Determining Ramsey numbers on a quantum computer

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We present a quantum algorithm for computing the Ramsey numbers whose computational complexity grows superexponentially with the number of vertices of a graph on a classical computer. The problem is mapped to a decision problem on a quantum computer, and a probe qubit is coupled to a register that represents the problem and detects the energy levels of the problem Hamiltonian. The decision problem is solved by detecting the decay dynamics of the probe qubit.

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I. INTRODUCTION

Ramsey numbers are part of the Ramsey theory [1] which is a branch of mathematics that studies the occurrence of order in large disordered structures, and has a wide application in mathematics, information theory, and theoretical computer science [2–4]. The computation of Ramsey numbers are extremely difficult; only nine of them are currently known. Here, we focus on the two-color Ramsey numbers which can be described as follows [1]: In an n-vertex graph, the x vertices form an x clique, and the y vertices form a y-independent set. An x clique is a set of x vertices in which any two of the vertices are connected by an edge, while a y-independent set is a set of y vertices in which no two of the vertices are connected by an edge [5]. According to the Ramsey theory [1], there exists a threshold value $R(x,y)$ for given integers x and y, such that every graph of n vertices contains either an x clique or a y-independent set as long as $n \geqslant R(x,y)$. The task is to compute the Ramsey number—the threshold value $R(x,y)$ for given x and y.

A total number of $2^{n(n-1)/2}$ different graphs can be formed by n vertices. To determine whether n is the Ramsey number $R(x,y)$ or not, one has to check all the $2^{n(n-1)/2}$ graphs, which grow superexponentially with n, and the task quickly becomes intractable. Mathematically, bounds for Ramsey numbers have been given, but it is still a challenge to determine the exact Ramsey numbers in most cases.

In Ref. [6], the problem of computing a Ramsey number is mapped to a combinatorial optimization problem, and an adiabatic quantum optimization (AQO) algorithm [7] is applied for solving the problem. The algorithm has been implemented experimentally on the D-Wave One device [8]. In this algorithm, a Hamiltonian for the problem is constructed, and the system is evolved to the ground state of the problem Hamiltonian through adiabatic quantum evolution. Then one can check whether the ground-state energy of the problem Hamiltonian equals or is larger than zero. Here, in this paper, we propose a different quantum algorithm for determining Ramsey numbers and obtaining all the corresponding graphs that have the minimum number of x cliques or y-independent sets. In this algorithm, a Ramsey number can be determined without knowing the ground state of the Hamiltonian.

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II. THE ALGORITHM

We use the scheme introduced in Ref. [6] to transform the problem of computing a Ramsey number $R(x,y)$ to a decision problem on a quantum computer.

In an n-vertex graph G, there are $L = n(n-1)/2$ ways of choosing a pair of vertices (v,v'), a bit variable $a_{v,v'}$ is associated for each pair of (v,v'), $a_{v,v'} = 1$ if v and v' are connected with an edge and $a_{v,v'} = 0$ otherwise. Then a bit vector $\mathbf{a} = (a_{1,2},\ldots,a_{1,n},a_{2,3},\ldots,a_{2,n},\ldots,a_{n-1,n})$ of length L uniquely represents an n-vertex graph G, and there are $N = 2^L$ vectors in total. For given integers n, x, and y, count the number of x cliques $C_x^n(\mathbf{a})$ and y-independent sets $I_y^n(\mathbf{a})$ in an n-vertex graph G represented by vector $\mathbf{a}$, and define an "energy" function $h_{x,y}^n(\mathbf{a}) = C_x^n(\mathbf{a}) + I_y^n(\mathbf{a})$. If $n < R(x,y)$, the minimum of the function $h_{x,y}^n(\mathbf{a})$ is zero, and if $n \geqslant R(x,y)$, Ramsey theory guarantees that $h_{x,y}^n(\mathbf{a}) > 0$.

On a quantum computer, each bit variable $a_{v,v'}$ is represented by a qubit. $|z_k\rangle$ with $z_k = 0$ or 1 represents the kth bit, and all the $N = 2^L$ vectors are represented by the L-qubit vectors $|\psi\rangle = |z_1 z_2 \cdots z_L\rangle$. These vectors form a complete computational basis of the L qubits. The problem Hamiltonian H_P is defined as

$$H_P|z_1 z_2 \cdots z_L\rangle = h_{x,y}^n(\mathbf{a})|z_1 z_2 \cdots z_L\rangle, \tag{1}$$

$H_P|\psi\rangle = 0$, if and only if the bit string $z_1 z_2 \cdots z_L$ does not contain either x cliques or y-independent sets.

