

Ant Colony Optimization

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ITI0210 Foundations of Artificial Intelligence and Machine Learning

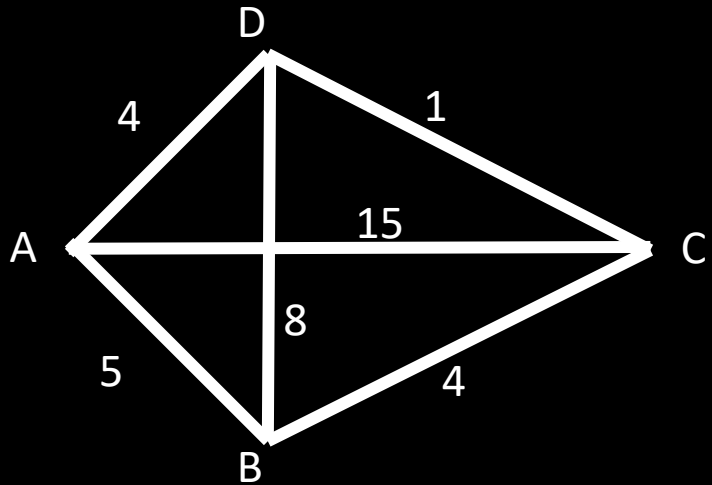
20.09.2022

Algortihm (1/X)

Prerequisites

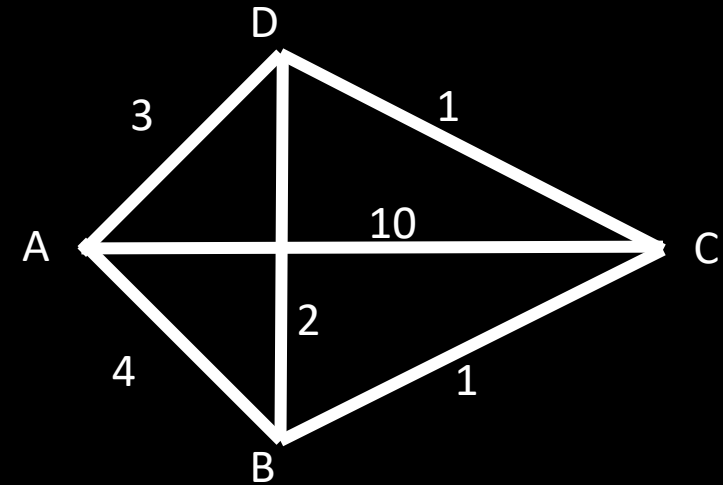
Cost matrix

	A	B	C	D
A	0	5	15	4
B	5	0	4	8
C	15	4	0	1
D	4	8	1	0



Pheromone matrix

	A	B	C	D
A	0	4	10	3
B	4	0	1	2
C	10	1	0	1
D	3	2	1	0



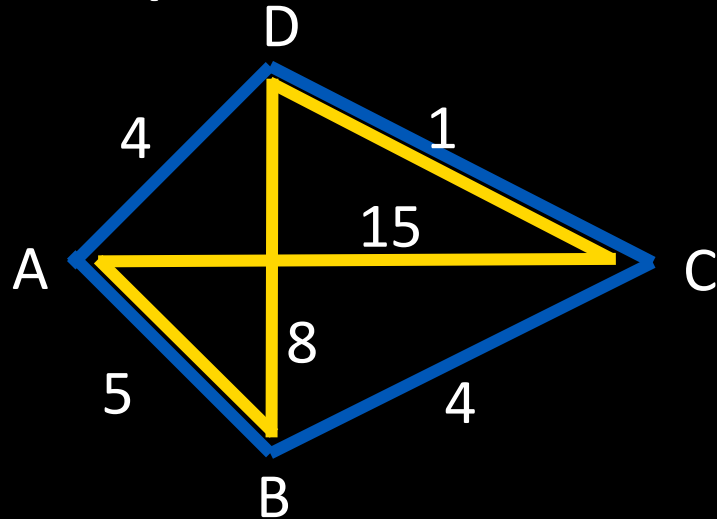
Algorithm (2/X)

