Ant Colony Optimization

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ITI0210 Foundations of Artificial Intelligence and Machine Learning

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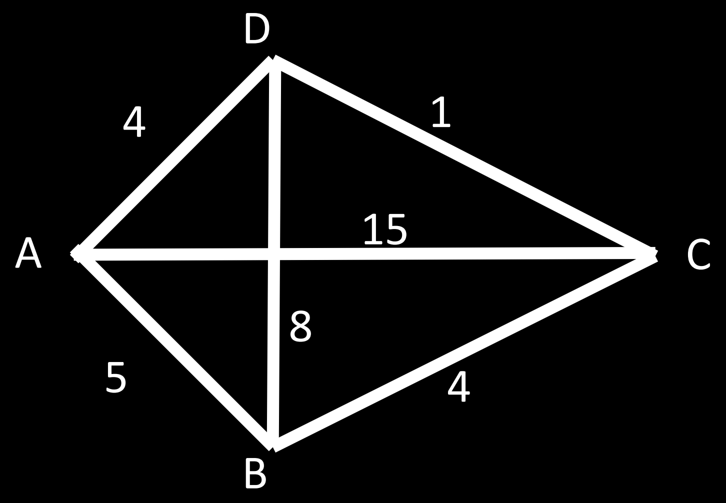
Algortihm (1/9)
Prerequisites

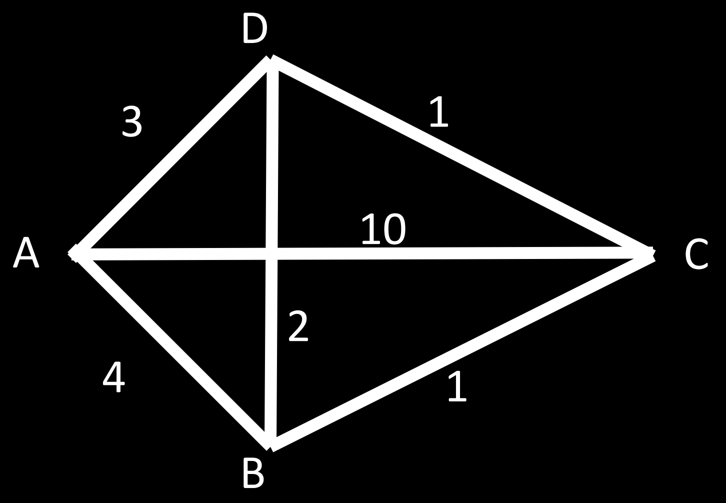
Cost matrix

	A	B	C	D
A	0	5	15	4
B	5	0	4	8
C	15	4	0	1
D	4	8	1	0



Pheromone matrix

	A	B	C	D
A	0	4	10	3
B	4	0	1	2
C	10	1	0	1
D	3	2	1	0



Algortihm (2/9)
Formulas

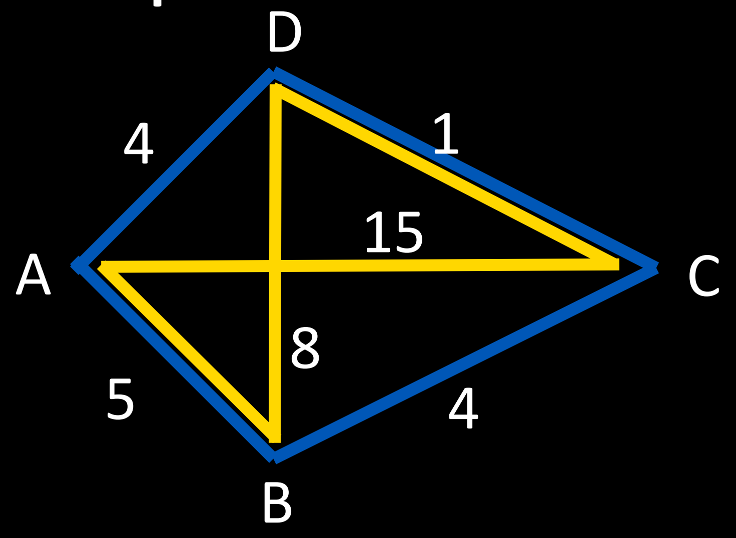
- $\Delta\tau_{i,j}^{k} = \begin{cases} \frac{1}{L_k} \; k\ th\ ant\ travels\ on\ the\ edge\ i,j \\ 0\ otherwise \end{cases}$