Ramsey numbers are integers in a bounded range. The computation of Ramsey numbers begins by setting n equal to a lower bound for $R(x,y)$ which can be found in a table of two-color Ramsey numbers [5]. Increase n by one each time, and check if the ground-state energy of H_P is zero or not. The first integer n for which the ground-state energy E_1 of H_P is larger than zero is the Ramsey number $R(x,y)$ for given x and y.

From the analysis above, we can see that computation of a Ramsey number is transformed to a problem of determining whether the ground-state energy E_1 of H_P is zero or not. We proposed the following quantum algorithm for this problem.

First, we construct a quantum register Q which contains one ancilla qubit and an L-qubit quantum register that represents the state space of the problem Hamiltonian of dimension N. A probe qubit is coupled to Q and the Hamiltonian of the

entire $(L+2)$-qubit system is constructed as

$$H = -\tfrac{1}{2}\omega\sigma_z \otimes I_2^{\otimes(L+1)} + I_2 \otimes H_Q + c\sigma_x \otimes A, \qquad (2)$$

where I_2 is the two-dimensional identity operator, and σ_x and σ_z are the Pauli matrices. The first term in the above equation is the Hamiltonian of the probe qubit, the second term is the Hamiltonian of the register Q, and the third term describes the interaction between the probe qubit and Q. ω is the frequency of the probe qubit ($\hbar = 1$) and c is the coupling coefficient and $c \ll \omega$. The Hamiltonian of register Q is in the form,

$$H_Q = |0\rangle\langle 0| \otimes [\varepsilon_0(|0\rangle\langle 0|)^{\otimes L}] + |1\rangle\langle 1| \otimes H_P, \qquad (3)$$

where ε_0 is a parameter that is set as a reference point to the ground-state energy E_1 of H_P. The register Q is prepared in a reference state $|\Phi\rangle = |0\rangle^{\otimes(L+1)}$, which is an eigenstate of H_Q with eigenvalue ε_0. The operator A is constructed as $A = \sigma_x \otimes H_d^{\otimes L}$, where H_d is the Hadamard matrices. It acts on the reference state $|\Phi\rangle$ and generates a superposition of the basis states with equal probability.

Suppose the problem Hamiltonian H_P has r energy levels and the ith energy level is m_i-fold degenerate, then in the basis of $\{|\Psi_0\rangle = |1\rangle|0\rangle|0\rangle^{\otimes n}, |\Psi_i\rangle = |0\rangle|1\rangle|\varphi_i\rangle, i = 1,2,\cdots,r\}$, where $|\varphi_i\rangle = \frac{1}{\sqrt{m_i}}\sum_{s_i=0}^{m_i-1}|k_{s_i}\rangle$ are the eigenstates of the ith energy level of H_P, the Hamiltonian H in matrix form is $H_{00} = \frac{1}{2}\omega + \varepsilon_0$, $H_{0i} = H_{i0} = c\sqrt{m_i/N}$, and $H_{ii} = -\frac{1}{2}\omega + E_i$, for $i \geqslant 1$, and $H_{ij} = 0$ for $i,j \geqslant 1$ and $i \neq j$.

Let the entire system evolve for time t; the probe qubit will exhibit resonance dynamics when $H_{00} = H_{11}$, or $E_1 - \varepsilon_0 = \omega$, which means the transition frequency between the reference state and the state $|1\rangle|\varphi_1\rangle$ of the register Q matches the frequency of the probe qubit, and $E_2 - E_1 \geqslant 1 \gg c$; therefore by appropriately setting the parameters ε_0 and ω and detecting the dynamics of the probe, one can determine if the ground-state energy E_1 is zero or not, thus tell whether or not the integer n is the Ramsey number. The detailed procedures are as follows.

