MECA482 - Furuta Pendulum Model

This is the model from the Student Workbook by Quanser developed by Jacob Apkarian, Paul Karam and Michel Lévis. This script starts from the definition of the non-linear model. It solves the ODE numerically to quickly verify its correctness. It reduced the system of 2 seocnd order equations to 4 first equation in order to linearize it about the unstable equilibrium position to get the matrixes A,B and C. All the results are exported in Matlab using the library CodeGeneration.

```
> restart;
> with(plots):
  with(CodeGeneration):
  with(VectorCalculus):
  with(LinearAlgebra):
  with(ArrayTools):
> merge := proc(x,y)
    [op(x), op(y)]
  end proc;
```

$$merge := \mathbf{proc}(x, y)\ \ [op(x), op(y)]\ \mathbf{end\ proc} \tag{1.1}$$

```
> linearize := proc(eqs, lin_point)
  local var, vardot, f, var_sub_eqs,var_sub,lin_point_sub,f_sub,
  J,deltax,f_subx0,f_lin_sub,var_sub_eqs_inv,f_lin,
  eqs_space_state;

      var := [seq(lhs(lin_point[i]),i=1..numelems(lin_point))];
      vardot := diff(var,t);
      f:= map(rhs,map(op,solve(eqs,vardot)));
      var_sub_eqs := [seq(var[i]=cat(x,__,i), i = 1..numelems
   (var))];
      var_sub := [seq(rhs(var_sub_eqs[i]), i=1..numelems
   (var_sub_eqs))];
      lin_point_sub := subs(var_sub_eqs, lin_point);
      f_sub := subs(var_sub_eqs, f);
      J := Matrix(evalf(subs(lin_point_sub,Jacobian(f_sub,
  var_sub))));
      deltax:= Transpose(Matrix([seq(var_sub[i] - rhs
   (lin_point_sub[i]), i=1..numelems(var_sub))]));
      f_subx0 := Transpose(Matrix(evalf(subs(lin_point_sub,
   (f_sub)))));
      f_lin_sub := simplify(f_subx0 + J.deltax);
      var_sub_eqs_inv := [seq(rhs(var_sub_eqs[i]) = lhs
   (var_sub_eqs[i]), i =1..numelems(var_sub))];
      f_lin := subs(var_sub_eqs_inv,f_lin_sub);
      eqs_space_state := [seq(vardot[i] = f_lin[i,1],i=1..
  numelems(var))];

  end proc;
```

$linearize := \mathbf{proc}(eqs, lin\_point)$ **(1.2)**

$\quad \mathbf{local}\ var, vardot, f, var\_sub\_eqs, var\_sub, lin\_point\_sub, f\_sub, J, deltax, f\_subx0, f\_lin\_sub, var\_sub\_eqs\_inv, f\_lin, eqs\_space\_state;$

$\quad var := [seq(lhs(lin\_point[i]), i = 1 ..numelems(lin\_point))];$

$\quad vardot := VectorCalculus{:-}diff(var, t);$

$\quad f := map(rhs, map(op, solve(eqs, vardot)));$

$\quad var\_sub\_eqs := [seq(var[i] = cat(x, \_\_, i), i = 1 ..numelems(var))];$

```
var_sub := [seq(rhs(var_sub_eqs[i]), i = 1 ..numelems(var_sub_eqs))];
lin_point_sub := subs(var_sub_eqs, lin_point);
f_sub := subs(var_sub_eqs, f);
J := Matrix(evalf(subs(lin_point_sub, VectorCalculus:-Jacobian(f_sub, var_sub))));
deltax := LinearAlgebra:-Transpose(Matrix([seq(VectorCalculus:-`+`(var_sub[i],
VectorCalculus:-`-`(rhs(lin_point_sub[i]))), i = 1 ..numelems(var_sub))]));
f_subx0 := LinearAlgebra:-Transpose(Matrix(evalf(subs(lin_point_sub, f_sub))));
f_lin_sub := simplify(VectorCalculus:-`+`(f_subx0, VectorCalculus:-`.`(J, deltax)));
var_sub_eqs_inv := [seq(rhs(var_sub_eqs[i]) = lhs(var_sub_eqs[i]), i = 1
 ..numelems(var_sub))];
f_lin := subs(var_sub_eqs_inv, f_lin_sub);
eqs_space_state := [seq(vardot[i] = f_lin[i, 1], i = 1 ..numelems(var))]
end proc
```

Equations of Motion with and without parameters

```
> eq1 := (m__p*L__r^2+1/4*m__p*L__p^2-1/4*m__p*L__p^2*cos(alpha
  (t))^2+J__r)*diff(theta(t),t,t) -(1/2*m__p*L__p*L__r*cos
  (alpha(t)))*diff(alpha(t),t,t) +(1/2*m__p*L__p^2*sin(alpha(t)
  *cos(alpha(t))))*diff(alpha(t),t)*diff(theta(t),t)+(1/2*m__p*
  L__p*L__r*sin(alpha(t)))*diff(alpha(t),t)^2 = eta__g*k__g*
  eta__m*k__t*((V__m - k__g*k__m*diff(theta(t), t))/r__m) -
  B__r*diff(theta(t),t);
```

