Resolution Algorithm

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From form with ∨ and ∧ to CNF

1.

$$\neg\neg G \iff G$$

$$\neg(G \wedge H) \iff (\neg G \vee \neg H)$$

$$\neg(G \vee H) \iff (\neg G \wedge \neg H)$$

2.

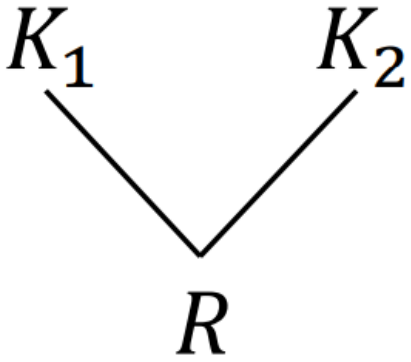
$$(F \vee (G \wedge H)) \iff ((F \vee G) \wedge (F \vee H))$$

$$((F \wedge G) \vee H) \iff ((F \vee H) \wedge (G \vee H))$$

Basics

- Conjunctive Normal Form
  - $\{a, b, \neg c\} == (\ a\ \vee b \vee \neg c)$
  - $\{\quad\} == false$
  - $\{\{a, b\}, \{c\}\} == (a\ \vee b)\ \wedge c$

Algorithm



while F does not contain { } {

if F contains two clauses $K1$, $K2$ with resolvent R,

with $R \notin F$(= R is not clause of F)

then add R as new clause to F;

else return „ fulfillable”;

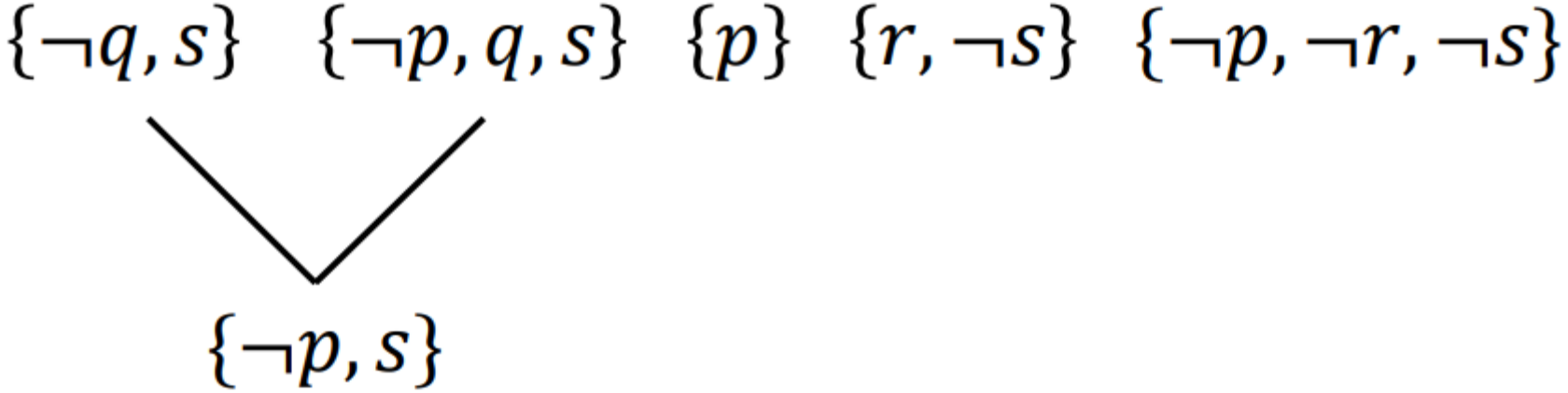
}

return „not fulfillable“;

