J. Deepika
10MSD012A
CAT 1

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Mathematical Foundation for Comp. Science

1. a) $(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r)$.

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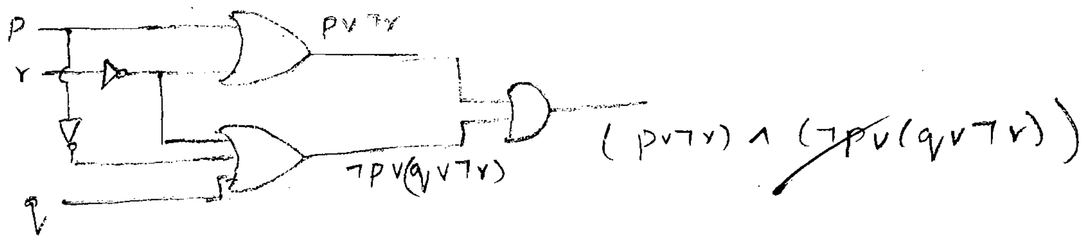

b)

Consistent premises

→ If from a given set of premises we are able to come to any conclusion other than contradiction (F), then the premises are consistent.

eg: Let P1, P2 be 2 premises

P1 : $p \to q$

P2 : p

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Step	Derivation	Rule
1.	$p \to q$	Rule P
2.	p	Rule P
3.	q	1,2 Modus Ponens.

Here the conclusion is q i.e. not F, so the premises are consistent.

Inconsistent premises

→ If from a given set of premises we come to a conclusion of F i.e. a contradiction then the premises are inconsistent.

eg: Let p1 and p2 be 2 premises.

P1: $p \wedge q$

P2: $\neg p$

Step	Derivation	Rule
1.	$p \wedge q$	Rule P
2.	$\neg p$	Rule P.
3.	p	1 - Conjunction Simplification
4.	F	2,3 $p \wedge \neg p \equiv F$

Here the conclusion is a contradiction so the premises are inconsistent.

c) Let us consider the following prepositions

S: Diagnostic message is stored in buffer.

r: Diagnostic message is retransmitted.

From the given question we can consider the following premises.

P_1 : $S \vee r$.

P_2 : $\neg S$

P_3 : $S \rightarrow r$.

$p \to q$, $\neg p \vee q$: $\neg S \vee r$

Step	Derivation	Rule.
1.	$S \vee r$	Rule P.
2.	$\neg S \rightarrow r$	1. Rule T $p \rightarrow q \equiv \neg p \vee q$
3.	$\neg S$	Rule P.
4.	r	1, 2, Modus Ponens, Rule
5.	$S \rightarrow r$	Rule P.
6.	$\neg S \vee r$	Rule T, 5, $p \rightarrow q \equiv \neg p \vee q$.
7.	$\neg S$	6, Disjunction simplification.
8.	$(\neg S \vee r) \wedge r$	4, 6.
9.	$(\neg S \wedge r) \vee r$	Distribution. 8.

Since the conclusion is not a contradiction the premises are consistent

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2. $(p\vee\neg s)\wedge(\neg p\vee(q\vee\neg r))$.

$$p\vee\neg s = p\vee\neg s\vee F$$
$$= p\vee\neg s\vee(q\wedge\neg q)$$
$$= (p\vee\neg s\vee q)\wedge(p\vee\neg s\vee\neg q)$$
$$= (p\vee\neg s\vee q)\vee F\wedge(p\vee\neg s\vee\neg q)\vee F$$
$$= [(p\vee\neg s\vee q)\vee(r\wedge\neg r)]\wedge[(p\vee\neg s\vee\neg q)\vee(r\wedge\neg r)]$$
$$= (p\vee q\vee r\vee\neg s)\wedge(p\vee q\vee\neg r\vee\neg s)\wedge(p\vee\neg q\vee r\vee\neg s)\wedge(p\vee\neg q\vee$$

$$\neg p\vee(q\vee\neg r) = \neg p\vee q\vee\neg r\vee F$$
$$= \neg p\vee q\vee\neg r\vee(s\wedge\neg s)$$
$$= (\neg p\vee q\vee\neg r\vee s)\wedge(\neg p\vee q\vee\neg r\vee\neg s)$$

$\therefore$ The PCNF of $(p\vee\neg s)\wedge(\neg p\vee(q\vee\neg r))$ is

$(p\vee q\vee r\vee\neg s)\wedge(p\vee q\vee\neg r\vee\neg s)\wedge(p\vee\neg q\vee r\vee\neg s)\wedge(p\vee\neg q\vee\neg r\vee\neg s)\wedge(\neg p\vee q\vee\neg r\vee s)\wedge(\neg p\vee q\vee\neg r\vee\neg s)$.