- $\tau_{i,j}^{k} = \sum_{k=1}^{m} \Delta\tau_{i,j}^{k}$ without vaporization

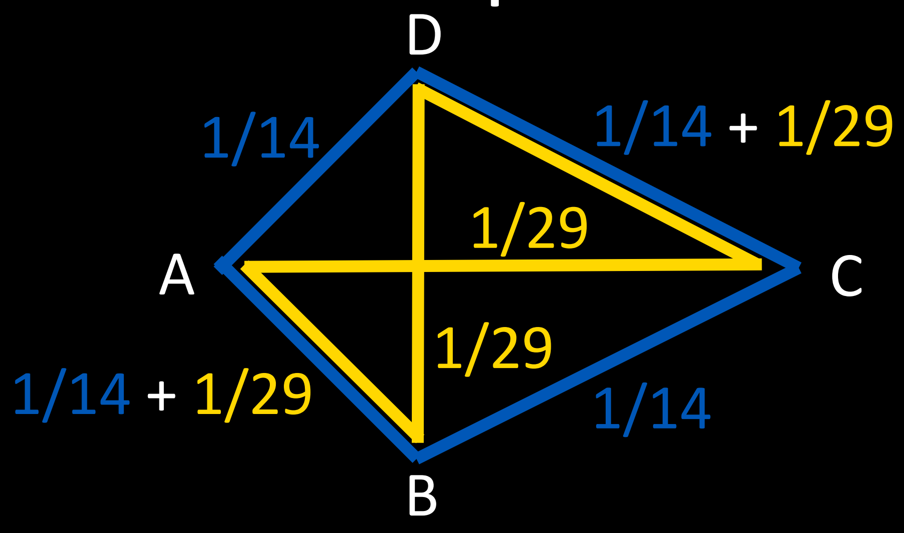
- $\tau_{i,j}^{k} = (1-\rho)\tau_{i,j} \sum_{k=1}^{m} \Delta\tau_{i,j}^{k}$ with vaporization

Algortihm (3/9)