$$eq1 := \left(m_p L_r^2 + \frac{m_p L_p^2}{4} - \frac{m_p L_p^2 \cos(\alpha(t))^2}{4} + J_r\right)\left(\frac{d^2}{dt^2}\,\theta(t)\right) - \frac{m_p L_p L_r \cos(\alpha(t))\left(\frac{d^2}{dt^2}\,\alpha(t)\right)}{2} + \frac{m_p L_p^2 \sin(\alpha(t)\cos(\alpha(t)))\left(\frac{d}{dt}\,\alpha(t)\right)\left(\frac{d}{dt}\,\theta(t)\right)}{2} + \frac{m_p L_p L_r \sin(\alpha(t))\left(\frac{d}{dt}\,\alpha(t)\right)^2}{2} = \frac{\eta_g k_g \eta_m k_t \left(V_m - k_g k_m \left(\frac{d}{dt}\,\theta(t)\right)\right)}{r_m} - B_r\left(\frac{d}{dt}\,\theta(t)\right) \tag{1.1.1}$$

```
> eq2 := -1/2*m__p*L__p*L__r*cos(alpha(t))*diff(theta(t),t,t)+
  (J__p+1/4*m__p*L__p^2)*diff(alpha(t),t,t)-1/4*m__p*L__p^2*cos
  (alpha(t)) * sin(alpha(t)) * diff(theta(t),t)^2 - 1/2*m__p*
  L__p*g*sin(alpha(t)) = -B__p*diff(alpha(t),t)
```

$$eq2 := -\frac{m_p L_p L_r \cos(\alpha(t))\left(\frac{d^2}{dt^2}\,\theta(t)\right)}{2} + \left(J_p + \frac{m_p L_p^2}{4}\right)\left(\frac{d^2}{dt^2}\,\alpha(t)\right) \tag{1.1.2}$$

$$-\frac{m_p L_p^2 \cos(\alpha(t)) \sin(\alpha(t)) \left(\frac{d}{dt}\theta(t)\right)^2}{4} - \frac{m_p L_p g \sin(\alpha(t))}{2} = -B_p \left(\frac{d}{dt}\alpha(t)\right)$$

```
> data_electrical := [eta__g = 0.85, eta__m = 0.87, k__g = 70,
  k__m = 0.0076 , k__t=0.0076, r__m=2.6, V__m = 10];
```

$$data\_electrical := \left[\eta_g = 0.85, \eta_m = 0.87, k_g = 70, k_m = 0.0076, k_t = 0.0076, r_m = 2.6, V_m = 10\right] \quad \textbf{(1.1.3)}$$

```
> data_mechanical := [J__p = 0.0023, m__p = 0.125, L__r =
  0.215, m__r = 0, L__p = 0.335, J__r = 0.0023, B__p = 0.000,
  tau__1 = 0, g=9.81, tau__2 = 0, B__r = 0.000];
```

$$data\_mechanical := \left[J_p = 0.0023, m_p = 0.125, L_r = 0.215, m_r = 0, L_p = 0.335, J_r = 0.0023, B_p = 0., \tau_1 = 0, g = 9.81, \tau_2 = 0, B_r = 0.\right] \quad \textbf{(1.1.4)}$$

```
> data := merge(data_mechanical, data_electrical);
```

$$data := \left[J_p = 0.0023, m_p = 0.125, L_r = 0.215, m_r = 0, L_p = 0.335, J_r = 0.0023, B_p = 0., \tau_1 = 0, g = 9.81, \tau_2 = 0, B_r = 0., \eta_g = 0.85, \eta_m = 0.87, k_g = 70, k_m = 0.0076, k_t = 0.0076, r_m = 2.6, V_m = 10\right] \quad \textbf{(1.1.5)}$$

```
> subs(data, eq1);
  subs(data, eq2);
```

$$\left(0.01158515625 - 0.003507031250 \cos(\alpha(t))^2\right) \left(\frac{d^2}{dt^2}\theta(t)\right) - 0.004501562500 \cos(\alpha(t)) \left(\frac{d^2}{dt^2}\alpha(t)\right) + 0.007014062500 \sin(\alpha(t) \cos(\alpha(t))) \left(\frac{d}{dt}\alpha(t)\right) \left(\frac{d}{dt}\theta(t)\right) + 0.004501562500 \sin(\alpha(t)) \left(\frac{d}{dt}\alpha(t)\right)^2 = 1.513130769 - 0.08049855691 \left(\frac{d}{dt}\theta(t)\right)$$

$$-0.004501562500 \cos(\alpha(t)) \left(\frac{d^2}{dt^2}\theta(t)\right) + 0.005807031250 \left(\frac{d^2}{dt^2}\alpha(t)\right) - 0.003507031250 \cos(\alpha(t)) \sin(\alpha(t)) \left(\frac{d}{dt}\theta(t)\right)^2 - 0.2053968750 \sin(\alpha(t)) = -0. \quad \textbf{(1.1.6)}$$

```
> eqs := [eq1 ,eq2];
```

$$eqs := \left[\left(m_p L_r^2 + \frac{m_p L_p^2}{4} - \frac{m_p L_p^2 \cos(\alpha(t))^2}{4} + J_r\right) \left(\frac{d^2}{dt^2}\theta(t)\right) \right. \quad \textbf{(1.1.7)}$$

$$-\frac{m_p L_p L_r \cos(\alpha(t)) \left(\frac{d^2}{dt^2}\alpha(t)\right)}{2}$$

$$+\frac{m_p L_p^2 \sin(\alpha(t)\cos(\alpha(t)))\left(\frac{d}{dt}\alpha(t)\right)\left(\frac{d}{dt}\theta(t)\right)}{2}$$

$$+\frac{m_p L_p L_r \sin(\alpha(t))\left(\frac{d}{dt}\alpha(t)\right)^2}{2} = \frac{\eta_g k_g \eta_m k_t \left(V_m - k_g k_m \left(\frac{d}{dt}\theta(t)\right)\right)}{r_m}$$

$$-B_r\left(\frac{d}{dt}\theta(t)\right), -\frac{m_p L_p L_r \cos(\alpha(t))\left(\frac{d^2}{dt^2}\theta(t)\right)}{2} + \left(J_p + \frac{m_p L_p^2}{4}\right)\left(\frac{d^2}{dt^2}\right.$$

