Practical Course Matlab/Simulink

Symbolic Math Toolbox

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0 General Information and Advice

The following exercises cover MATLAB's Symbolic Math Toolbox. They increase in difficulty and they also build upon each other, so you should start with the first exercise, continue with the second and so on. The first five exercises make you familiar with the most important functions and functionalities of the Symbolic Math Toolbox. In the sixth and last exercise, you have to apply some of those functions to model the movement of the small line-tracking robot considered in this course.

It is recommended that you write the MATLAB code you produce during this session into an M-File (MATLAB script). This makes it easier for your supervisor to help you in case of problems. You can also save and keep the file for your records.

At the beginning of each exercise, delete all existing variables.

1 Symbolic Objects

This first exercise introduces symbolic objects and basic functions of the toolbox.

Exercise (6 Points, 1 Point each)

(1) Create three symbolic variables `p`, `q` and `r`.

(2) Create an expression $K = \ln(p^q)$

(3) Evaluate K for $q = 0$

RESULT:	

(4) Create a function $L = \sin^2(r) + \cos^2(p + q)$

(5) Evaluate L for $p = r - q$

RESULT:	

(6) Simplify this result

RESULT:	

2 Symbolic vs. Numeric

This exercise illustrates the difference between exact symbolic and approximate numeric computations.

Exercise (6 Points, 1 Point each)

(1) Save the current digits setting as a variable `digits_old`

(2) Make a new digits setting: 20

(3) Instantiate a symbolic constant `e_exact` with the exact value of the Euler number.

(4) Instantiate a symbolic constant `e_approx` with a decimal approximation of the Euler number, using VPA. According to your previous setting, this will be a 20 digit approximation.

(5) Subtract `e_exact` from `e_approx` and convert the result to numeric using **`double`**. How large is the difference?

NUMERIC RESULT:	

(6) Re-set the digits setting to its original value

3 Calculus

Now that you are familiar with symbolic objects and symbolic-numeric conversions, this exercise lets you apply this knowledge to perform calculus.

Exercise (7.5 Points, 1.5 Points each)

(1) Compute the definite integral $\int_{-1}^{1}(27 \cdot x \cdot \ln(x+3) + \cos(2^x))dx$

NUMERIC RESULT:	

(2) Compute the derivative $\frac{d}{dz}\left(\frac{z}{y}\tan(yz) + \frac{1}{y^2}\ln(\cos(yz))\right)$. Do not forget to simplify!

RESULT:	

Note that the result could be simplified further…

(3) Compute the rotation (= curl) of the vector field $\left(\alpha\frac{z}{x}, \quad \frac{z}{y}, \quad \ln(xy)\right)^T$ in (x, y, z)

RESULT:	

(4) What is the inverse Laplace transform of $1/\sqrt{s+a}$, if s is the Laplace variable?

RESULT:	

(5) Use the Taylor tool to determine the 2nd-order Taylor series approximation of e^x around $x = 1$

RESULT:	

Feel free to experiment with the tool.

4 Assumptions and Algebraic Equation Solving

This exercise introduces functions for algebraic equation solving. It also shows how assumptions on symbolic variables can simplify equation solving.

Exercise (6 Points, 1.5 Point each)

(1) Find <u>all</u> solutions to $|e^x| = 1$ without making any assumptions on x.

RESULT:	

(2) Now, assume that $x \in \mathbb{R}$ and solve the same equation $|e^x| = 1$.

RESULT:	

(3) Solve the following system of equations for a and b:

$$a^2 b^2 = 0$$

$$a - \frac{b}{2} = c$$

RESULT:	a:	
	b:	

(4) Solve the same system again, assuming $c > 1$ and $b \geq 0$

RESULT:	a:	
	b:	

5 ODE Solving and Graphics

Consider a pendulum subject to gravity and damping, as shown in Figure 5-1.

