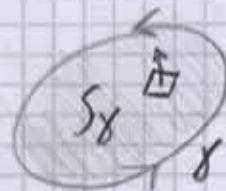


EQUAZIONI DI MAXWELL

1) FARADAY-NEUMAN-LENZ

$$\oint_{\gamma} \underline{E} \cdot \hat{t} dl = - \frac{d}{dt} \left[\iint_{S_{\gamma}} \underline{B} \cdot \hat{n} ds \right]$$



2) AMPERE-MAXWELL

$$\oint_{\gamma} \underline{B} \cdot \hat{t} dl = \mu_0 I_{S_{\gamma}} + \mu_0 \epsilon_0 \frac{d}{dt} \left[\iint_{S_{\gamma}} \underline{E} \cdot \hat{n} ds \right]$$

3) GAUSS-ELETTRICO

$$\oiint_S \underline{E} \cdot \hat{n} ds = Q_S / \epsilon_0$$

4) GAUSS-MAGNETICO

$$\oiint_S \underline{B} \cdot \hat{n} ds = 0$$

5) CONTINUITA' CARICA

$$I_S = - \frac{d}{dt} Q_S$$

6) FORZA DI LORENZ

$$\underline{F} = q(\underline{E} + \underline{v} \wedge \underline{B})$$

$$2 + 3 \Rightarrow 5$$

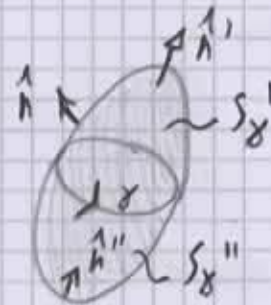
$$2 + 5 \Rightarrow 3$$

$$1 \Rightarrow 4$$

$$1 \Rightarrow 4$$

$$\oint_{\gamma} \underline{E} \cdot \hat{t} dl = - \frac{d}{dt} \iint_{S_{\gamma}'} \underline{B} \cdot \hat{n}' ds$$

$$\oint_{\gamma} \underline{E} \cdot \hat{t} dl = - \frac{d}{dt} \iint_{S_{\gamma}''} \underline{B} \cdot \hat{n}'' ds$$



$$\Rightarrow 0 = \frac{d}{dt} \left[\oiint_{S_{\gamma}} \underline{B} \cdot \hat{n} ds \right] \Rightarrow \text{se } \underline{B} = 0 \big|_{t=-\infty} \Rightarrow \oiint_{S_{\gamma}} \underline{B} \cdot \hat{n} ds = 0$$

TEOREMA DI GAUSS

$$\int_{\Omega} \nabla \cdot \underline{W} d\tau = \oiint_{S_{\Omega}} \underline{W} \cdot \hat{n} ds$$



$$\nabla \cdot \underline{W} = \partial_x W_x + \partial_y W_y + \partial_z W_z$$

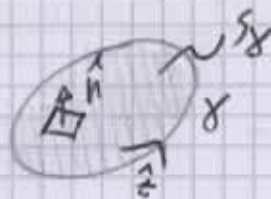
DEFINIZIONE INDIRETTA:

$$\lim_{V \rightarrow \Omega} \frac{\oiint_S \underline{W} \cdot \hat{n} ds}{V} = \nabla \cdot \underline{W}$$

TEOREMA DI STOKES

$$\oint_{\gamma} \underline{w} \cdot \hat{t} \, dl = \iint_{S_{\gamma}} \bar{\nabla} \wedge \underline{w} \cdot \hat{n} \, ds$$

$$\bar{\nabla} \wedge \underline{w} = \begin{vmatrix} u_x & u_y & u_z \\ \partial_x & \partial_y & \partial_z \\ w_x & w_y & w_z \end{vmatrix}$$



$$\lim_{\Delta \gamma \rightarrow 0} \left(\frac{\oint_{\gamma} \underline{w} \cdot \hat{t} \, dl}{\Delta \gamma} \right) = \bar{\nabla} \wedge \underline{w} \cdot \hat{n}$$

GRADIENTE

$$\lim_{\Delta \rightarrow 0} \frac{\varphi(\underline{r} + \Delta \hat{n}) - \varphi(\underline{r})}{\Delta} = \bar{\nabla} \varphi \cdot \hat{n} = \frac{\partial \varphi}{\partial \hat{n}}$$

$$\Rightarrow \int_{A \rightarrow B} \bar{\nabla} \varphi \cdot \hat{t} \, dt = \varphi(A) - \varphi(B)$$

DENSITÀ DI CARICA

VOLUMETRICA

$$\rho = \frac{\Delta Q}{\Delta \tau}$$

$$[\rho] = \text{COULOMB/METRO}^3$$

SUPERFICIALE

$$\sigma = \frac{\Delta Q}{\Delta S}$$

$$[\sigma] = \text{COULOMB/METRO}^2$$

LINEARE

$$\lambda = \frac{\Delta Q}{\Delta l}$$

$$[\lambda] = \text{COULOMB/METRO}$$

DENSITÀ DI CORRENTE

VOLUMETRICA



$$\Delta Q = \rho \cdot \Delta \tau = \rho \cdot \Delta s \cdot h = \\ = \rho \cdot \Delta s \cdot \Delta t \cdot \underline{v} \cdot \hat{n}$$

$$\frac{\Delta Q}{\Delta t \Delta s} = \rho \underline{v} \cdot \hat{n}$$

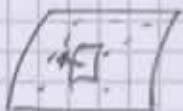
\vec{J} DENSITÀ VOLUMETRICA
DI CORRENTE ELETTRICA

$$\text{MA } \rho = n q$$

NUMERO DI PORTATORI
PER UNITÀ DI VOLUME

$$\text{QUINDI } \vec{J} = n q \vec{v}$$

SUPERFICIALE



$$\Delta I = \underline{K} \cdot \hat{n} \cdot \Delta l$$

LINEARE



$$\Delta I = \underline{L} \cdot \hat{n} \cdot \Delta l$$

MAXWELL DIFFERENZIALE

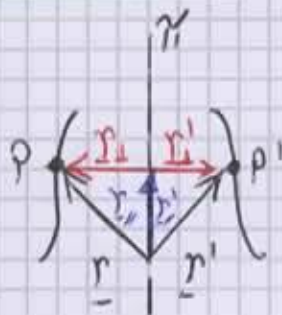
$$\oint_S \underline{E} \cdot \hat{n} \, ds = \frac{Q_E}{\epsilon_0}$$

$$\vec{\nabla} \cdot \underline{E} \cdot \Delta \tau = \int_{\Delta \tau} \vec{\nabla} \cdot \underline{E} \cdot d\tau = \int_{\Delta \tau} \rho / \epsilon_0 \, d\tau = \frac{\rho}{\epsilon_0} \Delta \tau$$

$$\Rightarrow \vec{\nabla} \cdot \underline{E} = \rho / \epsilon_0$$

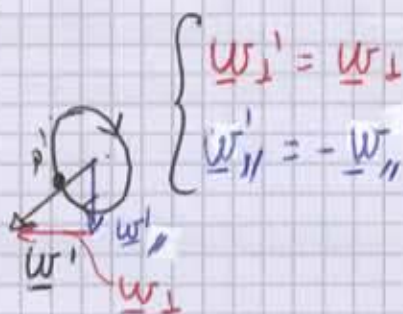
VETTORI E PSEUDOVETTORI

VETTORI



$$\begin{cases} r'_\perp = -r_\perp \\ r'_\parallel = r_\parallel \end{cases} \quad \begin{cases} \underline{v}'_\perp = -\underline{v}_\perp \\ \underline{v}'_\parallel = \underline{v}_\parallel \end{cases} \quad \begin{cases} \underline{a}'_\perp = -\underline{a}_\perp \\ \underline{a}'_\parallel = \underline{a}_\parallel \end{cases}$$

PSEUDOVETTORI



$$\begin{cases} \underline{w}'_\perp = \underline{w}_\perp \\ \underline{w}'_\parallel = -\underline{w}_\parallel \end{cases}$$

N.B. PRODOTTO SCALARE:

$$\begin{aligned} -\underline{v} \times \underline{v} &= \underline{v} \cdot \underline{v} \\ \underline{v} \times \underline{p} \cdot \underline{v} &= \underline{p} \cdot \underline{v} \\ -\underline{v} \times \underline{p} \cdot \underline{v} &= \underline{v} \\ -\underline{p} \cdot \underline{v} \times \underline{v} &= \underline{v} \end{aligned}$$

ESEMPI

E VETTORE $\Leftarrow \underline{F} = q \underline{E}$

\uparrow \uparrow
 VETT. VETT.

P, D VETTORI $\Leftarrow \underline{D} = \epsilon_0 \underline{E} + \underline{P}$

\uparrow \uparrow \uparrow
 VETT. VETT. VETT.