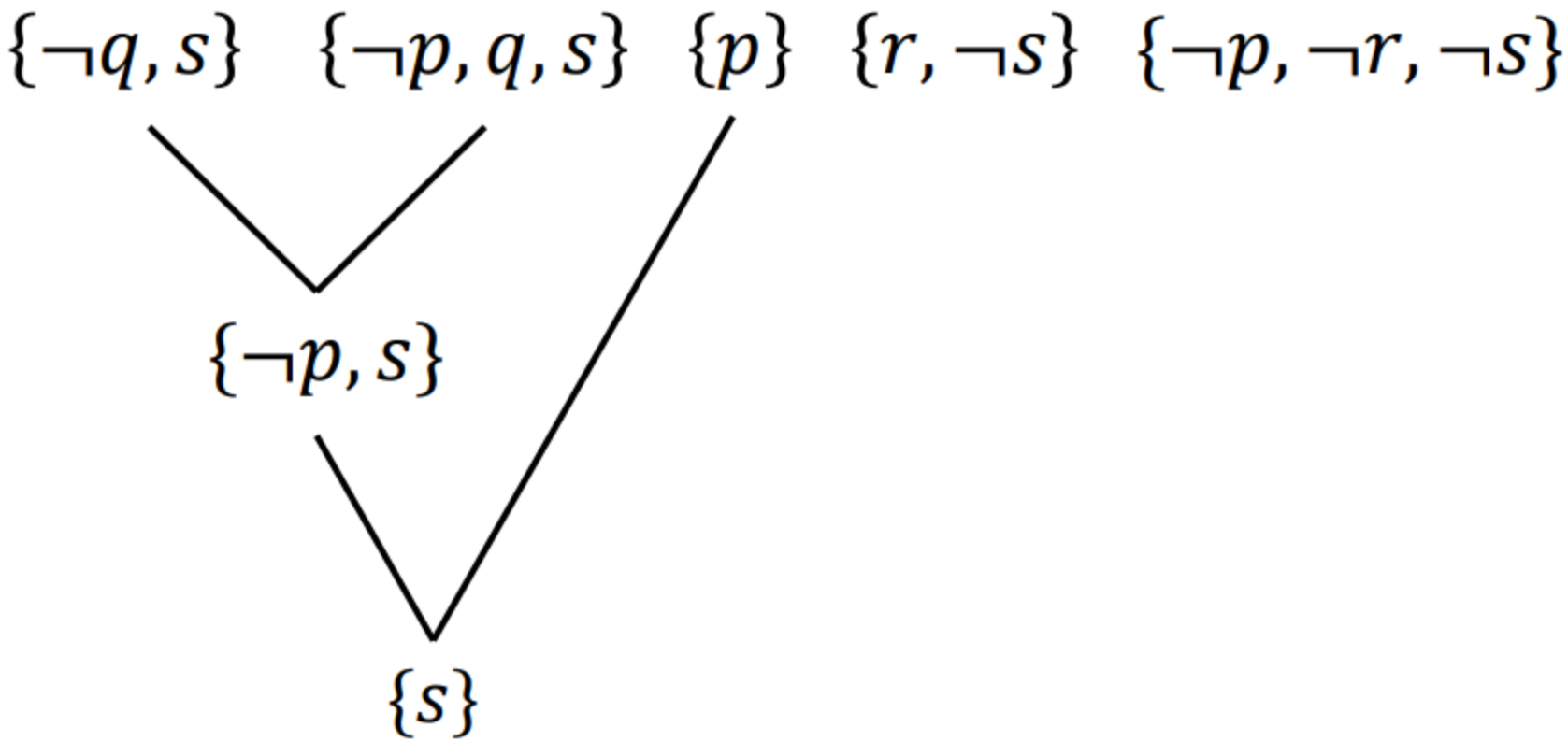
Example

$$\{\neg q, s\} \quad \{\neg p, q, s\} \quad \{p\} \quad \{r, \neg s\} \quad \{\neg p, \neg r, \neg s\}$$