Cost Graph



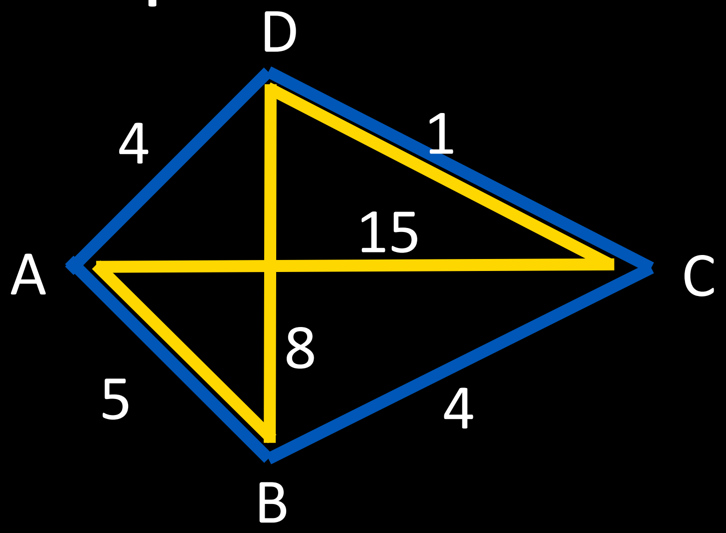
Pheromone Graph



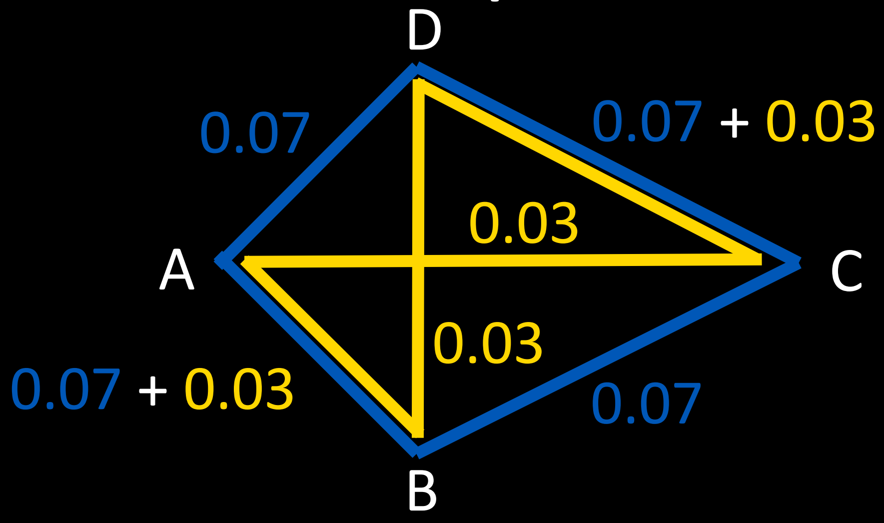
- Ant1: $L_1 = 14 \rightarrow \Delta\tau_{i,j}^{1} = \frac{1}{14}$
- Ant2: $L_2 = 29 \rightarrow \Delta\tau_{i,j}^{2} = \frac{1}{29}$
- $\tau_{i,j}^{k} = \sum_{k=1}^{m} \Delta\tau_{i,j}^{k}$

Algortihm (4/9)

Cost Graph



Pheromone Graph

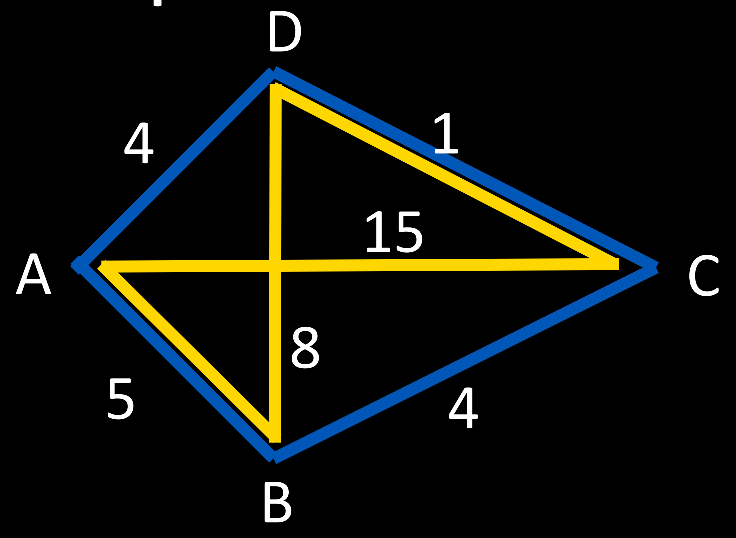


- Ant1: $L_1 = 14 \rightarrow \Delta\tau_{i,j}^{1} = \frac{1}{14}$
- Ant2: $L_2 = 29 \rightarrow \Delta\tau_{i,j}^{2} = \frac{1}{29}$

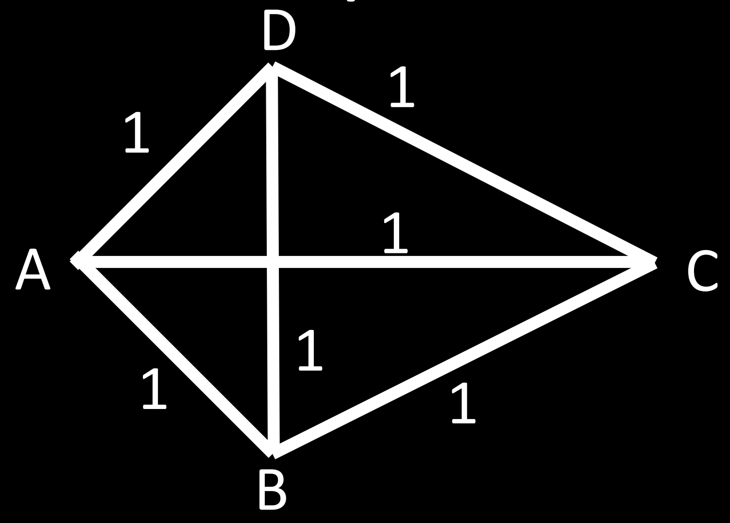
- $\tau_{i,j}^{k} = \sum_{k=1}^{m} \Delta\tau_{i,j}^{k}$