B PSEUDO $\Leftarrow \underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$

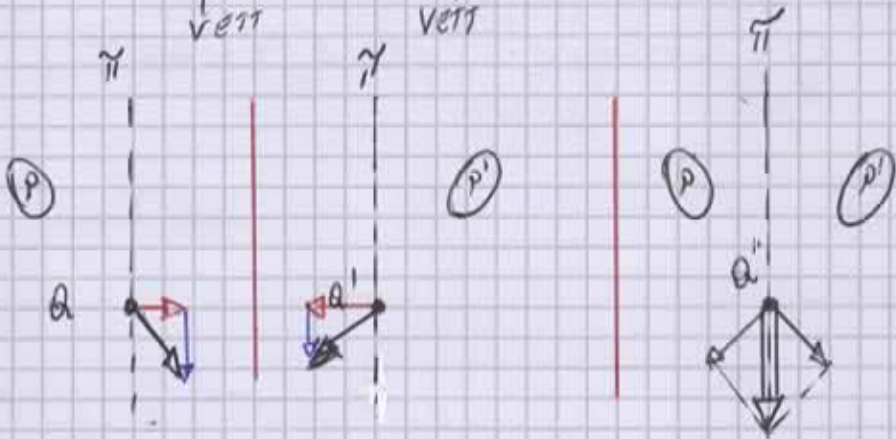
\uparrow \uparrow \uparrow \uparrow
 VETT. VETT. VETT. PSEUDO

H, M PSEUDO $\Leftarrow \underline{B} = \mu_0 \underline{H} + \mu_0 \underline{M}$

\uparrow \uparrow \uparrow
 PSEUDO PSEUDO PSEUDO

J VETTORE $\Leftarrow \underline{J} = \sum_k m_k q_k \underline{v}_k$

\uparrow \uparrow
 VETT. VETT.

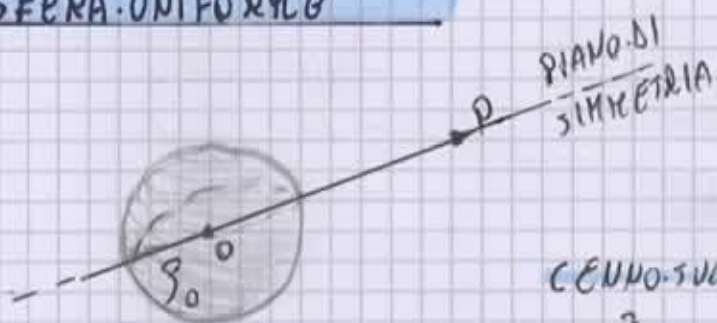


SE HO UNA DISTRIBUZIONE DI CARICHE SIMMETRICHE, SU QUALSIASI PUNTO DEL PIANO DI SIMMETRIA AVRO' UN VETTORE DI CAMPO ELETTRICO PRIVO DI COMPONENTE PERPENDICOLARE

ELETTROSTATICA DELLE DISTRIBUZIONI DI CARICA

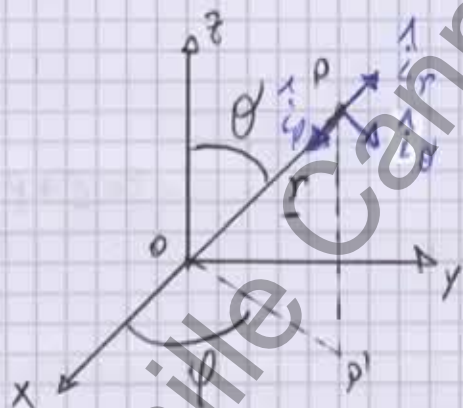
$$\left\{ \begin{array}{l} \oint \underline{E} \cdot \underline{\hat{e}} \, d\Omega = 0 \\ \oint_S \underline{E} \cdot \underline{\hat{n}} \, dS = Q_S / \epsilon_0 \\ \text{CONDIZIONE DI REGOLARITÀ A } +\infty \end{array} \right.$$

SFERA UNIFORME



IL CAMPO ELETTRICO NEL PUNTO P
HA UN'UNICA COMPONENTE CHE
PASSA PER O E P.

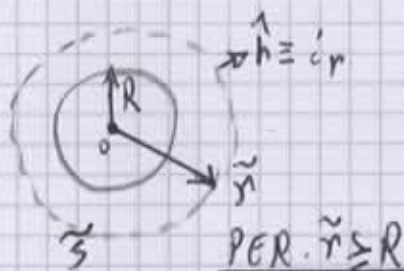
CENNO SULLE COORDINATE SFERICHE



I VERSORI SONO ORTOGONALI
ALLE SUPERFICI COORDINATE
QUANDO LA LORO COMPONENTE
 \underline{E} È FISSA.

QUINDI CAPISCO CHE:

$$\underline{E}(\underline{r}, \theta, \phi) = E_r(r) \underline{\hat{e}}_r \quad \text{HA SOLO COMPONENTE RADIALE}$$



$$Q_{\tilde{r}} = \rho_0 \frac{4\pi R^3}{3} = Q_0 \quad \text{PER } \tilde{r} \geq R$$

$$\Rightarrow E_r(\tilde{r}) \cdot (4\pi \tilde{r}^2) = Q_0 / \epsilon_0$$

$$\oint_{\tilde{S}} \underline{E} \cdot \underline{\hat{n}} \, dS = \oint_{\tilde{S}} \underline{E} \cdot \underline{\hat{e}}_r \, dS = E_r(\tilde{r}) (4\pi \tilde{r}^2)$$

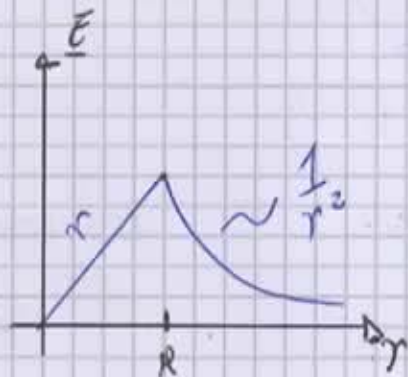
PER $\tilde{r} \leq R$

$$Q_{\tilde{r}} = \rho_0 \frac{4\pi \tilde{r}^3}{3} = Q_0 \cdot \left(\frac{\tilde{r}}{R}\right)^3$$

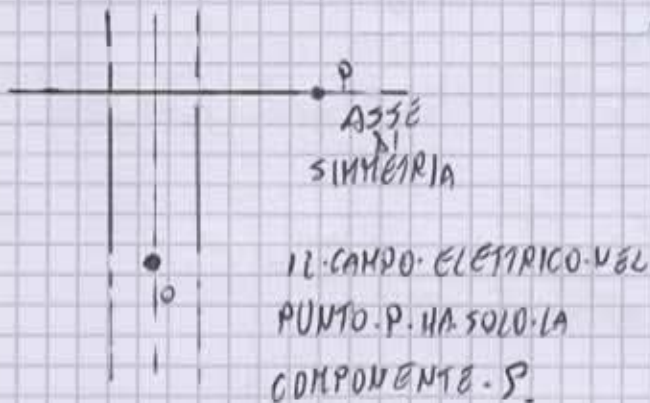
$$\Rightarrow E_r(\tilde{r}) (4\pi \tilde{r}^2) = Q_0 \left(\frac{\tilde{r}}{R}\right)^3 / \epsilon_0$$

QUINDI:

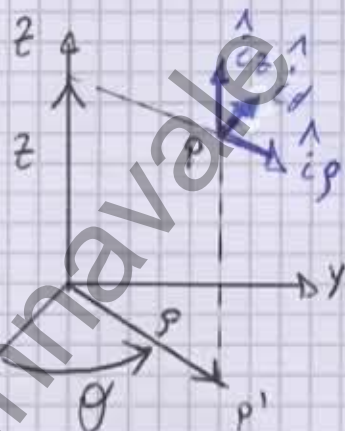
$$E_r(\tilde{r}) = \begin{cases} \frac{Q_0}{4\pi \tilde{r}^2 \epsilon_0} & , \tilde{r} \leq R \\ \frac{Q_0}{4\pi R^2 \epsilon_0} & , \tilde{r} \geq R \end{cases}$$



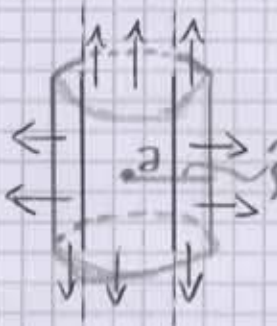
CILINDRO UNIFORME



CONVIGUE COORDINATE CILINDRICHE



$$\underline{E}(r, \theta, z) = E_S(r) \cdot \hat{e}_S$$



$$\oint_S \underline{E} \cdot \hat{n} dS = \oint_{\text{SOPRA}} \underline{E} \cdot \hat{n} dS + \oint_{\text{SOTTO}} \underline{E} \cdot \hat{n} dS + \oint_{\text{LATO}} \underline{E} \cdot \hat{n} dS =$$

$$= E_S(\tilde{r}) \cdot 2\pi \tilde{r} \cdot h$$

PER. $\tilde{r} \geq a$

$$Q_{\tilde{r}} = \rho_0 \cdot \pi a^2 h$$

$$E_S(\tilde{r}) \cdot 2\pi \tilde{r} h = \frac{\rho_0 \pi a^2 h}{\epsilon_0}$$

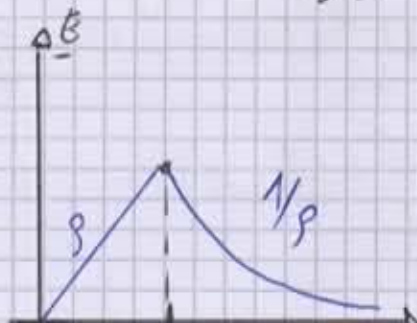
$$E_S(\tilde{r}) = \frac{\lambda_0}{2\pi \tilde{r} \epsilon_0}$$