$$\left.\alpha(t)\right) - \frac{m_p L_p^2 \cos(\alpha(t))\sin(\alpha(t))\left(\frac{d}{dt}\theta(t)\right)^2}{4} - \frac{m_p L_p g \sin(\alpha(t))}{2} =$$

$$-B_p\left(\frac{d}{dt}\alpha(t)\right)\Bigg]$$

Simulation

Initial condition --> Vertical Arm at Pi/4

```
> ics1 := [diff(alpha(t), t) = 0, diff(theta(t), t) = 0,
  alpha(t) = Pi/2, theta(t) = 0]
```

$$ics1 := \left[\frac{d}{dt}\alpha(t) = 0, \frac{d}{dt}\theta(t) = 0, \alpha(t) = \frac{\pi}{2}, \theta(t) = 0\right] \quad \textbf{(1.2.1.1)}$$

```
> ics1 := subs(t=0,convert(ics1, D))
```

$$ics1 := \left[D(\alpha)(0) = 0, D(\theta)(0) = 0, \alpha(0) = \frac{\pi}{2}, \theta(0) = 0\right] \quad \textbf{(1.2.1.2)}$$

```
> ODEs1 := merge((subs(data, eqs)), ics1);
```

$$ODEs1 := \left[\left(0.01158515625 - 0.003507031250\cos(\alpha(t))^2\right)\left(\frac{d^2}{dt^2}\theta(t)\right)\right. \quad \textbf{(1.2.1.3)}$$

$$-0.004501562500\cos(\alpha(t))\left(\frac{d^2}{dt^2}\alpha(t)\right)$$

$$+0.007014062500\sin(\alpha(t)\cos(\alpha(t)))\left(\frac{d}{dt}\alpha(t)\right)\left(\frac{d}{dt}\theta(t)\right)$$

$$+0.004501562500\sin(\alpha(t))\left(\frac{d}{dt}\alpha(t)\right)^2 = 1.513130769$$

$$- 0.08049855691 \left(\frac{d}{dt}\,\theta(t)\right), -0.004501562500 \cos(\alpha(t)) \left(\frac{d^2}{dt^2}\,\theta(t)\right)$$

$$+ 0.005807031250 \left(\frac{d^2}{dt^2}\,\alpha(t)\right)$$

$$- 0.003507031250 \cos(\alpha(t)) \sin(\alpha(t)) \left(\frac{d}{dt}\,\theta(t)\right)^2$$

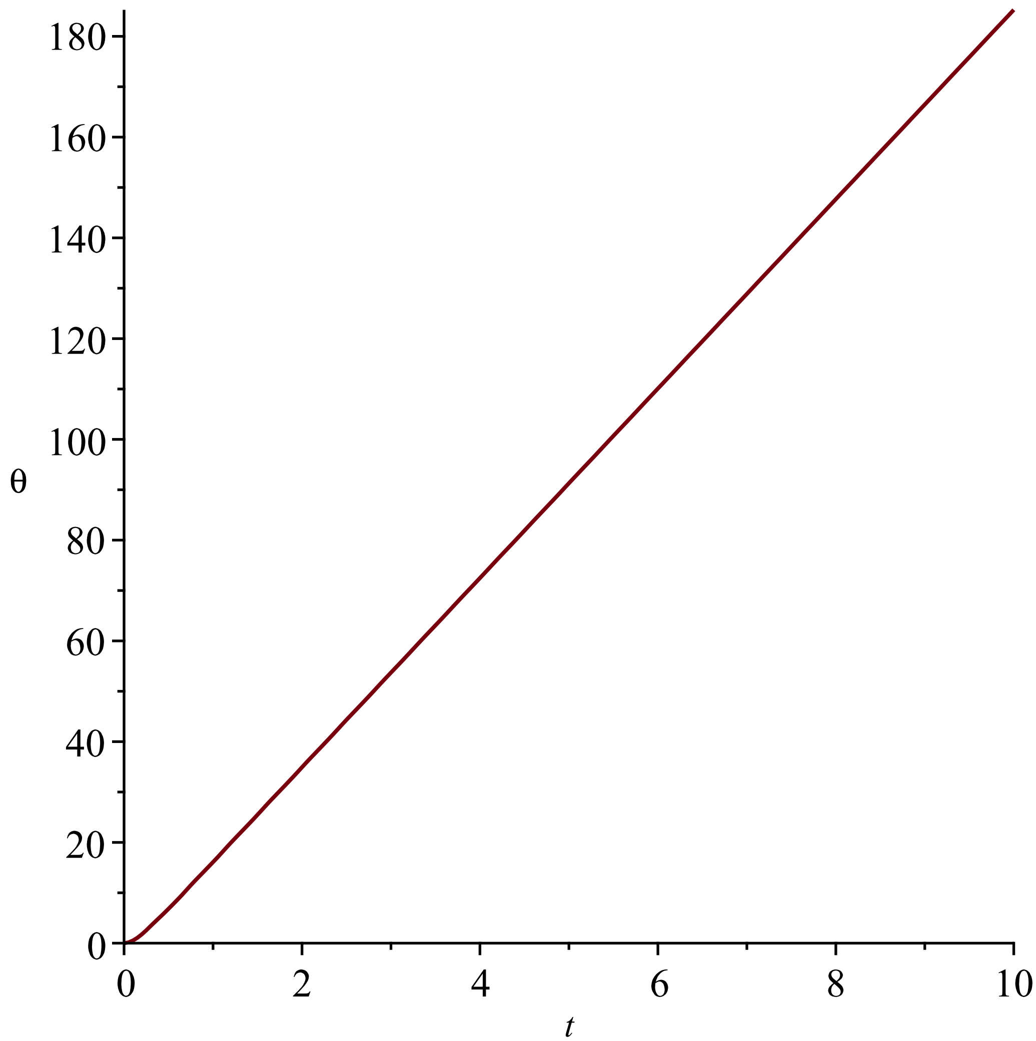
$$- 0.2053968750 \sin(\alpha(t)) = -0., D(\alpha)(0) = 0, D(\theta)(0) = 0, \alpha(0) = \frac{\pi}{2},$$