Algortihm (5/9)

Cost Graph



Pheromone Graph

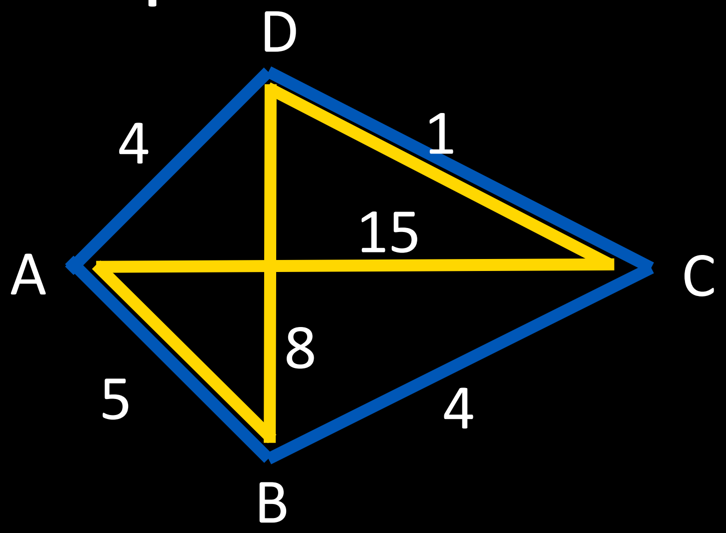


- Ant1: $L_1 = 14 \rightarrow \Delta\tau_{i,j}^{1} = \frac{1}{14}$
- Ant2: $L_2 = 29 \rightarrow \Delta\tau_{i,j}^{2} = \frac{1}{29}$

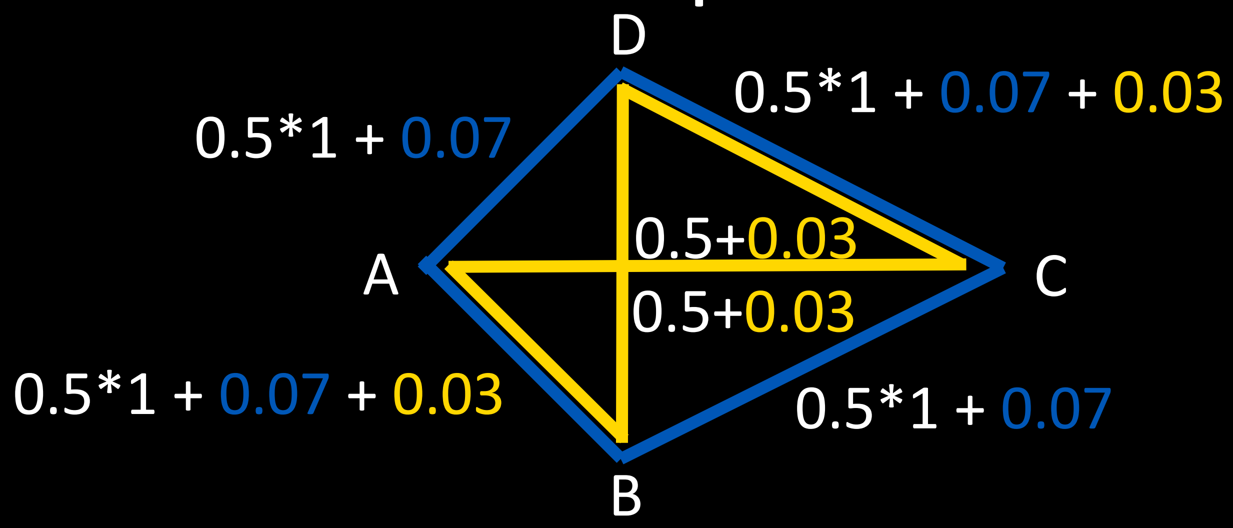
- $\tau_{i,j}^{k} = (1-\rho)\tau_{i,j} \sum_{k=1}^{m} \Delta\tau_{i,j}^{k}$

Algortihm (6/9)

