0.2e1 * l__2 * m__2 * L__1 ^ 2 * (-theta2dot * b__2 +
tau__2) * sin(theta2) + ((0.4e1 * L__1 * theta1dot *
theta2dot + g) * l__2 ^ 2 - g * L__1 ^ 2) * l__2 ^ 2 *
m__2 ^ 2 + ((-0.4e1 * L__1 * theta2dot * (J__2xx - J__2yy
- J__2zz) * theta1dot + g * (m__1 * l__1 ^ 2 + J__1zz +
J__2yy + J__2zz)) * l__2 ^ 2 + g * J__2zz * L__1 ^ 2) *
m__2 + J__2zz * (-0.4e1 * L__1 * theta2dot * (J__2xx -
J__2yy) * theta1dot + g * (m__1 * l__1 ^ 2 + J__1zz +
J__2yy))) * r__m * ((L__1 ^ 2 + l__2 ^ 2) * m__2 + m__1 *
l__1 ^ 2 + J__1zz + J__2yy) * cos(theta2) - (l__2 ^ 2 *
m__2 + J__2zz) * (m__2 * l__2 * (r__m * theta1dot * b__1 +
eta__g * eta__m * k__g * k__t * (k__g * k__m * theta1dot -
V__m)) * L__1 * sin(theta2) + r__m * (l__2 ^ 2 * (L__1 ^ 2
* theta1dot ^ 2 - L__1 ^ 2 * theta2dot ^ 2 + l__2 ^ 2 *
theta1dot ^ 2) * m__2 ^ 2 + ((m__1 * l__1 ^ 2 + J__1zz -
J__2xx + 0.2e1 * J__2yy) * l__2 ^ 2 - L__1 ^ 2 * (J__2xx -
J__2yy)) * theta1dot ^ 2 * m__2 - theta1dot ^ 2 * (J__2xx
- J__2yy) * (m__1 * l__1 ^ 2 + J__1zz + J__2yy))) * ((L__1
^ 2 + l__2 ^ 2) * m__2 + m__1 * l__1 ^ 2 + J__1zz +
J__2yy)) / r__m / (((L__1 ^ 2 * l__2 ^ 2 + l__2 ^ 4) *
m__2 ^ 2 - l__2 ^ 2 * (J__2xx - J__2yy - J__2zz) * m__2 -
J__2zz * (J__2xx - J__2yy)) * cos(theta2) ^ 2 - (l__2 ^ 2
* m__2 + J__2zz) * ((L__1 ^ 2 + l__2 ^ 2) * m__2 + m__1 *
```

```
l__1 ^ 2 + J__1zz + J__2yy)) ^ 2 -0.2e1 * ((theta1dot * (-
l__2 ^ 2 * m__2 + J__2xx - J__2yy) * cos(theta2) ^ 2 +
L__1 * theta2dot * l__2 * m__2 * cos(theta2) + theta1dot *
((L__1 ^ 2 + l__2 ^ 2) * m__2 + m__1 * l__1 ^ 2 + J__1zz +
J__2yy)) * r__m * (-l__2 ^ 2 * m__2 + J__2xx - J__2yy) *
sin(theta2) - m__2 * l__2 * (k__t * k__m * k__g ^ 2 *
eta__m * eta__g + b__1 * r__m) * L__1 / 0.2e1) * cos
(theta2) / r__m / (((-L__1 ^ 2 * l__2 ^ 2 - l__2 ^ 4) *
m__2 ^ 2 + l__2 ^ 2 * (J__2xx - J__2yy - J__2zz) * m__2 +
J__2zz * (J__2xx - J__2yy)) * cos(theta2) ^ 2 + (l__2 ^ 2
* m__2 + J__2zz) * ((L__1 ^ 2 + l__2 ^ 2) * m__2 + m__1 *
l__1 ^ 2 + J__1zz + J__2yy)) (0.2e1 * (l__2 * m__2 * L__1
* theta1dot * sin(theta2) + b__2 / 0.2e1) * (l__2 ^ 2 *
m__2 - J__2xx + J__2yy) * cos(theta2) ^ 2 - 0.2e1 * l__2 ^
2 * L__1 ^ 2 * theta2dot * m__2 ^ 2 * sin(theta2) * cos
(theta2) - ((L__1 ^ 2 + l__2 ^ 2) * m__2 + m__1 * l__1 ^ 2
+ J__1zz + J__2yy) * b__2) / (((-L__1 ^ 2 * l__2 ^ 2 -
l__2 ^ 4) * m__2 ^ 2 - l__2 ^ 2 * (-J__2xx + J__2yy +
J__2zz) * m__2 + J__2zz * (J__2xx - J__2yy)) * cos(theta2)
^ 2 + (l__2 ^ 2 * m__2 + J__2zz) * ((L__1 ^ 2 + l__2 ^ 2)
* m__2 + m__1 * l__1 ^ 2 + J__1zz + J__2yy));];
```

Reducing to state space model and exporting to Matlab