Formulas

- $\Delta\tau_{i,j}^k = \begin{cases} \frac{1}{L_k} & k \text{ th ant travels on the edge } i,j \\ 0 & \text{otherwise} \end{cases}$
- $\tau_{i,j}^k = \sum_{k=1}^m \Delta\tau_{i,j}^k$ without vaporization
- $\tau_{i,j}^k = (1 - \rho)\tau_{i,j} + \sum_{k=1}^m \Delta\tau_{i,j}^k$ with vaporization

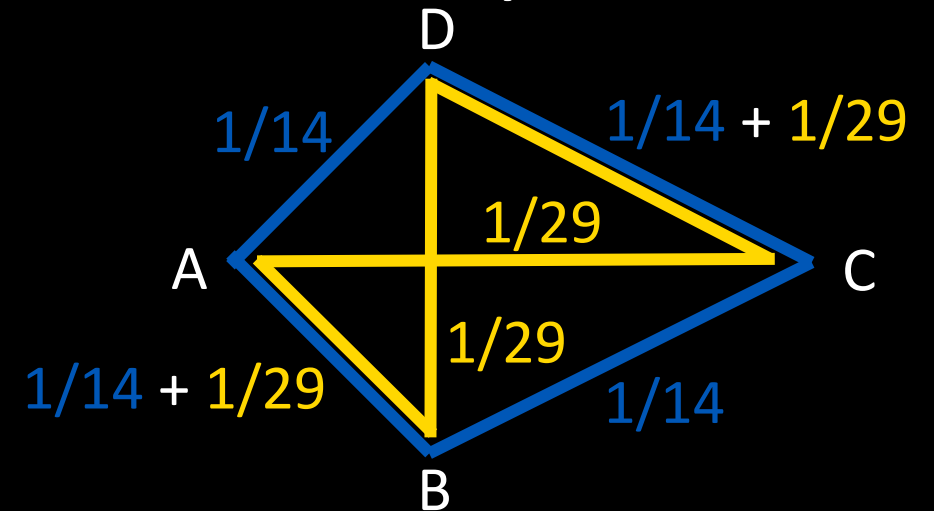
Algorithm (1/X)

Cost Graph



- Ant1: $L_1 = 14 \rightarrow \Delta\tau_{i,j}^1 = \frac{1}{14}$
- Ant2: $L_2 = 29 \rightarrow \Delta\tau_{i,j}^2 = \frac{1}{29}$

