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Design and Analysis of a Control System Using Root Locus and Frequency Response Methods

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ABSTRACT

Control systems play a very important role in the domain of Electrical Engineering. Without them, it is impossible to comprehensively analyze and design electrical systems. This paper successfully attempts to model a practical real control system using root locus (time domain) and frequency response (Bode Plots) techniques. A brief review of root locus and Bode plots is given. Major focus has been placed on controller design and how the required goal criteria can be achieved. MATLAB has been used exclusively for simulation and design purpose.

Keywords: *Control Systems, Root Locus, Frequency Response, Bode Plots, Controller, MATLAB*

1. INTRODUCTION

The paper aims to control a real time practical system using root locus and frequency response techniques. The device chosen to control is a commercial car. Point of interest is controlling the overshoot produced by shock absorbers (shock dampers) in case of bumps the car may experience. The less the overshoot, the more the ability of car to get to stable condition in case of external disturbances in the form of sudden bumps on the road. The parameter which shall be controlled is the position. We can also choose velocity or acceleration as the desired variable but “position” has been chose for the sake of simplicity. The control can be hydraulic or pneumatic. It must be pointed out that we are not interested in the way it is controlled but rather how it meets the desired performance parameters.

Pneumatic and hydraulic shock absorbers are often used in combination with cushions and springs. An automobile shock absorber has spring-loaded check valves and openings to regulate the oil flow with the aid of an internal piston. One design thought, when designing a shock absorber, is where that extra energy will go. In many of these shock absorbers, energy is dissipated as heat inside the viscous fluid. In hydraulic cylinders, the hydraulic fluid heats up, while in air cylinders, the hot air is usually exhausted to the atmosphere. In other types of shock absorbers, such as electromagnetic types, the dissipated energy can be stored and used later. In general terms, shock absorbers help boost vehicles on uneven bumpy roads [1-2].

2. ROOT LOCUS BACKGROUND

The root locus technique or method is a very handy graphical method for sketching the locus of roots in the s-plane as a parameter is varied. The parameter may be gain, phase, overshoot or any other suitable control parameter. This method has been applied broadly in control engineering design problems. It equips the control engineer with a degree of the sensitivity of roots of the system a disparity in parameter under observation [3].

Consider Figure 1 which shows a closed loop feedback system. Gain of the system is given by ‘K’. By feedback, we mean that the “loop” is closed. The output “feeds” back to the input.

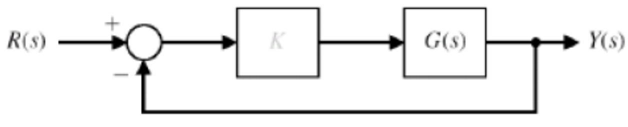

Figure 1 Feedback Control System

The closed loop transfer function of the system can be found by using:

$$T = \frac{KG(s)}{1 + KG(s)}$$

Closed loop poles are found by finding the roots of the denominator in the above mentioned equation.

Some key points or rules regarding root locus are mentioned for easy reference [4]:

Rule 1: Number of branches are equal to number of closed loop poles

Rule 2: Root locus is always symmetrical about the real axis

Rule 3: Real-axis segments lie to the left of an odd number of real-axis finite poles/zeros.

Rule 4: Root locus originates at poles and terminates at zeros.

3. BODE PLOTS BACKGROUND

A Bode plot is a standard technique used for plotting frequency response of Linear Time-Invariant (LTI) systems. Becoming acquainted with this arrangement is valuable because firstly it is a standard procedure, thereby using it enables to reduce communication gap between engineers and helps to remove any ambiguity which may arise due to working on different formats. Secondly, many common system behaviors produce simple graphical shapes (e.g. straight lines, circles, hyperbolas etc.) on a Bode plot, so it is relatively simpler to either look at a plot and identify the system behavior, or to draw a plot from what information you have regarding the system behavior. The standard format of a typical Bode plot consists of a log frequency scale on the X-axis (horizontal axis) and on the Y-axis (vertical axis), phase in degrees (for phase plot) and magnitude in decibels (for gain plot) [5].

To accurately plot Bode Graphs, we need to collect gain and phase responses for a large range of frequencies, mostly in the order of kilohertz (kHz.). After gathering the phase and amplitude (gain) responses for a large number of frequencies, one of the most instinctive ways to show it is to plot the Gain Bode Plot (amplitude response vs. frequency) and Phase Bode Plot (phase response vs. frequency) and display it on a single diagram [6].