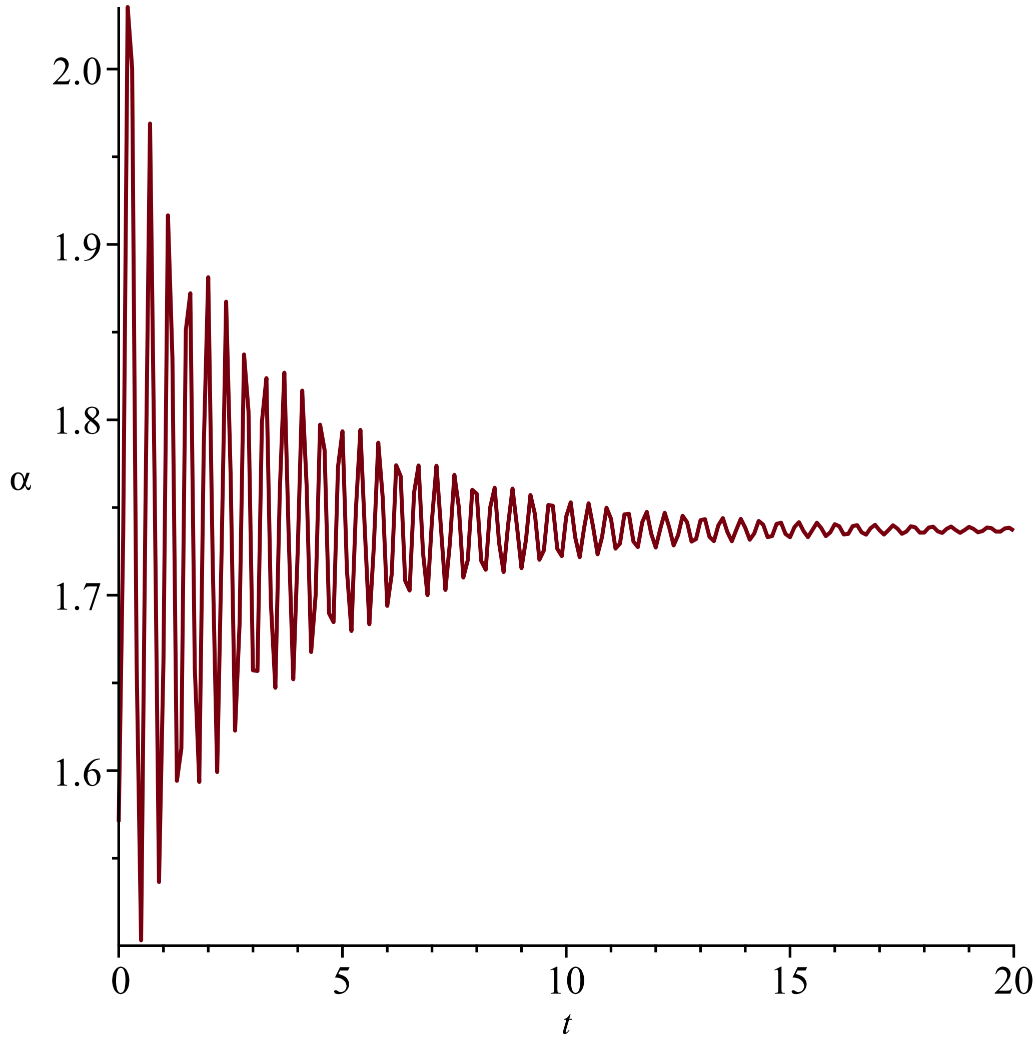
$$\theta(0) = 0\Big]$$

```
> ode_sol1 := dsolve(ODEs1, numeric);
```

$ode\_sol1 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc}$ **(1.2.1.4)**

```
> odeplot(ode_sol1, [t, theta(t)], t=0..10);
  odeplot(ode_sol1, [t, alpha(t)], t=0..20);
```





Reducing to first order

```
> add_diff := [diff(theta(t), t) = thetadot(t), diff(alpha(t), t) = alphadot(t)];
```

$$add\_diff := \left[\frac{d}{dt}\, \theta(t) = thetadot(t),\ \frac{d}{dt}\, \alpha(t) = alphadot(t) \right] \quad \textbf{(1.3.1)}$$

```
> eqs_first_order := merge(subs(add_diff, eqs), add_diff);
```

$$eqs\_first\_order := \left[\left(m_p L_r^2 + \frac{m_p L_p^2}{4} - \frac{m_p L_p^2 \cos(\alpha(t))^2}{4} + J_r \right) \left(\frac{d}{dt}\, thetadot(t) \right) \quad \textbf{(1.3.2)} \right.$$

$$- \frac{m_p L_p L_r \cos(\alpha(t)) \left(\frac{d}{dt}\, alphadot(t) \right)}{2}$$

$$+ \frac{m_p L_p^2 \sin(\alpha(t) \cos(\alpha(t)))\, alphadot(t)\, thetadot(t)}{2}$$

$$+\frac{m_p L_p L_r \sin(\alpha(t))\ alphadot(t)^2}{2} = \frac{\eta_g k_g \eta_m k_t (V_m - k_g k_m\ thetadot(t))}{r_m}$$