PCNF of the $\neg[(p\vee\neg s)\wedge(\neg p\vee(q\vee\neg r))]$ is $(\neg p\vee\neg q\vee r\vee s)\wedge(\neg p\vee q\vee r\vee s)\wedge(\neg p\vee\neg q\vee\neg r\vee\neg s)\wedge(\neg p\vee\neg q\vee\neg r\vee s)\wedge(\neg p\vee\neg q\vee r\vee\neg s)\wedge(p\vee q\vee r\vee s)\wedge(\neg p\vee q\vee r\vee\neg s)\wedge(p\vee\neg q\vee\neg r\vee s)\wedge(p\vee\neg q\vee r\vee s)\wedge(p\vee q\vee\neg r\vee s)$

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$\therefore$ the PDNF of the term $(p\vee\neg s)\wedge(\neg p\vee(q\vee\neg r))$ is $(p\wedge q\wedge r\wedge s)\vee(p\wedge q\wedge r\wedge\neg s)\vee(p\wedge q\wedge\neg s\wedge s)\vee(p\wedge q\wedge\neg r\wedge\neg s)\vee(p\wedge\neg q\wedge\neg r\wedge\neg s)\vee(p\wedge\neg q\wedge\neg r\wedge s)\vee(\neg p\wedge q\wedge r\wedge\neg s)\vee(\neg p\wedge q\wedge\neg r\wedge\neg s)\vee(\neg p\wedge\neg q\wedge r\wedge\neg s)\vee(\neg p\wedge\neg q\wedge\neg r\wedge\neg s)$.

3. $wx + w\bar{x}z + \bar{w}xyz + yz +$ **Grip-** $\bar{w}\bar{x}y\bar{z}$

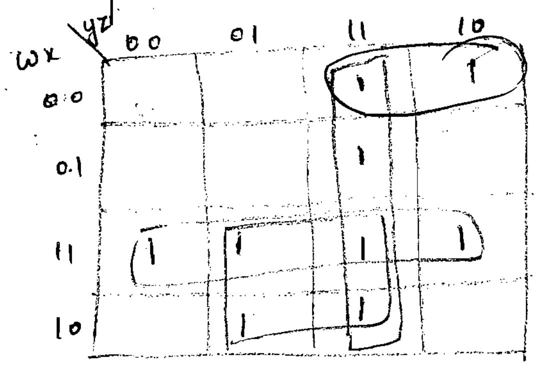
$= wxy + wx\bar{y} + w\bar{x}yz + w\bar{x}\bar{y}z + \bar{w}xyz + wyz + \bar{w}yz + \bar{w}\bar{x}y\bar{z}$

$= wxyz + wxy\bar{z} + wx\bar{y}z + wx\bar{y}\bar{z} + w\bar{x}yz + w\bar{x}\bar{y}z + \bar{w}xyz +$

$wxyz + w\bar{x}yz + \bar{w}xyz + \bar{w}\bar{x}yz + \bar{w}\bar{x}y\bar{z}$

=

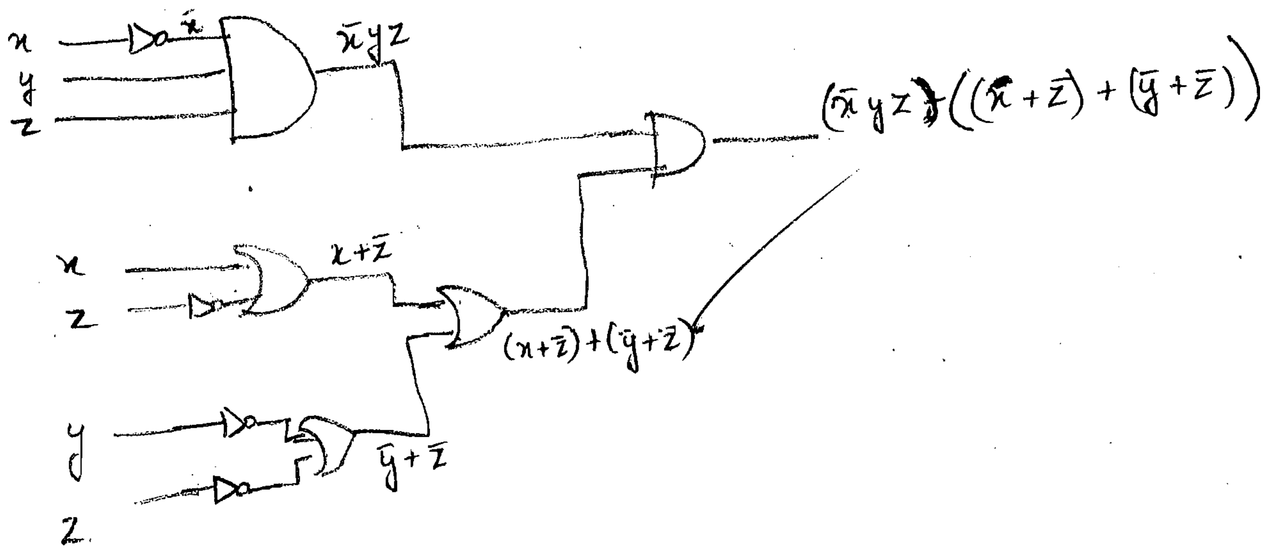
Using K map.



Simplified form is

$wx + wz + yz + \bar{w}\bar{x}y\bar{z}$

4.