Figure 2 below shows an example of Bode Gain and Phase plots on a single diagram for an ideal motor.

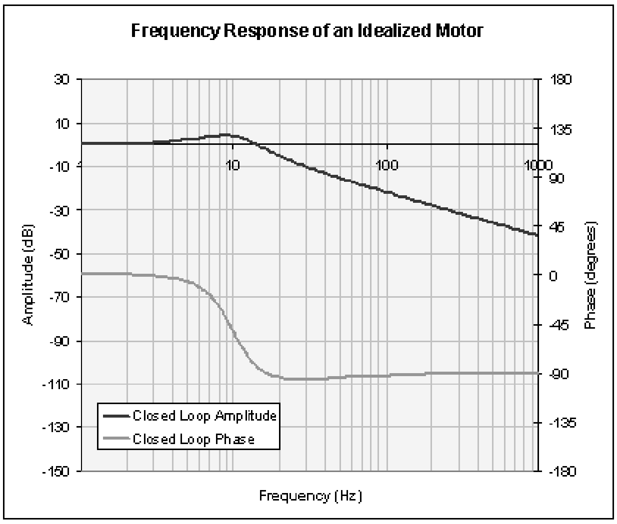

Figure 2 Frequency Response of an Ideal Motor

The term 'ideal motor' implies that the data used in evaluating frequency response is based on simplification of real data for understanding.

Referring to Figure 2, observe the frequency response at the frequencies which are less than the value of 5 Hz. The response is quite near to 0 dB. This implies that the response(output) of the motor will be like its input. If there is input of, say, 2 Hz position command, this motor with this control system can follow it with nearly equal amplitude (gain). Observing the phase, for the same range of frequency values, it is evident that the phase is about 0 degrees, which shows that the output will not lag the input. 0 dB at 0 degrees infers flawless arrangement between the output and input to the system. This is the basic theme which has been implemented in this paper for a commercial car control system. Now, let us look at the amplitude and phase response near 10 Hz frequency point. Observing closely, the amplitude response is about 4 dB and phase angle at that point is -60 degrees. From this, it can be inferred that the output will be 1.6 times the input and 60 degrees lagging the input at 10 Hz. Obviously, this is not our requirement. We want the designed system to move as required i.e. to accurately adhere the position control. Lastly, observe the frequency response at frequency value of 200 Hz. The amplitude response is -40 dB and phase angle is -90 degrees. This corresponds to the fact that the output amplitude is 0.01 times the input gain (amplitude) and the output lags 90 degrees from the input. This condition does not conform to our requirement [6-8].

4. TYPES OF CONTROL SYSTEMS

Control systems can be broadly classified per various parameters. The most common and simple classification is based on the value of damping factor denoted by ς (zeta). The following conditions show the system type per zeta (ς) value [6].

ς >1 (Overdamped)

ς =1 (Critically Damped)

0< ς <1 (Under Damped)

ς =0 (Undamped)

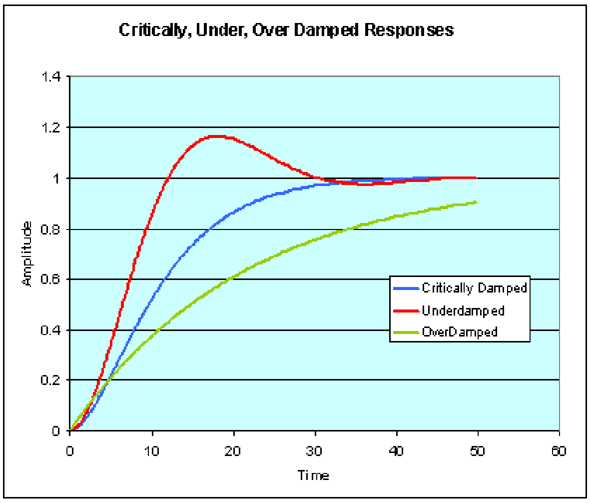

Figure 3 Types of Control Systems based on Damping Factor

5. BANDWIDTH

The term "Bandwidth" is used often to describe "efficacy" or "efficiency" of a control system. The bandwidth of a system is simply the "value" of frequency at which the closed loop gain response declines to about -3 dB. As an example, consider a control system's Bode Plot in Figure 4. According to the definition mentioned above, the bandwidth of the system is 17 Hz [7].