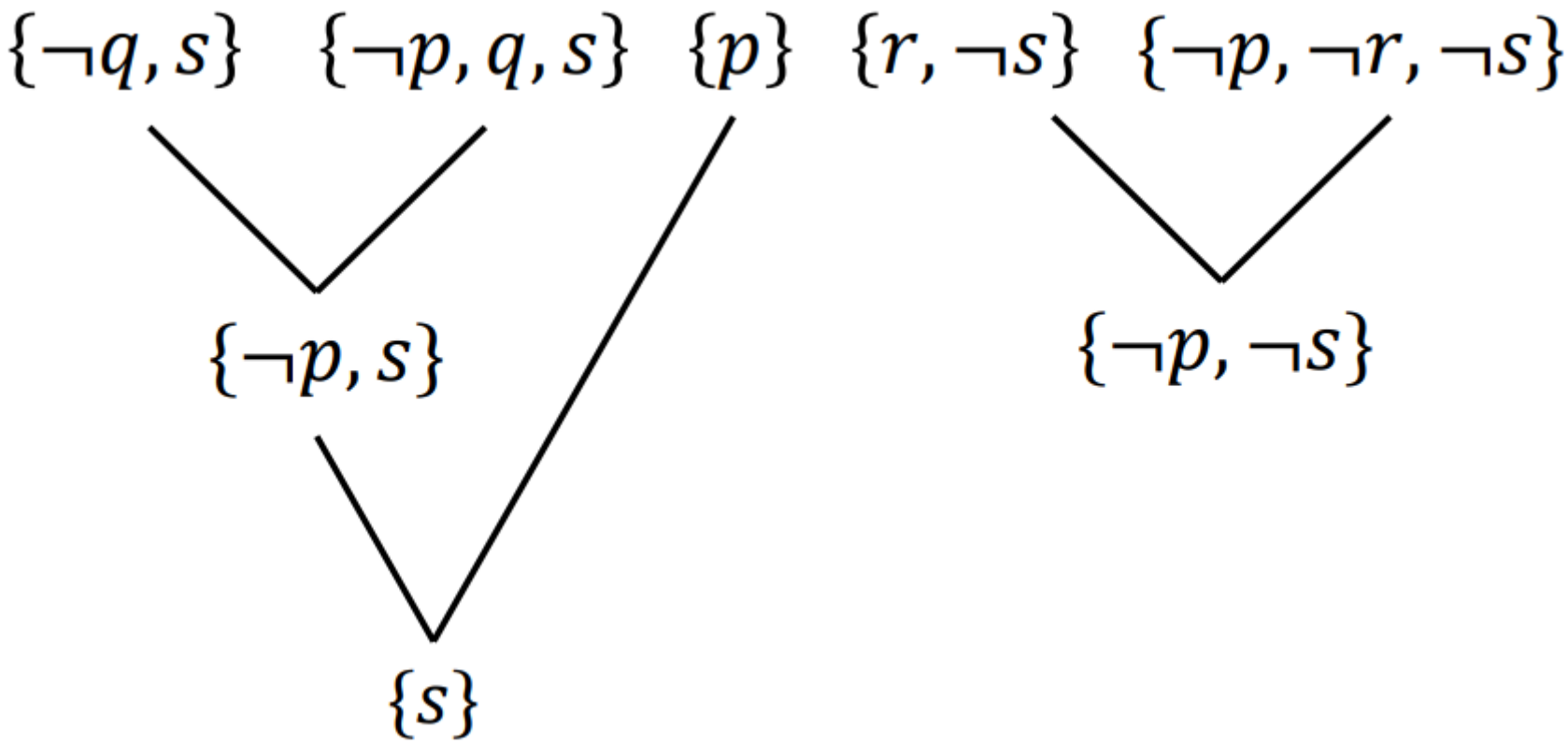
Example



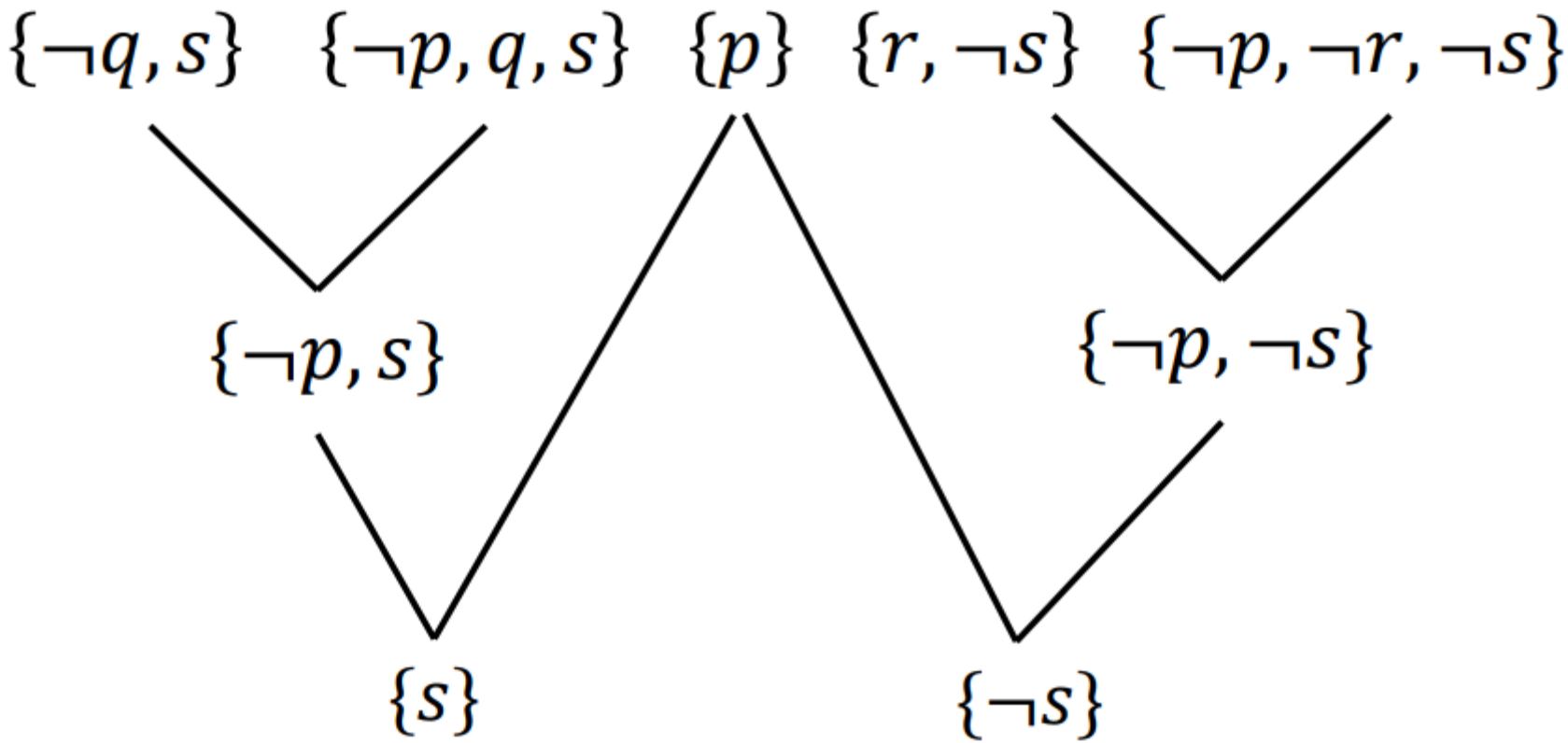
Example



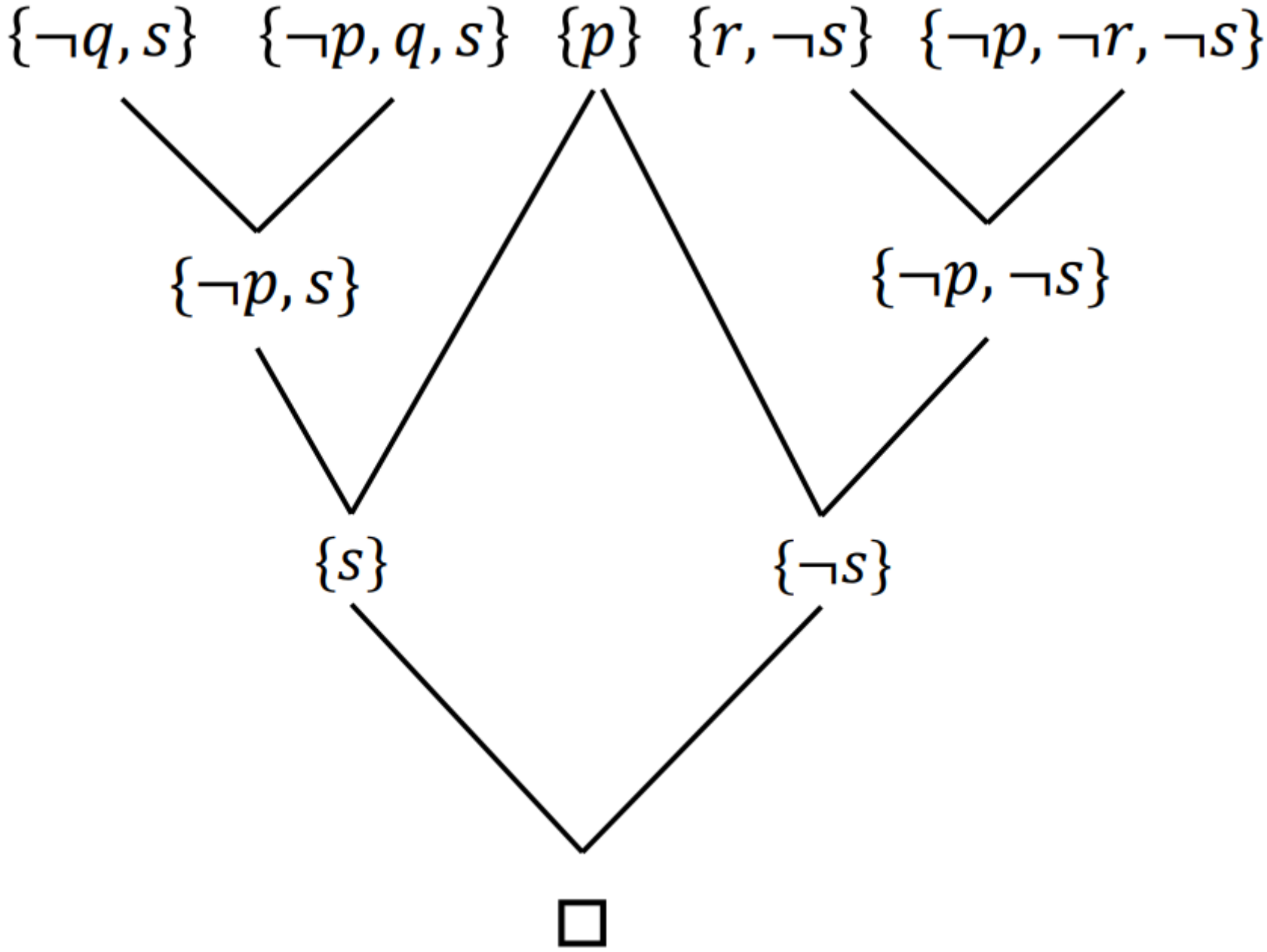
