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Fingerprint Segmentation

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ABSTRACT

In this thesis, a new algorithm has been proposed to segment the foreground of the fingerprint from the image under consideration. The algorithm uses three features, mean, variance and coherence. Based on these features, a rule system is built to help the algorithm to efficiently segment the image. In addition, the proposed algorithm combine split and merge with modified Otsu. Both enhancements techniques such as Gaussian filter and histogram equalization are applied to enhance and improve the quality of the image. Finally, a post processing technique is implemented to counter the undesirable effect in the segmented image.

Fingerprint recognition system is one of the oldest recognition systems in biometrics techniques. Everyone have a unique and unchangeable fingerprint. Based on this uniqueness and distinctness, fingerprint identification has been used in many applications for a long period. A fingerprint image is a pattern which consists of two regions, foreground and background. The foreground contains all important information needed in the automatic fingerprint recognition systems. However, the background is a noisy region that contributes to the extraction of false minutiae in the system. To avoid the extraction of false minutiae, there are many steps which should be followed such as preprocessing and enhancement. One of these steps is the transformation of the fingerprint image from gray-scale image to black and white image. This transformation is called segmentation or binarization. The aim for fingerprint segmentation is to separate the foreground from the background. Due to the nature of fingerprint image, the segmentation becomes an important and challenging task.

The proposed algorithm is applied on FVC2000 database. Manual examinations from human experts show that the proposed algorithm provides an efficient segmentation results. These improved results are demonstrating in diverse experiments.

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Last but not least, thank God for my life through all difficult times since the past nine years when I came to Sweden. He has made my life more bountiful. May his name be exalted, honored, and glorified.

To

My husband Atef Chit,

my son Ali Chit and

my daughter Fatima Chit

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1 Biometric

Humans recognize each other by identifying the characteristic of the body such as face, voice or gait. Company, firm, hospital, and all kind of organization recognize their customer by establishing traditional methods such as recording a personal password or using ID cards. However this representation of a person's identity is not reliable. It can be easily lost, stolen or discovered from imposters. To prevent these imposters accessing our resources, we need a reliable identification system. Biometric propose a solution to obtain a secure system based on the physical, chemical and behavioural attributes of the person. By applying Biometrics, the recognition system does not identify a person in what he posses such as ID cards, or what he remembers such as password. However the biometric system depends on who he is.

1.1 Biometric System

A biometric system provides the following three functions:

- Enrollment: In the enrollment phase, biometric information from individuals is stored in a system database. First, a biometric reader scans the biometric characteristic and creates a raw data representation from this characteristic. Afterwards, a quality checker pre-processes the data to confirm the reliability of this data. Next, this raw data is processed by feature extractor to get an optimal data called template. Therefore this template is recorded in the database to future use in the verification or identification phase.
- Identification: A biometric identification system has the ability of searching the entire biometric database to determine if there are any database entries that resemble the input sample [1]. One by one the database template is matched to the input sample. The output of the matching phase is the closest match to the user's identity otherwise the match fails if the user is not enrolled in the database.

- **Verification:** A biometric verification system differs from the identification system in that the user's is compared to a single enrolled biometric entity. Unlike identification system, each template in the system database is stored with a distinct identifier. Therefore the input sample is not only a biometric sample as in the case of identification system, but it is associated to some identifier such as ID, smart cards or usernames. The output of this system is Accept or reject.

1.2 Biometric Characteristic

Every biometric characteristic require satisfying the following properties:

- *Universality:* the characteristic should be encountered in each person.
- *Uniqueness:* the characteristic should be unique between individuals.
- *Permanence:* the characteristic should be sufficiently constant over long time.
- *Collectability:* the characteristic can be measured from a quantity point of view.
- *Performance:* the characteristic give an accurate result under different environment.
- *Acceptability:* the entire people accept to give their traits to the system without any problem.
- *Circumvention:* The characteristic must be hard to deceive and imitate.

Not all biometric traits require having all these properties. Fingerprint, face, hand/finger geometry, iris, retina, signature, gait, palm print, voice or DNA information are examples of biometrics characteristics. These characteristics are used in different applications. And to select which traits suitable to a particular application depend on the problems within this application. For example, in a supermarket they have arranged fingerprint scanners to help customer's payroll checks. In some airport, they have enrolled automated hand geometry-based identification or iris scans to validate the identity of the traveler.

1.3 Description of the different types of biometrics

Biometrics can be physiological or behavioral. Physiological biometrics is more common and accurate than behavioral. Some of physiological biometrics, that was the most commonly used in many fields, is facial scan, iris scan, hand scan and fingerprint scan. However keystroke, signature and voice recognition are some of behavioral biometrics. A brief introduction of some of this biometrics, are described below.

Face: Facial features are the most normal feature used by human to recognize one another. Face recognition is based on both the shape and location of the eyes, eyebrows, nose, lips, and chin or on the overall analysis of the face image that represent a face as a number of recognized faces. In a face recognition system, it is hard to match face images taken from two different views and under different illumination conditions. Moreover the face of individual can be changed by times. All this criteria make face recognition system to be uncertain if really the face itself is enough to recognize a person from a large number of identities.

Hand geometry: This method is simple, easy and inexpensive. It has been established in many locations in the world. Hand geometry recognition systems are based on a number of measurements taken from the human hand. It measures the shape of hand, the size of palm, and the lengths and widths of the fingers. Many environmental or anomalies factors don't affect any change in the accuracy of this recognition system. However this system can't be generalized to recognize from a large number of population. In addition the geometry of the hand is not constant; it can be changed in proportion of the age. Moreover the size of the hand is big and it is not currently in wide deployment for computer security applications primarily because it requires a large scanner.

Iris: Iris recognition is based on the features that exist in the coloured tissue surrounding the pupil which has many interesting points that can be used for comparison, including rings, rows and spots. The texture of the iris is very complex and distinctive which is very useful for the recognition system. Even the irises of identical twins are different. Although based on this complexity and this distinctness, the system is more accurately deployed and supports the probability of extensive identification systems.

Fingerprint: Fingerprint recognition system is the oldest recognition system among all the biometrics techniques. Everyone have a unique and unchangeable fingerprint. Like iris, fingerprints of the twins are even different. Based on this uniqueness and distinctness, fingerprint identification is used in many applications for a long period. A fingerprint is the pattern of ridges and valleys on the surface of the finger. It can be changeable only by some environmental and job-related factors such as cuts or injuries on the finger. These factors cause to be the system unsuitable in some degree. Generally the accuracy of the fingerprint recognition is sufficient in many applications especially in Forensics. To allow great identification systems for a large number of identities, the systems require having a multiple fingerprint from the same person to give additional information.

Keystroke: The way and the manner of typing on computer keyboard vary from individual to individual. This biometric is not considered as unique but it can be sufficient for some applications. Identification of this behavioral biometrics is basically what a person types is less important than how he types it. Using this approach several things can be analyzed: time between key-pressed and key-released, type of keyboard used or the emotional and physical state of the person. So no special hardware is required for keystroke analysis, just the usual computer keyboard.

Signature: Signature is the way a person signs his name. Depending on this sign, the individual can be identified. Signature recognition examines the unique way in which the signature is written. In the signature recognition system, the signature is compared by examining how the signature was written or it is verified by estimating how the signature was created [2]. Sometimes, this type of biometric can be changed over time. The same person can sign in different way. In addition, it is affected by physical conditions such as sickness or sentimental condition such as individual's feeling.

Voice: Voice recognition is the identification of a person based on unique characteristic on their voice. Voice characteristic is the combination between physical and behavioral biometric. For the physical part of view, voice is constant because it depends on the size or shape of the mouth, lips, vocal tracts and nasal cavities and so on. However for the behavioral part, voice is not constant. It can be changed based on individual's emotion, sickness or age [3]. Due to

this behavioral effect, voice recognition system can't not be considered as a distinctive biometric.

Gait: Gait recognition is a particular type of biometric due to its capability to identify a person at distance. Gait is related to the way of the person walking. The gait recognition system use standard camera in any conditions and develop algorithms to extract the silhouette of the person in case he is moving. Therefore the system can track the person over time. However the algorithm is not very efficient for this trait is affected by many conditions such as the type of cloth's or shoes the individual's wearing, the walking surface or the health.

All these biometrics are acceptable in different environment and none of them is optimal. However the most accurate ones are iris and fingerprint techniques. Due to the fact iris recognition is expensive and it requires advance requirement, fingerprint is one of the most mature biometrics and suitable for many applications [4]. Fingerprint biometrics is very distinctive, not expensive, unique and permanent and has a very good balance from all the properties.

The rest of this thesis is organized as follows. Chapter 2 presents fingerprint including automatic fingerprint recognition system that consist of image acquisition, pre-processing, feature extraction and matching part. In chapter 3, illustrates fingerprint segmentation and methodology which contains different segmentation methods, image enhancement and features for fingerprint segmentation. The implementation part is described in chapter 4. At the end and in chapter 5, results and discussion are presented and the thesis will be completed with a conclusion and future work part.

2 Fingerprint

Fingerprint is fully created at about seven months of fetus development and it is unique and unchangeable during individual's life excluding the situation of accidents in the finger such as cuts or injuries. Fingerprint consists of a sequence of ridges and valleys on the surface of the finger. In image point of view, ridges are actually the dark part in the image except the presence of different kind of noise, whereas valleys are the bright part. Frequently, ridges and valleys go in parallel but sometimes they bifurcate or terminate. Bifurcation is present when the ridge suddenly split into two ridges and termination exist when the ridge abruptly stop. In fact, bifurcation and termination represent some of important minutiae points. Minutiae mean small details that can determine important local features in the fingerprint. The most common minutiae types are the following [4] (see figure 2.1):

- Termination
- Bifurcation
- Lake
- Independent ridge
- Point or island
- Spur
- crossover








Minutia	Meaning
	Termination
	Bifurcation
	Lake
	Independent ridge
	Point or island
	Spur
	Crossover

Figure 2.1: Common types of minutiae

At the global level, fingerprint show some regions that represent distinctive shapes. These regions may be classified into three topologies (see **figure 2.2**):

- Loop , characterized by \cap shape
- Delta, characterized by Δ shape
- Whorl, characterized by \circ shape

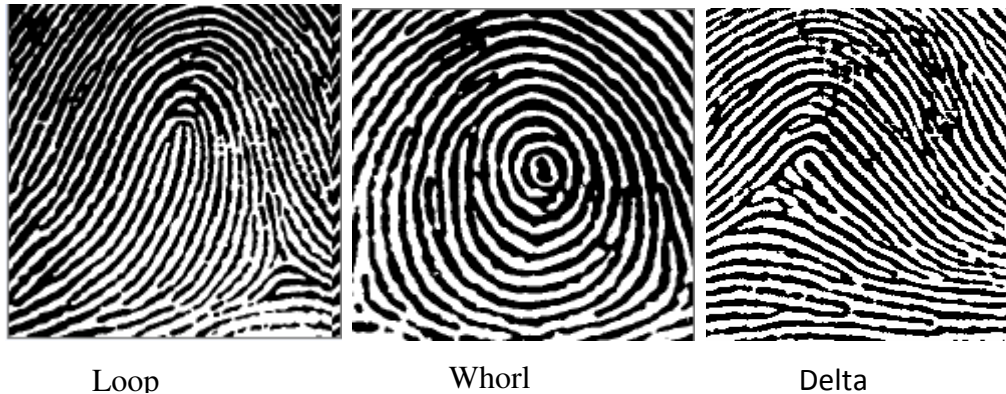


Figure 2.2: Common topologies of fingerprint

Therefore, the uniqueness of this kind of biometric depends on either these special shapes of ridges in the pattern or on these minutiae points.

2.1 Automatic Fingerprint Recognition System

Among all biometric techniques, fingerprint recognition system is the oldest recognition system. In the early twentieth century, fingerprint recognition becomes accepted as a personal identification system in forensic. Afterwards, different fingerprint recognition techniques, like latent fingerprint acquisition, fingerprint classification, and fingerprint matching were developed. At present, automatic fingerprint recognition is in progress day after day not just in forensic applications, but even in civilian applications.

Fingerprint recognition systems consist of the following parts (see **figure 2.3**):

- Sensing or Image acquisition
- Pre-processing
- Feature or minutiae extraction
- Matching

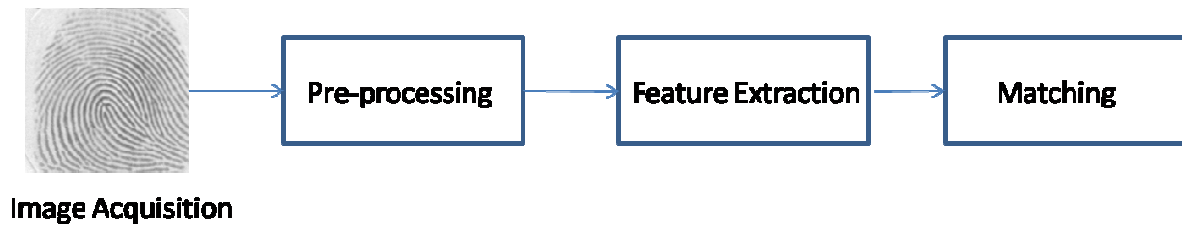


Figure 2.3: Fingerprint Recognition System

2.1.1 Sensing or Image Acquisition

The acquisition of a fingerprint images was accomplished by using off-line sensing or live-scan. Off-line sensing is defined as ink-technique. An individual place his finger in black ink then his finger is pressed in a paper card. After that the paper is scanned in a scanner to produce the digital image. This type of scanning is common in crime scene to obtain a latent fingerprint. However, live-scan scanners become presently more frequent, because of its simplicity in usage. There is no need for ink. The digital image is directly acquired by pressing against the surface of the scanner. The development of live-scan doesn't delete some disadvantages including:

- The difficulty of managing wet and dry fingers
- The misrepresentation of the fingerprint image in case the individual press hardly or slightly against the surface of the scanner
- The inability to detect false finger. Both techniques are involved by some factors that make

Therefore the quality of a fingerprint scanner, the size of its sensing area and the resolution of the acquired image can extremely affect the performance of a fingerprint recognition algorithm.

2.1.2 Pre-processing

To simplify the task of minutiae extraction and make it more easy and reliable, some preprocessing techniques are applied to the raw input image. Enhancement and segmentation of the fingerprint are the most commonly methods performed in the preprocessing step.

The principal aim of enhancement is to improve the clarity of ridge in the recoverable area in the image and to assign the unrecoverable ridges as a noisy area. Recoverable region is considered when ridges and valleys are corrupted by a small amount of dirt, ceases, or other kind of noise. Unrecoverable region are the regions which are impossible to recover them from a very corrupted and noisy image [5]. The most famously enhancement technique is contextual filter. This filter depends on changing filter parameter in relation to the local characteristic of the image. This parameter can be local ridge orientation or local ridge frequency [6].

However the primary purpose of segmentation is to avoid extraction of feature in the background that is in reality considered as a noisy area [6]. Segmentation indicates the separation of fingerprint area or foreground from the image background. Due to the streaked nature of the fingerprint area, a simple thresholding technique is not sufficient. In addition to the presence of noise in a fingerprint image, fingerprint segmentation requires more robust and strong techniques [7].

2.1.3 Feature extraction

After preprocessing step, the segmented and enhanced fingerprint is further processed to identify the main and distinctive minutiae. Most of the minutiae extraction methods necessitate the fingerprint gray-scale image to be transformed into a binary image. The acquired binary image is forwarded to a thinning stage to reduce the thickness of the ridge to one pixel ridge. Afterwards, the minutiae are simply detected by a simple image scan.

Due to the characteristic of the pixel that corresponds to minutiae, the simple scan image is one of many methods developed for minutiae detection. It depends on calculating crossing number of a pixel. The crossing number is the half sum of the differences between pairs of

adjacent pixels in the 8-neighborhood of p . Since the minutiae pixel can be bifurcation, crossover, termination, and so on. Therefore, the crossing number for minutiae must be different from 2 [4].

To avoid the problems related to fingerprint binarization and thinning, many methods have been proposed. Direct gray-scale minutiae extraction is one of these methods. The basic idea of this algorithm is to track the ridge lines in the gray-scale image by going according to the local orientation of the ridge. When a ridge line terminates or intersects another line, the algorithm detects this location as a minutiae point [4].

2.1.4 Matching

Algorithms that extract important and efficient minutiae, will improve the performance of the fingerprint matching techniques. The features extracted of the input image are compared to one or more template that was previously stored in the system database. Therefore the system returns either a degree of similarity in case of identification or a binary decision in case of verification. Due to many factors that affect the variability of fingerprint image of the same finger, matching techniques get to be a hard problem. Some of these factors are mentioned below:

- Fingerprint pressure, dryness, sweat, dirt, humidity
- Placement of the finger in different locations on the sensor
- Rotation of the finger at different angles to the sensor
- Residues from the previous fingerprint acquisition
- Feature extraction errors

Minutiae-based and correlation-based matching techniques are the most common techniques in fingerprint matching. In Minutiae-based techniques, first systems extract the minutiae in both images then the decision is based on the correspondence of the two sets of minutiae locations. However in correlation-based techniques compare two fingerprints based on their gray level intensities. First it selects relevant templates in the primary fingerprint then it uses template matching to locate them in the secondary image and compare positions of both fingerprints [8]. In correlation-based techniques, the propagation of errors in minutiae

extraction step is avoided. It doesn't require many preprocessing steps. Nonetheless, minutiae-based technique is the most widely used technique in fingerprint matching.

3 Fingerprint Segmentation and Methodology

Most automatic fingerprint identification or verification systems are derived from minutiae matching. The most important step in minutiae-based matching is to extract reliably the minutiae points from the input image. This extraction depends on the efficiency of the detection of ridges and valleys in the fingerprint segmentation step. Therefore an accurate segmentation of fingerprint images produces an effective result in automatic fingerprint recognition system.

Fingerprint image consist of two parts, foreground part that is the essential area for extracting minutiae points, and background part that is the noisy area. Segmentation of the fingerprint image is to decide which part from the image is associated with the foreground and which part is associated with the background. Due to the nature of fingerprint image and the presence of noise, the decision for separation these two regions is critical. The fingerprint image can be affected by many conditions that perform the segmentation to be a challenging task. The first problem is the presence of dust and grease in the scanner's sensor. The second one is the presence of some traces from previous image acquisition. The last one is the contrast of fingerprint that can be influenced by the dryness or the wetness of the finger. For dry finger, fingerprint contrast is low and for wet finger, the contrast is high.