$$- B_r\ thetadot(t), -\frac{m_p L_p L_r \cos(\alpha(t)) \left(\frac{d}{dt}\ thetadot(t)\right)}{2} + \left(J_p + \frac{m_p L_p^2}{4}\right)\left(\frac{d}{dt}\right.$$

$$\left. alphadot(t)\right) - \frac{m_p L_p^2 \cos(\alpha(t)) \sin(\alpha(t))\ thetadot(t)^2}{4} - \frac{m_p L_p g \sin(\alpha(t))}{2} =$$

$$-B_p\ alphadot(t), \frac{d}{dt}\ \theta(t) = thetadot(t), \frac{d}{dt}\ \alpha(t) = alphadot(t)\Big]$$

```
> var := [theta(t), alpha(t), thetadot(t), alphadot(t)];
```

$$var := [\theta(t), \alpha(t), thetadot(t), alphadot(t)] \quad \mathbf{(1.3.3)}$$

```
> vardot := diff(var,t);
```

$$vardot := \left[\frac{d}{dt}\ \theta(t), \frac{d}{dt}\ \alpha(t), \frac{d}{dt}\ thetadot(t), \frac{d}{dt}\ alphadot(t)\right] \quad \mathbf{(1.3.4)}$$

```
> var_sub := [theta(t) = theta, alpha(t) = alpha, thetadot(t) =
  thetadot, alphadot(t) = alphadot]
```

$$var\_sub := [\theta(t) = \theta, \alpha(t) = \alpha, thetadot(t) = thetadot, alphadot(t) = alphadot] \quad \mathbf{(1.3.5)}$$

```
> solve(eqs_first_order, vardot)[1]:
  map(rhs, %):
  F := simplify(subs(var_sub, %));
```

$$F := \Big[thetadot, alphadot, \Big(2\, m_p r_m\, alphadot\ thetadot\, L_p^2 (m_p L_p^2 + 4 J_p) \sin(\alpha \cos(\alpha)) \quad \mathbf{(1.3.6)}$$

$$- 2 L_p^3 L_r \cos(\alpha)^2 \sin(\alpha)\ thetadot^2 m_p^2 r_m - 4 m_p r_m L_p L_r \big(g\, m_p \sin(\alpha) L_p$$

$$- 2\, alphadot\, B_p\big) \cos(\alpha) + 4 (m_p L_p^2 + 4 J_p) \Big(\frac{m_p r_m\, alphadot^2 \sin(\alpha) L_p L_r}{2}$$

$$+ r_m\, thetadot\, B_r + k_t \eta_g \eta_m k_g (k_g k_m\, thetadot - V_m)\Big)\Big) \Big/ \Big(\Big(L_p^2 m_p \Big(\Big(L_p^2$$

$$+4\,L_r^{2}\big)\,m_p+4\,J_p\big)\cos(\alpha)^{2}-\big(m_p\,L_p^{2}+4\,J_p\big)\,\big(\big(L_p^{2}+4\,L_r^{2}\big)\,m_p+4\,J_r\big)\big)\,r_m\big)$$

$$,\Bigg(4\,L_p^{3}\,L_r\cos(\alpha)\,\sin(\alpha\cos(\alpha))\;thetadot\;alphadot\;m_p^{2}\,r_m$$

$$+L_p^{4}\cos(\alpha)^{3}\sin(\alpha)\;thetadot^{2}\,m_p^{2}\,r_m+2\,m_p\,r_m\,L_p^{2}\,(g\,m_p\sin(\alpha)\,L_p$$

$$-2\,alphadot\,B_p)\cos(\alpha)^{2}$$

$$+8\,L_p\left(\frac{1}{2}\left(L_p\left(\left(-\frac{thetadot^{2}\,L_p^{2}}{4}+L_r^{2}\,(alphadot^{2}-thetadot^{2})\right)m_p\right.\right.\right.$$

$$\left.\left.-thetadot^{2}\,J_r\right)r_m\sin(\alpha)\right)+L_r\,\big(r_m\;thetadot\;B_r+k_t\,\eta_g\,\eta_m\,k_g\,(k_g\,k_m\;thetadot$$

$$\left.-V_m)\big)\right)m_p\cos(\alpha)-2\,r_m\,(g\,m_p\sin(\alpha)\,L_p-2\,alphadot\,B_p)\,\big(\big(L_p^{2}+4\,L_r^{2}\big)\,m_p$$

$$\left.+4\,J_r\big)\right)\Bigg/\Big(\big(L_p^{2}\,m_p\,\big(\big(L_p^{2}+4\,L_r^{2}\big)\,m_p+4\,J_p\big)\cos(\alpha)^{2}-\big(m_p\,L_p^{2}+4\,J_p\big)\,\big(\big(L_p^{2}$$