PER. $\tilde{r} \leq a$

$$Q_{\tilde{r}} = \rho_0 \cdot \pi \tilde{r}^2 h$$

$$E_S(\tilde{r}) \cdot 2\pi \tilde{r} h = \frac{\lambda_0 \tilde{r} h}{\epsilon_0}$$

$$E_S(\tilde{r}) = \frac{\lambda_0}{2\pi \epsilon_0} \frac{\tilde{r}}{a^2}$$

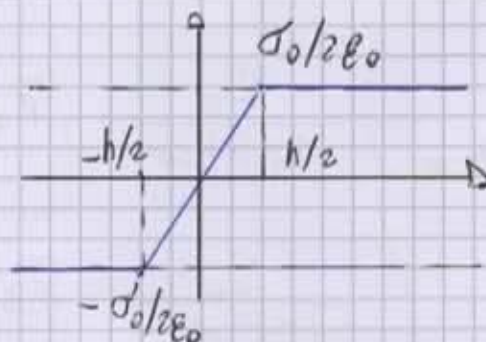


PIANO UNIFORME

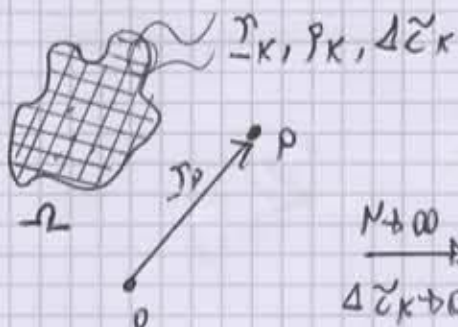


PER $h \rightarrow 0$

$$E = \frac{\sigma_0}{2\epsilon_0}$$



INTEGRALE DI SOVRAPP.



$$E_p = \sum_k^N (\Delta E_k)_p = \sum_k^N \frac{\rho_k \Delta \tilde{z}_k}{4\pi\epsilon_0 (r_p - r_k)^2} \hat{r}_{k,p}$$

$$\xrightarrow[N \rightarrow \infty]{\Delta \tilde{z}_k \rightarrow 0} \int_{\Omega} \frac{\rho(r') \cdot \hat{r}'_p}{4\pi\epsilon_0 (r_p - r')^2} d\tilde{z}' = E_p$$

PER $r_p \rightarrow \infty \Rightarrow E_p = \frac{\rho|\Omega|}{4\pi\epsilon_0 r_p^2} \sim \frac{1}{r^2}$ CONDIZIONE DI REGOLARITÀ.

POTENZIALE SCALARE ELETTROSTATICO

DEFINIAMO IL POTENZIALE SCALARE ELETTROSTATICO IN QUESTO MODO:

$$\varphi_p = K + \int_p^0 \underline{E} \cdot \hat{e} dl$$

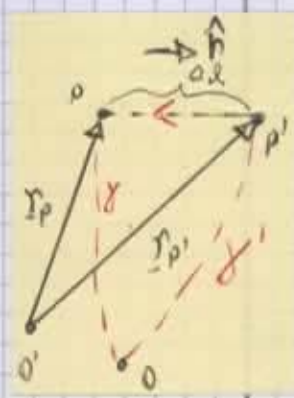
INOLTRE AFFERMIAMO CHE; SE $\oint_{\gamma} \underline{E} \cdot \hat{e} dl = 0$, OVVERO SE \underline{E} È CONSERVATIVO

ALLORA LA CIRCUITAZIONE NON DIPENDE DALLA CURVA γ , QUINDI DICIAMO CHE;

$$\underline{E} = -\nabla \varphi$$

DIMOSTRAZIONE

$$\begin{aligned} \nabla \varphi_p \cdot \hat{h} &= \frac{\partial \varphi}{\partial h} = \frac{\varphi(r_p') - \varphi(r_p)}{\Delta l} = \frac{\varphi(r_p + \Delta l \hat{h}) - \varphi(r_p)}{\Delta l} \\ &= \frac{1}{\Delta l} \left(\int_{p'}^p \underline{E} \cdot \hat{e} dl \right) = \frac{1}{\Delta l} \left(\int_{p'}^p \underline{E} \cdot (-\hat{h}) dl \right) = -\underline{E}_p \cdot \hat{h} = \frac{\partial \varphi}{\partial h} = \nabla \varphi_p \cdot \hat{h} \end{aligned}$$

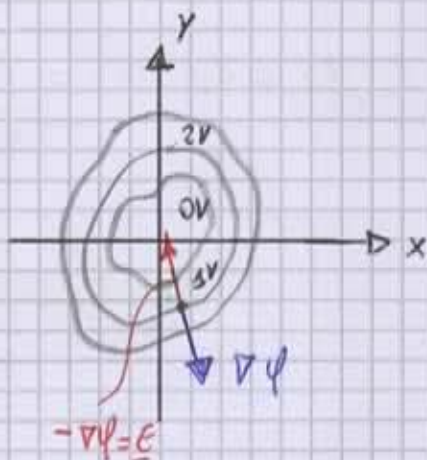


QUINDI PER TUTTE LE DIREZIONI AVREMO $\underline{E} = -\nabla \varphi$

CURVE EQUIPOTENZIALI

SI DEFINISCE CURVA EQUIPOTENZIALE UNA CURVA SULLA QUALE IL POTENZIALE SCALARE ELETTROSTATICO È COSTANTE.

$$R(x, y) = V_0$$

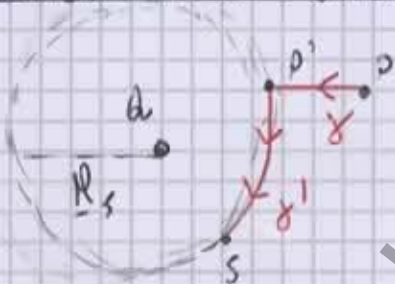


$$0 = \varphi(P') - \varphi(P) = \frac{\partial \varphi}{\partial \epsilon} \cdot \Delta l = \nabla \varphi \cdot \hat{\epsilon} \Delta l = -\underline{E} \cdot \hat{\epsilon} \Delta l$$

$\Delta l \rightarrow 0$

QUINDI IL CAMPO ELETTRICO NELLA DIREZIONE TANGENTE È 0.

POTENZIALE SCALARE PER CARICA PUNT



$$\varphi_P = K + \int_P^S \underline{E} \cdot \hat{\epsilon} dl$$

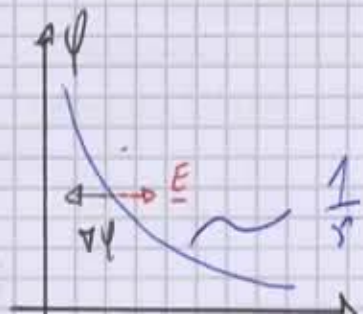
$$\underline{E} = \frac{Q}{4\pi\epsilon_0 r^2} \cdot \hat{c}_r$$

$$\varphi_P = K + \int_{P \times P'} \underline{E} \cdot \hat{\epsilon} dl + \int_{P' \times S} \underline{E} \cdot \hat{\epsilon} dl = K + \int_{P \times P'} \underline{E} \cdot (-\hat{c}_r) dl =$$

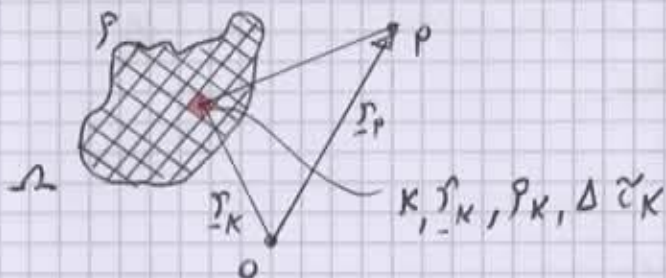
$$= K - \frac{Q}{4\pi\epsilon_0} \int_{r_P}^{R_s} \frac{1}{r^2} \hat{c}_r dl = K - \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R_s} - \frac{1}{r_P} \right) =$$

$$= K - \frac{Q}{4\pi\epsilon_0} \frac{1}{R_s} + \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{r} = \frac{Q}{4\pi\epsilon_0 r}$$

$$SE K = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{R_s}$$



INTEGRALE DI SOVRAPPOSIZIONE PER IL POT. SCAL.



$$\varphi_P = \sum_{k=1}^N \varphi_{k,P} = \sum_{k=1}^N \frac{\rho_k \cdot \Delta \tau_k}{4\pi\epsilon_0 (r_P - r_k)}$$

$\Delta \tau_k \rightarrow 0$
 $N \rightarrow \infty$

$$\frac{1}{4\pi\epsilon_0} \int_{\Omega} \frac{\rho(r')}{|r_P - r'|} d\tau' = \varphi_P$$

INOLTRE NOTIAMO CHE SE:

1) Ω LIMITATO

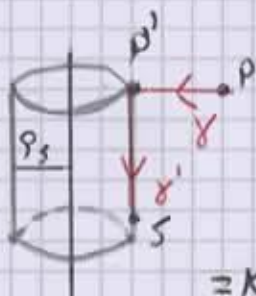
2) $|\rho(r)| \leq \rho_0$ IN Ω

QUINDI VA AD INFINITO

ALLORA $|\varphi_P|$ PER $r_P \rightarrow \infty \leq \frac{\rho_0}{4\pi\epsilon_0 |r_P|} \cdot |\Omega| \Rightarrow$ COME $\frac{1}{r}$, E QUESTO

VUOL DIRE CHE A ∞
 Ω È VISTA COME UNA
CARICA PUNTIFORME.