We start with an integer $n < R(x,y)$ and construct the corresponding problem Hamiltonian H_P for the n-vertex graph. Prepare the probe qubit in its excited state $|1\rangle$ and the register Q in the reference state $|\Phi\rangle$, set $\omega = 1$ and $\varepsilon_0 = -1$. The entire system of the $(L+2)$ qubits is in state $|\Psi_0\rangle = |1\rangle|\Phi\rangle = |1\rangle|0\rangle|0\rangle^{\otimes L}$. Evolving the entire system with the Hamiltonian H for time t, then perform a measurement on the probe qubit in the computational basis of the probe qubit. Run this procedure a number of times to obtain the probability of the probe qubit staying in its initial state $|1\rangle$. Repeat the above steps for different evolution time to obtain the dynamics of the probe qubit. The circuit for the algorithm is shown in Fig. 1.

If n is not the Ramsey number, we have $E_1 = 0$ and the condition $E_1 - \varepsilon_0 = \omega$ is satisfied; the probe qubit will exhibit resonance dynamics. Then we can increase the integer n by 1 and repeat the above procedures until the probe qubit shows no resonance dynamics. If n is the Ramsey number, we have $E_1 \geqslant 1$ and $E_1 - \varepsilon_0 \geqslant 2$, there is a finite gap between the two transition frequencies, and no resonance dynamics will be observed.

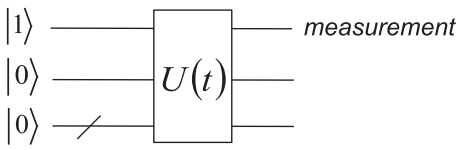

FIG. 1. Quantum circuit for obtaining dynamics of a probe qubit. The first line represents the probe qubit. $U(t)$ is a time evolution operator driven by the Hamiltonian as shown in Eq. (2). The measurement is performed in the computational basis of the probe qubit.

III. EFFICIENCY OF THE ALGORITHM

In the following, we show that the resonance and the nonresonance dynamics of the probe qubit can be well distinguished and discuss the efficiency of the algorithm. In the case where n is not the Ramsey number, the probe qubit will exhibit resonance dynamics. In the algorithm, the excitation from the reference state to the ground state of H_P contributes the most to the decay dynamics of the probe qubit, while the transitions from the reference state to the excited states of H_P also contribute to the decay of the probe qubit. The instantaneous effect of these transitions on the decay of the probe can be mimicked by assuming that all the excited states are degenerate with eigenvalue E', and E' should be $1 \leqslant E' \leqslant v$, where $v = \binom{n}{x} + \binom{n}{y}$, and $\binom{n}{x}$ is the largest possible number of x cliques in the n-vertex graphs, while $\binom{n}{y}$ is the largest possible number of the y-independent set in the n-vertex graphs. Suppose all the excited states of H_P are $(N-m_1)$-fold degenerate with eigenvalue E', let $|\Psi_2\rangle = |0\rangle|1\rangle\frac{1}{\sqrt{N-m_1}}\sum_{s_i=m_1}^{N-1}|k_{s_i}\rangle$; the Hamiltonian of the algorithm H in the basis of $\{|\Psi_0\rangle = |1\rangle|0\rangle|0\rangle^{\otimes n}, |\Psi_1\rangle = |0\rangle|1\rangle|\varphi_1\rangle, |\Psi_2\rangle\}$ can be written as

$$H = \begin{pmatrix} -\frac{1}{2} & c\sqrt{\frac{m_1}{N}} & c\sqrt{\frac{N-m_1}{N}} \\ c\sqrt{\frac{m_1}{N}} & -\frac{1}{2} & 0 \\ c\sqrt{\frac{N-m_1}{N}} & 0 & E' - \frac{1}{2} \end{pmatrix}. \qquad (4)$$

The Schrödinger equation of the above Hamiltonian can be solved exactly for given parameters N, m_1, and E'.

Let $|\Psi(t)\rangle = c_0(t)|\Psi_0\rangle + c_1(t)|\Psi_1\rangle + c_2(t)|\Psi_2\rangle$; the Schrödinger equation with the above Hamiltonian can be solved exactly and

$$c_0(t) = 2\sum_x \frac{[2x^2 + (2ia - i)x + a]e^{xt}}{12x^2 + 8i(a-1)x + 4c^2 + 4a - 1}, \qquad (5)$$

where x satisfies the equation,

$$4x^3 + 4i(a-1)x^2 + (4c^2 + 4a - 1)x + i(2c^2 m_1/N + 4c^2 a m_1/N - a - 2c^2) = 0, \qquad (6)$$

where $a = E' - \frac{1}{2}$. The probability of the system staying in the initial state $|\Psi_0\rangle$ is $P(t) = |c_0(t)|^2$. It depends on the evolution time t, the coupling coefficient c, m_1/N, and the eigenvalue E' of the state $|\Psi_2\rangle$. It can be expressed as $P(c, m_1/N, E', t)$ and reaches maximal value at the runtime $t \sim \frac{1}{c\sqrt{m_1/N}}$.