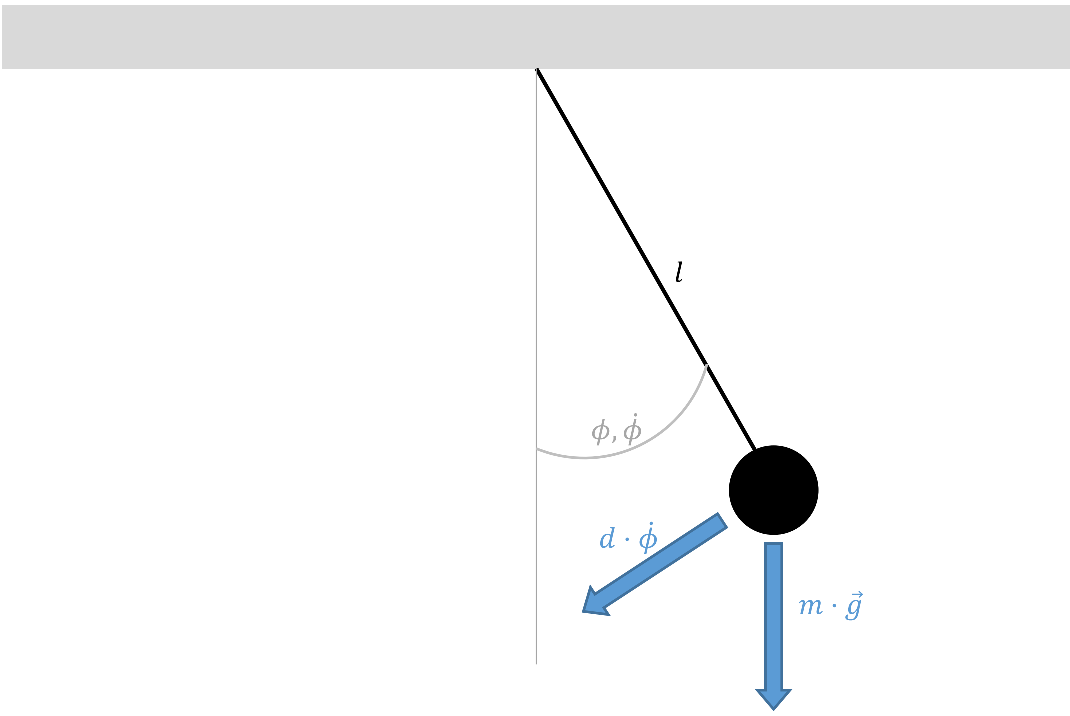

Figure 5-1: Schematic of the pendulum

The pendulum's nonlinear dynamics can be written as follows

$$\ddot{\phi} = -\frac{g}{l}\sin\phi - \frac{d}{ml^2}\dot{\phi} \tag{5-1}$$

with the Earth's gravitation g, the mass m, the length of the rod l and a damping constant d. For small angles ϕ, we can assume $\sin\phi \approx \phi$, which leads to the following linear equation.

$$\ddot{\phi} = -\frac{g}{l}\phi - \frac{d}{ml^2}\dot{\phi} \tag{5-2}$$

Exercise (7.5 Points, 1.5 Point each)

(1) Solve the linear pendulum differential equation in MATLAB symbolic language. Assume $\phi(0) = 1rad$ and $\dot{\phi}(0) = 0$.

(2) Take the solution obtained and insert the following values:

$$l = 10\,m \qquad m = 5\,kg \qquad g = 9.81\,m\,s^{-2} \qquad d = 50\,kg\,m^2 s^{-1}$$

(3) Compute the pendulum angle $\phi(5s)$ and give a 40-digit numeric representation. Note down the first three and the last three digits. (Hint: use **vpa**)

NUMERIC RESULT:	…

(4) Make a 3D-plot for $t \in [0,40]$.
Plot $l \sin\phi$ on the x-axis, t on the y-axis and $-l\cos\phi$ on the z-axis.

(5) Look at the plot from different angles. Note that when you look along the z-axis onto the xy-plane, you can see the pendulum oscillation.