Expression is :

$(\bar{x}yz) \cdot ((\bar{x}+\bar{z})+(\bar{y}+\bar{z}))$

$= (\bar{x}yz)(\bar{x}+\bar{z}+\bar{y}+\bar{z})$.

$= (\bar{x}yz) \cdot (\bar{x}+\bar{y}+\bar{z})$

PCNF of $(\bar{x}+\bar{y}+\bar{z}) \equiv \bar{x}+\bar{y}+\bar{z}$

PCNF of the $\neg(\bar{x}+\bar{y}+\bar{z})$ is

$(x+\bar{y}+z)(x+y+\bar{z})(x+y+z)$.

$(\bar{x}+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})(\bar{x}+y+z)$

$\therefore$ PDNF of $(x+\bar{y}+\bar{z})$ is

$(xyz)+(x\bar{y}z)$

$\overline{\bar{x}yz}$

PDNF of $\bar{x}yz$.

PDNF of $\overline{(\bar{x}yz)}$ $= (\bar{x}\bar{y}\bar{z})+(\bar{x}\bar{y}z)+(\bar{x}y\bar{z})+(x\bar{y}\bar{z})+(x\bar{y}z)$

$(xy\bar{z})+(xyz)$

PCNF of $\bar{x}yz$ $= (x+y+z)(x+y+\bar{z})(x+\bar{y}+z)\cdot(\bar{x}+y+z)\cdot(\bar{x}+y$

$\cdot(\bar{x}+\bar{y}+z)\cdot(\bar{x}\bar{y}+\bar{z})$.

$\therefore$ The term $(\bar{x}yz)(x+\bar{y}+\bar{z})$ can be written as.

$(x+y+z)(x+y+\bar{z})(x+\bar{y}+z)(\bar{x}+y+z)(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z}$

$(x+\bar{y}+\bar{z})$

There is no PDNF

then??

[$\Rightarrow$ Quine Mcklickey method can't be used as there is no PDNF]

5 a) $W(x,y)$: x wrote y

$L(x,y)$: x is longer than y

$N(x)$: x is a novel.

h : Hardy a : Austen j : Jude the Obsure p : Pride and Pre

1) Hardy wrote a novel which is longer than any of Austens.

$\exists x \exists y (L(W(H, N(x)), W(A, N(y))))$

$\cancel{\forall y\, N(x)\; L(W(H, x))}$

2) Jude the Obscure is not longer than Pride and Prejudice.

$L(P, J)$ (or) $\neg L(J, P)$

3) Austen wrote Jude the Obscure.

$W(a, N(j))$

b) i) Proposition :

It is a simple, declarative sentence with truth value either true or false but not both.
It is indicated by lower case alphabets.
eg : p: Dog is black.

Predicate Logic:

It is an extension of propositional logic ~~using~~ which permits consisely the extension of a concept over a related set of objects.

It is denoted by upper case letters.

eg: $P(x)$: x is black.

The predicate logic separates the subject from the predicate.

When x is replaced by a specific subject it becomes a proposition.

Proposition is the atomic level of predicate

eg: $P(x)$: x is black.
when x is a dog, it becomes the proposition dog is black.

b) i) Conditional Proof:

If we are able to coclude 'S' from a given set of premises and an extra premise 'R', then we can come the conclusion $R \rightarrow S$ from the given set of premises.

eg: ~~P⊢Q~~ Let there be 2 premises.

P1: $p \rightarrow q$

P2: $q \rightarrow r$

Conclusion: $p \rightarrow r$

If we take p as an extra premise

P3: p

Then the conclusion is r.

Step	Derivation	Rule.
1.	$p \rightarrow q$	Rule P.
2.	p	Rule CP.
3.	q	1, 2. Modus Ponens
4.	$q \rightarrow r$	Rule P.
5.	r	3, 4 Rule T Modus Ponen.

Hence r is a valid conclusion.

∴ We can say $(p \rightarrow r)$ logically follows from $(p \rightarrow q)$ and $q \rightarrow r$.

(ii) Indirect Method:

If we take the negation of the conclusion as an extra premise and prove that the conclusion is a contradiction then we can say the extra premise (Conclusion) follows the given set of premises

$P2 : q \to r$

$C : p \to r$.

If we consider $\neg(p \to r)$ as a premise

$P3 : \neg(p \to r)$

If the Conclusion is F, then we can say

$p \to r$ is a valid conclusion from $(p \to q)$ and $q \to r$.

Step	Derivation	Rule
1.	$p \to q$	Rule P.
2.	$\neg(p \to r)$	Rule P.
3.	$\neg(\neg p \vee r)$	$p \to r \equiv \neg p \vee r$.
4.	$p \wedge \neg r$	Rule T. De Morgans.
5.	p	Rule T. Conjunction Simplicatn
6.	q	1,5. Modus ponens.
7.	$q \to r$	Rule P.
8.	r	6,7 Modus Ponens
9.	$\neg r$	4, Conjunction Simpli
10	F	8,9. $r \wedge \neg r = F$.

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