BABCOCK UNIVERSITY

SCHOOL OF SCIENCES AND TECHNOLOGY

DEPARTMENT OF BASIC SCIENCES

SECOND SEMESTER EXAMINATION, 2016/2017

COURSE CODE: MATH 102 **COURSE TITLE:** GENERAL MATHEMATICS II

CREDIT UNITS: 3 **TOTAL MARKS:** 60 **TIME ALLOWED:** 2 HOURS

INSTRUCTION: Attempt any **FOUR** questions

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QUESTION 1

a) With two arbitrary non empty sets A & B, define a mapping from set A into set B (3 marks)

b) Sketch the following functions;

(i) $f(x) = (x+2)^2$ (ii) $g(x) = x^2$ (iii) $h(x) = (x-1)^2$ (3 marks)

c) State the domain of the following functions

(i) $f(x) = \frac{1}{x}$ (ii) $f(x) = \frac{1}{\sin x}$ (2 marks)

d) Let $f(x) = ax^2 + bx + c$, where a, b and c are constants. If $f(0) = -2$, $f(1) = 2$, and $f(2) = 0$. Obtain f(x) explicitly, hence evaluate $f(2)$ (5 marks)

e) If $f(x) = x^{-1}$. Evaluate $\frac{1}{2}[f(x+1) - f(x-1)]$. (2 marks)

QUESTION 2

a) When is a function $f(t)$ said to be discontinuous at a point $t = t_0$ (3 marks)

b) Discuss the continuity of the functions

(i) $f(t) = \frac{t^3+t^2-17t+15}{t^2+2t-15}$ at $t = 3$ and $t = 5$ (6 marks)

(ii) $f(t) = \frac{t^3+t^2-4t-4}{t^2-t-2}$ at $t = 2$ and $t = -1$ (6 marks)

QUESTION 3

a) Use the first principle to find the differential coefficient of $y = 2x^2 - 5$ with respect to x. (4 marks)

b) (i) Given $y = ae^{mx} + be^{-mx}$, where a, b & m are constants. Show that

$$\frac{d^2y}{dx^2} = m^2y$$ (4 marks)

(ii) Differentiate $y = e^{\sin 2x}$ with respect to x (3 marks)

c) Find the gradient of the curve $x^2y + x^2 + 2y^2 - y = 0$ at the point (1,2) (4 marks)

QUESTION 4

a) Evaluate the following integrals

i. $\int 2x\sqrt{x^2+5}\ dx$ (4 marks)

ii. $\int 3x^2e^{x^3}dx$ (4 marks)

iii. $\int 3x\ sinx\ dx$ (4 marks)

b) Given $\frac{dy}{dx} = 6x^2$, find y in terms of x if $y = 5$ when $x = 1$ (3 marks)

QUESTION 5

a) Show that the tangents to the curves $x^2 + y^2 - 4x + 3y + 12 = 0$ and $x^2 + y^2 - 12x - 6y + 30 = 0$ do not coincide at the point (1,2) (5 marks)

b) Differentiate $(3x^4 + 7)^6$ with respect to x ($2\frac{1}{2}$ marks)

c) If $y = (sec\ x + \tan x)^n$, show that $\frac{dy}{dx} = ny\ sec\ x$ (3 marks)

d) If $y = e^{4x}cos3x$, prove that $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 25y = 0$ ($4\frac{1}{2}$ marks)

QUESTION 6

a) Evaluate the following limits;

(i) $\lim_{x\to-\frac{1}{2}} \frac{2x^2+x}{2x^2-x-1}$

(ii) $\lim_{x\to3} \frac{x-3}{1-\sqrt{4-x}}$

(iii) $\lim_{x\to0} \frac{e^x-e^{-x}}{\sin x}$ ($7\frac{1}{2}$ marks)

b) Integrate the following indefinite integrals;

i. $\int \frac{2x}{\sqrt{3+x^2}}dx$ (4 marks)

ii. $\int xe^x dx$ ($3\frac{1}{2}$ marks)