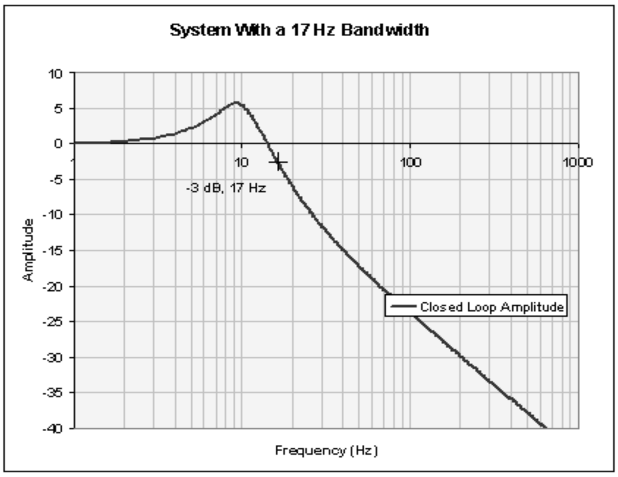

Figure 4 Finding Bandwidth using Bode Plot

It must, however, be mentioned that bandwidth is not a very exact measure of system performance. It can be used only for getting approximate idea of system performance.

6. GAIN AND PHASE MARGINS

Stability of control systems is commonly described by using the terms "Gain Margin" and "Phase Margin" These terms are used more frequently as compared to bandwidth as they are ideal indicators of stability.

Gain margins and phase margins are measured using the open loop frequency response of the system. This is a significant point which must be considered. Gain and phase margins cannot be developed directly from a closed loop frequency response. To find the value gain margin of a typical control system, we find the point at which the open loop phase response intersects "-180 degrees" phase angle. At the same frequency as this point, we calculate the open loop gain response. The distance below 0 dB that the open loop amplitude response displays at this frequency value gives us the gain margin. In the example (Figure 5) below, the gain margin comes out to be about 32.5 dB. To measure the phase margin of the same control system, we locate the point at which open loop amplitude intersects 0 dB. At the same frequency as this point, we calculate the open loop phase response. The distance above -180 degrees that the open loop phase response is at this frequency value is the phase margin. Using same figure, the phase margin is 33 degrees (180 degrees - 147 degrees = 33 degrees) [7].

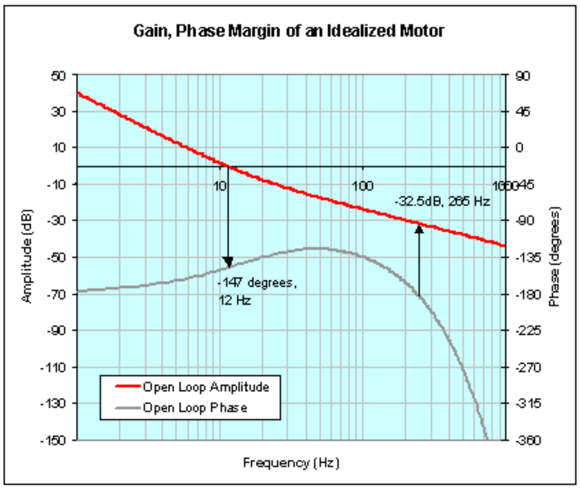

Figure 5 Finding Gain and Phase Margins

7. PLANT CONTROL

To begin designing our control requirement, we first implement the uncompensated closed loop system block diagram in Simulink (MATLAB) using "Control Systems Toolbox". The diagram is shown in Figure 6. It is drawn using Simulink tool.

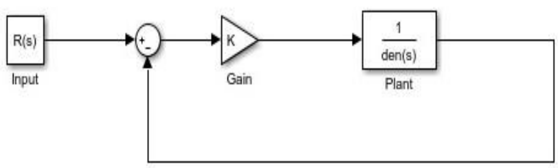

Figure 6 Uncompensated System Block Diagram

Next, we need to derive the closed loop transfer function. Using the MATLAB command, **T=feedback(K*G,1),** we get the desired closed loop transfer function:

T(s)=K/[s3+27s2+218s+504+K]

The root locus of open loop system is shown in Figure 7. It has been plotted using the simple MATLAB command rlocus(G).