Cost Graph



Pheromone Graph



- Ant1: $L_1 = 14 \rightarrow \Delta\tau_{i,j}^{1} = \frac{1}{14}$
- Ant2: $L_2 = 29 \rightarrow \Delta\tau_{i,j}^{2} = \frac{1}{29}$

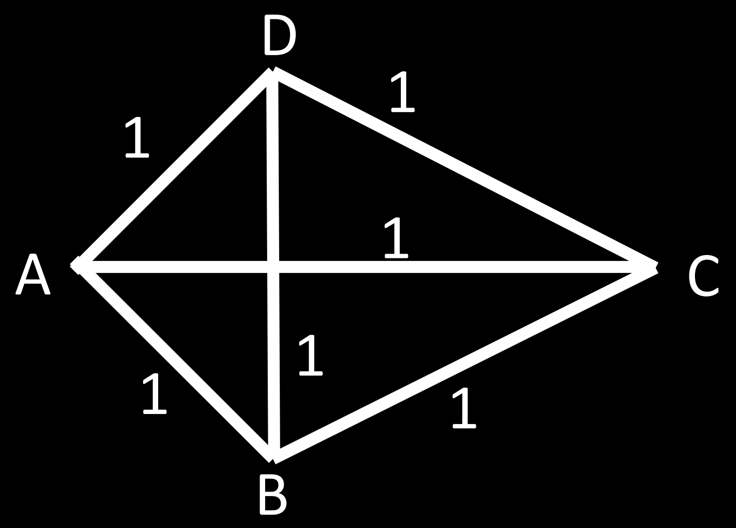
- $\tau_{i,j}^{k} = (1-\rho)\tau_{i,j} \sum_{k=1}^{m} \Delta\tau_{i,j}^{k}$
- $\rho = 0.5$

Algortihm (7/9) Probalility

- $P_{i,j} = \dfrac{(\tau_{i,j})^{\alpha}\,(\eta_{i,j})^{\beta}}{\sum\left((\tau_{i,j})^{\alpha}\,(\eta_{i,j})^{\beta}\right)}$