POTENZIALE SCALARE FILO

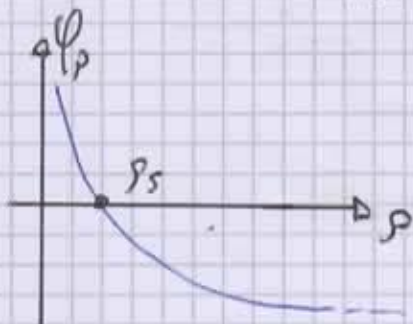


$$\underline{E} = \frac{\lambda_0}{2\pi r \epsilon_0} \hat{r}_P$$

$$\varphi_P = K + \int_{r_P}^{\infty} \underline{E} \cdot \hat{r} dl + \int_{r_P}^{\infty} \underline{E} \cdot \hat{r} dl =$$

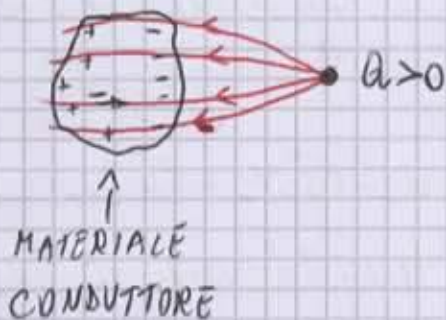
$$= K - \int_{r_P}^{\infty} \frac{\lambda_0}{2\pi \epsilon_0 s} (-ds) = K + \frac{\lambda_0}{2\pi \epsilon_0} \left[\ln(s) \right]_{r_P}^{\infty} =$$

$$= K + \frac{\lambda_0}{2\pi \epsilon_0} \ln(r_P) - \frac{\lambda_0}{2\pi \epsilon_0} \ln(r_P) = K - \frac{\lambda_0}{2\pi \epsilon_0} \ln\left(\frac{r_P}{r_0}\right)$$



GLI ARGOMENTI
DEI LOGARITMI DEVONO
ESSERE ADIMENSIONALI

CONDUTTORI



IL CAMPO ELETTRICO ALL'INTERNO FA 0!!
LO POSSIAMO COPIRE DALLA FORZA DI
LORENZ CHE INSISTE SU UN ELETTRONE:

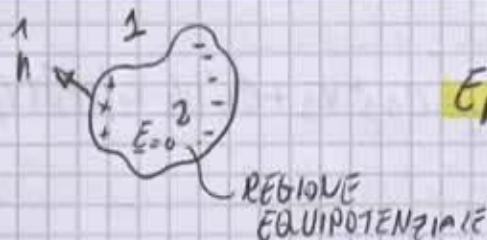
$$\underline{F} = q \underline{E}_{TOT} - \gamma \underline{V}$$

FORZA DI ATTRAZIONE

$$m \underline{\dot{v}} = q \underline{E} - \gamma \underline{V} \Rightarrow \underline{E} = 0$$

UN ELETTRONE DOPO UN CERTO
LATTO DI TEMPO PERDERÀ LA
SUA ENERGIA CINETICA E
SI FERMRÀ.

TEOREMA DI COULOMB



$$E_{n1} - E_{n2} = \frac{\sigma}{\epsilon_0} \Rightarrow E_n = \frac{\sigma}{\epsilon_0}$$

CAPACITÀ

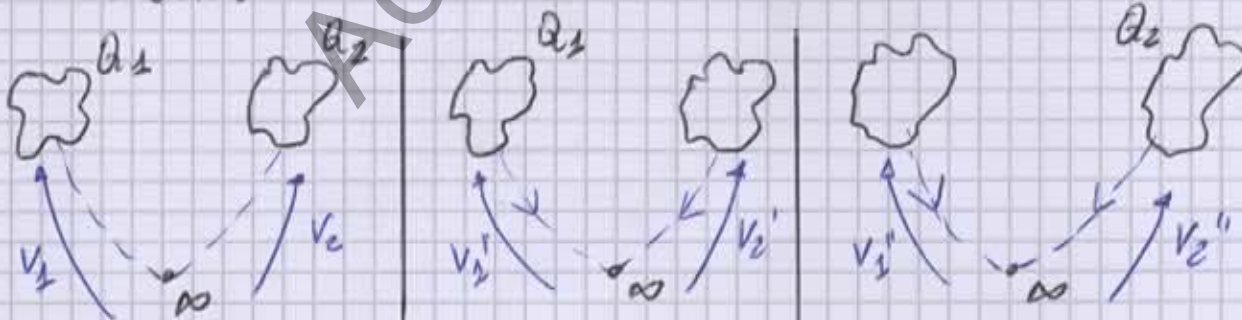
DEFINIAMO LA CAPACITÀ COME:



$$V = \int_P^\infty \underline{E} \cdot d\underline{l}$$

$$C = \frac{Q}{V} = \text{CONSTANTE}$$

ORA GENERALIZZIAMO NEL CASO DI DUE ELETTRODI, USANDO LA SOVRAPPOSIZIONE
DEGLI EFFETTI:



$$V_1' = A_{11} Q_1$$

$$V_2' = A_{21} Q_1$$

$$V_1'' = A_{12} Q_2$$

$$V_2'' = A_{22} Q_2$$

$$\Rightarrow \begin{cases} V_1 = A_{11} Q_1 + A_{12} Q_2 \\ V_2 = A_{21} Q_1 + A_{22} Q_2 \end{cases}$$

IN FORMA MATRICIALE DIVENTA:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \Rightarrow \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \underline{A}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{cases} Q_1 = C_{11} V_1 + C_{12} V_2 \\ Q_2 = C_{21} V_1 + C_{22} V_2 \end{cases} \xrightarrow{\text{RISCRIVO}} \begin{cases} Q_1 = C_{11}^* V_1 + C_{12}^* (V_1 - V_2) \\ Q_2 = C_{21}^* (V_2 - V_1) + C_{22}^* V_2 \end{cases}$$

$$C_{11}^* = C_{11} + C_{12}$$

$$C_{12}^* = -C_{12}$$

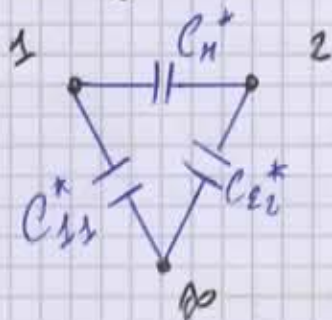
$$C_{21}^* = C_{11} + C_{12}$$

$$C_{22}^* = C_{11} + C_{12}$$

$$\downarrow C_{12}^* = C_{21}^* = C_H^*$$

$$\begin{cases} Q_1 = C_{11}^* V_1 + C_H^* (V_1 - V_2) \\ Q_2 = C_H^* (V_2 - V_1) + C_{22}^* V_2 \end{cases}$$

TRADUCIAMO LE DUE EQUAZIONI IN UN
MODELLO CIRCUITALE:



SE ORA IMPONIAMO LA CONDIZIONE AFFINE CHE
I DUE ELETTRODI SI COMPORTINO COME UN
CONDENSATORE:

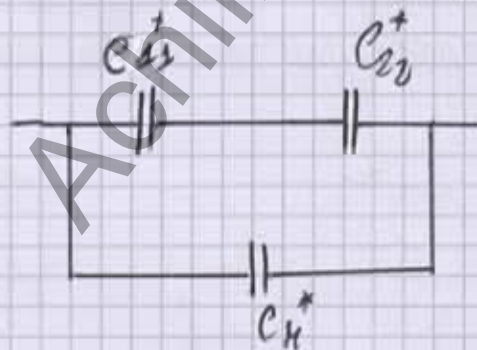
$$Q_1 + Q_2 = 0$$

ALLORA:

$$0 = C_{11}^* V_1 + C_H^* (V_1 - V_2) + C_H^* (V_2 - V_1) + C_{22}^* V_2$$

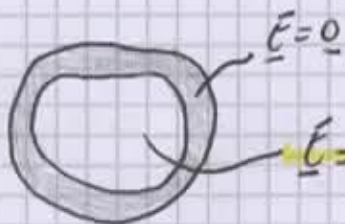
$$\Rightarrow 0 = C_{11}^* V_1 + C_{22}^* V_2$$

$$\frac{Q_1}{V_1 - V_2} = \frac{C_{11}^* \cdot V_1}{V_1 - V_2} + C_H^* = \frac{C_{11}^*}{1 - \frac{V_2}{V_1}} + C_H^* = \frac{C_{11}^*}{1 + \frac{C_{11}^*}{C_{22}^*}} + C_H^* = \frac{C_{22}^* \cdot C_{11}^* + C_H^*}{C_{11}^* + C_{22}^*}$$



ANCORA LA GENERALIZZAZIONE

SCHEMA ELETTROSTATICO



$E = ? \Rightarrow E = 0$, CERCHIAMO DI DIMOSTRARLO:

SUPPONIAMO PER ASSURDO CHE NELLA CAVITÀ CI SIA UN CAMPO ELETTRICO DIVERSO DA 0 $\Rightarrow \vec{\nabla} \cdot \vec{E} = 0$



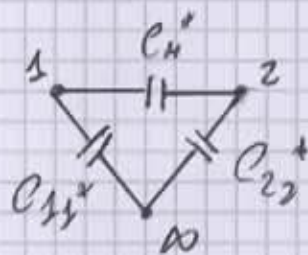
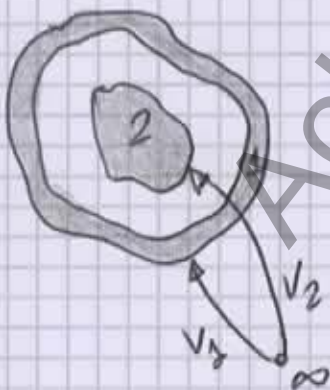
LE LINEE DI FORZA DEL CAMPO ELETTRICO ATTRAVERSAVO LA CAVITÀ.