$$\left.+4\,L_r^{2}\big)\,m_p+4\,J_r\big)\big)\,r_m\Big)\Bigg]$$

```
> Matlab(F[1], resultname="dtheta_dt");
dtheta_dt = thetadot;
> Matlab(F[2], resultname="dalpha_dt");
dalpha_dt = alphadot;
> Matlab(F[3], resultname="d2theta_dt2");
d2theta_dt2 = (0.2e1 * m__p * r__m * alphadot * thetadot *
(L__p ^ 2) * (m__p * L__p ^ 2 + 4 * J__p) * sin(alpha * cos
(alpha)) - 0.2e1 * (L__p ^ 3) * L__r * cos(alpha) ^ 2 * sin
(alpha) * thetadot ^ 2 * (m__p ^ 2) * r__m - 0.4e1 * m__p *
r__m * L__p * L__r * (g * m__p * sin(alpha) * L__p - 0.2e1 *
alphadot * B__p) * cos(alpha) + 0.4e1 * (m__p * L__p ^ 2 + 4
* J__p) * (m__p * r__m * alphadot ^ 2 * sin(alpha) * L__p *
L__r / 0.2e1 + r__m * thetadot * B__r + k__t * eta__g *
eta__m * k__g * (k__g * k__m * thetadot - V__m))) / ((L__p ^
2) * m__p * (((L__p ^ 2) + 0.4e1 * L__r ^ 2) * m__p + (4 *
J__p)) * cos(alpha) ^ 2 - (m__p * L__p ^ 2 + 4 * J__p) * ((
```

```
(L__p ^ 2) + 0.4e1 * L__r ^ 2) * m__p + (4 * J__r))) / r__m;
> Matlab(F[4], resultname="d2alpha_dt2");
d2alpha_dt2 = (0.4e1 * L__p ^ 3 * L__r * cos(alpha) * sin
(alpha * cos(alpha)) * thetadot * alphadot * m__p ^ 2 * r__m
+ L__p ^ 4 * cos(alpha) ^ 3 * sin(alpha) * thetadot ^ 2 *
m__p ^ 2 * r__m + 0.2e1 * m__p * r__m * L__p ^ 2 * (g * m__p
* sin(alpha) * L__p - 0.2e1 * alphadot * B__p) * cos(alpha)
^ 2 + 0.8e1 * L__p * (L__p * ((-thetadot ^ 2 * L__p ^ 2 /
0.4e1 + L__r ^ 2 * (alphadot ^ 2 - thetadot ^ 2)) * m__p -
thetadot ^ 2 * J__r) * r__m * sin(alpha) / 0.2e1 + L__r *
(r__m * thetadot * B__r + k__t * eta__g * eta__m * k__g *
(k__g * k__m * thetadot - V__m))) * m__p * cos(alpha) -
0.2e1 * r__m * (g * m__p * sin(alpha) * L__p - 0.2e1 *
alphadot * B__p) * ((L__p ^ 2 + 0.4e1 * L__r ^ 2) * m__p +
0.4e1 * J__r)) / (L__p ^ 2 * m__p * ((L__p ^ 2 + 0.4e1 *
L__r ^ 2) * m__p + (4 * J__p)) * cos(alpha) ^ 2 - (m__p *
L__p ^ 2 + (4 * J__p)) * ((L__p ^ 2 + 0.4e1 * L__r ^ 2) *
m__p + 0.4e1 * J__r)) / r__m;
```

Reducing to state space model and exporting to Matlab

```
> eqs_first_order;
```

$$\left[\left(m_p L_r^2+\frac{m_p L_p^2}{4}-\frac{m_p L_p^2\cos(\alpha(t))^2}{4}+J_r\right)\left(\frac{d}{dt}\,thetadot(t)\right)-\frac{m_p L_p L_r\cos(\alpha(t))\left(\frac{d}{dt}\,alphadot(t)\right)}{2}+\frac{m_p L_p^2\sin(\alpha(t)\cos(\alpha(t)))\,alphadot(t)\,thetadot(t)}{2}+\frac{m_p L_p L_r\sin(\alpha(t))\,alphadot(t)^2}{2}=\frac{\eta_g k_g \eta_m k_t\left(V_m-k_g k_m\,thetadot(t)\right)}{r_m}-B_r\,thetadot(t),\ -\frac{m_p L_p L_r\cos(\alpha(t))\left(\frac{d}{dt}\,thetadot(t)\right)}{2}+\left(J_p+\frac{m_p L_p^2}{4}\right)\left(\frac{d}{dt}\,alphadot(t)\right)-\frac{m_p L_p^2\cos(\alpha(t))\sin(\alpha(t))\,thetadot(t)^2}{4}-\frac{m_p L_p g\sin(\alpha(t))}{2}=-B_p\,alphadot(t),\ \frac{d}{dt}\,\theta(t)=thetadot(t),\ \frac{d}{dt}\,\alpha(t)=alphadot(t)\right] \tag{1.4.1}$$

```
> lin_point:=[theta(t) =2.99,alpha(t)=0,thetadot(t)=0,alphadot
(t)=0];
```

$$lin\_point := [\theta(t)=2.99,\ \alpha(t)=0,\ thetadot(t)=0,\ alphadot(t)=0] \tag{1.4.2}$$

```
> eqs_space_state := linearize(eqs_first_order, lin_point);
```

$$eqs\_space\_state := \left[\frac{d}{dt}\,\theta(t) = 1.\,thetadot(t),\ \frac{d}{dt}\,\alpha(t) = 1.\,alphadot(t),\ \frac{d}{dt}\,thetadot(t) = \frac{1}{r_m\left(4.\,J_p L_r^2 m_p + J_r L_p^2 m_p + 4.\,J_p J_r\right)}\left(L_p^2 L_r g\, m_p^2\,\alpha(t)\,r_m + \left(\left(-1.\,B_r\,thetadot(t)\,r_m + \left(-1.\,thetadot(t)\,k_g^2 k_m k_t + V_m k_g k_t\right)\eta_m\eta_g\right)L_p^2 - 2.\,B_p L_p L_r\,alphadot(t)\,r_m\right)m_p + \left(-4.\,B_r\,thetadot(t)\,r_m + \left(-4.\,thetadot(t)\,k_g^2 k_m k_t + 4.\,V_m k_g k_t\right)\eta_m\eta_g\right)J_p\right),\ \frac{d}{dt}\,alphadot(t) = \frac{1}{r_m\left(4.\,J_p L_r^2 m_p + J_r L_p^2 m_p + 4.\,J_p J_r\right)}\left(2.\,L_p g\, m_p^2 r_m\,\alpha(t)\,L_r^2 + \left(\left(\left(-2.\,B_r\,thetadot(t)\,r_m + \left(-2.\,thetadot(t)\,k_g^2 k_m k_t + 2.\,V_m k_g k_t\right)\eta_m\eta_g\right)L_r + 2.\,g\,r_m\,\alpha(t)\,J_r\right)L_p - 4.\,B_p L_r^2\,alphadot(t)\,r_m\right)m_p - 4.\,B_p J_r\,alphadot(t)\,r_m\right)\right] \quad \textbf{(1.4.3)}$$

```
> eqs_space_state2 := solve(eqs_space_state, vardot):
  A, RES := GenerateMatrix(map(rhs,eqs_space_state2[1]),var):
> B, RES := GenerateMatrix(map(rhs, eqs_space_state2[1]),[V__m]
  ):
```