6 Moving Robot

In this final exercise, you apply most of what you have learned so far to the problem of a moving robot, like the line-tracking robot considered in the scope of this lab course. Figure 6-1 shows the geometry of the robot. It has two wheels driven by one motor each and a third, ball-shaped support.

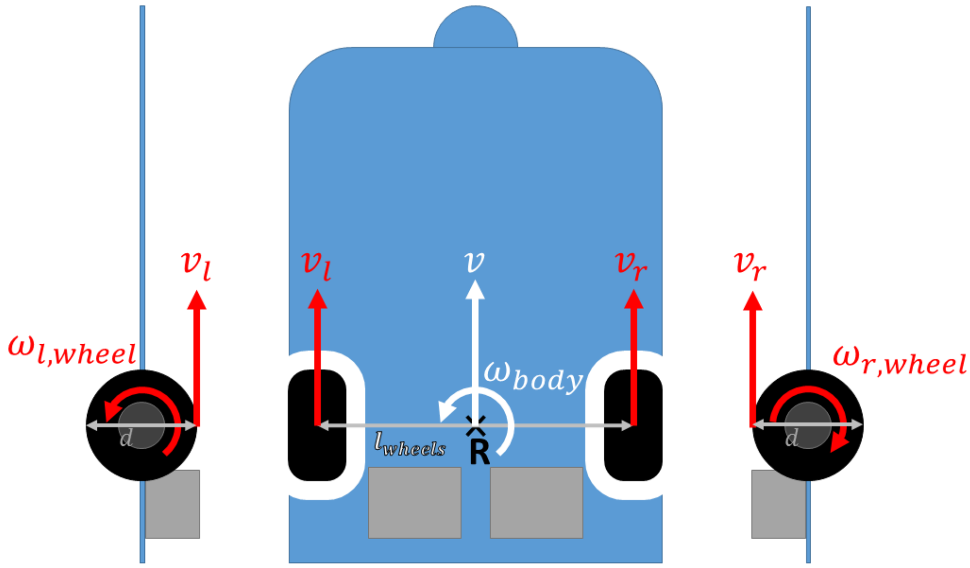

Figure 6-1: Geometry of the robot

Consider the left and right wheel rotation rates $\omega_{lw}(t)$ and $\omega_{rw}(t)$ given. Using the wheels diameter d, the linear wheel speed can be calculated.

$$v_r = \omega_{r,wheel}(t) \cdot \frac{d}{2}$$
$$v_l = \omega_{l,wheel}(t) \cdot \frac{d}{2} \tag{6-1}$$

The forward speed at the reference point can then be calculated as

$$v = \frac{1}{2}(v_l + v_r) \tag{6-2}$$

and the rotational rate of the robot is

$$\omega = \frac{1}{l_{wheels}}(v_r - v_l) \tag{6-3}$$

Using these quantities, the nonlinear equations of motion of the robot can be stated as

$$\begin{aligned} \dot{\psi} &= \omega \\ \dot{x} &= v \cdot \cos\psi \\ \dot{y} &= v \cdot \sin\psi \end{aligned} \tag{6-4}$$

Exercise (8 Points, 2 Point each)

(1) Rewrite equations (6-1), (6-2) and (6-3) in MATLAB symbolic language. Let t be positive.

(2) Consider $\omega_{l,wheel} = 8\sin^2(t)$ and $\omega_{r,wheel} = 8\cos^2(t)$. What are the resulting robot speed v^* and robot rotational rate ω^*? (The star denotes this particular solution.)

RESULT:	v^*:
	ω^*:

(3) When is the first moment $t > 0$ at which the robot does not rotate ($\omega^* = 0$)? Assume $d = 0.03m$ and $l_{wheels} = 0.1m$. Give a 3-digit approximation. (Hint: make a first guess based on a symbolic plot, then determine the result by numeric equation solving.)

NUMERIC RESULT:	

(4) Determine the robot's position after 50 seconds $x(50), y(50)$. (Hint: Make the right choice between definite and indefinite integration.)

NUMERIC RESULT:	x:
	y: