

# Group assignment

TL and Waveguides

March 12, 2024

### Group Members

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### Assignment

a). Standing wave ratio(SWR) and the reflection coefficient

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1+T_L}{1-T_L}$$

$$I_{max} = \frac{V_{max}}{Z_0}$$

$$I_{min} = \frac{V_{min}}{Z_0}$$

The voltage reflection coefficient at any point on the line is the ratio of the magnitude of the reflected voltage to that of the incident voltage waves. That is;

$$T(z) = \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^- e^{2\gamma z}}{V_0^+}$$

But,  $Z = l - l^1$

$$T(z) = \frac{V_0^- e^{2\gamma l} e^{-2\gamma l}}{V_0^+} = T_L e^{-2\gamma l}$$

The current reflection coefficient at any point on the line is negative of the voltage reflection coefficient at that point. That is,

$$\frac{I_0^- e^{\gamma l}}{I_0^+ e^{-\gamma l}} = -T_L$$

b). Short circuit, open circuit and matched line characteristics

For short circuit,  $Z_L = 0$ ,

$$Z_{sc} = Z_{in} \text{ for } Z_L = 0$$

$$Z_{sc} = jZ_0 \tan \beta l$$

Transmission coefficient=-1, s=infinity

For open circuit,

$$Z_{os} = -jZ_0 \cot \beta l$$

For matched line,

$$Z_{in} = Z_0$$

Transmission coefficient=0, s=1

c). Transmission lines impedance matching(including the quarter wave transformers)

### Quarter wave transformer matching

$Z_0$  cannot be equal to  $Z_L$  as the load is mismatched and a reflected wave exists. Maximum power transfer cannot take place

We recall that,

$$l = \frac{\lambda}{4} \text{ or } \beta l = \frac{2\pi}{\lambda} x \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \frac{\pi}{2}}{Z_0 + jZ_L \tan \frac{\pi}{2}} = \frac{Z_0^2}{Z_L}$$

that is,

$$\frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L}$$

$$\text{or, } Z_{in} = \frac{1}{Z_L}, y_{in} = Z_L$$

thus by adding a  $\frac{\lambda}{4}$  line on our smith chart, we obtain the input admittance corresponding to a given load impedance.

$$Z_0 = \sqrt{Z_0 Z_L}$$

The main disadvantage of the quarter wave transformer is that it is a narrow band or frequency sensitive device.

### Single stub turner(Matching)

It eliminates the major drawback of using a quarter wave transformer.

$$Z_0 = Z_{in} = 1$$

First, we draw the locus  $y = 1 + jb$  ( $r = 1$  circle) on the smith chart. If a shunt stub of admittance  $y_s = -jb$  is introduced at A, then,  $y_{in} = 1 + jb + y_s = 1 + jb - jb = 1 + j0$

1. A  $100\Omega$  transmission line is connected to a load consisting of  $50\Omega$  resistors in series with a  $10\text{pF}$  capacitor. Find the reflection coefficient at the load for a  $100\text{MHz}$  signal

$$\text{Reflection coefficient, } T_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$T_L = \frac{50 - j1590 - 100}{50 - j159 + 100}$$

$$= 0.76 \angle -60.70$$

Find the impedance at the input end of the transmission line if its length is  $0.125\lambda$   $Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$

$$\beta(l) = \frac{2\pi}{\lambda} 1.25\lambda$$

$$= \frac{\pi}{4}$$

$$Z_{in} = 100 \frac{50 - j159 + j100}{100 + j150 - j159}$$

$$Z_{in} = 29.32 \angle -60.65\Omega$$

2. A lossless transmission line with  $Z_0 = 50\Omega$  and  $d = 1.5\text{m}$  connects a voltage source to a terminal load of  $Z_L = (50 + j50)\Omega$ . If  $V_g = 60\text{V}$ , operating frequency  $f = 100\text{MHz}$ , and  $Z_g = 50\Omega$  and assuming that the speed of the wave along the transmission line equals to the speed of light,  $C$ , find the distance of the first voltage maximum from the load.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^8} = 3\text{m}$$

$$T_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$T_L = \frac{50 + j50 - 50}{50 + j50 + 50} = 0.45 \angle 1.11 \text{ radians}$$

$$l_m = \frac{\theta_L \lambda}{4\pi} + \frac{n\lambda}{2}$$

when  $n=0$ ,

$$l_m = \frac{1.11}{4\pi} \lambda = 0.09\lambda$$

$$l_m = 0.09 \times 3 = 0.27 \text{ (from one load)}$$

What is the power delivered to the load  $P_L$ ?

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$\beta = \frac{2\pi}{3}$$

$$l = 1.5$$

$$Z_{in} = 50 + j50\Omega$$

$$I_{in} = \frac{V_g}{Z_g + Z_{in}} = \frac{6}{50 + 50 + j50} = 0.536656 \angle -26.56$$

$$P_L = P_{in} = 0.5 * 0.536656^2 * 50 = 7.2W$$

3. A 40m long transmission line has  $V_g = 15 \angle 0 V_{rms}$ ,  $Z_0 = 30 + j60\Omega$ , and  $V_L = 5 \angle -48 V_{rms}$ . If

the line is matched to the load, calculate: The input impedance  $Z_{in}$

Hints:  $Z_0 = 30 + j60\Omega$ , b).  $0.112 \angle -63.43 A$ ,  $7.5 \angle 0 V_{rms}$ , c).  $0.0101 + j0.2094$

$$Z_{in} = 30 + j60$$

The sending end current and voltage

$$V_{in} = \frac{Z_{in} V_g}{Z_{in} + Z_0} = \frac{V_g}{2} \text{ as } Z_{in} = Z_0$$

$$V_{in} = 7.5 \angle 0 V_{rms}$$

$$I_{in} = \frac{V_g}{2Z_0} = \frac{15 \angle 0}{2(30 + j60)}$$

$$I_{in} = 0.112 \angle -63.43$$

The propagation constant,  $\gamma$

$$e^{\alpha} l e^{j\beta l} = 1.5 \angle 48$$

$$e^{-\gamma} l = \frac{V_0^+}{V_L} = \frac{7.5 \angle 0}{5 \angle -48} = 1.5 \angle 48$$

$$\ln[e^{\alpha} l e^{j\beta l}] = \ln[1.5 \angle 48]$$

$$\alpha l + j\beta l = \ln[1.5 \angle 48]$$

$$\alpha + j\beta = \frac{\ln[1.5 \angle 48]}{l}$$

$$\alpha = \frac{\ln 1.5}{40} = 0.0101 N_p/m$$

$$\beta = \frac{\pi}{150} = 0.02094 \text{rad}/m$$

$$\gamma = \alpha + j\beta = 0.0101 + j0.02094/m$$