Generally, Image segmentation methods are classified into two categories, discontinuity and similarity of intensity value. In the discontinuity-based categories, segmentation can be defined as edge-based segmentation that subdivides an image based on abrupt changes in the intensity. In the similarity-based categories, the segmentation is related to partitioning an image into regions according to their similarity. The similarity is a measure that is defined in advance depending on the fundamental problem in the image. This measure can be a specific intensity level, mean value, variance value, and so on. Point, line and edge detection are examples of discontinuity methods. Also, threshold, Otsu, splitting and merging and region growing are examples of similarity-based methods. The combination between different methods can give an improvement in segmentation performance.

3.1 Previous work

Many algorithms have been developed and all face nearly same problems. The background was not good distinguishable from the foreground. Chunxiao et al. [9] propose a hybrid algorithm based on block-wise classifier to separate the foreground from the background and pixel-wise classifier to deal with pixels accurately. Marques and Thome [10] partitioned the image into various sub blocks, and then extract a feature vector based on its Fourier descriptors. Each one of these vectors is passed to a neural network that classifies it. In [11] Ghassemian investigates a new on-line unsupervised ridges detection method that is based on fuzzy classification techniques. Chengpu et al. [12] present a novel algorithm that firstly uses the method of gradient projection, secondly adopt gradient coherence and finally carry out morphological operation to get the exact foreground region. Zhu et al. [13] propose a scheme for systematically estimating fingerprint ridge orientation and segmenting fingerprint image by evaluating the correctness of ridge orientation based on neural network. The neural network is used to learn the correctness of the estimated orientation by gradient-based method. Helfrroush and Mohammadpour [14] use a combination of three variance mean and ridge orientation features and also employs the median filter as a post processing step. Akram et al. [15] present a modified gradient based method to extract region of interest. This method compute the local gradient values for fingerprint images which detect sharp change in the gray level value of background. In [16], Bazen and Gerez propose also an algorithm that uses three pixel features, being the coherence, the mean and the variance. An optimal linear classifier is trained for the classification per pixel, while morphology is applied as post processing. Yin et al. [17] show two steps for fingerprint segmentation to exclude the remaining ridge region from the background. The non-ridge regions and unrecoverable low quality ridge regions are removed as background in the first step, and then the foreground produced by the first step is further analyzed so as to remove the remaining ridge region.

3.2 Segmentation Methods

In this thesis, Otsu and Split and Split and Merge are proposed to segment fingerprint images. The combination between these traditional segmentation methods is adequate to discriminate foreground and background. These methods are described in detail in the next section.

3.2.1 Threshold

The main idea in threshold methods is to select a threshold T that can separate objects from the background. This threshold can be specified according to the intensity histogram. Histogram of an image displays the gray-level values versus the number of pixels at that value.

Any pixel with gray level $f(x, y) > T$ is assigned as a foreground; otherwise the pixel is assigned as background see formula **3.1**.

$$\begin{cases} G(x, y) = 255 & \text{If } f(x, y) > T \\ G(x, y) = 0 & \text{If } f(x, y) \leq T \end{cases} \quad (3.1)$$

For fingerprint images, the histogram shows the contrast of the image and the distribution of the gray level. As shown below in **figure.3.1** the image is a bright image and no obvious gray level point can be as thresholding point. Because of the nature of fingerprint images, this algorithm cannot apply a simple thresholding technique.



Figure 3.1: Histogram for bright fingerprint

3.2.2 Optimum Global Thresholding, Otsu's method

The basic idea of Otsu's method is that an optimum threshold maximizes the separation between classes with respect to intensity value. According to maximizing the between-class variance, this method is optimum. Otsu's algorithm can be presented as the follows [9]:

- Compute the normalized histogram of the input image by using equation (3.2)

$$P_i = \frac{n_i}{MN} \quad (3.2)$$

Where n_i is the number of pixels with intensity i , MN is total number of pixel

- Set through possible threshold as shown in equation (3.3)

$$T(k) = k, \quad 0 < k < L-1, \quad (3.3)$$

Where L is the maximum intensity level in the image

- Compute the cumulative sum by using equation (3.4)

$$P(k) = \sum_{i=0}^k P_i \quad (3.4)$$

- Compute the cumulative means by using equation (3.5)

$$m(k) = \sum_{i=0}^k (iP_i) \quad (3.5)$$

- Compute the global intensity mean by applying equation (3.6)

$$m_G = \sum_{i=0}^{L-1} (iP_i) \quad (3.6)$$

- Compute the between class variance by using equation (3.7)

$$\sigma_B^2 = \frac{(P(k).(1-P(k)))^2}{P(k).(1-P(k))} \quad (3.7)$$

- Compute the Global variance by using equation (3.8)

$$\sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2 P_i \quad (3.8)$$

- Obtain the Otsu threshold as in equation (3.9)

$$K = \max(\sigma_B^2) \quad (3.9)$$

- Obtain the separability measure by using equation (3.10)

$$\eta = \frac{\sigma_B^2}{\sigma_G^2} \quad (3.10)$$

As explained before, the objective of this method is to find one threshold to give the best separation between classes. Due to the quality and nature of fingerprint, Otsu's method cannot produce a good separation between the foreground and the background. In addition, the presence of dark contrast in the foreground, the algorithm can destroy the details for ridges and valleys. Also the formal Otsu's Algorithm may not preserve the content of the foreground (see figure 3.2). This created a need to modify the algorithm.

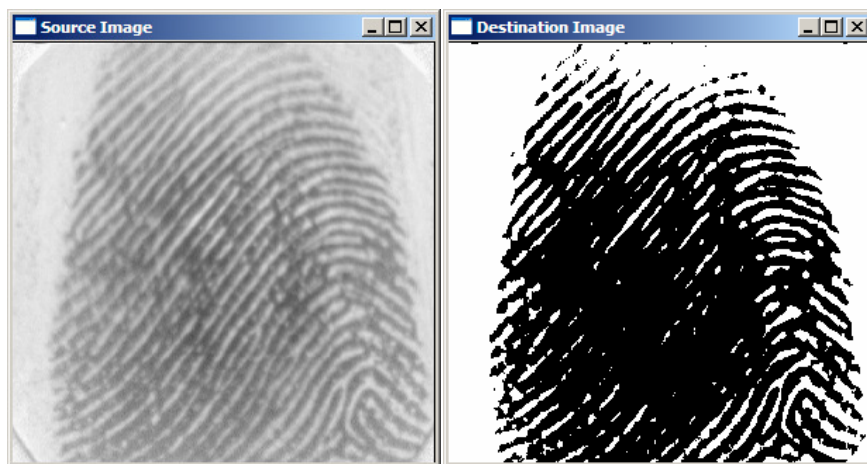


Figure 3.2: Segmentation by using original Otsu

3.2.3 Modified Otsu' method

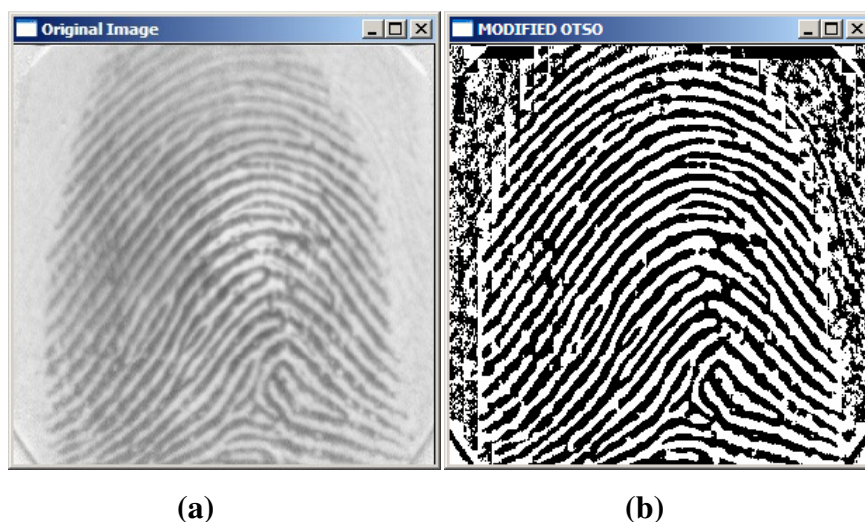
Noise and non uniform illumination in the foreground and background play a principal role in the performance of original Otsu's algorithm. Also, the algorithm goes through the entire image to find an optimal threshold to further use in segmentation. In this thesis a new version of Otsu method is developed. The developed algorithm seeks locally in small regions of the

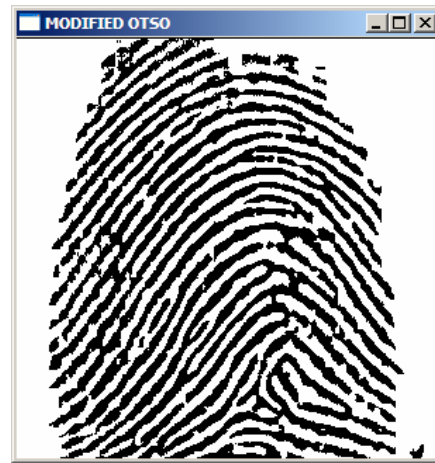
image and find corresponding optimal threshold for every region. The modification is illustrated as follow:

- Apply original Otsu to the image and save the obtained segmented image into **Otsu's image**
- Subdivide **Otsu's image** into no overlapping rectangles
- Subdivide the original image with the same previous subdivision and save into **original partitioned image**
- Calculate the mean gray level for each subdivision in **Otsu's image**
- Check in every partition the mean in **Otsu's image**
- If the mean for this **Otsu's image** partition is 90% white, assign the whole partition as background
- Otherwise, apply again original Otsu's global method in the subdivision in the **original partitioned image** to get the optimal threshold to this partition.