PER CALCOLARE LA CIRCUITAZIONE DEL CAMPO ELETTRICO CHE DEVE FARE 0, SCELGO LA CURVA X.

$$0 = \oint_X \vec{E} \cdot \hat{e} dl = \int_{\gamma_{\text{CAVITÀ}}} \vec{E} \cdot \hat{e} dl + \int_{\gamma_{\text{LATERALE}}} \vec{E} \cdot \hat{e} dl =$$

$$0 = |\vec{E}| \cdot \int_{\gamma_{\text{CAVITÀ}}} \hat{e} dl \Rightarrow |\vec{E}| = 0 \text{ SU } \gamma_{\text{CAVITÀ}}$$

APPLICAZIONE



$$C_{22}^* = 0, \text{ DIMOSTRIAMO: } C_{22}^* = \frac{Q_2}{V_2} \Big|_{V_1=V_2}$$



PER LA DIMOSTRAZIONE DI PRIMA LE LINEE DI FORZA DEVONO ATTRAVERSARE LA CAVITÀ.

$$V_1 - V_2 = 0 = \int_1^2 \vec{E} \cdot \hat{e} dl \Rightarrow \vec{E} = 0$$

$$\Rightarrow Q_2 = \oint E \cdot \hat{n} \cdot dS = 0$$

$$|E| > 0$$

OSSERVAZIONE



$$Q_2 = -Q_1^I$$

$$\oint_{\Sigma} \vec{E} \cdot \hat{n} dS = \frac{Q_1^I + Q_2}{\epsilon_0} = 0 \Rightarrow Q_2 = -Q_1^I$$

PERCHÉ - ABBIAMO DETTO

CHÉ PELLA CONCAVITÀ

IL CAMPO ELETTRICO È NULO.

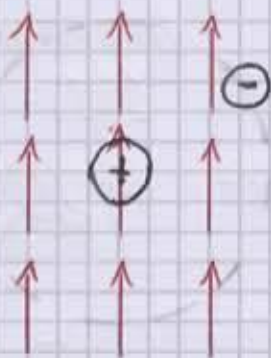
MANCA IL PIANO INFINITO

Achille Cannavale

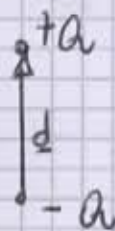
MATERIALI DIELETTRICI

SI DEFINISCONO MATERIALI DIELETTRICI QUELLI IN CUI È DIFFICILE "STRAPPARE" ELETTRONI, DAI PROPRI NUCLEI, CON UN CAMPO ELETTRICO.

DI CONSEGUENZA, PER LO SFASAMENTO DEI BARI CENTRI TRA NUCLEO E ELETTRONI VIENE CREATO UN DIPOLLO ELETTRICO.



DIPOLLO ELETTRICO



DEFINIAMO

MOMENTO DI DIPOLLO ELETTRICO

DENSITA' DI
POLARIZZAZIONE

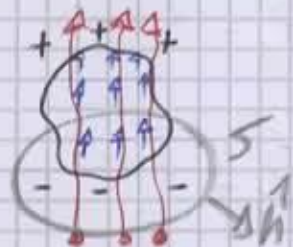
$$\underline{P} = \sum_i p_i / \Delta z$$

IN SENSO MACROSC.

$$\underline{P} = q \cdot \underline{d}$$

DEFINIAMO INOLTRE LA QUANTITA' DI CARICA ELETTRICA DI POLARIZZAZIONE:

$$Q_p = - \oint_S \underline{P} \cdot \underline{\hat{n}} dS$$



QUINDI L'IDEA È CHE CONOSCO LE CARICHE

SORGENTI CHE METTO E DI CONSEGUENZA CONOSCO IL

CAMPO ELETTRICO CHE GENERANDO, MA QUESTO CAMPO ELETTRICO CREA DEI DIPOLI ELETTRICI NEL MATERIALE CHE A LORO VOLTA GENERANO UN CAMPO ELETTRICO IN SOTTO-ILNOTO. \Rightarrow QUINDI MI OCCORRONO UN SET DI EQUAZIONI:

PER FARE PIÙ SPECIALIZZIAMO LA LEGGE DI GAUSS PER IL CAMPO ELETTRICO:

$$\epsilon_0 \oint_S \underline{E} \cdot \underline{\hat{n}} dS = Q_o + Q_p = Q_o - \oint_S \underline{P} \cdot \underline{\hat{n}} dS \Rightarrow$$

\uparrow CARICHE SORGENTI \uparrow CARICHE DI POLARIZZAZIONE

$$\Rightarrow \oint_S \epsilon_0 \underline{E} \cdot \underline{\hat{n}} dS + \oint_S \underline{P} \cdot \underline{\hat{n}} dS = Q_o \rightarrow \oint_S \underline{D} \cdot \underline{\hat{n}} dS = Q_o, \text{ DOVE}$$

$\underline{D} = \epsilon_0 \underline{E} + \underline{P}$ CHE HA IL NOME DI SPOSTAMENTO ELETTRICO

QUINDI PARTIAMO DA QUESTE EQUAZIONI:

$$\oint_{\gamma} \underline{E} \cdot \underline{\hat{t}} dl = 0$$

$$\oint_S \underline{D} \cdot \underline{\hat{n}} ds = Q_0$$

C.R. ∞

QUINDI COMPLETIAMO TUTTE LE CIRCUITAZIONI DI \underline{E}
E TUTTI I FLUSSI DI \underline{D} .

QUESTO VUOL DIRE CHE CI OCCORRE UNA LEGGE
CHE LEGA QUESTI DUE VETTORI:

$$\underline{P}(\underline{r}) = \epsilon_0 \chi(\underline{r}) \cdot \underline{E}(\underline{r})$$

χ SUSCETTIVITÀ
DIELETTICA

NEL CASO IN CUI:

$$\chi(\underline{r}) = \chi(\underline{r}) \underline{I}$$

IL MEZZO SI DICE
ISOTROPO

$$\underline{D} = \epsilon_0 \underline{E} + \epsilon_0 \chi \cdot \underline{E} = \epsilon_0 (1 + \chi) \underline{E}$$

$\epsilon_0 (1 + \chi)$ COSTANTE DIELETTICA RELATIVA ϵ_r

$\epsilon_0 \cdot \epsilon_r = \epsilon$ COSTANTE DIELETTICA ASSOLUTA.

QUINDI IL NOSTRO SET DI EQUAZIONI FINALI SARÀ:

$$- \oint_{\gamma} \underline{E} \cdot \underline{\hat{t}} dl = 0$$

$$- \oint_S \underline{D} \cdot \underline{\hat{n}} ds = Q_0$$

$$- \underline{D} = \epsilon \underline{E}$$

$$- \text{C.R. } \infty$$

RICORDANDO CHE:

$$\oint_S \underline{E} \cdot \underline{\hat{n}} ds = \frac{Q_S}{\epsilon_0}$$

$$\begin{cases} \nabla \cdot \underline{E} = \frac{\rho}{\epsilon_0} \\ \underline{E}_{n1} - \underline{E}_{n2} = \sigma / \epsilon_0 \end{cases}$$

LA POLARIZZAZIONE
 \underline{P} È PARALLELA
AL CAMPO ELETTRICO
SORGENTE.

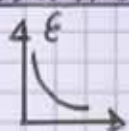
ALLORA POSSIAMO PROCEDERE SIMILMENTE PER \underline{P}

$$\oint_S \underline{P} \cdot \underline{\hat{n}} ds = -Q_p$$

$$\begin{cases} \nabla \cdot \underline{P} = -\rho_p \\ \underline{P}_{n1} - \underline{P}_{n2} = -\sigma_p \end{cases}$$

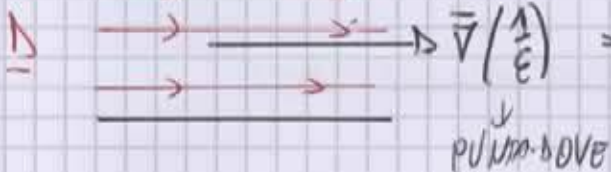
$$\oint_S \underline{P} \cdot \underline{\hat{n}} ds = -Q_p \Leftrightarrow \begin{cases} \nabla \cdot \underline{P} = -\rho_p \\ \underline{P}_{n1} - \underline{P}_{n2} = -\sigma_p \end{cases}$$

OSSERVAZIONI



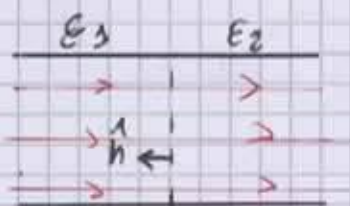
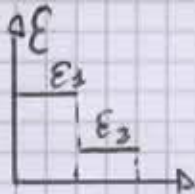
$$\rho_0 = 0 \Rightarrow \nabla \cdot \underline{D} = 0$$

$$\rho = \epsilon_0 \cdot \nabla \cdot \underline{E} = \epsilon_0 \cdot \nabla \cdot \left(\frac{1}{\epsilon} \underline{D} \right) = \epsilon_0 \nabla \cdot \left(\frac{1}{\epsilon} \right) \cdot \underline{D} + \frac{1}{\epsilon} \nabla \cdot \underline{D}$$



$$\Rightarrow \rho = \epsilon_0 \nabla \cdot \left(\frac{1}{\epsilon} \right) \cdot \underline{D} > 0$$

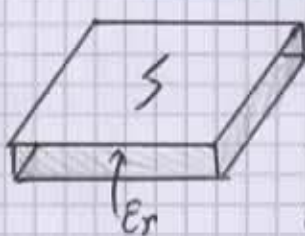
QUINDI STANNO NASCENDO
CARICHE POSITIVE



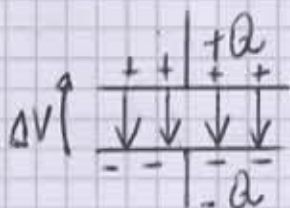
$$\sigma = \epsilon_0(\epsilon_{N1} - \epsilon_{N2}) = \epsilon_0 \left(\frac{1}{\epsilon_1} D_N - \frac{1}{\epsilon_2} D_N \right) = \epsilon_0 \left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2} \right) D_N$$

CAPACITÀ CONDENSATORE CON DIELETTRICO

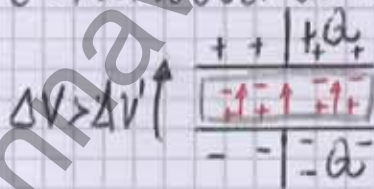
$$C = \epsilon_0 \epsilon_r \frac{S}{d}$$



QUINDI INSERENDO UN MATERIALE DIELETTRICO NEL CONDENSATORE DIMINUISCE IL CAMPO ELETTRICO E DI CONSEGUENZA AUMENTA LA CAPACITÀ.



$$C = \frac{Q}{\Delta V}$$



$$C' = \frac{Q}{\Delta V'}$$

MODELLO DELLA CONDUZIONE STAZION.

SET DI EQUAZIONI:

$$-\oint \vec{E} \cdot \hat{t} dl = 0$$

$$-\oint \vec{J} \cdot \hat{n} ds = -\frac{d}{dt} Q_s = 0$$

$$\vec{J} = \sigma (\vec{E} + \vec{E}_n)$$

↑
CONDUCIBILITÀ
ELETTRICA

$$[\sigma] = S/m$$



* ELETTRONI

IN CUI LA CONDUCEBILITÀ ELETTRICA VA ALL'∞.

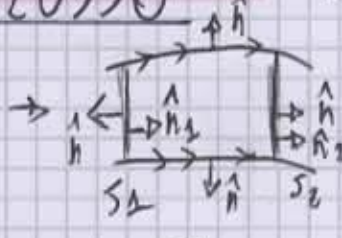
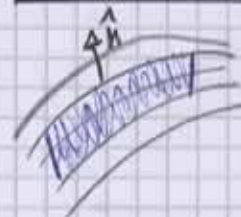
$$\left(\vec{E} = \frac{1}{\sigma} \vec{J} \right) \rightarrow 0$$

⇒ E = 0 SUGLI ELETT.

⇒ QUINDI SONO REGIONI

EQUIPOTENZIALI.

TUBO DI FLUSSO



$$0 = \iint_{S_1} \vec{J} \cdot \hat{n} ds + \iint_{S_2} \vec{J} \cdot \hat{n} ds + \iint_{S_L} \vec{J} \cdot \hat{n} ds$$

LINEE DI FORZA CHE ATTRAVERSSANO UNA SUPERF.

CAMPO ELETTROMOTORE

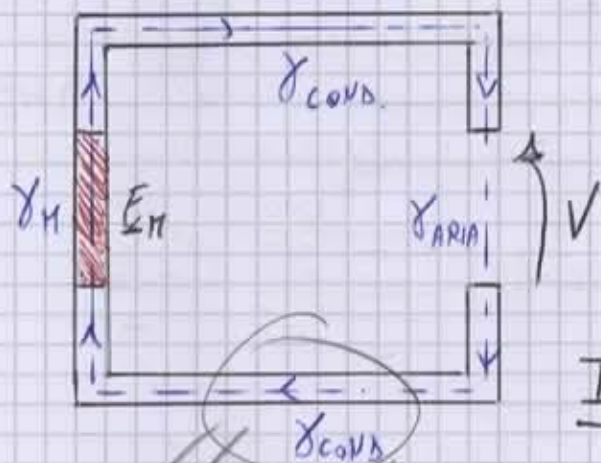


VOLGO DIMOSTRARE CHE SENZA IL CAMPO "EM" NON C'È DENSITÀ DI CORRENTE ELETTRICA \underline{J} .

$$0 = \oint \underline{E} \cdot d\underline{\ell} = \oint \left(\frac{1}{\sigma} \cdot \underline{J} \right) \cdot d\underline{\ell} \Rightarrow$$

$$\Rightarrow \frac{1}{\sigma} \cdot \underline{J} \cdot d\underline{\ell} = 0 \Rightarrow |\underline{J}| = 0$$

LEGGE DEL CIRCUITO SEMPLICE



$$0 = \int_{\text{ARIA}} \underline{E} \cdot d\underline{\ell} + \int_{\text{COND}} \underline{E} \cdot d\underline{\ell} + \int_{\text{COND}} \underline{E} \cdot d\underline{\ell} =$$

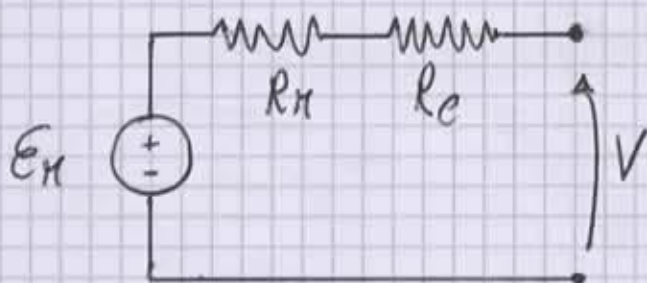
$$= V + \int_{\text{COND}} \left(\frac{1}{\sigma} \underline{J} \right) \cdot d\underline{\ell} + \int_{\text{COND}} \left(\frac{1}{\sigma} \underline{J} \right) \cdot d\underline{\ell}$$

$$\underline{I} = \iint_{S_L} \underline{J}_L \cdot d\underline{S} = \underline{J}_L \cdot \underline{\hat{e}}_L \cdot S_L$$

$$\Rightarrow \underline{J}_L \cdot \underline{\hat{e}}_L = \frac{I}{S_L}$$

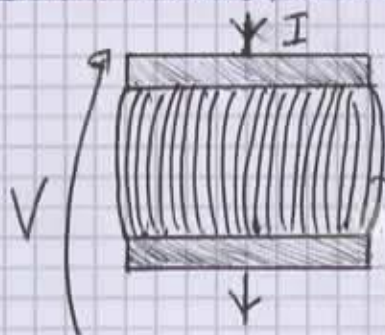
$$\Rightarrow V + \int_{\text{COND}} \left(\frac{1}{\sigma_L} \frac{I}{S_L} \right) d\underline{\ell} + \int_{\text{COND}} \left(\frac{1}{\sigma_H} \frac{I}{S_H} \right) d\underline{\ell} - \int_{\text{COND}} \underline{E}_H \cdot d\underline{\ell} = 0$$

$$\Rightarrow V = E_H - R_H I - R_e I$$



MODELLO CIRCUITALE
EQUIVALENTE PER
RICAVARE IL LEGAME
TENSIONE-CORRENTE

GENERALIZZAZIONE CONDOTTORE MASSICCIO



L'IDEA È DI APPROSSIMARLO COME L'INSIEME DI TANTI TUBI DI FLUSSO.

PRIVO DI CAMPO ELETTROMOTORE

$$I = \sum_k^N \Delta I_k = \sum_k^N \frac{\Delta V_k}{R_k} = V \sum_k^N \frac{1}{R_k} \quad \square$$

DATO CHE LA PUNTA
È LA CODA. POSSIAMO SUGLI
ELETTRODI EQUIPOTENZIALI
 ΔV_k È COSTANTE!

DENSITÀ DI POTENZA ELETTRICA DISSIPATA

$$\Sigma = \rho V = q \cdot n \cdot V$$

$$\Rightarrow \Delta P = q \cdot E \cdot V \cdot n \cdot \Delta \tau = \Sigma \cdot E \cdot \Delta \tau$$



$$\Delta V = n \cdot \Delta \tau$$

$$(q \cdot E) \cdot V = \text{POTENZA SUZ PUNTO}$$

QUINDI LA POTENZA SPESA DA E SARÀ:

$$\frac{\Delta P_e}{\Delta \tau} = \Sigma \cdot E$$

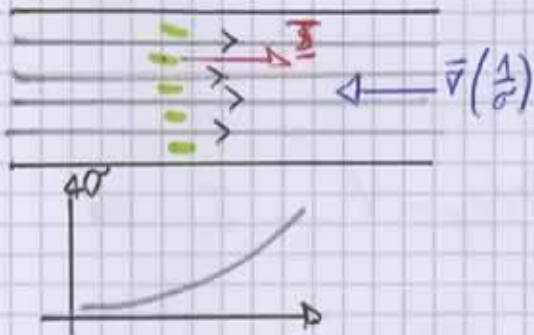
$$P_e = \int_{\Omega} \Sigma \cdot E \, d\tau \quad \left(P_H = \int_{\Omega} \Sigma \cdot \underline{E}_H \, d\tau \right)$$

SE IN Ω $\underline{E}_H = 0$, ALLORA:

$$P_e = \int_{\Omega} \sigma E^2 \, d\tau > 0$$

DENSITĂȚI CARICĂ ÎN MATERII ÎN CUI VARIA σ

$$E_N = 0$$



$$\rho = \epsilon_0 \nabla \cdot \vec{E}$$

$$= \epsilon_0 \nabla \cdot \left(\frac{1}{\sigma} \vec{\Sigma} \right) =$$

$$= \epsilon_0 \left(\nabla \frac{1}{\sigma} \cdot \vec{\Sigma} + \frac{1}{\sigma} \nabla \cdot \vec{\Sigma} \right) =$$

$$= \epsilon_0 \left(\nabla \left(\frac{1}{\sigma} \right) \cdot \vec{\Sigma} \right) < 0$$