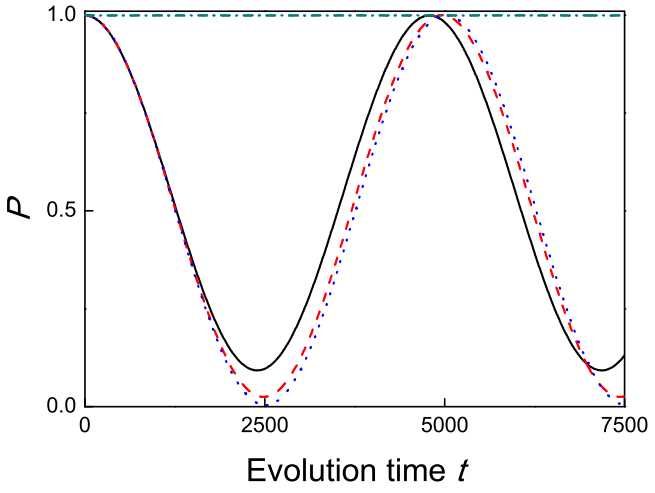

FIG. 2. The probability of the probe qubit staying in its initial excited state $|1>$ vs the evolution time t. The parameters are set as $\omega = 1$, $\varepsilon_0 = -1$, $c = 0.02$, $N = 2^{10}$, and $m_1 = 1$. The black solid line shows the results for $E' = 1$; the red dashed line shows the results for $E' = 2$; and the blue dotted line shows the results for $E' = 5$, respectively. The cyan dash dot line shows the results for the nonresonance case with $E'' = 1$. Atomic units are used in the figure.

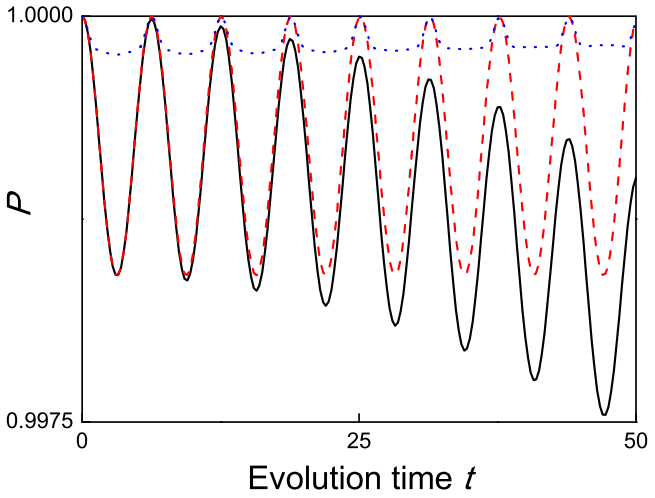

FIG. 3. Same as in Fig. 2. The black solid line shows the results for the resonance case with $E' = 1$; the red dashed line shows the results for the nonresonance case with $E'' = 1$; and the blue dotted line shows the results for the case $n = R(2,4) = 4$. Atomic units are used in the figure.

In Fig. 2, by setting $N = 2^{10}$, $m_1 = 1$, and $c = 0.02$, we show the dynamics of the probe qubit for $E' = 1,2,5$, respectively. We can see that the probe qubit shows clear resonance dynamics.