The size of subdivision is not simple to recognize. By looking to the histogram doesn't give any help. Therefore the need of experiments is required to decide suitable size of this subdivision.

The main idea of this method is to extract the foreground from the background by applying the global Otsu's method and to preserve ridges lines in the foreground by using Otsu's in image partitioning (see figure 3.3).





(c)

Figure 3.3: (a) Original image, (b) Image segmented into 900 regions using Otsu thresholds in each region, (c) Segmentation of (a) using modified Otsu

3.2.4 Split and Merge

The previous method show the efficiency of segmentation when the algorithm partitions an image into regions. The segmentation was accomplished via thresholds based on the distribution of pixels properties such as intensity value. In this section, Region Splitting and Merging technique is introduced here to segment the image by finding the regions directly.

The principal idea of this method is to iteratively split no similar regions and merge similar regions. The approach for this algorithm is to subdivide first the image into smaller and smaller quadrant regions. This type of splitting is called quadtrees, each node has exactly four descends. The root for this tree corresponds to the entire image and each node corresponds to the subdivision. The region of interest has some clear characteristics that should be taken as a predicate for this region segmentation. This predicate can be considered as mean, variance or color. The splitting takes place if the region doesn't satisfy the predicate, otherwise these splitting stops (**see figure 3.4**). If only splitting used, the algorithm contains only adjacent regions with same properties. Therefore, the merging technique is needed to combine two adjacent regions with identical properties.

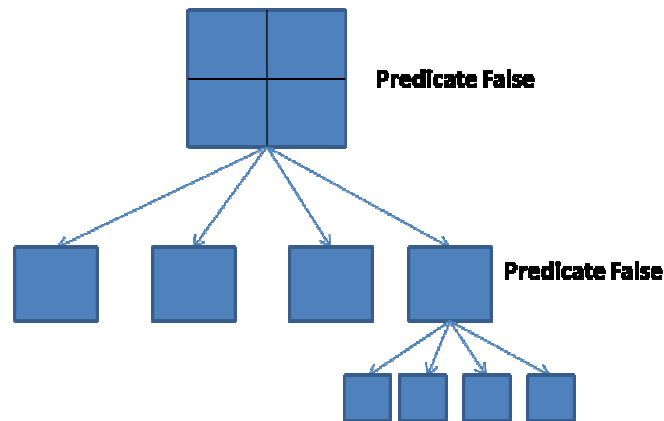


Figure 3.4: Quad tree splitting

Split and merge algorithm is summarized by the following steps [18]:

- Define a typical predicate to segment the image
- Split the image into four disjoint quadrants in the case of the region doesn't satisfy the predicate
- Merge two adjacent regions if they satisfy the predicate.
- Merge two regions in different level if they satisfy the predicate
- Merge small region with the most similar adjacent region
- Stop merging and splitting until no regions remain unchecked.

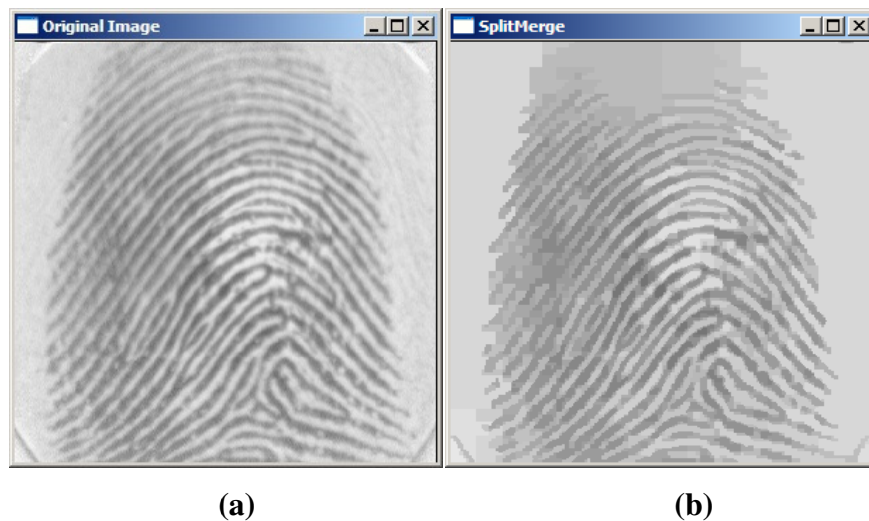


Figure 3.5: (a) Original image, (b) Segmented image by using Split and merge with variance 100.0 as a predicate

In the figure above (**figure 3.5**), it shows the result obtained by applying Split and Merge algorithm in a fingerprint image. This algorithm segment the image by using the variance as a predicate. The algorithm starts with image. If the variance of the image is greater than 100.0, the algorithm divides the image into smaller quadrant regions. If the variance is greater than 100.0 for any quadrant, the algorithm subdivides that quadrant into sub quadrant and so on. After splitting the image, merging technique starts by combining two regions if their variance is less than 100.0. As seen in the resulting image, Split and Merge helps to nearly extract the foreground from the background. Due to the predicate used in this segmentation, the foreground has not effectively partitioned. Variance is not a sufficient predicate to segment an image without any problems. Looking for efficient feature or suitable predicate is needed. For that reason, the explanation of some important feature in fingerprint segmentation is important to discuss.

3.3 Image Enhancement

As described in the previous section, when the image is acquired, the quality of fingerprint image can be affected by different factors. However, fingerprint images with low contrast or false traces ridges or noisy complex background cannot be segmented correctly by segmentation methods. Therefore, it is required to improve the quality of the image by applying some enhancement techniques. Some of these techniques that were used in this thesis are Gaussian Filter, Histogram Equalization and morphological reconstruction.

3.3.1 Gaussian Filter

Gaussian Noise appears in the image caused by factors such as poor illumination and high temperature for sensor. Gaussian filter is used to smooth the image and remove these noises. This filter is similar to mean filter which is called averaging filter. The degree of smoothing of Gaussian is expressed in term of σ which the standard deviation of the distribution.

In 2-D the Gaussian distribution has the form of **(3.11)**

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (3.11)$$

The goal of Gaussian Filter is to use this distribution as a point spread function which can be performed by convolution mask. The meaning of convolution is the process of moving the mask from the upper-left corner to the lower-right corner and replacing the value of the center pixel in the image by the value of $g(x,y)$. $G(x,y)$ is calculated by the sum of products of the filter coefficients and the corresponding image pixels in the area spanned by the filter mask. Normally, the filter mask is a two dimensional array in which the values of the mask coefficients affect the nature of the image [18]. Therefore Gaussian filters smooths the image by using the following mask based on discrete approximation to the Gaussian function (**figure 3.6**).

$$\frac{1}{273}$$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

Figure 3.6: Gaussian mask

Figure 3.7 shows the result of using Gaussian filter with $\sigma = 5$. This filter blurs the fingerprint image by reducing some noises in the background.

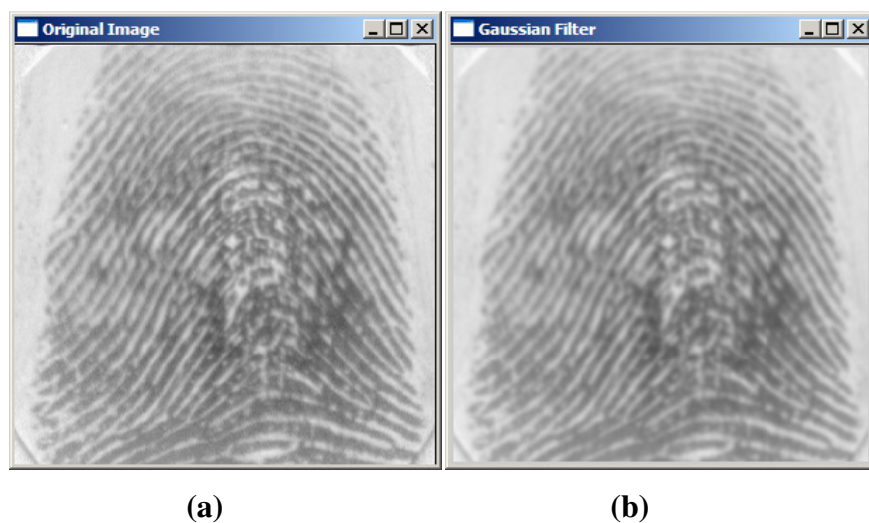


Figure 3.7: (a) Original image, (b) Noise reduction with Gaussian Filter

3.3.2 Histogram Equalization

The usage of Gaussian Filter in fingerprint images help efficiently to enhance the image in case of the presence of noise such that image acquisition noises. However, this filter cannot enhance an image which is affected by contrast problem. Therefore Histogram Equalization is a good solution for this problem. Histogram Equalization is the most common technique for improving the appearance of a poor image. It is the technique to get the histogram for the destination image as flat as possible. Histogram Equalization defines a mapping of gray level p into gray level q such that the distribution of gray level q is uniform. This mapping stretches the contrast of gray level near the maxima in the histogram [19]. The probability density function of a pixel intensity level r_k is yield by formula (3.12)

$$p_r(r_k) = \frac{n_k}{n} \quad (3.12)$$

Where r_k is between 0 and 1, $k=0, 1 \dots 255$, n_k is the number of pixels at intensity level r_k and n is the total number of pixels.