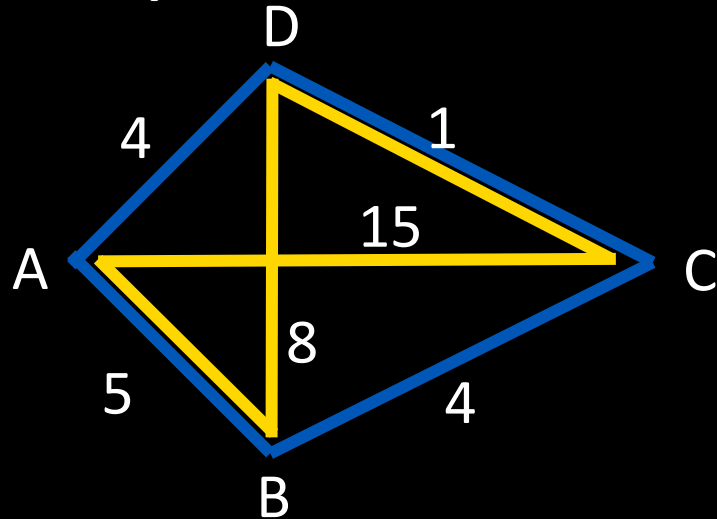
Pheromone Graph



- $\tau_{i,j}^k = \sum_{k=1}^m \Delta\tau_{i,j}^k$

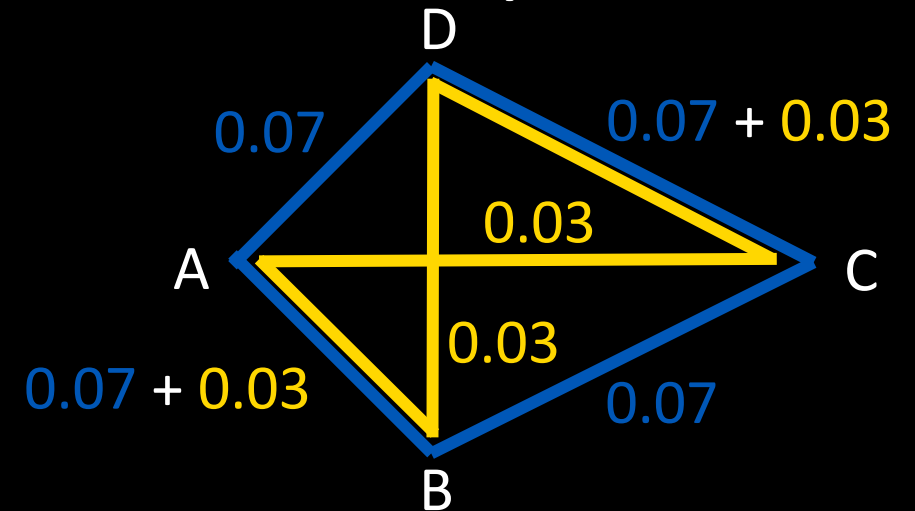
Algoritihm (1/X)

Cost Graph



- Ant1: $L_1 = 14 \rightarrow \Delta\tau_{i,j}^1 = \frac{1}{14}$
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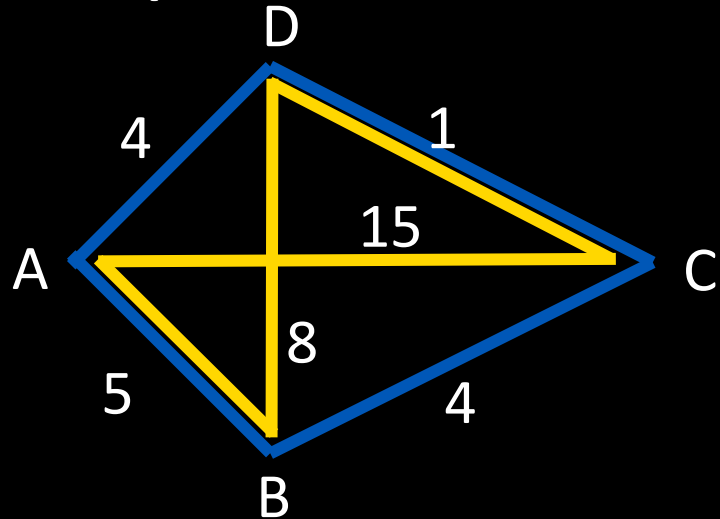
Pheromone Graph



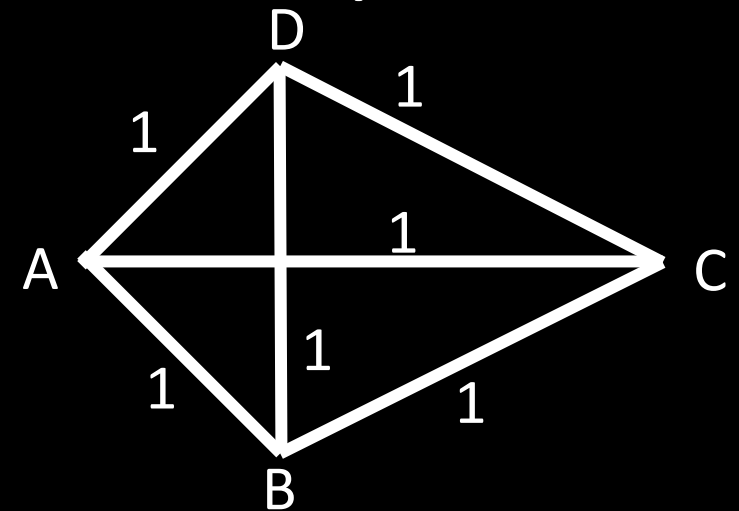
- $\tau_{i,j}^k = \sum_{k=1}^m \Delta\tau_{i,j}^k$