In the case where n is the Ramsey number, the ground-state energy of H_P is $E_1 \geqslant 1$. By setting $\omega = 1$ and $\varepsilon_0 = -1$, the transition frequency between the reference state and the ground state of H_P is $E_1 - \varepsilon_0 \geqslant 2$; it does not match the frequency of the probe qubit. Therefore no resonance dynamics will be observed on the probe qubit. The instantaneous effect of transitions from the reference state to all the eigenstates of H_P on the decay dynamics of the probe qubit can be mimicked by assuming that all the eigenstates of H_P are degenerate with eigenvalue $E'' \geqslant 1$. Then the Hamiltonian H in the basis of $\{|\Psi_0\rangle = |1\rangle|0\rangle|0\rangle^{\otimes n}, |\Psi_1\rangle = |0\rangle|1\rangle\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}|k\rangle\}$ can be written as

$$H = \begin{pmatrix} -\frac{1}{2} & c \\ c & E'' - \frac{1}{2} \end{pmatrix}. \quad (7)$$

With the initial state being set as $|\Psi_0\rangle$, the probability of the probe qubit being in its initial state $|1\rangle$ is

$$P_{\text{non-res}}(t) = \frac{1}{b^2 + 4c^2}[b^2 + 2c^2(1 + \cos\sqrt{b^2 + 4c^2}t)], \quad (8)$$

where $b = E'' - \frac{1}{2}$.

From Eq. (8), we can see that the minimum of $P_{\text{non-res}}(t)$ is $\frac{(E''-1/2)^2}{(E''-1/2)^2+4c^2}$. The contribution to the decay probability of the probe qubit from the transitions between the reference state and all the eigenstates of H_P must be less than $\frac{4c^2}{(E''-1/2)^2+4c^2}$. And since $E'' - 1/2 \geqslant 1/2$, this contribution can be controlled to be very small by setting a small coupling coefficient c.

In Fig. 2, we show the dynamics of the probe qubit by setting $E'' = 1$, which is the lowest possible eigenenergy of H_P in the nonresonance case, and therefore considers the largest possible contribution to the decay of the probe qubit through transitions from the reference state to all the eigenstates of H_P. It provides an upper bound for the effect of the transitions on the decay dynamics of the probe qubit. Even in this case, we can see that the system almost stays in its initial state during the time evolution. It can be well distinguished from the resonance dynamics of the probe qubit as shown in Fig. 2.

The resonance and nonresonance dynamics of the probe qubit can be distinguished even in very short evolution time. In Fig. 3, we show the dynamics of the probe qubit for both the resonance and the nonresonance cases in very short evolution time with the same parameters as in Fig. 2. We also show the dynamics of the probe qubit for the case $n = R(2,4) = 4$. From the figure we can see that these three cases can be well distinguished. In the resonance case, the probability of the probe qubit being in its excited state decreases globally while keeping small oscillations in the first half period. We can also see that the decay probability of the probe qubit in the case $n = 4$ is much smaller than that of the case described by Eq. (8) while setting $E'' = 1$.

Compare with the nonresonance case the dynamics of the probe qubit showing clear decline in short evolution time in the resonance case; therefore this tells whether the integer n is the Ramsey number or not. In Fig. 4, we show that in the resonance case, the dynamics of the probe qubit shows clear negative slope by setting $N = 10^6$, $m_1 = 1$, $c = 0.01$, and $c = 0.001$, respectively. And this can be observed even at runtime $t = 10^3 \sim \sqrt{N}$.

The probe qubit decays much faster in the resonance case than in the nonresonance case. Using $R(2,4) = 4$ as an example, when the integer $n = 3$, the ground-state eigenvalue of the problem Hamiltonian is zero, and the probe qubit exhibits resonance dynamics. While as $n = 4$, the ground-state eigenvalue of the problem Hamiltonian is one, and the probe qubit exhibits nonresonance dynamics. In Fig. 5(a), by setting

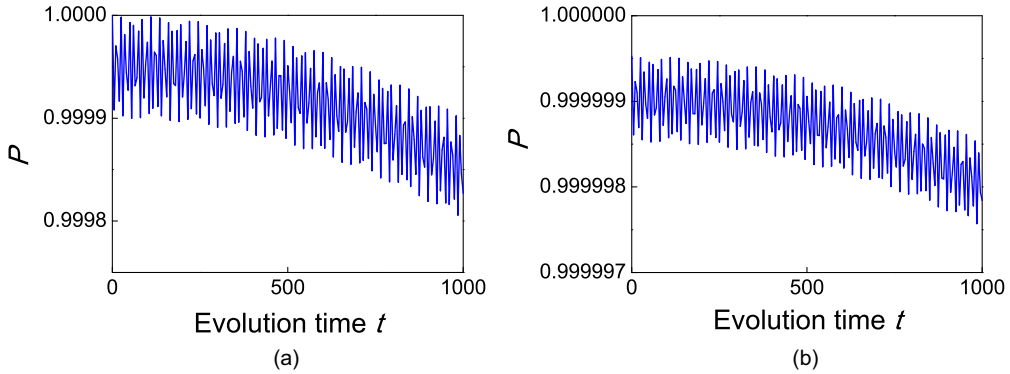

FIG. 4. The probability of the probe qubit staying in its initial state vs the evolution time t. The parameters are set as $\omega = 1$, $\varepsilon_0 = -1$, $N = 10^6$, and $m_1 = 1$. In (a) $c = 0.01$, while in (b) $c = 0.001$. Atomic units are used in the figure.