The new intensity value s_k for level k is derived by formula (3.13)

$$s_k = \sum_{j=0}^k \frac{n_j}{n} = \sum_{j=0}^k p_r(r_j) \quad (3.13)$$

By applying histogram equalization in a fingerprint image, the contrast is increased in most of fingerprint pixel. The first column in **Figure 3.8** shows the original image with its corresponding histogram. The second column shows the result of the equalized image to the original one with its histogram. As expected, the histogram of the original image (b) is concentrated on the light side of the intensity scale. That is because of the light contrast in the original image. The result of histogram equalization show important improvement in the contrast. In the equalized histogram (d), the intensity values cover the entire gray scale. Therefore, the significant contrast differences between the original histogram and the equalized one, illustrate the power of histogram equalization as a principal contrast enhancement tool.

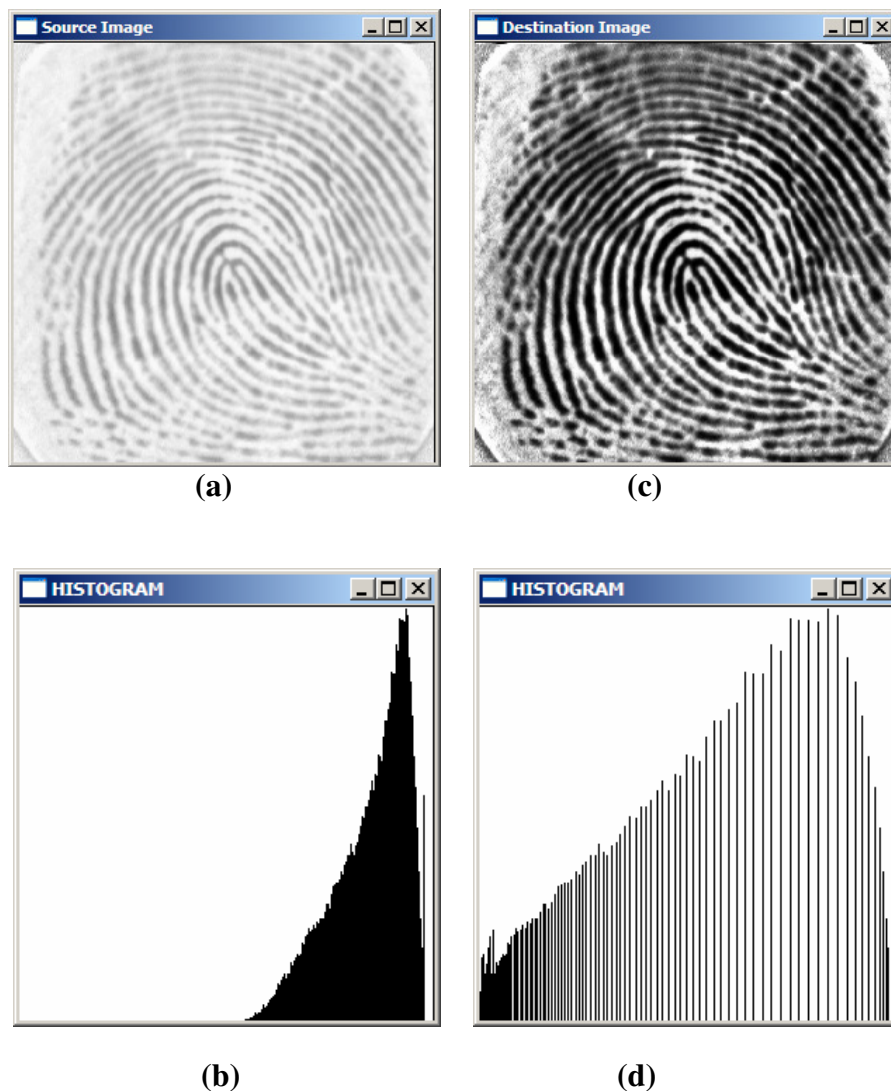


Figure 3.8: (a) Original image, (b) histogram of the original image, (c) histogram equalized image, (d) histogram of the equalized image

3.3.3 Post processing

The two enhancement techniques described before are not expected to perform correctly in all cases. Especially after applying segmentation techniques, there are new gaps between the fingerprint ridges. To counter this undesirable effect in the segmented image, a simple post processing technique is proposed to detect the gaps in the image. After detection of gaps, the corresponding area would be filled from the original image (see figure 3.9).



Figure 3.9: (a) segmented image before filling the missing part, (b) the post processed after filling the missing part

The post processing technique is simple and it is described in more detail in the following algorithm (see figure 3.10):

1. Calculate the mean for the center pixel (i, j) in each block in the image
2. If the center pixel is background
 - 2.1. check the mean for its four neighbors pixels ($i, j-1$), ($i, j+1$), ($i-1, j$) and ($i+1, j$)
 - 2.2. If two or more of these neighbors are foreground, change all the pixel in the block back to their original value before any segmentation

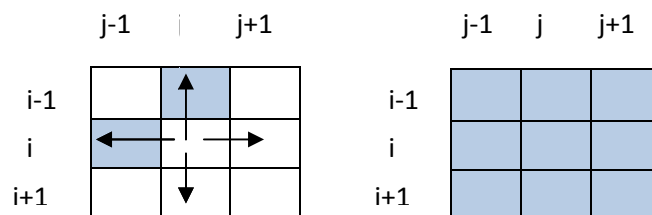


Figure 3.10: filling gaps in a block image

3.4 Features for Fingerprint Segmentation

As explained before, the fingerprint pattern is composed of ridges and valleys. The gray level of ridges is dark and the gray level of valleys is bright. Due to that, the use of mean and variance as principal feature to segment the foreground from background is logical. Mean of an image is simply the arithmetic average of the intensity values. Also, Variance of an image is the arithmetic average of the squared differences between the intensity values and the mean. In fact, mean is a measure of the centre of distribution but variance is a measure of the spread of the distribution about the mean. Mean and variance are feasible features for some good quality image but the algorithm fails in the following cases:

- Segment some regions from background as foreground if the background is full of noise
- Segment some regions from foreground as background if the contrast is small but the orientation is strong

However, the complicated construction of fingerprint pattern and the imbalance in the contrast, require local feature instead of the global feature.

Local mean and local variance are calculated as the following formulas (3.14, 3.15):

$$mean = \sum_w I \quad (3.14)$$

$$variance = \sum_w (I - mean)^2 \quad (3.15)$$

Where I is the intensity and w is the window size centered on the processed pixels.

Additionally, fingerprint features must reflect both the gray level of fingerprint and the direction of ridge lines. The direction can be detected by calculating the coherence. The coherence feature indicates the strength of the local window gradients centered on the processed point along the same dominant orientation.

Usually the coherence is also higher in the foreground than in the background, but it may be influenced significantly by boundary signal and noise. Therefore, a single coherence feature is not sufficient for robust segmentation. Systematic combination of those features is necessary. Therefore in a window w , the coherence feature is defined as (3, 4):

$$coherence = \frac{\sqrt{(g_x - g_y)^2 + 4(g_{xy})^2}}{(g_x + g_y)} \quad (3.16)$$

$$g_x = \sum_w G_x^2 \quad g_y = \sum_w G_y^2 \quad g_{xy} = \sum_w G_x G_y \quad (3.17)$$

Where G_x and G_y are corresponding horizontal and vertical gradient components. There exist many operators to approximate the gradient components, but the simplest operator used in this work, is the first order derivative of a one-dimensional function. The first order derivative can be approximated by

$$G_x = f(i+1, j) - f(i, j) \quad (3.18)$$

$$G_y = f(i, j+1) - f(i, j) \quad (3.19)$$

Where $f(x, y)$ is the gray level value in the image at position (x, y) (see figure 3.11).

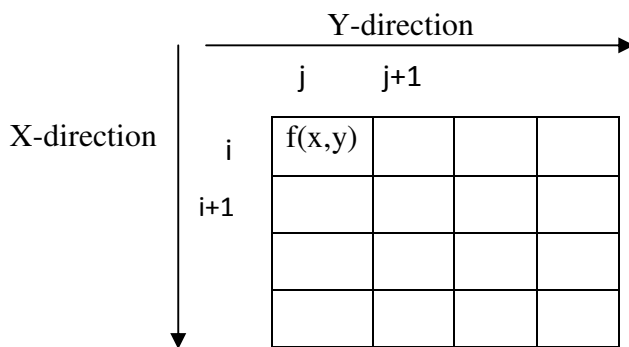


Figure 3.11: Image in x, y direction

4 Implementation

As explained in previous sections, based on the nature of the fingerprint a simple threshold technique, doesn't work. Using the proposed modified Otsu's method can be sufficient for some good quality images but the most of fingerprint images have problem in the contrast which contributes this method to fail. Split and merge technique can separate the foreground from the background but it destroys the contents of the foreground. In addition a simple predicate such as variance is not enough to apply. In this thesis, an efficient algorithm is proposed. The goal of this algorithm is to combine modified Otsu with split and merge by using variance, mean and coherence as threshold features.

