Design and Simulation of an Elevator System

Control Engineering 1

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0.1 Question

You are hired as a team of 5 engineers to design, model and simulate an elevator system to transport employees of an upcoming lab to several floors of a cleanroom. Your job is to ensure safety and comfort of the passenger and the payload. To this end, you will be required to model and simulate the elevator system and determine safe parameters for safe and smooth operation.

System Description:

The system is driven by a permanant magnet DC motor. The back emf generated in the armature winding is proportional to the rotor speed, $e_a = k_f \cdot \theta$ and the motor torque τ_m is proportional to the armature current, $\tau_m = k_t i_a$ where k_t and k_f are constants.

0.2 Tasks

0.2.1 motor dynamics

Question 1: Using KVL On the rotor circuit, derive the voltage equation.

Applying KVL,

$$V_a = e_a + i_a R_a + L_a \left(\frac{di_a}{dt} \right) \tag{1}$$

Question 2: Find the laplace transform of the voltage equation (1)

$$V_a(s) = E_a(s) + I_a(s)R_a(s) + sL_a(s)I_a(s) \tag{2}$$

Question 3: Express the armature current in terms of the supply voltage and the back emf

$$V_a(s) - E_a(s) = I_a R_a(s) + sL_a I_a(s) \tag{3}$$

making armature current the subject,

$$I_a(s) = \frac{V_a(s) - E_a(s)}{R_a(s) + sL_a(s)} \tag{4}$$

Question 4: Draw a standard block diagram representation of the system yielding the armature current.

0.2.2 motor dynamics 2

Question 1: Laplace transform of the expression for emf generated and motor torque

given,

back emf, $e_a = k_f \theta$ where k_f is a constant, motor torque, $\tau_m = k_t i_a$ where k_t is a constant

$$E_a(s) = L(k_f \theta) \tag{5}$$

$$E_a(s) = k_f L\theta \tag{6}$$

$$L(\theta) = S\theta(s) \tag{7}$$

$$E_a(s) = k_f s\theta(s) \tag{8}$$

$$\tag{9}$$

laplace transorm of motor torque,

$$T_m(s) = Lk_t I_a T_m(s) = ktLI_a \tag{10}$$

Question 2: Draw a standard block diagram represention of the system yielding the back emf and motor torque

0.2.3 rotational load dynamics

Ignoring belt elasticity, given that the belt ratio is k and that the winch and pulley B have a combined moment of Inertia J_w and damping factor B_w , forces acting of the pulley-winch arrangement are as in the manual. A rope having damping constant B_r and spring constant k_r is used to hoist the car. The rope is therefore exerting a force component F_b and F_k as a result. Assuming $x > r_w \theta_m$

Question 1: Derive the net force equation on the flywheel in time domain

The net force equation on the flywheel can be expressed by the sum of the torques acting on it

$$J_\omega \frac{d^2\theta_m}{dt^2} + B_w \frac{d\theta_m}{dt} = T_m - T_b - Tr \tag{11}$$

the force, F_k is related to the rope dynamics,

$$F_k = k_r x - B_r \frac{dx}{dt} \tag{12}$$

where;

J_w is the moment of inertia of the winch and the pulley B

B_w is the damping factor of the winch and pulley B

θ_m is the angle of the flywheel

k_t is the motor torque coefficient

I_a is the armature current

k is the belt ratio

F_b is the component due to the belt

r_ω is the product of the radius and belt ratio

F_k is the force component due to the rope

X is the displacement of the rope

k_r is the rope spring constant

B_r is the rope damping constant

flywheel smooths out delivery of power from the motor to the machine

$$T_m = k_t l_a \tag{13}$$

$$T_b = \frac{F_b}{k} \tag{14}$$

$$T_r = \frac{F_k}{r\omega} \tag{15}$$

Question 2: Laplace transform the equation

$$J_\omega s^2 \theta_m(s) + B_\omega s \theta_m(s) = k_t I_a(s) - \frac{f_b(s)}{k} - \frac{f_k(s)}{r\omega} \tag{16}$$

Question 3: Express the motor torque in terms of the balancing forces

$$k_t I_a(s) = J_\omega s^2 \theta_m(s) + B_\omega s \theta_m(s) + \frac{F_k(s)}{k} + \frac{f_b(s)}{r\omega} \tag{17}$$