```
> eqs_first_order;
```

$$\Bigg[\left(\frac{d}{dt}\, theta1dot(t)\right)\left(m_2 L_1^2 + m_1 l_1^2 + J_{1zz} + \left(l_2^2 m_2 + J_{2yy}\right)\sin(\theta 2(t))^2 + J_{2xx}\cos(\theta 2(t))^2\right) + \left(\frac{d}{dt}\, theta2dot(t)\right) m_2 L_1 l_2 \cos(\theta 2(t)) - m_2 L_1 l_2 \sin(\theta 2(t))\, theta2dot(t)^2 + theta1dot(t)\, theta2dot(t) \sin(2\,\theta 2(t))\left(l_2^2 m_2 - J_{2xx} + J_{2yy}\right) + b_1\, theta1dot(t) = \frac{\eta_g k_g \eta_m k_t \left(V_m - k_g k_m\, theta1dot(t)\right)}{r_m}, \left(\frac{d}{dt}\, theta1dot(t)\right) m_2 L_1 l_2 \cos(\theta 2(t)) + \left(\frac{d}{dt}\, theta2dot(t)\right)\left(l_2^2 m_2 + J_{2zz}\right) + \frac{theta1dot(t)^2 \sin(2\,\theta 2(t))\left(-l_2^2 m_2 + J_{2xx} - J_{2yy}\right)}{2} + b_2\, theta2dot(t) + g\, m_2 l_2 \sin(\theta 2(t)) = \tau_2, \frac{d}{dt}\,\theta 1(t) = theta1dot(t), \frac{d}{dt}\,\theta 2(t) = theta2dot(t)\Bigg] \quad \textbf{(1.4.1)}$$

```
> lin_point := [theta1(t) = 2.99, theta2(t) = Pi, theta1dot(t)
  = 0, theta2dot(t) = 0];
```

$$lin\_point := [\theta 1(t) = 2.99, \theta 2(t) = \pi, theta1dot(t) = 0, theta2dot(t) = 0] \quad \textbf{(1.4.2)}$$

```
> eqs_space_state := simplify(linearize(eqs_first_order,
```

```
lin_point));
```

$$eqs\_space\_state := \left[\frac{d}{dt}\, \theta 1(t) = theta1dot(t),\ \frac{d}{dt}\, \theta 2(t) = theta2dot(t),\ \frac{d}{dt}\right. \quad (1.4.3)$$

$$theta1dot(t) = \left(-\left(m_2 l_2^2 + J_{2zz}\right)\left(\eta_g \eta_m k_g^2 k_m k_t + r_m b_1\right) theta1dot(t)\right.$$

$$- L_1 b_2 l_2 m_2\, theta2dot(t)\, r_m + L_1 g l_2^2 m_2^2 r_m\, \theta 2(t) - L_1 g l_2^2 m_2^2 r_m \pi$$

$$+ l_2 \left(V_m \eta_g \eta_m k_g k_t l_2 + L_1 r_m \tau_2\right) m_2 + J_{2zz} V_m \eta_g \eta_m k_g k_t\Big) \Big/ \Big(r_m \Big(\Big(\left(m_1 l_1^2 + J_{1zz}\right.$$

$$\left. + J_{2xx}\right) l_2^2 + J_{2zz} L_1^2\Big) m_2 + J_{2zz}\left(m_1 l_1^2 + J_{1zz} + J_{2xx}\right)\Big)\Big),\ \frac{d}{dt}\, theta2dot(t) = \Big($$

$$-r_m b_2 \left(m_2 L_1^2 + m_1 l_1^2 + J_{1zz} + J_{2xx}\right) theta2dot(t) + g l_2 m_2 r_m \left(m_2 L_1^2 + m_1 l_1^2 + J_{1zz}\right.$$

$$\left. + J_{2xx}\right) \theta 2(t) - l_2 m_2 L_1 \left(\eta_g \eta_m k_g^2 k_m k_t + r_m b_1\right) theta1dot(t) - \left(m_2 L_1^2 + m_1 l_1^2\right.$$

$$\left. + J_{1zz} + J_{2xx}\right)\left(\pi g l_2 m_2 - \tau_2\right) r_m + L_1 V_m \eta_g \eta_m k_g k_t l_2 m_2\Big) \Big/ \Big(r_m \Big(\Big(\left(m_1 l_1^2 + J_{1zz}\right.$$

$$\left.\left. + J_{2xx}\right) l_2^2 + J_{2zz} L_1^2\Big) m_2 + J_{2zz}\left(m_1 l_1^2 + J_{1zz} + J_{2xx}\right)\Big)\Big)\right]$$