5.1 Proposed algorithm

The algorithm consists of the following steps (see figure 4.2):

1. Apply Gaussian mean Filter to the image.
2. Compute Global mean and Global variance for the whole image
3. Define a threshold T based on the calculated global mean. If the global mean is greater than T , then go to step 4, otherwise, go to step 5.
4. Apply Histogram Equalization to the image
5. Apply Split and Merge
 - a. Define a threshold T based on variance
 - b. Divide the image into four regions to get two parent regions by the height and two parent regions by the weight
 - c. Calculate the variance for every parent region. If the variance for each parent is greater than the predefined threshold T , split recursively the region into 4 child regions and so on
 - d. Calculate the sum of the variance for two regions for the same or different parent. If the sum is less or equal to the threshold T , merge these two regions into one region and so on
 - e. Repeat step c until the splitting reach the pixel size in the image
 - f. Repeat step d until the merging cannot reach any region or pixel satisfy the predicate

- g. Save the splitter and merged image into a *temp* image
6. Divide the image into non overlapping sub-images. If fingerprint image have the size $N \times N$, the size for every sub-image is $w \times w$. Therefore the number of sub-images is $N/w * N/w$ (see figure 4.1).

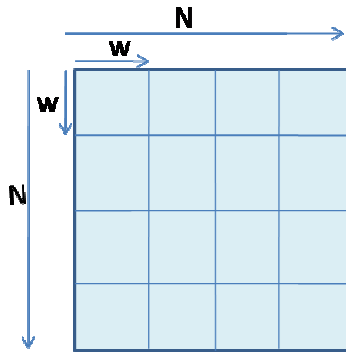


Figure 4.1: an image subdivision

7. Calculate the local mean, local variance and local coherence for each sub-image in the image
8. Define a set of rules based on Global mean for the image and local mean, local variance and local coherence for each block. Use this rules to segment *temp* image as multiple thresholds
9. Enhance *temp* image by filling the missing parts in the foreground. It checks the center pixel in the sub-image with its four neighbors. If at least two of these neighbors is foreground, therefore fill all the pixels in the blocks to the previous value in the original fingerprint image
10. In the last step, apply modified Otsu to the *temp* image and save it in the destination image

The proposed algorithm is shown in the diagram below.

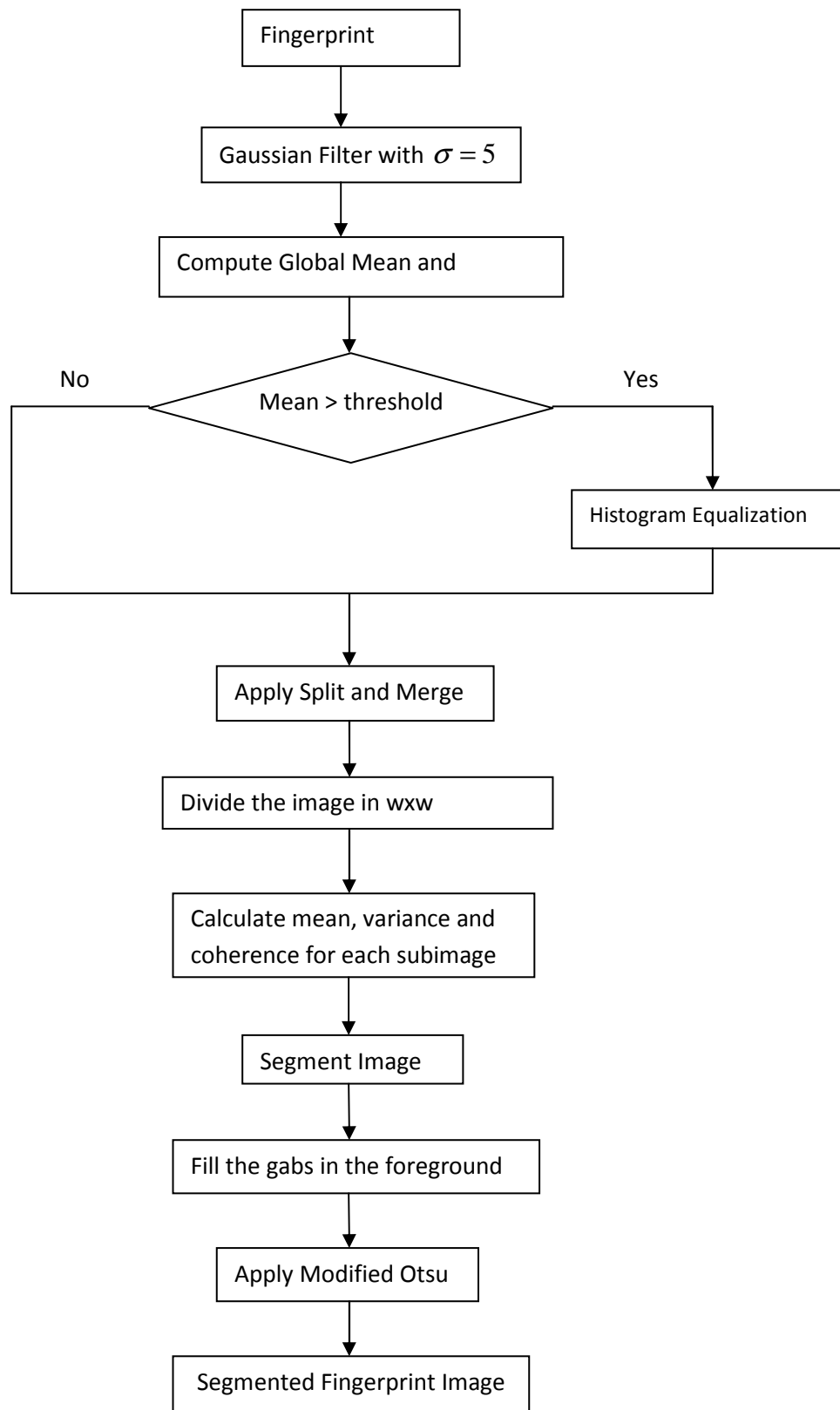


Figure 4.2: Diagram of proposed algorithm

5.2 Distribution of mean, variance and coherence

Mean and variance are the features for gray-level based methods. Coherence is the feature for direction based method. The combination for these feature in one algorithm show efficiently the distribution of the pixels for ridges and valleys in the image. The main idea of gray level and direction based methods is to derive a threshold T_1 , T_2 and T_3 from these features. If the mean of block is less than T_1 , the variance is less than T_2 and/or the coherence is less than T_3 , the block is foreground; otherwise, the block is background. The values of these thresholds are based on the distributions of these three pixel features in both the foreground and the background area.

The segmentation method described above has been implemented on the fingerprint images from the FVC200 database. 1_4 image has been segmented manually by dividing the fingerprint image into 900 sub images (see **figure 4.3**). Every block is classified as a foreground or background.

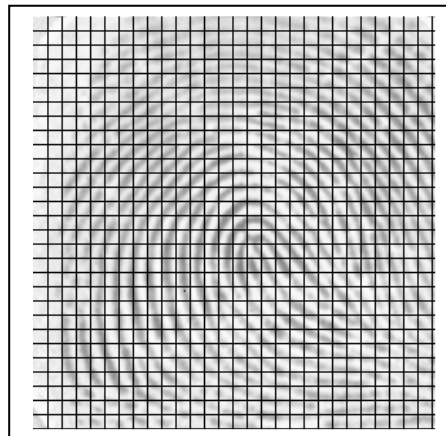


Figure 4.3: Subdivision of 1_4 image

Afterwards, the values of *Coherence*, *Mean* and *Variance* are calculated for every block. These values are normalized between 0 and 1. The distributions of coherence, mean and variance in x and y directions, are shown in the figures below. As mentioned before, the coherence measure how well the gradient is indicated in the same direction. In the previous segmented image, it is clearly seen that the foreground part is nearly spread in whole image. Therefore the coherence in the figure below (**figure 4.4**) is increasing, except in the right and

left borders of the image which is the background area. Due to the poor image quality, in some area, the coherence of foreground decreases while the coherence of background increases. Moreover, the quality for ridges lines in the foreground is disrupted. Therefore, the graph doesn't give a clear clue. The coherence of foreground has nearly considered a value between 0.2 and 0.8.