Space State Matrixes. Checking for Stability and Reachability

```
> A := Matrix(A);
  A_data := subs(data, A);
```

$$A := \left[\left[0, 0, 1, 0\right],\ \left[0, 0, 0, 1\right],\ \left[0, \frac{1.\,L_p^2 L_r g\, m_p^2}{4.\,J_p L_r^2 m_p + J_r L_p^2 m_p + 4.\,J_p J_r}, -\frac{1.\,\left(L_p^2\eta_g\eta_m k_g^2 k_m k_t m_p + 4.\,J_p\eta_g\eta_m k_g^2 k_m k_t + B_r L_p^2 m_p r_m + 4.\,B_r J_p r_m\right)}{r_m\left(4.\,J_p L_r^2 m_p + J_r L_p^2 m_p + 4.\,J_p J_r\right)}, -\frac{2.\,B_p L_p L_r m_p}{4.\,J_p L_r^2 m_p + J_r L_p^2 m_p + 4.\,J_p J_r}\right],\right.$$

$$\left[0, -\frac{2.\left(-L_p L_r^2 g m_p^2 r_m - J_r L_p g m_p r_m\right)}{r_m\left(4.\, J_p L_r^2 m_p + J_r L_p^2 m_p + 4.\, J_p J_r\right)}, -\frac{2.\left(L_p L_r \eta_g \eta_m k_g^2 k_m k_t m_p + B_r L_p L_r m_p r_m\right)}{r_m\left(4.\, J_p L_r^2 m_p + J_r L_p^2 m_p + 4.\, J_p J_r\right)}, -\frac{2.\left(2.\, B_p L_r^2 m_p r_m + 2.\, B_p J_r r_m\right)}{r_m\left(4.\, J_p L_r^2 m_p + J_r L_p^2 m_p + 4.\, J_p J_r\right)}\right]\right]$$

$$A\_data := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 34.69983298 & -17.54334995 & -0. \\ 0 & 62.26939834 & -13.59945949 & -0. \end{bmatrix} \tag{1.4.1.1}$$

```
> B := Matrix(B);
  B_data := subs(data, Matrix(B));
```

$$B := \begin{bmatrix} 0 \\ 0 \\ -\frac{1.\left(-L_p^2 \eta_g \eta_m k_g k_t m_p - 4.\, J_p \eta_g \eta_m k_g k_t\right)}{r_m\left(4.\, J_p L_r^2 m_p + J_r L_p^2 m_p + 4.\, J_p J_r\right)} \\ \frac{2.\, L_p L_r \eta_g \eta_m k_g k_t m_p}{r_m\left(4.\, J_p L_r^2 m_p + J_r L_p^2 m_p + 4.\, J_p J_r\right)} \end{bmatrix}$$

$$B\_data := \begin{bmatrix} 0 \\ 0 \\ 32.97622171 \\ 25.56289378 \end{bmatrix} \tag{1.4.1.2}$$

```
> C_output := Matrix([[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 
  0], [0, 0, 0, 1]]);
```

$$C\_output := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.4.1.3}$$

```
> state := var;
```

$$state := [\theta(t), \alpha(t), thetadot(t), alphadot(t)] \tag{1.4.1.4}$$

Point locally not aymptotically stable

```
> Eigenvalues(A_data);
```

$$\begin{bmatrix} 0.\,I \\ -19.1026211185592 + 0.\,I \\ 6.53211884198625 + 0.\,I \\ -4.97284767342700 + 0.\,I \end{bmatrix} \tag{1.4.1.5}$$

```
> A.B
```

$$\left[\left[-\frac{1.\,\left(-L_p^2\,\eta_g\,\eta_m\,k_g\,k_t\,m_p - 4.\,J_p\,\eta_g\,\eta_m\,k_g\,k_t\right)}{r_m\,\left(4.\,J_p\,L_r^2\,m_p + J_r\,L_p^2\,m_p + 4.\,J_p\,J_r\right)}\right],\right. \tag{1.4.1.6}$$

$$\left[\frac{2.\,L_p\,L_r\,\eta_g\,\eta_m\,k_g\,k_t\,m_p}{r_m\,\left(4.\,J_p\,L_r^2\,m_p + J_r\,L_p^2\,m_p + 4.\,J_p\,J_r\right)}\right],$$

$$\left[\frac{1}{r_m^2\,\left(4.\,J_p\,L_r^2\,m_p + J_r\,L_p^2\,m_p + 4.\,J_p\,J_r\right)^2}\left(1.\,\left(L_p^2\,\eta_g\,\eta_m\,k_g^2\,k_m\,k_t\,m_p\right.\right.\right.$$

$$\left. + 4.\,J_p\,\eta_g\,\eta_m\,k_g^2\,k_m\,k_t + B_r\,L_p^2\,m_p\,r_m + 4.\,B_r\,J_p\,r_m\right)\left(-L_p^2\,\eta_g\,\eta_m\,k_g\,k_t\,m_p\right.$$