POICHE $\vec{\Sigma}$
EST DIVERGENTE



$$\sigma_2 < \sigma_1$$

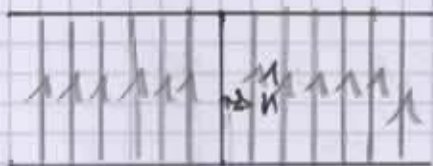
$$\rho_s = \epsilon_0 (\epsilon_{N1} - \epsilon_{N2}) =$$

$$= \epsilon_0 \left(\frac{1}{\sigma_1} \cdot \vec{\Sigma}_{N1} - \frac{1}{\sigma_2} \cdot \vec{\Sigma}_{N2} \right) =$$

$$= \epsilon_0 \left(\frac{1}{\sigma_1} - \frac{1}{\sigma_2} \right) \cdot \vec{\Sigma}_N < 0$$

NEL CASO IN CUI $\Sigma_N = 0$

\Rightarrow NON SI CREA DENSITÀ SUPERFICIALE



MAGNETOSTATICA

SET DI EQUAZIONI:

$$- \oint_{\gamma} \frac{1}{\mu_0} \underline{B} \cdot \hat{e} \, dl = I_{S\gamma}$$

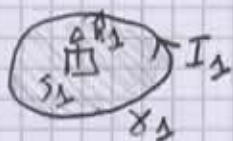
$$- \oint_S \underline{B} \cdot \hat{n} \, dS = 0 \quad [\omega] = [\tau]/[m]$$

- C.R. ∞

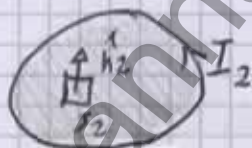
DATO CHE \underline{B} È SOLENOIDALE
LE SUE LINEE DI FORZA SARANNO
CIRCOLARI!



COEFF. DI AUTO E MUTUA INDUZIONE



$$\phi_1 = \iint_{S_1} \underline{B} \cdot \hat{n}_1 \, dS$$



$$\phi_2 = \iint_{S_2} \underline{B} \cdot \hat{n}_2 \, dS$$

DA CUI:

$$\phi_1 = \iint_{S_1} (\underline{B}_1 + \underline{B}_2) \cdot \hat{n}_1 \, dS = \iint_{S_1} \underline{B}_1 \cdot \hat{n}_1 \, dS + \iint_{S_1} \underline{B}_2 \cdot \hat{n}_1 \, dS = L_1 I_1 + M_{12} I_2$$

$$\phi_2 = \iint_{S_2} (\underline{B}_1 + \underline{B}_2) \cdot \hat{n}_2 \, dS = \iint_{S_2} \underline{B}_1 \cdot \hat{n}_2 \, dS + \iint_{S_2} \underline{B}_2 \cdot \hat{n}_2 \, dS = M_{21} I_1 + L_2 I_2$$

QUINDI:

$$\begin{cases} \phi_1 = L_1 I_1 + M_{12} I_2 \\ \phi_2 = M_{21} I_1 + L_2 I_2 \end{cases}$$

ORA DIMOSTRIAMO CHE:

$$L_1 \cdot L_2 \geq M^2$$

DEFINISCO IL FLUSSO DEL CAMPO MAGNETICO ATTRAVERSO N SPIRE:

$$\phi_1 = N_1 \cdot \phi_1^{\text{SINGOLA SPIRA}}$$

$$\phi_2 = N_2 \cdot \phi_2^{\text{SINGOLA SPIRA}}$$

NOTANDO CHE: $\frac{|\phi_1|}{N_1} \leq \frac{|\phi_2|}{N_2}$ E CHE $M_{12} = \frac{\phi_1}{I_2} \Big|_{I_1=0}$

ALLORA:

$$\frac{|M_{12}| |I_2|}{N_1} \leq \frac{L_2 |I_2|}{N_2}$$

$$M_{21} = \frac{\phi_2}{I_1} \Big|_{I_2=0}$$

QUINDI:

$$\Rightarrow \frac{|M_{12}|}{N_1} \leq \frac{L_2}{N_2}$$

$$\Rightarrow \frac{|M_{21}|}{N_2} \leq \frac{L_1}{N_1}$$

$\Rightarrow M_{12} = M_{21}$

$$\Rightarrow \frac{|M_{12}| |M_{21}|}{N_1 \cdot N_2} \leq \frac{L_1 \cdot L_2}{N_1 \cdot N_2} \Rightarrow M^2 \leq L_1 \cdot L_2 \quad \blacksquare$$

POTENZIALE VETTORE

EQUAZIONI DELLA MAGNETOSTATICA:

$$\begin{cases} \vec{\nabla} \times \underline{B} = \mu_0 \underline{J} \\ \vec{\nabla} \cdot \underline{B} = 0 \\ CR. \infty \end{cases}$$

CON $\underline{B} = \vec{\nabla} \times \underline{A}$ LA SECONDA EQUAZIONE È SODDISFATTA.

$$\underline{A}_1 \rightarrow \underline{B} \leftarrow \underline{A}_2 \Rightarrow \vec{\nabla} \times \underline{A}_1 = \underline{B} = \vec{\nabla} \times \underline{A}_2$$

$$\downarrow$$

$$\vec{\nabla} \times (\underline{A}_1 - \underline{A}_2) = 0$$

$$\begin{cases} \vec{\nabla} \times (\vec{\nabla} \psi) = 0 \\ \vec{\nabla} \cdot (\vec{\nabla} \times \underline{W}) = 0 \end{cases}$$

SE SCELGO UN POTENZIALE VETTORE CHE ABBA $\vec{\nabla} \cdot \underline{A} = 0$ VIENE DETTA GAUGE DI COULOMB.

$$\underline{A}_1 = \underline{A}_2 + \vec{\nabla} \psi$$

TRASFORMAZIONE DI GAUGE

$$\nabla^2 \psi = -\vec{\nabla} \cdot \underline{A}_2$$

POISSON

NEL CASO IN CUI NON SODDISFI QUESTA CONDIZIONE POSSO IMPORRE IN MODO TALE CHE IL NUOVO POTENZIALE VETTORE AVRA' DIVERGENZA NULLA LA SOLUZIONE DEL POTENZIALE VETTORE PER UNA DENSITA' DI CORRENTE VOLUMETRICA, NELLO SPAZIO LIBERO SARA' ANALOGA A QUELLA DEL POTENZIALE SCALARE:

$$\psi(\underline{r}) = \frac{1}{4\pi} \int_{\Omega} \frac{\rho(\underline{r}')}{\epsilon_0} \cdot \frac{1}{|\underline{r} - \underline{r}'|} d\tau'$$

$$\text{POICHE' } \vec{\nabla} \cdot \underline{J} = 0 \Rightarrow \vec{\nabla} \cdot \underline{A} = 0$$

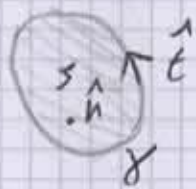
$$\underline{A}(\underline{r}) = \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\underline{J}(\underline{r}')}{|\underline{r} - \underline{r}'|} d\tau'$$

POSSIAMO ORA QUINDI RICAVARE L'INDUZIONE MAGNETICA:

$$\underline{B} = \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\underline{J}(\underline{r}') \times (\underline{r} - \underline{r}')}{|\underline{r} - \underline{r}'|^3} d\tau'$$

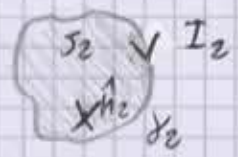
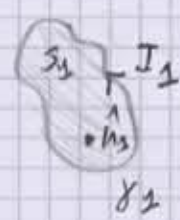
BIOT-SAVART

APPLICO IL CONCETTO DEL POT. VETT. AI COEFF. DI AUTO- E MUTUA-INDUZIONE



$$\phi = \iint_S \underline{B} \cdot \hat{n} dS = \iint_S \nabla \times \underline{A} dS \underset{\text{STOKES}}{=} \oint_{\gamma} \underline{A} \cdot \hat{t} dl$$

ORA CERCHIAMO DI DIMOSTRARE CHE: $M_{12} = M_{21} =$



$$M_{12} = \left. \frac{\phi_1}{I_2} \right|_{I_1=0}$$

$$\rightarrow \phi_1 = \oint_{\gamma_1} \underline{A} \cdot \hat{t}_1 dl_1 \rightarrow \underline{A} = \underline{A}_2 = \frac{\mu_0}{4\pi} \oint_{\gamma_2} \frac{I_2 \cdot \hat{t}_2}{|\underline{r}_1 - \underline{r}_2|} dl_2$$

$$\downarrow$$

$$\phi_1 = \oint_{\gamma_1} \underline{A}_2(\underline{r}_1) \cdot \hat{t}_1(\underline{r}_1) dl_1 =$$

$$= \oint_{\gamma_1} \left[\frac{\mu_0}{4\pi} \oint_{\gamma_2} \frac{I_2 \hat{t}_2(\underline{r}_2)}{|\underline{r}_1 - \underline{r}_2|} dl_2 \right] \cdot \hat{t}_1(\underline{r}_1) dl_1 =$$

$$\underline{A}_2(\underline{r}_1)$$

$$= \frac{\mu_0}{4\pi} I_2 \oint_{\gamma_1} \oint_{\gamma_2} \frac{\hat{t}_2 \cdot \hat{t}_1}{|\underline{r}_1 - \underline{r}_2|} dl_2 dl_1 \rightarrow M_{12} = \frac{\mu_0}{4\pi} \oint_{\gamma_1} \oint_{\gamma_2} \frac{\hat{t}_2 \cdot \hat{t}_1}{|\underline{r}_1 - \underline{r}_2|} dl_2 dl_1$$