Question 4: Draw a standard block diagram representation of the system yielding from the motor torque

0.2.4 translational load dynamics

An elevator car having bare weight of M_t and payload of M_p is secured at the end of the rope as shown in the figure below where $M = M_t + M_p$

Question 1: Derive the net force equation of the translational rope-car system in time domain

The net force can be derived from the Newton's second law,

$$m\frac{d^2x}{dt^2} + B_r\frac{dx}{dt} = f_t - f_b \tag{18}$$

where,

M is the total mass of the system, $M = M_t + M_p$

x is the desplacement of the elevator car

B_r is the damping factor of the rope

F_b is the force exerted by the balancing forces

Question 2: Laplace transform the equation

$$Ms^2x(s) + B_r s x_s = F_t(s) + F_b(s) \tag{19}$$

Question 3: Express the motor torque in terms of the balancing forces

$$k_t I_a(s) = Ms^2x(s) + B_r sx(s) + F_b - F_t(s) \tag{20}$$

Question 4: Draw a standard block diagram representation of the system yielding from the motor torque

0.2.5 entire system

Question 1: Combine the blocks developed so far to form the permanent magnet dc motor control system to the rope dynamics

Motor dynamics:

$$k_t I_a(s) = J_\omega s^2 \theta_m(s) + B_\omega s \theta_m(s) + \frac{F_b(s)}{k} + \frac{F_k(s)}{r\omega} \tag{21}$$

Translational load dynamics

$$M s^2 X(s) + B_r s X(s) = I_a(s) - f_b(s) \tag{22}$$

$$\tag{23}$$

combining the systems,

the motor torque $k_t I_a(s)$ contributes to the force on the rope $f_t(s)$

$$f_t = k_t I_a(s) \tag{24}$$

$$M s^2 X(s) + B_r S X_\omega = k_t I_a(s) - f_b(s) \tag{25}$$

$$M s^2 X(s) + B_r s X(s) = J_\omega s^2 \theta_m(s) + B_\omega s \theta_m(s) + \frac{f_b(s)}{k} + \frac{f_k(s)}{r\omega} - f_b(s) \tag{26}$$

to obtain the transfer function,express $f_t(s)$ in terms of motor input voltage, $V(s)$

$$f_t(s) = k_t(\frac{1}{sL_a + R_a})(E_a(s) + V(s)) \tag{27}$$

$$\tag{28}$$

substitute $f_t(s)$ into the translational load dynamics equation

$$Ms^2X(s) + B_rSX(s) = J_\omega s^2\theta_m(s) + B_\omega S\theta_m(s) + \frac{f_b(s)}{k} + \frac{f_k(s)}{r\omega} - f_b(s)$$

$$X(s) = \frac{G(s)}{1 + H(s)}V(s) \tag{29}$$

$$X(s) = \frac{J_\omega S^2 + B_\omega S + (\frac{k_t}{sl_a + R_a})E_a(s) + V(s) + (\frac{1}{r\omega})(-k_r - B_rS)}{Ms^2 + B_rS + \frac{k_t k}{(sl_a + R_a)r\omega} + \frac{k_t}{sl_a + R_a} + \frac{1}{(sl_a + R_a)r\omega}} \tag{30}$$

the above transfer function represents the relationship between input voltage to the motor and the displacement of the translational load in the laplace domain.

0.3 Matlab Simulation

0.4 Discussion

Elevator systems consists of components such as the elevator car, counterweights, guide rails, pulleys, motors, cables and control systems[1]. The dynamics of the motion of the elevator car are governed by Newtons laws of motion. It involves factors such as the mass of the elevator car, applied forces from the motor and graviational pull. To ensure passanger comfort and safety, the dynamics of accelaration and deccelaration of the elevator car are considered. The dynamics of elevator motors influence its speed, torque and acceleration. The comfort and safety of the elevator system depend on factors such as smooth operation, minimal vibrations, and adequate safety margins.

Since an elevator is a feedback controller, simulating its root locus helps achieve desired performance and characteristics[2]. As observed from the simulation in the controller gains(K) used render the system unstable and should therefore be adjusted to ensure safety and comfortability of the system.

Reccommendations:

To enhance comfort and safety, the controller parameters can be optimized to minimize overshoot and settling time. Vibration damping mechanisms could also be implemented and emergency breaking systems incorporated.