```
> dof := [theta1(t), theta2(t), theta1dot(t), theta2dot(t)];
```

$$dof := \left[\theta 1(t), \theta 2(t), theta1dot(t), theta2dot(t)\right] \quad (1.4.4)$$

```
> eqs_space_state2 := op(solve(eqs_space_state, var)):
  A, RES := GenerateMatrix(map(rhs,eqs_space_state2),dof):
> B, RES := GenerateMatrix([seq(RES[i],i=1..4)],[V__m]):
```

Space State Matrixes

```
> A := Matrix(A):
  A_data := subs(data, A);
```

$$A\_data := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 59.98558232 & -16.13923708 & -0. \\ 0 & 96.15182894 & -14.69336137 & 0. \end{bmatrix} \quad (1.4.1.1)$$

```
> B := Matrix(B):
  B_data := subs(data, B);
```

$$B\_data := \begin{bmatrix} 0 \\ 0 \\ -30.33691182 \\ -27.61910033 \end{bmatrix} \quad (1.4.1.2)$$

```
> C = Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], [0,
  0, 0, 1]]);
```

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.4.1.3}$$

```
> Eigenvalues(A_data);
```

$$\begin{bmatrix} 0.\,I \\ -19.3199170245686 + 0.\,I \\ 7.69202314634950 + 0.\,I \\ -4.51134320178092 + 0.\,I \end{bmatrix} \tag{1.4.1.4}$$

Exporting to Matlab

Matrixes