Algorithm (1/X)

Cost Graph



Pheromone Graph



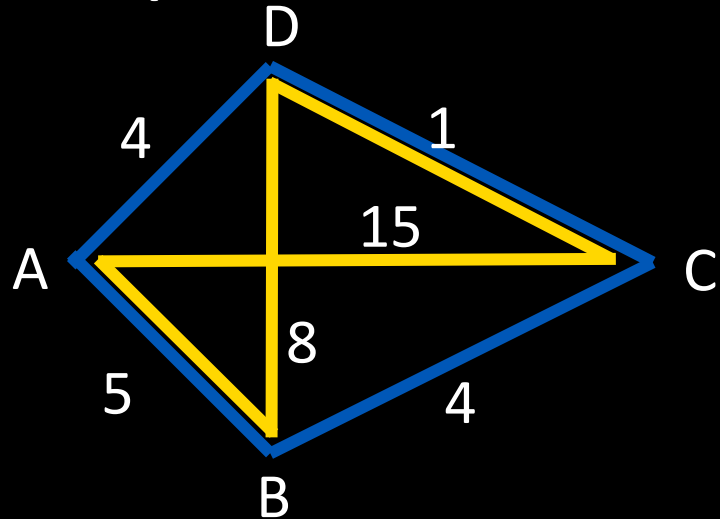
- Ant1: $L_1 = 14 \rightarrow \Delta\tau_{i,j}^1 = \frac{1}{14}$

- Ant2: $L_2 = 29 \rightarrow \Delta\tau_{i,j}^2 = \frac{1}{29}$

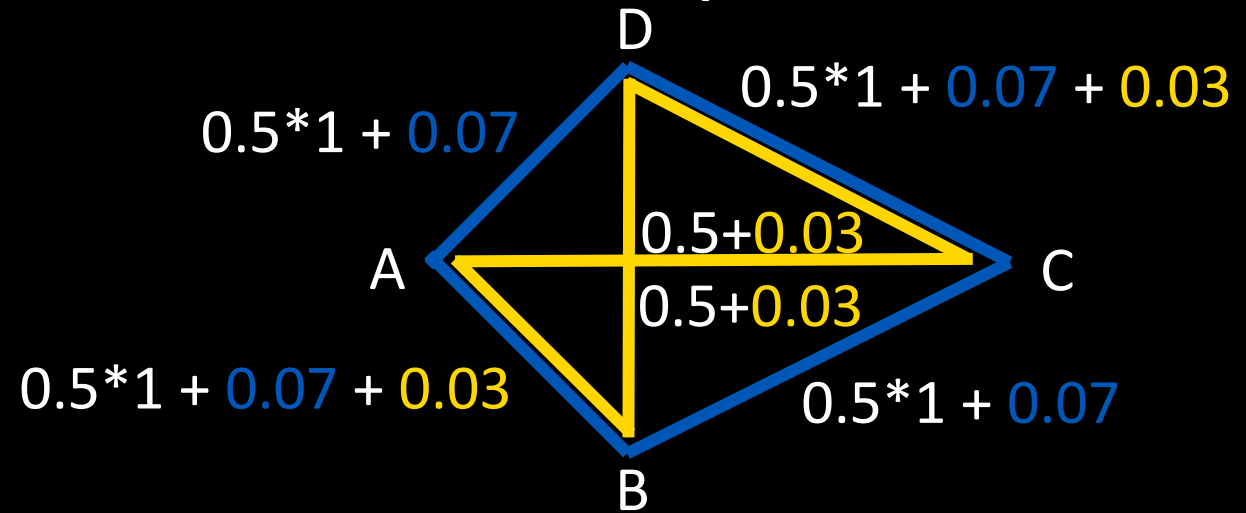
- $\tau_{i,j}^k = (1 - \rho)\tau_{i,j} \sum_{k=1}^m \Delta\tau_{i,j}^k$

Algoritmo (1/X)

