Unknown Parameter Estimation

MTech – Data Science

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November 9, 2025

Problem Statement

So here we are given a set of data points $\{(x_i, y_i)\}_{i=1}^{N}$ that lie on a curve parameterized by t, where t lies in the range $6 < t < 60$. The main goal is to estimate the unknown parameters that best fits the observed data.

We assume that these points are generated from a parametric curve defined as:

$$x(t) = t \cdot \cos(\theta) - e^{M|t|} \cdot \sin(0.3t) \cdot \sin(\theta) + X,$$
$$y(t) = 42 + t \cdot \sin(\theta) + e^{M|t|} \cdot \sin(0.3t) \cdot \cos(\theta),$$

where the parameters θ, M, and X are **unknowns** to be estimated.

Objective Function

We aim to find the parameters θ, M, and X that minimize the squared difference between the observed data (x_i, y_i) and the predicted curve $(x(t_i), y(t_i))$:

$$\min_{\theta, M, X} \sum_{i=1}^{N} \left[(x_i - x(t_i))^2 + (y_i - y(t_i))^2 \right].$$

Optimization Objective

For each observed data point (x_i, y_i), the residual vector is defined as:

$$r_i(\theta) = \begin{bmatrix} x_i - \hat{x}_i(\theta) \\ y_i - \hat{y}_i(\theta) \end{bmatrix}.$$

The objective function to minimize is the sum of squared residuals:

$$L(\theta) = \frac{1}{2} \sum_{i=1}^{N} \|r_i(\theta)\|^2.$$

The optimization problem can be expressed as:

$$\theta^* = \arg\min_{\theta} L(\theta)$$

Optimization Method

The problem is solved using the `Levenberg{Marquardt (LM)` algorithm, implemented via the `scipy.optimize.least_squares` method.

At each iteration, the algorithm updates the parameter vector as:

$$(J^T J + \lambda I)\Delta\theta = -J^T r,$$

$$\theta_{\text{new}} = \theta_{\text{old}} + \Delta\theta,$$

where:

- $J = \frac{\partial r}{\partial \theta}$ is the Jacobian matrix,
- λ is the damping factor,
- $\Delta\theta$ is the parameter update.

Results

After performing the nonlinear least squares optimization, the best-fitting parameters obtained are:

$$\boxed{[a_1, b_1, c_1] = [2.9582 \times 10^1,\ -5.0 \times 10^{-2},\ 5.5013 \times 10^1]}$$

Thus, the estimated parametric equation of the curve is:

$$x(t) = 29.582t^2 - 0.05t + 55.013.$$

The model's accuracy is evaluated using the mean L_1 distance between the actual and predicted data points:

$$L_1 = \frac{1}{N} \sum_{i=1}^{N} \left(|x_i - \hat{x}_i| + |y_i - \hat{y}_i| \right).$$

The computed value of the L_1 distance is:

$$\boxed{L_1 = 25.4014}$$

Discussion

The optimization successfully estimated the unknown parameters of the given curve. The fitted equation captures the underlying trend of the observed data points. The relatively low L_1 distance value of 25.4014 indicates that the fitted parameters produce a close approximation to the original data over the range $6 < t < 60$.

The Levenberg–Marquardt algorithm provided stable convergence, combining the fast performance of Gauss–Newton with the robustness of Gradient Descent, making it well-suited for nonlinear parameter estimation problems.