$c = 0.02$, we show the dynamics of both cases; we can see they can be well distinguished. For the same coupling strength, the probe qubit decays faster in the resonance case than in the nonresonance case. In Fig. 5(b), by setting $c = 0.2$, and running the algorithm, we can see that they can be distinguished even at runtime $t = 1$ through the decay speed of the probe qubit.

We can also find all the graphs that have the minimum number of x cliques or y-independent sets by obtaining the ground state of the problem Hamiltonian through the resonance dynamics of the probe qubit. We first obtain the ground-state energy of H_P by increasing either the frequency ω of the probe qubit or the eigenvalue of the reference state ε_0 by one each time, and run the algorithm. When the resonance dynamics on the probe qubit is observed, this indicates that the resonance condition $E_1 - \varepsilon_0 = \omega$ is satisfied; therefore the ground-state energy E_1 of H_P is obtained. Then we set $\omega = 1$ and $\varepsilon_0 = E_1 - \omega$; the system evolves from the initial state $|\Psi_0\rangle$ to the state $|\Psi_1\rangle = |0\rangle|1\rangle|\varphi_1\rangle$ and reaches maximal probability at time $t \sim \frac{\pi}{2} \times 1/(c|\langle\Psi_1|\sigma_x \otimes A|\Psi_0\rangle|) = \frac{\pi}{2}\frac{1}{c\sqrt{m_1/N}}$. The ground state $|\varphi_1\rangle = \frac{1}{\sqrt{m_1}}\sum_{s_1=0}^{m_1-1}|k_{s_1}\rangle$ of the problem Hamiltonian is obtained on the last L qubits of the register Q, and it consists of all the states $|k_{s_1}\rangle$, each of which corresponds to a graph that has the minimum number of x cliques or y-independent sets, and these basis states can be obtained with equal probability. As we have shown in Ref. [9], in the resonance case, the runtime of the algorithm for obtaining the ground state of the problem Hamiltonian scales as $O(\sqrt{N}) \leqslant t \leqslant O(N)$.

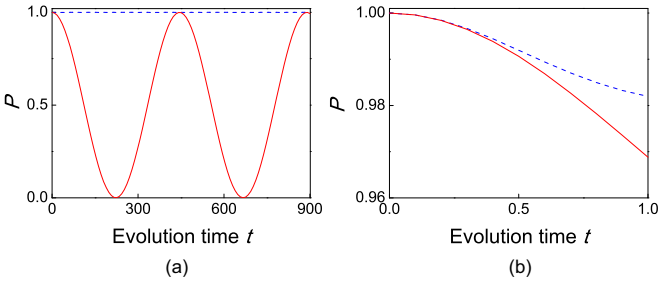

FIG. 5. The probability of the probe qubit staying in its initial state vs the evolution time t for determining the Ramsey number $R(2,4) = 4$. The red solid line shows the results for $n = 3$, while the blue dashed line shows the results for $n = R(2,4) = 4$. In (a) the coupling strength is set as $c = 0.02$, while in (b) $c = 0.2$. Atomic units are used in the figure.