Example



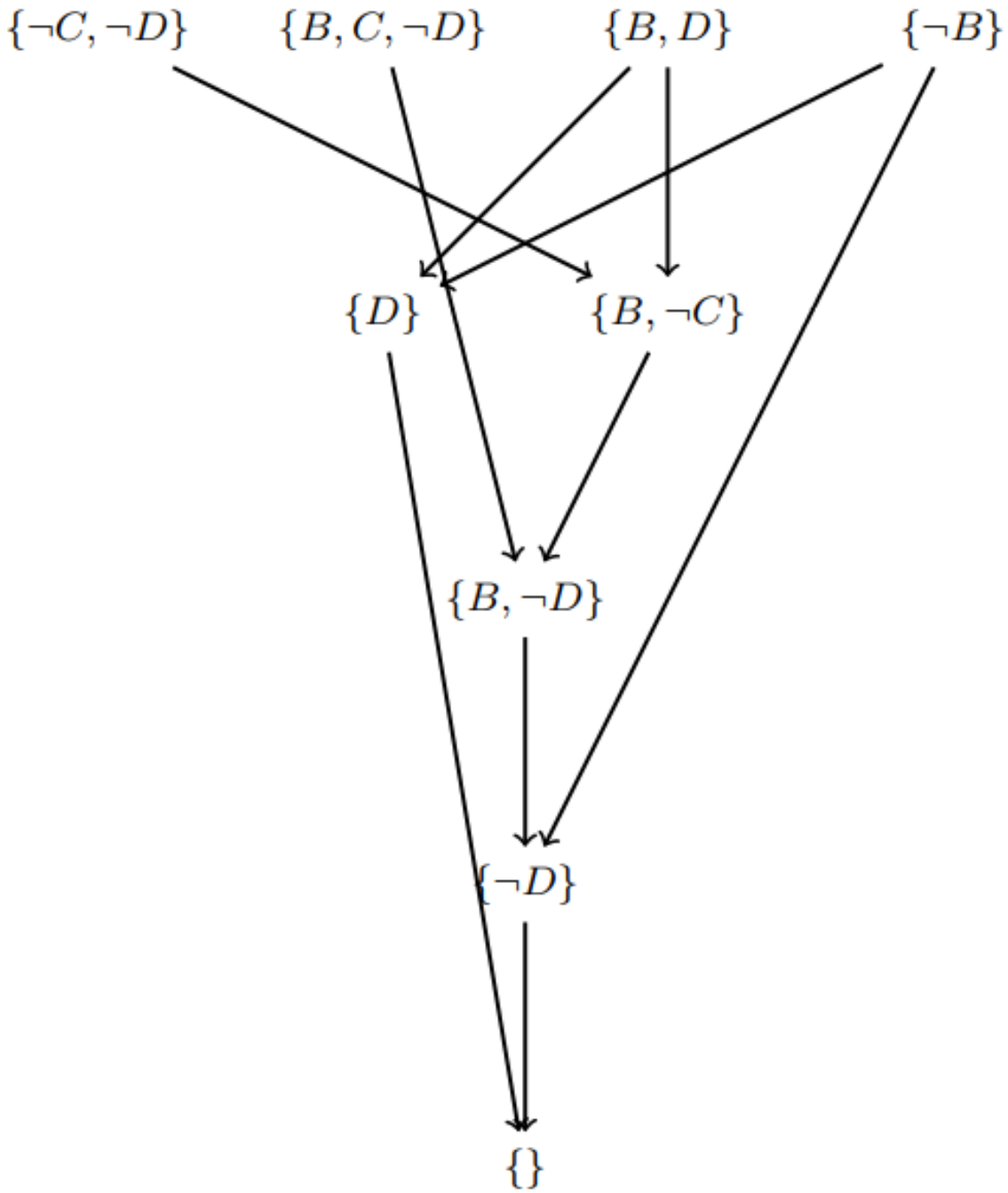
Example



Example


$\{\neg q, s\}$
$\{\neg p, q, s\}$
$\{p\}$
$\{r, \neg s\}$
$\{\neg p, \neg r, \neg s\}$
$\{\neg p, s\}$
$\{\neg p, \neg s\}$
$\{s\}$
$\{\neg s\}$
$\square$

Example



Outcome

1. Terminates for every formular
2. If formular is fulfillable then the algorithm returns „fulfillable“
3. If formular is NOT fulfillable then the algorithm returns „not fulfillable“