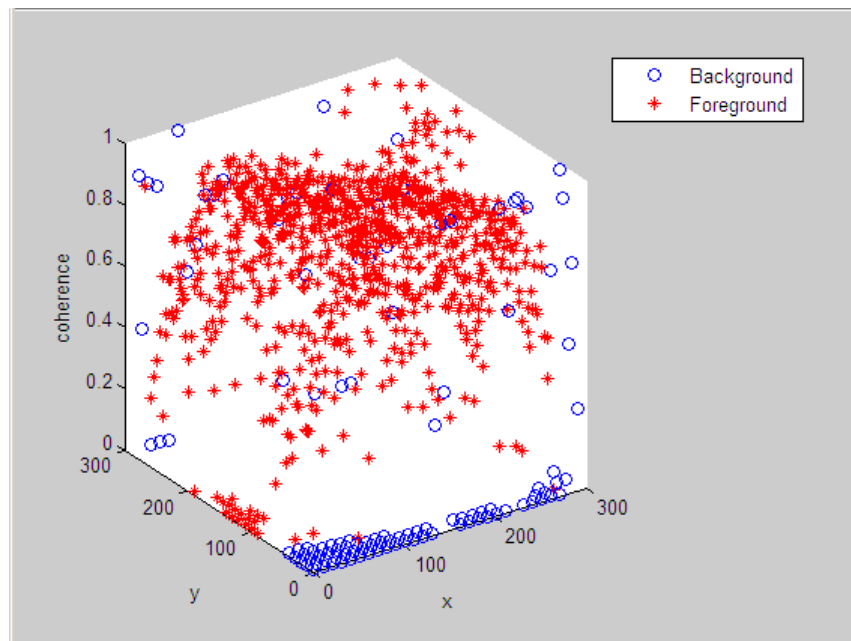


Figure 4.4: distribution of coherence in 1_4 image

Normally, the variance is high in the foreground and low in the background. In *figure 5.5*, the variance is low in the border of the image as in the case for coherence. However the variance of background is not distributed in whole image as in the case of coherence. In addition, when X and Y direction increase, the variance increases. As result, there is a big part in the foreground have a low variance that depends on the quality of 1_4 image.

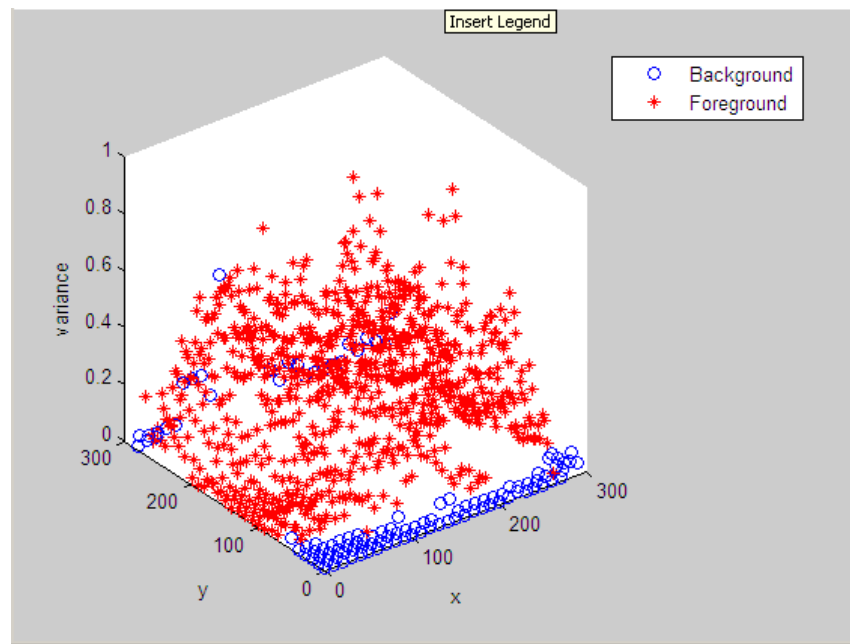


Figure 4.5: distribution of variance in 1_4 image

The background is considered when the finger does not touch the sensor for fingerprint scanner. Therefore the background is the white pixel in the image. Because of intensity of gray value, the distribution of mean requires being high in the background which is bright and low in the foreground which is darker (see figure 4.6).

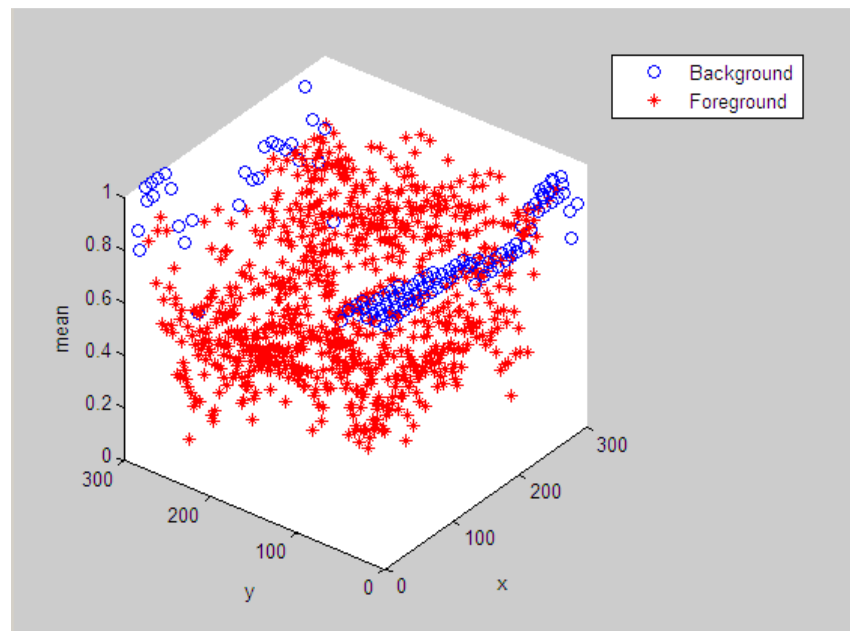


Figure 4.6: distribution of mean in 1_4 image

Mean, variance and coherence are important features for fingerprint segmentation. Based on these features, rules have been derived to segment fingerprint images. Therefore, the common distributions of combination of two features simplify to extract these rules.

The relation between the mean and variance are shown in **figure 4.7**. For foreground area, the mean is low and the variance is high. However, for background area, the mean is high and the variance is low. Therefore, the mean decreases when the variance increases.

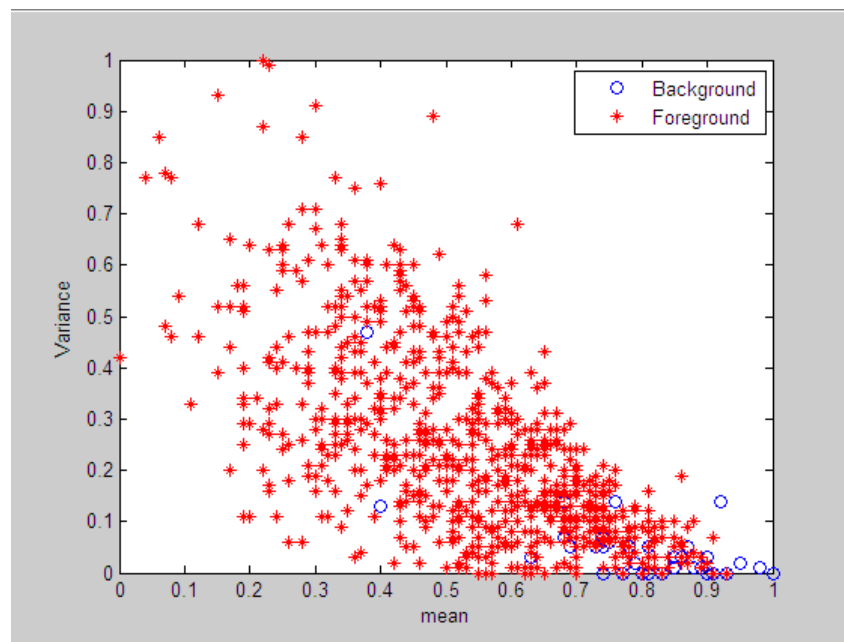


Figure 4.7: the relation between mean and variance

Figure 4.8 displays the combination between the variance and coherence. Normally, the variance increase when the coherence increases.

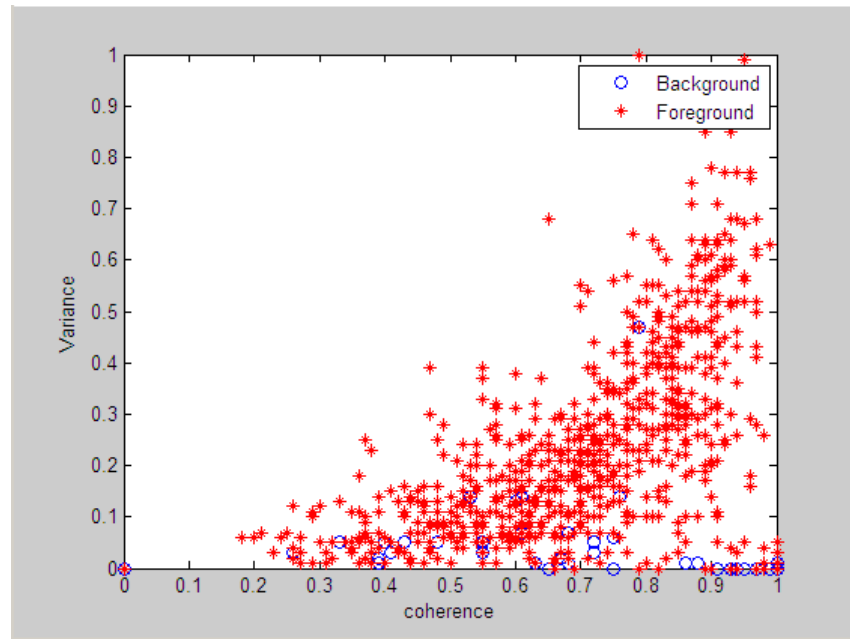


Figure 4.8: the relation between coherence and variance

Logically, if the coherence is high then the mean is low in the case for foreground pixels. And in the case for background, if the coherence is low then the mean is high. In **figure 4.9**, the mean for background is always high but the mean for foreground is slowly decreasing when the coherence increase.