IV. IMPLEMENTATION OF THE ALGORITHM

The time evolution operator $U(\tau) = \exp(-iH\tau)$ can be implemented through the Trotter formula [10], $U(\tau) = [e^{-i(\frac{1}{2}\omega\sigma_z + H_Q)\tau/M} e^{-i(c\sigma_x \otimes A)\tau/M}]^M + O(\frac{1}{M})$, where M is a large number. The operator $e^{-i(\frac{1}{2}\omega\sigma_z + H_Q)\tau/M}$ is diagonal and can be implemented efficiently. For the unitary operator $e^{-i(c\sigma_x \otimes A)\tau/M}$, the operator $A = \sigma_x \otimes H_d^{\otimes L}$ can be written as $A = (I_2 \otimes S^{\otimes L})\sigma_x^{\otimes(L+1)}(I_2 \otimes S^{\dagger\otimes L})$, where $H_d = S\sigma_x S^\dagger$ and S is an unitary operator. The time slices $e^{-i(c\sigma_x \otimes A)\tau/M}$ of the unitary operator can be written as $e^{-i(c\sigma_x \otimes A)\tau/M} = (I_2^{\otimes 2} \otimes S^{\otimes L}) e^{-ic\tau/M\sigma_x^{\otimes(L+2)}} (I_2^{\otimes 2} \otimes S^{\dagger\otimes L})$. The operator $e^{-ic\tau/M\sigma_x^{\otimes(L+2)}}$ involves many-body interaction and can be simulated by a Hamiltonian with two-body interactions [11].

The interaction Hamiltonian in Eq. (2) involves $(L+2)$-body interactions, which leads to the increase of implementation complexity of the algorithm. In some physical system, such as trapped ions, the many-body interactions can be implemented efficiently through the Mølmer-Sørensen gates [12,13].

V. DISCUSSION

The problem of computing a Ramsey number $R(x,y)$ for given x and y can be transformed to a decision problem: to determine if the ground-state energy of the problem Hamiltonian H_P is zero or not. In our algorithm, we can conclude whether the ground-state energy of H_P is zero or not without knowing the ground state of H_P. The algorithm is based on the resonance phenomena and the decision problem is solved by detecting whether the probe qubit exhibits resonance dynamics. By appropriately setting the parameters ε_0 and ω in the algorithm, the probe qubit will exhibit resonance dynamics if the ground state of H_P is zero, which means the integer n for the corresponding H_P is not the Ramsey number; the probe qubit will exhibit nonresonance dynamics if the ground-state energy of H_P is not zero, which means the corresponding integer $n \geqslant R(x,y)$. Therefore beginning

with an integer $n < R(x,y)$, one can find the Ramsey number $R(x,y)$ for given x and y from the dynamics of the probe qubit.

In this algorithm, the probe qubit is used to detect the energy spectrum of the problem Hamiltonian H_P by varying the probe frequency or the reference state energy. The dynamics of the probe is the "fingerprint" of the problem Hamiltonian. Especially for H_P that has discrete energy spectrum, the dynamics of the probe qubit are very different in the resonance and the nonresonance cases. For the same coupling strength, the probe qubit decays faster in the resonance case than in the nonresonance case. Therefore, in practice, the resonance and the nonresonance dynamics of the probe qubit can be distinguished even in very short runtime for finite N in determining a Ramsey number. The ground state of the problem Hamiltonian can also be obtained. And since we consider all possible excitations from the reference state to the basis states of the problem Hamiltonian with equal probability, the ground state we obtained is an equal superposition of all the basis states that represent the graphs having the minimum number of x cliques and y-independent sets.

Using the similar approach, we proposed an algorithm for obtaining the energy spectrum of a physical system in Ref. [14] and an algorithm for solving some discrete mathematical problems on a quantum computer in Ref. [15]. In the present algorithm, one can verify if the final solution to the problem has been obtained by checking whether or not the probe qubit exhibits resonance dynamics. Therefore this algorithm can also be applied for solving some combinatorial optimization problems and some other decision problems, such as the 3-SAT problems.

The Grover's search algorithm can also be applied for determining Ramsey numbers. We start with an integer $n < R(x,y)$; each graph state is mapped to a computational basis state on a quantum computer and "labeled" by its eigenvalue, which can be calculated as described in the algorithm. We can introduce an oracle f: For a computational basis state with eigenvalue of zero, map it to one, and for a computational basis with eigenvalue larger than zero, map it to zero, and run the procedures of the Grover's algorithm. If we can find a computational basis state with eigenvalue of zero, this means the ground-state energy of the problem Hamiltonian is zero and the integer n is not a Ramsey number. Then we can increase n by one and repeat the procedure, until we cannot find a computational basis state with eigenvalue of zero; the corresponding integer is the Ramsey number. The number of times the oracle has to be called scales as $O(\sqrt{N})$. The Grover's algorithm for determining the Ramsey number requires extra mapping steps. In our algorithm, we use the original problem Hamiltonian directly.

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