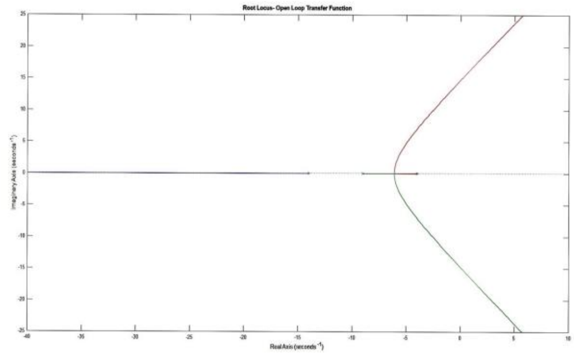

Figure 7 Root locus of Open Loop System

The Bode magnitude and phase plots (for K=1) are shown in Figure 8. They have been plotted using the MATLAB commands bode(G) and grid(on). The latter command is very helpful in visualizing the exact values of gains and phase angles. To force the plot to start at 0 dB, we set K=504 (DC gain) in the open loop function. The desired bode plots in this case are shown in Figure 9.

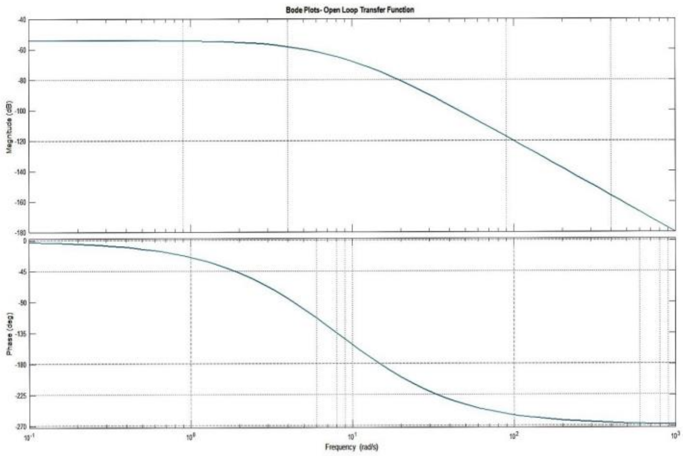

Figure 8 Bode Gain and Phase Plots for K=1

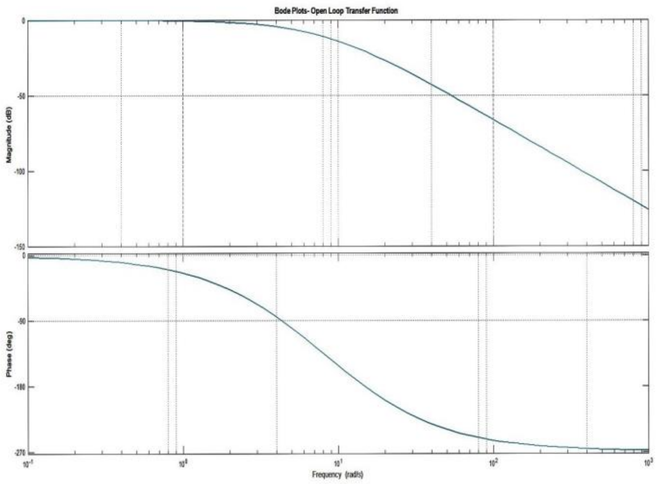

Figure 9 Bode Gain and Phase Plots for K=504

8. ANALYSIS

For finding the range of gain 'K' for stability, we inspect the open loop root locus plot (Figure 7). Using the command, [K, p] =rlocfind(G), we find the gain at the point where the root locus crosses the imaginary (jw) axis. Using this we find the range of gain for stability is: 0<K<5290

Frequency response technique can also be used to find this range. Using bode plots of Figure 9, we find the gain margin. It comes out to be 10.67. We multiply this by K=504 to get the value of K for the range of stability. Hence, for stability 0<K<5290 which agrees with the value found using root locus method

Next, we can use the command [Gm, Pm] =margin(G) to find the gain and phase margins. Alternatively, for finding gain margin, we see -180^0 frequency line on phase plot and see the corresponding frequency. Then on that frequency, we read the magnitude plot. The gain margin means that we can shift the magnitude plot by this gain to keep the system in stability range. In other words, gain margin is the change in open loop gain, usually expressed in dBs, required at 180^0 of phase shift to make the closed loop system in stable range. For finding phase margin, we see the 0-dB crossing of gain plot, and see the corresponding phase on phase plot. Then we subtract 180^0 from this value to get the phase margin. In other words, this is the change in open loop phase shift required at unity gain (or 0 dB) to make the closed loop system stable.