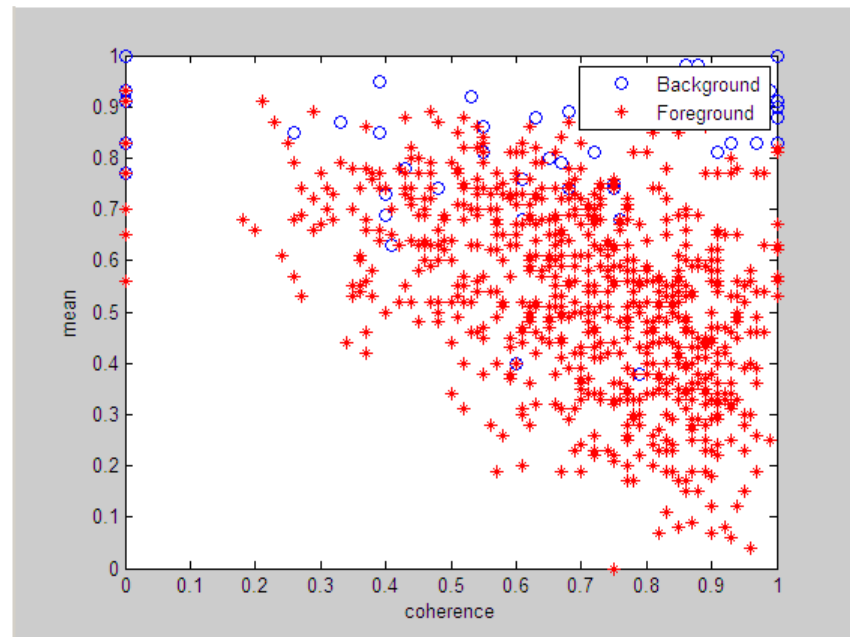


Figure 4.9: the relation between coherence and mean

5.3 Proposed rules

The distribution of pixels features in the foreground and background is not constant. It depends on the type of the image. The image may be of high contrast, low contrast, bright or dark contrast. Therefore, this huge change in the quality of every image require to have a rule system based on four thresholds, global mean, local mean, local variance and local coherence. First of all the algorithm checks the global mean for the whole image, if it satisfy the condition, then the algorithm look at the local features. The value of global mean is in interval $[0, 255]$. This value can give us the general contrast of the image. For that reason, we divide this interval to 11 intervals that are $[0, 159]$, $[160, 169]$, $[170, 179]$, $[180, 189]$, $[190, 199]$, $[200, 209]$, $[210, 219]$, $[220, 229]$, $[230, 239]$, $[240, 249]$, $[250, 255]$. In every interval, a characteristic value of local mean, local variance and local coherence are examined. All the examined cases for foreground pixels are summarized in Table I.

Table I: Examined value of mean, variance and coherence for different value of Global mean

Global Mean	Local mean	Local variance	Local coherence
$[0, 159]$	$m < 0.8$	$v > 0.15$	$\text{coh} > 0.3$
$[160, 169]$	$m < 0.8$	$v > 0.15$	$\text{coh} > 0.3$
$[170, 179]$	$m < 0.8$	$v > 0.15$	$\text{coh} > 0.3$
$[180, 189]$	$m < 0.7$	$v > 0.2$	$\text{coh} > 0.3$
$[190, 199]$	$m < 0.7$	$v > 0.2$	$\text{coh} > 0.5$
$[200, 209]$	$m < 0.85$	$v > 0.2$	$\text{coh} > 0.5$
$[210, 219]$	$m < 0.85$	$v > 0.2$	$\text{coh} > 0.5$
$[220, 229]$	$m < 1.1 \frac{G_{\text{mean}}}{256}$	$v > 0.2$	$\text{coh} > 0.5$
$[230, 239]$	$m < 1.1 \frac{G_{\text{mean}}}{256}$	$v > 0.2$	$\text{coh} > 0.5$
$[240, 249]$	$m < 0.8$	$v > 0.02$	$\text{coh} > 0.3$
$[250, 255]$	$m < 0.5$	$v > 0.2$	$\text{coh} > 0.2$

6 Result and Discussion

The proposed algorithm is tested on 100 fingerprints that are selected randomly from FVC200 database DB1. In fact, database 1 is collected by using two small-size and low-cost optical sensors. This database contained 880 fingerprints from 110 different fingers. The size of images in DB1 is 300x300 and each having a resolution of 500 dpi.

To evaluate the efficiency of this algorithm, human experts examine the result from these random images and show that the performance is satisfactory.

Hence some images have problem in the contrast; the use of histogram equalization improves the quality for most of low and light contrast images. Gaussian filter enhance the image by reducing some noises which make it difficult to be removed later in segmentation methods.

By applying Split and Merge with various rules based on mean, variance and coherence give better result than traditional split and merge. This method separates the foreground from the background but it produces some holes in the segmented image. To keep and enhance the foreground as in the original image, post processing technique has been implemented.

In addition, the combination of Split and Merge with Modified Otsu's in this algorithm performs a great advantage in comparison with a single method. Modified Otsu provides the binarization process. Efficiently it transforms the image in black and white.

All the mentioned steps are needed in this algorithm to reach a good solution to the most of problems that encounter the fingerprint images.

The results are classified into four categories:

- **Good**, where the result is perfectly segmented
- **Mostly Good**, where some part of foreground are missing or little part of background segmented as foreground
- **Mostly Bad**, where some part of the background are badly segmented as foreground
- **Bad**, where the background is not segmented from the foreground

Some experimental results of all categories are presented in figures below (**figure 6**).



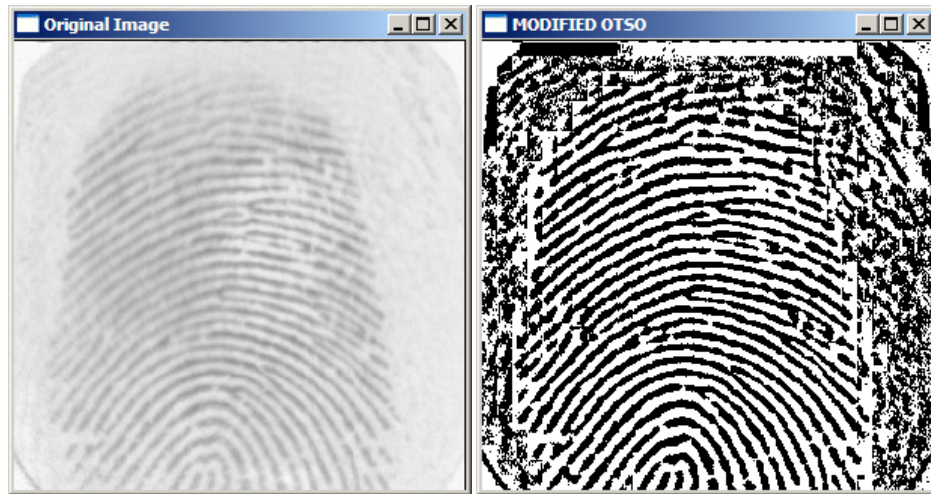


Figure 6: 1st column shows fingerprint images from FVC2000. 2nd columns show the segmented image detected with proposed algorithm. In 1st, 2nd, third and fourth show respectively the good, mostly good, mostly bad and bad category.

The results of the image segmentation with these different categories are summarized in Table II.

Table II: Results of different categories

Result	Percentage
Good	66%
Mostly Good	16%
Mostly Bad	4%
Bad	14%

7 Conclusion and Future Work

The aim of this work is to develop an algorithm for segmenting the foreground of the fingerprint from the image under consideration. This task is not an easy one as the foreground area is not easily extracted because of the quality of the image. An algorithm based on the combination of Split and Merge technique with Modified Otsu is presented in this work. Split and Merge is applied to separate the foreground from the background. However Modified Otsu is implemented to transform the image in black and white image.

The proposed algorithm use three local features that are mean, variance and coherence. From these feature, a rule system is built to segment diverse images.

Some fingerprint images have problem in their contrast and in their noisy background, therefore histogram equalization and Gaussian filter are included to enhance these images. This enhancement technique is efficient especially for the images with low and light contrast. Therefore histogram equalization is only applied in the case for poor images but Gaussian filter is implemented to all the images.

After splitting the image, there are new gaps in the segmented foreground. To counter this undesirable effect in this segmented image, a simple post processing technique is proposed to fill this missing part.

Experimental results show that the proposed algorithm is efficient and produce an improved performance.

For future work,

- The segmentation rules can be replaced by fuzzification techniques to the mean, variance and coherence.
- More test and experiments must be done to other Databases.
- Visual examination is a good qualitative evaluation for fingerprint segmentation. Error rate need to be more generalized by segmenting all images which is tested manually by human expertise.

- Mean, variance and coherence features values can be presented to a neural network to be trained by the target class. A target class can be obtained by classifying the image manually. After that diverse images can be tested.

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