TERM PAPER OF NTCC ON CARDINALITY CONSTRAINED PORTFOLIO OPTIMIZATION PROBLEM

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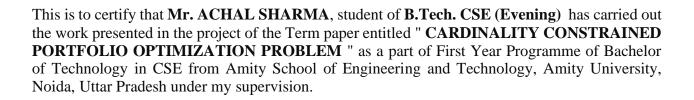
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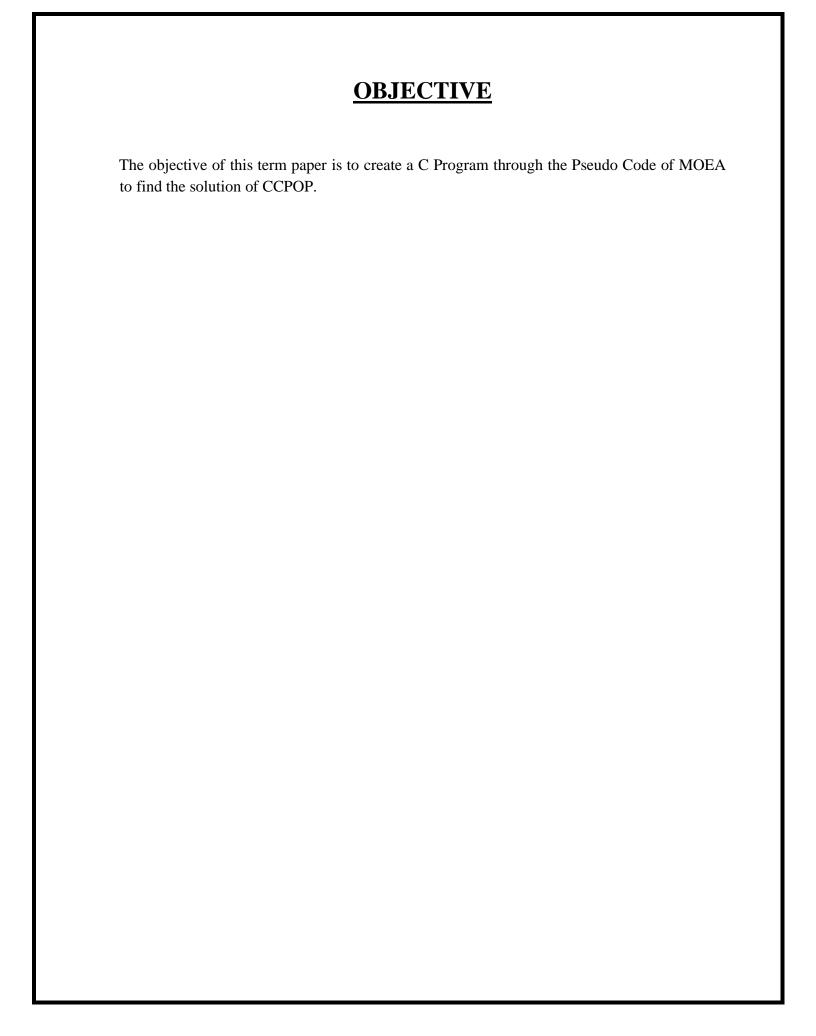
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ABSTRACT

This term paper implements the use of MOEA (Multiobjective Evolutionary Algorithm) to find the solution of CCPOP (Cardinality Constrained Portfolio Optimization Problem) by studying the Pseudo Code and using it to create a C Program. According to multiple sources, this MOEA is used in implementing an efficient encoding technique designed specifically to deal with CCPOP. There are many researches conducted that applied MOEAs to solve the constrained portfolio optimization problem. It may be used to predict the variation of the price of stocks of different companies.

INTRODUCTION

The portfolio optimization problem targets to find the ideal distribution of a restricted capital among a finite set of risky assets by concurrently maximizing the estimated return of the portfolio and minimalizing its related risk. The portfolio optimization problem can be resolved by quadratic programming. Nonetheless, if we apply some real-world nonlinear (deviating) constraints like Cardinality Constraint that apply upper and lower bound to number of assets stored in the portfolio, then the problem becomes NP-hard. The added deviating constraints made the problem more difficult to be solved with exact procedure. We present a different mutation operator specially designed to work well with the proposed representation scheme. The proposed genetic operators have been intended to satisfy the need of improving the already found good solutions and simultaneously to be able to explore other capable areas of the search landscape. This much asked balance between exploration & exploitation is achieved through the introduction of innovative mechanisms.



DISCUSSION

Typically, the mutation operator takes responsibility for the preservation of population's diversity. In this paper, we use a two-phase mutation operator (TPMO) specially designed to work well with the proposed representation scheme for solving the cardinality constrained portfolio optimization problem (CCPOP).