Using the command of [Gm, Pm] =margin(G), we attain Gain margin= 20.56 dBs or 10.67, Phase margin= -180^0

A critical point to consider is that systems possessing larger gain and phase margins can endure greater changes in system parameters before entering unstable mode of operation.

In this case, the gain can be increased by about 20 dBs before the shock dampers of car take the car into unstable region where it cannot be controlled by the driver. So, for shock dampers to properly limit the external disturbances, gain should not be increased more than the calculated gain margin so that the vehicle remains in stable condition. Similarly, phase margin tells us the maximum phase that the device can attain to remain in the region of stability. After that phase margin is exceeded, the system will go into instability mode.

Let us assume that our goal is to achieve damping ratio of 0.5 which implies a percent overshoot of 16.3%. The corresponding gain can be found by finding 'K' at the point where root locus crosses the damping ratio (0.5) line. Using the command [K,p] =rlocfind(G) to get the desired gain, we get corresponding gain 'K' to be 703.

The importance of this gain cannot be overestimated. It implies that the shock dampers of cars must be operated at this gain to keep within the desired overshoot of 16.3%. If the gain is less or greater than this value, the desired transient overshoot specification criterion will not be satisfied.

The step response of the uncompensated system is shown in Figure 10

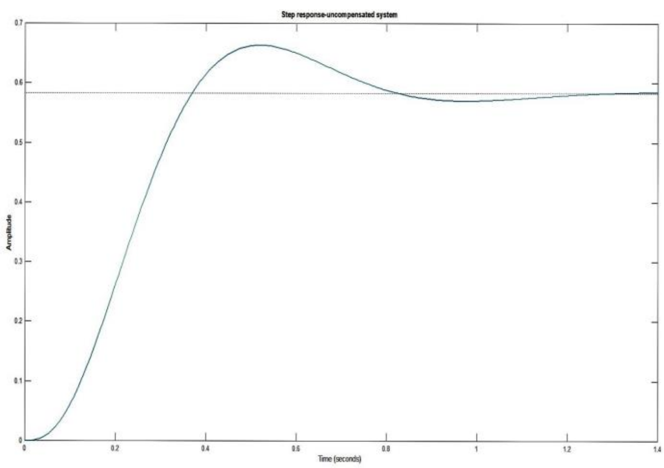

Figure 10 Step Response of Uncompensated System

To attain the transient response parameters, we right click on the response, go to characteristics and see the required values. The values found are given below:

Settling Time (Ts)= 1.03s, percent overshoot 13.9%, Peak Time (Tp)=0.518 s

At selected point (where root locus intersects zeta (ς=0.5) line), we have these poles:

-18.8293 + 0.0000i, -4.0853 + 6.8892i, -4.0853 - 6.8892i

Expected values: Settling time, Ts= 4/4.0853=0.979 s
Peak time, Tp=π/6.8892=0.456 s

2nd order approximation does not hold as real pole (-18.82) is not at least 5 times farther than the real part of dominant 2nd order pole pair. Moreover, there also exists difference between expected and actual values of settling and peak times.

Closed loop poles are found by using **pole(T)** MATLAB command. They are as follows:

-18.8293 + 0.0000i, -4.0853 + 6.8892i, -4.0853 - 6.8892i

9. DESIGN

Let us assume that our requirement is improvement in steady state error by a factor of 10 using a step input. It shall be achieved using a lag compensator.

Let E1 denoted the new required error and E the previous error (error before compensation)

E1= 1/(1+Kp.Zc/Pc) =0.0417

Putting Kp= 1.39 (703/504) and solving for Zc/Pc, we get

Zc/Pc= 19.16

Arbitrarily choosing compensator pole Pc=0.01. This implies compensator zero Zc= 0.19

Now the net open loop function is denoted by Gcomp(s) and is given by:

Gcomp(s)=K(s+0.19)/[(s+4) (s+9) (s+14) (s+0.01)]

We plot the root locus and find the point where it intersects ς=0.5 line. The value of gain K is noted at this point which comes out to be 669.