Cost Graph



Pheromone Graph



- Ant1: $L_1 = 14 \rightarrow \Delta\tau_{i,j}^1 = \frac{1}{14}$

- Ant2: $L_2 = 29 \rightarrow \Delta\tau_{i,j}^2 = \frac{1}{29}$

- $\tau_{i,j}^k = (1 - \rho)\tau_{i,j} \sum_{k=1}^m \Delta\tau_{i,j}^k$

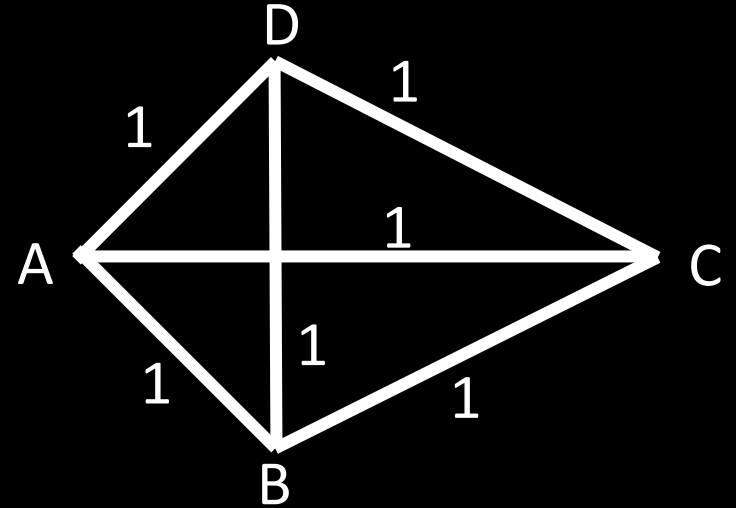
- $\rho = 0.5$

Algorithm (1/X) Probability

$$\bullet P_{i,j} = \frac{(\tau_{i,j})^\alpha (\eta_{i,j})^\beta}{\Sigma((\tau_{i,j})^\alpha (\eta_{i,j})^\beta)}$$

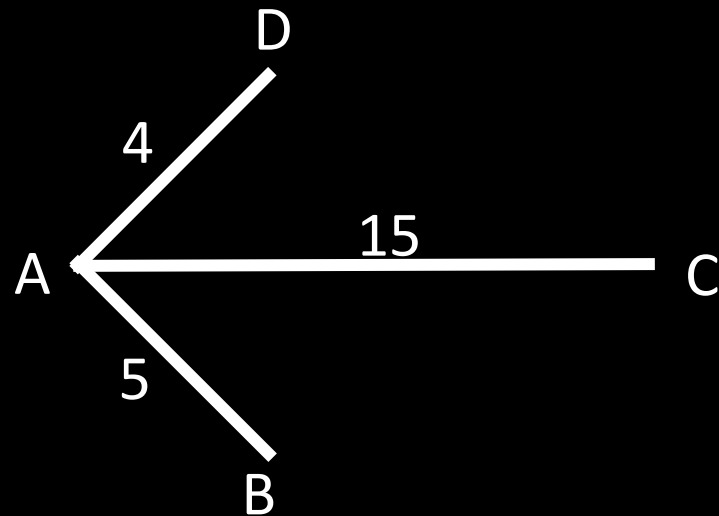
$$\bullet \eta_{i,j} = \frac{1}{L_{i,j}}$$