```
> Matlab(A, resultname="A");
  Matlab(A_data, resultname="dtheta1_dt");
A = [0 0 1 0; 0 0 0 1; 0 0.1e1 / (l__1 ^ 2 * l__2 ^ 2 *
m__1 * m__2 + J__1zz * l__2 ^ 2 * m__2 + J__2xx * l__2
^ 2 * m__2 + J__2zz * L__1 ^ 2 * m__2 + J__2zz * l__1 ^
2 * m__1 + J__1zz * J__2zz + J__2xx * J__2zz) * L__1 *
g * l__2 ^ 2 * m__2 ^ 2 -(eta__g * eta__m * k__g ^ 2 *
k__m * k__t * l__2 ^ 2 * m__2 + J__2zz * eta__g *
eta__m * k__g ^ 2 * k__m * k__t + b__1 * l__2 ^ 2 *
m__2 * r__m + J__2zz * b__1 * r__m) / r__m / (l__1 ^ 2
* l__2 ^ 2 * m__1 * m__2 + J__1zz * l__2 ^ 2 * m__2 +
J__2xx * l__2 ^ 2 * m__2 + J__2zz * L__1 ^ 2 * m__2 +
J__2zz * l__1 ^ 2 * m__1 + J__1zz * J__2zz + J__2xx *
J__2zz) -0.1e1 / (l__1 ^ 2 * l__2 ^ 2 * m__1 * m__2 +
J__1zz * l__2 ^ 2 * m__2 + J__2xx * l__2 ^ 2 * m__2 +
J__2zz * L__1 ^ 2 * m__2 + J__2zz * l__1 ^ 2 * m__1 +
J__1zz * J__2zz + J__2xx * J__2zz) * L__1 * b__2 * l__2
* m__2; 0 (L__1 ^ 2 * g * l__2 * m__2 ^ 2 * r__m + g *
l__1 ^ 2 * l__2 * m__1 * m__2 * r__m + J__1zz * g *
l__2 * m__2 * r__m + J__2xx * g * l__2 * m__2 * r__m) /
r__m / (l__1 ^ 2 * l__2 ^ 2 * m__1 * m__2 + J__1zz *
l__2 ^ 2 * m__2 + J__2xx * l__2 ^ 2 * m__2 + J__2zz *
L__1 ^ 2 * m__2 + J__2zz * l__1 ^ 2 * m__1 + J__1zz *
J__2zz + J__2xx * J__2zz) (-L__1 * eta__g * eta__m *
k__g ^ 2 * k__m * k__t * l__2 * m__2 - L__1 * b__1 *
l__2 * m__2 * r__m) / r__m / (l__1 ^ 2 * l__2 ^ 2 *
m__1 * m__2 + J__1zz * l__2 ^ 2 * m__2 + J__2xx * l__2
^ 2 * m__2 + J__2zz * L__1 ^ 2 * m__2 + J__2zz * l__1 ^
2 * m__1 + J__1zz * J__2zz + J__2xx * J__2zz) (-L__1 ^
2 * b__2 * m__2 * r__m - b__2 * l__1 ^ 2 * m__1 * r__m
- J__1zz * b__2 * r__m - J__2xx * b__2 * r__m) / r__m /
(l__1 ^ 2 * l__2 ^ 2 * m__1 * m__2 + J__1zz * l__2 ^ 2
* m__2 + J__2xx * l__2 ^ 2 * m__2 + J__2zz * L__1 ^ 2 *
m__2 + J__2zz * l__1 ^ 2 * m__1 + J__1zz * J__2zz +
J__2xx * J__2zz);];
dtheta1_dt = [0 0 1 0; 0 0 0 1; 0 0.5998558232e2
-0.1613923708e2 0; 0 0.9615182894e2 -0.1469336137e2 0;]
;
```

```
> Matlab(B, resultname="B");
  Matlab(B_data, resultname="B");
B = [0; 0; (-eta__g * eta__m * k__g * k__t * l__2 ^ 2 *
m__2 - J__2zz * eta__g * eta__m * k__g * k__t) / r__m /
(l__1 ^ 2 * l__2 ^ 2 * m__1 * m__2 + J__1zz * l__2 ^ 2
* m__2 + J__2xx * l__2 ^ 2 * m__2 + J__2zz * L__1 ^ 2 *
m__2 + J__2zz * l__1 ^ 2 * m__1 + J__1zz * J__2zz +
J__2xx * J__2zz); -L__1 * eta__g * eta__m * k__g * k__t
* l__2 * m__2 / r__m / (l__1 ^ 2 * l__2 ^ 2 * m__1 *
m__2 + J__1zz * l__2 ^ 2 * m__2 + J__2xx * l__2 ^ 2 *
m__2 + J__2zz * L__1 ^ 2 * m__2 + J__2zz * l__1 ^ 2 *
m__1 + J__1zz * J__2zz + J__2xx * J__2zz);];
B = [0; 0; -0.3033691182e2; -0.2761910033e2;];
```

Data

```
> Matlab(data_electrical, resultname="data_e");
eta__g = 0.85e0;
eta__m = 0.87e0;
k__g = 70;
k__m = 0.76e-2;
k__t = 0.76e-2;
r__m = 0.26e1;
V__m = 10;
> Matlab(data_mechanical, resultname="data_m");
J__1zz = 0.23e-2;
m__1 = 0;
l__1 = 0.215e0;
m__2 = 0.2e0;
L__1 = 0.215e0;
J__2yy = 0.23e-2;
l__2 = 0.1675e0;
J__2xx = 0;
J__2zz = 0.23e-2;
b__1 = 0;
Vm = 0;
g = 0.981e1;
tau__2 = 0;
b__2 = 0;
```