10. DISCUSSIONS

The selection of pole of lag compensator is done in such a way that compensator pole and zero are close together so that the angular contribution to point X (the dominant pole) is nearly zero degrees.

One important point to consider is that after adding the compensator the gain 'K' is nearly the same. This is because the length of the vectors drawn from the lag compensator are approximately equal and all other vectors have not changed much. In short, the lag compensator improves the static error contact by a factor of Zc/Pc.

There is no need of testing the design with other responses like ramp or parabolic as the desired steady error improvement has already been achieved using the step response and lag compensator. As it is a type zero system (no pure integrations in forward path), the error due to ramp and parabolic inputs shall be infinity (Refer to Table 1 below). However, if the system was a type 1 or type 2, we can plot the ramp and parabolic responses [9].

		Type 0		Type 1		Type 2	
Input	Steady-state error formula	Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p = \text{Constant}$	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v = \text{Constant}$	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = \text{Constant}$	$\frac{1}{K_a}$

Table 1 Relation Between Various System Types and Steady State Errors

11. DESIGN SUMMARY

Overall the design was successful as we achieved the required steady state error improvement. It must be noted, however, that the lag compensator has reduced the steady state error but has not reduced it to zero. For our system (the car), the improvement is sufficient to make the system operating in a good stable condition. With a steady state error improvement by a factor of 10, the car will have much better performance in terms of damping ratio and will sustain disturbances more efficiently. Before this improvement, the car would not have operated and behaved much better in steady state conditions.

12. TIME DOMAIN VS. FREQUENCY DOMAIN

It is important to mention a comparison of two different techniques that were used in this paper to achieve the designed control criteria. Frequency and time domain techniques are both very useful in meeting the desired transient performance characteristics. Each has its own pros and cons. Time domain techniques are less rigorous and involves less calculations (as far as the system is 2nd order) as compared to frequency techniques. In frequency techniques, we need to solve preliminary equations to get close loop bandwidth ω_{BW} and zeta (ς) value from phase margin. However, in time domain analysis, the analysis becomes cumbersome for systems of higher order. In frequency domain analysis, the order of the system has a little effect on the time or effort of analysis. Moreover, in frequency domain, pole-zero concept give an intuitive feel for the design problem. A simple example can be used to illustrate the difference: determining the stability of a system using Routh-Hurwitz criteria becomes increasingly time-consuming and tiresome when the system order increases. However, in a Bode plot, the effect of increased order is very less or negligible. Moreover, time domain usually put the equations in the form of state space, which allows to look "inside" system in the form of internal (state) variables that are invisible when using frequency domain. On the contrary, in frequency domain we can establish a relationship between input and output by Laplace transform. Summarizing the both approaches is very valuable when designing a complex control system.

13. CONCLUSION AND FUTURE WORK

This paper successfully attempts to model a practical control system using root locus and frequency response techniques. The system of interest is a commercial car whose damping ratio must be controlled and designed effectively. Major focus has been placed on controller design and how the required goal criteria can be achieved. Bode magnitude and phase plots were very helpful in getting the required design criteria. Root locus methods were also used simultaneously to get the required results. MATLAB software has been used exclusively for simulation and design purpose. This work can be continued in future on different software such as *MapleSoft* and using a different design criterion for a different model of the same commercial car.

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AUTHOR'S PROFILE

Mr. Umair Shahzad was born in Faisalabad, Pakistan on 28th September, 1987. He has completed his M.Sc. Electrical Engineering Degree, with Distinction, from The University of Nottingham (U.K.) in 2012. Prior to that, he completed his B.Sc. Electrical Engineering Degree from University of Engineering & Technology, Lahore (Pakistan) in 2010.

He has worked at The University of Faisalabad, Faisalabad (Pakistan) as a Lecturer for two years. During this tenure, he has taught various subjects on Control and Power Engineering to B.Sc. Electrical Engineering students. On his outstanding teaching abilities, he was presented the Best Teacher Award in 2014. Moreover, he has also taught various Electrical Engineering modules including Network Analysis and Power Generation at Riphah International University, Faisalabad, Pakistan. His research interests mainly consist of power systems, load flow studies, renewable energy, distributed generation, power system protection and microgrids. Presently, he is pursuing his Ph.D. in Electrical Engineering at University of Nebraska-Lincoln, USA in the capacity of a Fulbright Scholar.