$$\left.\left.\left. - 4.\,J_p\,\eta_g\,\eta_m\,k_g\,k_t\right)\right) - \frac{4.\,B_p\,L_p^2\,L_r^2\,m_p^2\,\eta_g\,\eta_m\,k_g\,k_t}{\left(4.\,J_p\,L_r^2\,m_p + J_r\,L_p^2\,m_p + 4.\,J_p\,J_r\right)^2\,r_m}\right],$$

$$\left[\frac{1}{r_m^2\,\left(4.\,J_p\,L_r^2\,m_p + J_r\,L_p^2\,m_p + 4.\,J_p\,J_r\right)^2}\left(2.\,\left(L_p\,L_r\,\eta_g\,\eta_m\,k_g^2\,k_m\,k_t\,m_p\right.\right.\right.$$

$$\left.\left. + B_r\,L_p\,L_r\,m_p\,r_m\right)\left(-L_p^2\,\eta_g\,\eta_m\,k_g\,k_t\,m_p - 4.\,J_p\,\eta_g\,\eta_m\,k_g\,k_t\right)\right)$$

$$\left.\left. - \frac{4.\,\left(2.\,B_p\,L_r^2\,m_p\,r_m + 2.\,B_p\,J_r\,r_m\right)L_p\,L_r\,\eta_g\,\eta_m\,k_g\,k_t\,m_p}{r_m^2\,\left(4.\,J_p\,L_r^2\,m_p + J_r\,L_p^2\,m_p + 4.\,J_p\,J_r\right)^2}\right]\right]$$

The system is reachable

```
> RR_data := Concatenate(2, B_data, A_data.B_data, A_data^(2).B_data, A_data^(3).B_data);
  Rank(RR_data);
```

$RR\_data :=$

[[0, 32.9762217100000, −578.513397487317, 11036.0911275349],
[0, 25.5628937800000, −448.458791278403, 9459.25552906097],
[32.97622171, −578.513397487317, 11036.0911275349, −209171.453886209
],
[25.56289378, −448.458791278403, 9459.25552906097, −178010.133330050
]]

4 **(1.4.1.7)**

Exporting to Matlab

Matrices

```
> Matlab(A, resultname="A");
A = [0 0 1 0; 0 0 0 1; 0 0.1e1 / (0.4e1 * J__p * L__r ^
2 * m__p + J__r * L__p ^ 2 * m__p + 0.4e1 * J__p *
J__r) * L__p ^ 2 * L__r * g * m__p ^ 2 -0.1e1 / r__m /
(0.4e1 * J__p * L__r ^ 2 * m__p + J__r * L__p ^ 2 *
m__p + 0.4e1 * J__p * J__r) * (L__p ^ 2 * eta__g *
eta__m * k__g ^ 2 * k__m * k__t * m__p + 0.4e1 * J__p *
eta__g * eta__m * k__g ^ 2 * k__m * k__t + B__r * L__p
^ 2 * m__p * r__m + 0.4e1 * B__r * J__p * r__m) -0.2e1
/ (0.4e1 * J__p * L__r ^ 2 * m__p + J__r * L__p ^ 2 *
m__p + 0.4e1 * J__p * J__r) * B__p * L__p * L__r *
m__p; 0 -0.2e1 * (-0.1e1 * L__p * L__r ^ 2 * g * m__p ^
2 * r__m - 0.1e1 * J__r * L__p * g * m__p * r__m) /
r__m / (0.4e1 * J__p * L__r ^ 2 * m__p + J__r * L__p ^
2 * m__p + 0.4e1 * J__p * J__r) -0.2e1 * (L__p * L__r *
eta__g * eta__m * k__g ^ 2 * k__m * k__t * m__p + B__r
* L__p * L__r * m__p * r__m) / r__m / (0.4e1 * J__p *
L__r ^ 2 * m__p + J__r * L__p ^ 2 * m__p + 0.4e1 * J__p
* J__r) -0.2e1 * (0.2e1 * B__p * L__r ^ 2 * m__p * r__m
+ 0.2e1 * B__p * J__r * r__m) / r__m / (0.4e1 * J__p *
L__r ^ 2 * m__p + J__r * L__p ^ 2 * m__p + 0.4e1 * J__p
* J__r);];
> Matlab(B, resultname="B");
B = [0; 0; -0.1e1 / r__m / (0.4e1 * J__p * L__r ^ 2 *
m__p + J__r * L__p ^ 2 * m__p + 0.4e1 * J__p * J__r) *
(-0.1e1 * L__p ^ 2 * eta__g * eta__m * k__g * k__t *
m__p - 0.4e1 * J__p * eta__g * eta__m * k__g * k__t);
0.2e1 * L__p * L__r * eta__g * eta__m * k__g * k__t *
m__p / r__m / (0.4e1 * J__p * L__r ^ 2 * m__p + J__r *
L__p ^ 2 * m__p + 0.4e1 * J__p * J__r);];
> Matlab(C_output, resultname="C");
C = [1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1;];
```

Data

```
> Matlab(data_electrical, resultname="data_e");
eta__g = 0.85e0;
eta__m = 0.87e0;
k__g = 70;
k__m = 0.76e-2;
k__t = 0.76e-2;
r__m = 0.26e1;
V__m = 10;
> Matlab(data_mechanical, resultname="data_m");
J__p = 0.23e-2;
m__p = 0.125e0;
L__r = 0.215e0;
m__r = 0;
L__p = 0.335e0;
J__r = 0.23e-2;
B__p = 0.0e0;
tau__1 = 0;
g = 0.981e1;
tau__2 = 0;
```

```
B__r = 0.0e0;
```