$$\bullet \tau_{i,j} = 1$$



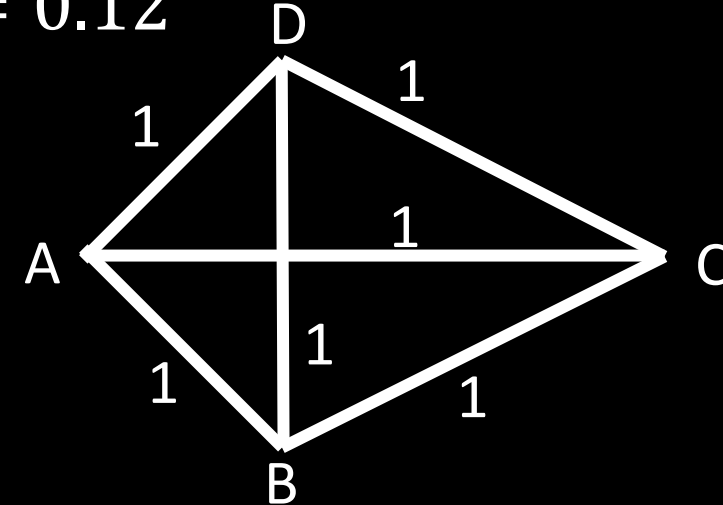
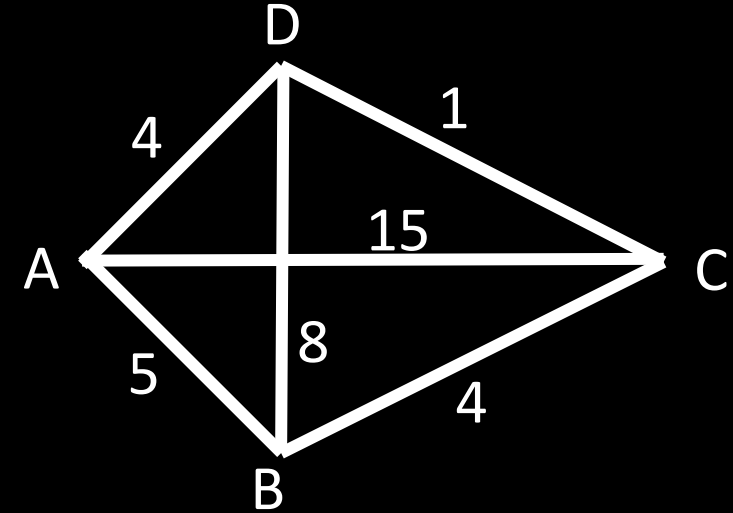
Algorithm (1/X)

$$P_{A,D} = \frac{1 * \frac{1}{4}}{\left(1 * \frac{1}{4}\right) + \left(1 * \frac{1}{15}\right) + \left(1 * \frac{1}{5}\right)} = 0.48$$



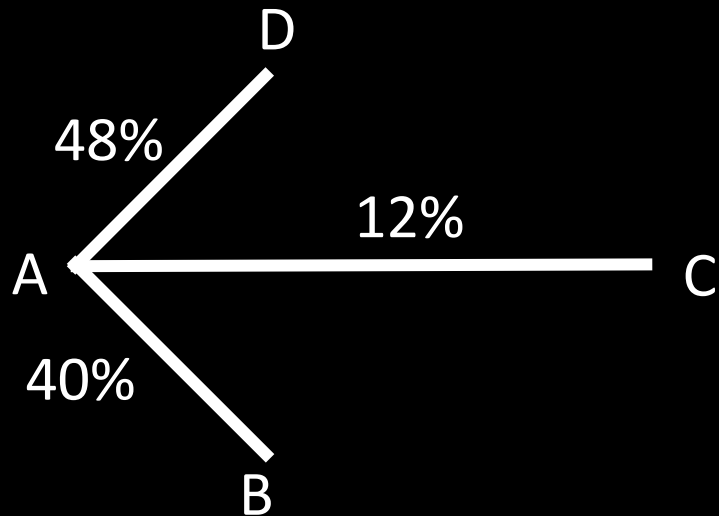
$$P_{A,C} = \frac{1 * \frac{1}{15}}{\left(1 * \frac{1}{4}\right) + \left(1 * \frac{1}{15}\right) + \left(1 * \frac{1}{5}\right)} = 0.12$$

$$P_{A,B} = \frac{1 * \frac{1}{5}}{\left(1 * \frac{1}{4}\right) + \left(1 * \frac{1}{15}\right) + \left(1 * \frac{1}{5}\right)} = 0.4$$



Algorithm (1/X)

For a random number r in $[0,1]$ $\left\{ \begin{array}{ll} 0.52 < r \leq 1.00 & \text{Go from A to D} \\ 0.12 < r \leq 0.52 & \text{Go from A to B} \\ 0.00 \leq r \leq 0.12 & \text{Go from A to C} \end{array} \right.$



Example

Slime mold form a map of the
Tokyo-area railway system