We employ a variation of the polynomial mutation operator (PLM) for the real-valued solution vector specially adapted to fit the needs of the proposed representation scheme and binary mutation for the discrete (i.e. binary-valued) solution vector. We start the presentation of the proposed TPMO by analysing the mutation process of the real-valued solution vector. The real-valued solution vector contains information about the selected assets and the corresponding budget capitalised in every single one of these assets.

Also, the size of both the real and binary solution vectors is restricted by the upper limit of the cardinality constraint (Kmax). Given the fact that the available pool of stocks for all examined test instances is considerably larger than Kmax, assuming a Kmax value equal to 10, we should devise an updating mechanism that allows the replacement of some assets of the real-valued solution vector with other assets from the available pool of securities.

At the same time, the mutation operator should be able to probe limited areas of the search landscape with the hope of improving a promising solution that has already been found. Below, we analyse step-by-step how we incorporate the aforementioned principals to the proposed mutation operator.

```
Begin
\eta_m = distribution index;
 for i=0 to P; // where P is the population size
           for z=0 to K_{max}; // where K_{max} is the upper limit of the cardinality constraint
                 if (rand <= mutation probability) then
                        r_p = \text{getValue}(z);
                        r_p = w_p + a_p //where w_p is the assigned weight and a_p the corresponding asset of the parent solution
                        w^L = \text{getLowerBound}(z);
                        w^U = \text{getUpperBound}(z);
                      rand_mut \longrightarrow [0, 1];
                      if (0.0 \le rand\_mut \le 0.2) // Phase 1
                        \delta_1 = \frac{w_p - w^L}{w^U - w^L} \qquad \delta_2 = \frac{w^U - w_p}{w^U - w^L}
                         r \longrightarrow [0, 1];
                        if (r \le 0.5) then
                               \delta_q = \left[2r + (1 - 2r)(1 - \delta_1)^{\eta_m + 1}\right]^{\frac{1}{\eta_m + 1}}
                               \delta_q = 1 - [2(1-r) + 2(r-0.5)(1-\delta_2)^{\eta_m+1}]^{\frac{1}{\eta_{m+1}}}
                           w^{child} = w_p + \delta_q(w^U - w^L)
                           w^{child'} = \text{normalized}(w^{child}) / w^{child'} satisfies the budget and floor and ceiling constraint
                          r_c = w^{child'} + a_p
                           child_solution_vector = parent_solution_vector.setValue(z, r<sub>c</sub>);
                       else if (0.2 < rand_mut \le 1.0) // Phase 2
                             rand\_asset \rightarrow [1, N]; //where N is the available pool of assets, e.g. for port5, N = 225
                        \delta_1 = \frac{w_p - w^L}{w^U - w^L} \qquad \delta_2 = \frac{w^U - w_p}{w^U - w^L}
                          r \longrightarrow [0, 1];
                         if (r \le 0.5) then
                               \delta_q = \left[2r + (1 - 2r)(1 - \delta_1)^{\eta_m + 1}\right]^{\frac{1}{\eta_m + 1}}
                               \delta_q = 1 - [2(1-r) + 2(r-0.5)(1-\delta_2)^{\eta_m+1}]^{\frac{1}{\eta_{m+1}}}
                           w^{child} = w_p + \delta_q(w^U - w^L)
                           w^{child'} = \text{normalized}(w^{child}) / w^{child'} satisfies the budget and floor and ceiling constraint
                           r_c = w^{child'} + rand \ asset
                           child_solution_vector = parent_solution_vector.setValue(z, r<sub>c</sub>);
```

endfor endfor

endif endif In particular, as seen in the pseudo code if rand $\leq P(m)$ then a decision variable is selected to be mutated. Suppose a hypothetical real-valued solution vector $\mathbf{r} = (r(1), r(2), \ldots, r(Kmax))$. Each real-valued decision variable $\mathbf{r}(i)$, $\mathbf{i} = 1, 2, \ldots$, Kmax contains information about the selected asset $\mathbf{a}(j)$, $\mathbf{j} = 1, 2, \ldots$, N, where N is the available pool of assets and the corresponding budget $\mathbf{w}(i)$, $\mathbf{i} = 1, 2, \ldots$, Kmax invested in this particular asset. The budget $\mathbf{w}(i)$ invested in each asset, can take values in the interval: $\mathbf{w}(L)(i) \leq \mathbf{w} \leq \mathbf{w}(U)(i)$, $\mathbf{i} = 1, 2, \ldots$, Kmax where $\mathbf{w}(L)(i)$ and $\mathbf{w}(U)(i)$ stand respectively for the floor and ceiling constraints.

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