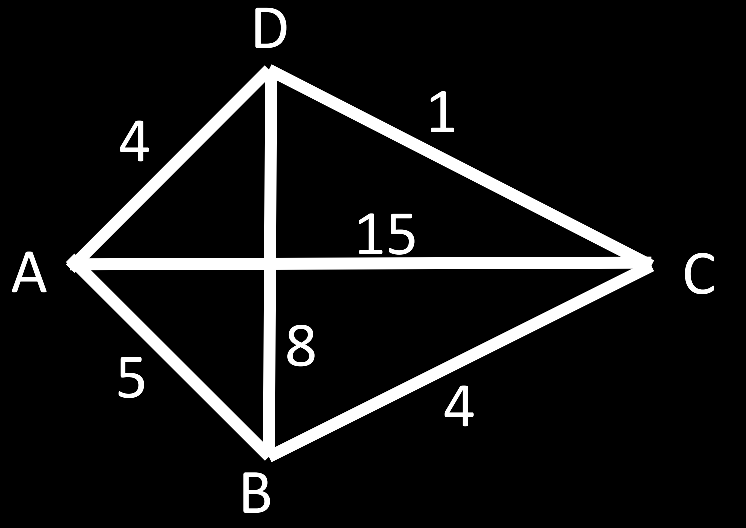
- $\eta_{i,j} = \dfrac{1}{L_{i,j}}$

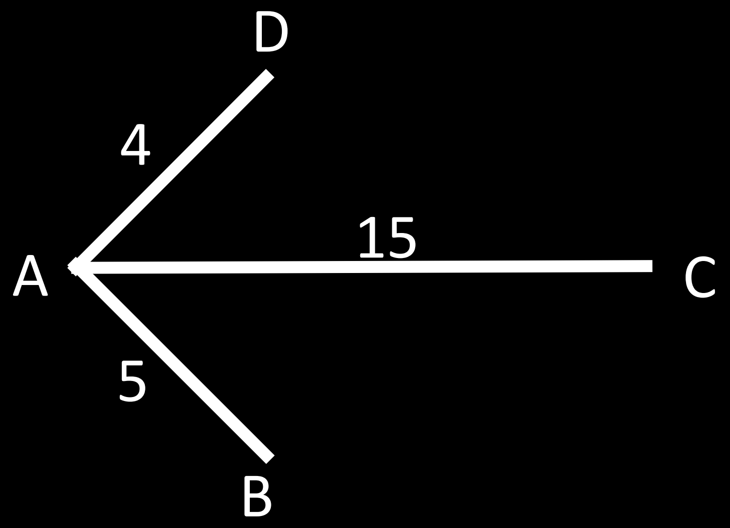
- $\tau_{i,j} = 1$



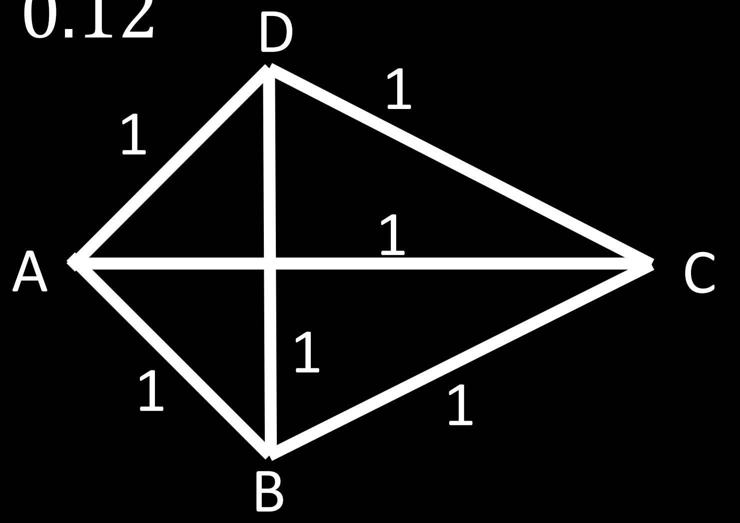
Algortihm (8/9)

$$P_{A,D} = \frac{1 * \frac{1}{4}}{\left(1 * \frac{1}{4}\right) + \left(1 * \frac{1}{15}\right) + \left(1 * \frac{1}{5}\right)} = 0.48$$





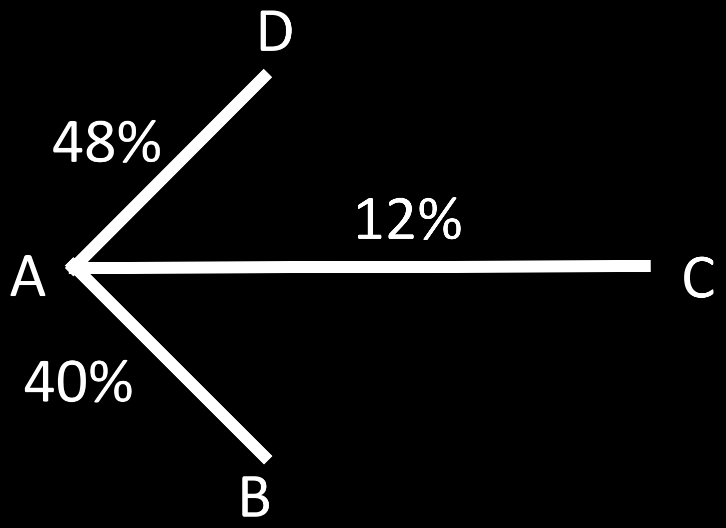
$$P_{A,C} = \frac{1 * \frac{1}{15}}{\left(1 * \frac{1}{4}\right) + \left(1 * \frac{1}{15}\right) + \left(1 * \frac{1}{5}\right)} = 0.12$$



$$P_{A,B} = \frac{1 * \frac{1}{5}}{\left(1 * \frac{1}{4}\right) + \left(1 * \frac{1}{15}\right) + \left(1 * \frac{1}{5}\right)} = 0.4$$

Algortihm (9/9)

For a random number r in [0,1] $\begin{cases} 0.52 < r \leq 1.00 & Go\ from\ A\ to\ D \\ 0.12 < r \leq 0.52 & Go\ from\ A\ to\ B \\ 0.00 \leq r \leq 0.12 & Go\ from\ A\ to\ C \end{cases}$



Example

Slime mold form a map of the Tokyo-area railway system