FORMULA DI NEUBAUER

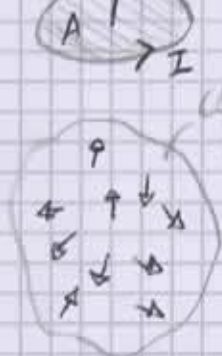
$$M_{21} = \frac{\mu_0}{4\pi} \oint_{\gamma_2} \oint_{\gamma_1} \frac{\hat{t}_1 \cdot \hat{t}_2}{|\underline{r}_2 - \underline{r}_1|} dl_1 dl_2 = M_{12} !!!$$



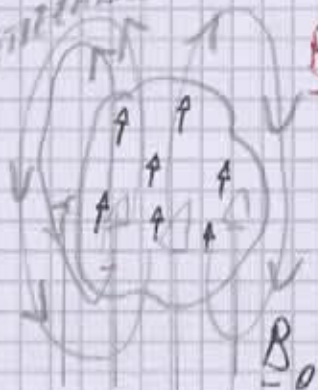
DIPOLO MAGNETICO



$$\underline{m} = A I \hat{m} \quad \text{MOMENTO MAGNETICO}$$



CORPO PARAMAGNETICABILE



QUANDO SI ATTIVA UN'INDUZIONE MAGNETICA ESTERNA \underline{B}_0 , I DIPOLI SI ORIENTANO E PRODUCONO UNA INDUZIONE MAGNETICA \underline{B}_R .

$$\Rightarrow \underline{B} = \underline{B}_0 + \underline{B}_R$$

I CAMPI MAGNETICI IN DIREZIONI STOCASTICHE SI ELIDONO.

RISCRIVIAMO LE EQUAZIONI DELLA MAGNETOSTATICA:

$$\begin{cases} \oint \underline{B} \cdot \hat{e} \, dl = \mu_0 I_{\gamma} = \mu_0 (I_{\gamma}^0 + I_{\gamma}^{mol}) \\ \oint \underline{B} \cdot \hat{n} \, dS = 0 \\ CR \infty \end{cases}$$

NON SIAMO IN GRADO DI CALCOLO

CALCOLIAMO IL VETTORE INTENSITA' DI MAGNETIZZAZIONE

SI DEFINISCE INTENSITA' DI MAGNETIZZAZIONE:

$$\underline{M} = \frac{\sum_K \underline{m}_K}{\Delta V} = \frac{\sum_K \underline{m}_K}{\Delta V} \cdot \frac{\Delta V}{\Delta V}$$

NUMERI DI DIPOLETTI IN ΔV

$M \times$ DIPOLETTI MEDIO

$$= \langle \underline{m} \rangle \cdot n$$

pongo $\Delta I_{\gamma}^{mol} = I \cdot m \cdot \Delta \gamma$, dove $\Delta \gamma = A \cdot \Delta h \cdot \hat{e} \cdot \hat{m}$ $\Delta h = \Delta l \cdot \hat{e} \cdot \hat{m}$

QUINDI:

$$\Delta I_{\gamma}^{mol} = A I m \Delta l \cdot \hat{e} \cdot \hat{m} = A I \hat{m} m \cdot \hat{e} \Delta l = \langle \underline{m} \rangle m \cdot \hat{e} \Delta l = \underline{M} \cdot \hat{e} \Delta l$$

$$\Rightarrow I_{\gamma}^{mol} = \oint_{\gamma} \underline{M} \cdot \hat{e} \, dl$$

SOSTITUIAMO ORA NELLE EQUAZIONI DELLA MAGNETOSTATICA:

$$\oint \underline{B} \cdot \underline{\hat{e}} dl = \mu_0 (I_{\gamma}^0 + \oint \underline{M} \cdot \underline{\hat{e}} dl)$$

$$\rightarrow \oint \left(\frac{1}{\mu_0} \underline{B} - \underline{M} \right) \cdot \underline{\hat{e}} dl = I_{\gamma}^0 \rightarrow \oint \underline{H} \cdot \underline{\hat{e}} dl = I_{\gamma}^0$$

\underline{H} CAMPO MAGNETICO

$$-\oint \underline{B} \cdot \underline{\hat{n}} dS = 0$$

$\epsilon R \infty$

$$-\underline{M} = \chi_m \underline{H}$$

PER MATERIALI LINEARI
 $\chi_m \ll 1$

SUE ETTIVITA' MAGNETICA
 (SE SCALARE = ISOTROPO)

$$\underline{B} = \mu_0 (\underline{H} + \underline{M})$$

$$= \mu_0 (\underline{H} + \chi_m \underline{H}) =$$

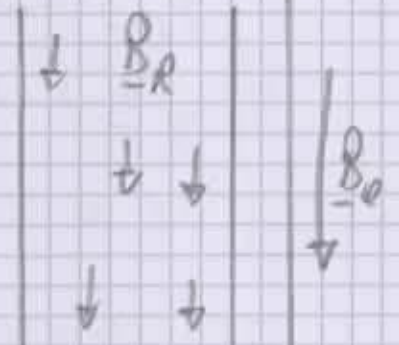
$$= \mu_0 (1 + \chi_m) \underline{H}$$

μ_r PERMEABILITA' MAGNETICA RELATIVA

μ PERMEABILITA' MAGNETICA ASSOLUTA



MATERIALI PARAMAGNETICI



MATERIALI DIAMAGNETICI

$$\oint \underline{B} \cdot \underline{\hat{e}} dl = \mu_0 (I_{\gamma}^0 + I_{\gamma}^{MOL}) \iff \begin{cases} \nabla \times \underline{B} = \mu_0 (\underline{J}^0 + \underline{J}^{MOL}) \\ \underline{\hat{n}} \times (\underline{B}_1 - \underline{B}_2) = \mu_0 (\underline{K}^0 + \underline{K}^{MOL}) \end{cases}$$

CERCO ORA DI RITROVARE \underline{J}^{MOL} E \underline{K}^{MOL} :

$$\oint \underline{M} \cdot \underline{\hat{e}} dl = I_{\gamma}^{MOL} \iff \begin{cases} \nabla \times \underline{M} = \underline{J}^{MOL} \\ \underline{\hat{n}} \times (\underline{M}_1 - \underline{M}_2) = \underline{K}^{MOL} \end{cases}$$

MODELLO QUASI-STAZIONARIO MAGNETICO

$$* \oint_{\gamma} \underline{E} \cdot \underline{\hat{t}} d\ell = - \frac{d}{dt} \iint_{S_{\gamma}} \underline{B} \cdot \underline{\hat{n}} dS$$

$$* \oint_{\gamma} \underline{B} \cdot \underline{\hat{t}} d\ell = \mu_0 I_{S_{\gamma}} + \mu_0 \epsilon_0 \frac{d}{dt} \iint_{S_{\gamma}} \underline{E} \cdot \underline{\hat{n}} dS$$

$$\iint_S \underline{E} \cdot \underline{\hat{n}} dS = Q_S / \epsilon_0$$

$$* \iint_S \underline{B} \cdot \underline{\hat{n}} dS = 0$$

$$* C.R. \omega$$

C.T.

$$* \underline{\Sigma} = \alpha \underline{E} + \alpha' \underline{E}_M \quad (\text{IN PRESENZA DI MATERIALI CONDUTTORI})$$

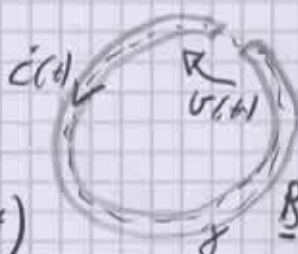
$\underline{\Sigma}_0$

POSSIAMO SCRIVERLO ANCHE IN FORMA LOCALE:

$$\left\{ \begin{array}{l} \underline{\nabla} \times \underline{E} = - \frac{\partial}{\partial t} \underline{B} \quad , \quad \underline{\hat{n}} \times (\underline{E}_1 - \underline{E}_2) = 0 \\ \underline{\nabla} \times \underline{B} = \mu_0 \underline{\Sigma} \quad , \quad \underline{\hat{n}} \times (\underline{B}_1 - \underline{B}_2) = \mu_0 \underline{K} \\ \underline{\nabla} \cdot \underline{B} = 0 \quad , \quad \underline{\hat{n}} \cdot (\underline{B}_1 - \underline{B}_2) = 0 \\ \underline{\Sigma} = \alpha \underline{E} + \underline{\Sigma}_0 \end{array} \right.$$

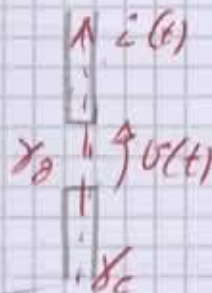
C.R. ω

CONSIDERIAMO ORA IL SEGUENTE CIRCUITO:



$$\underline{B}(\underline{r}, t) = \underline{B}_1(\underline{r}, t) + \underline{B}_0(\underline{r}, t)$$

$$\int_{\gamma_0} \underline{E} \cdot \underline{\hat{t}} d\ell + \underbrace{\int_{\gamma_0} \underline{E} \cdot \underline{\hat{t}} d\ell}_{= -V(t)} = - \frac{d}{dt} \iint_{S_{\gamma}} \underline{B} \cdot \underline{\hat{n}} dS$$



PER QUANTO RIGUARDA IL PRIMO MEMBRO:

$$\int_{\gamma_c} \underline{E} \cdot \underline{\hat{e}} dl = \int_{\gamma_c} \frac{1}{\sigma_l} \underline{J} \cdot \underline{\hat{e}} dl = \int_{\gamma_c} \frac{1}{\sigma_l} \frac{i(t)}{A_l} dl =$$

AREA SEZIONE TRASVERSA

$$= i(t) \int_{\gamma_c} \frac{1}{\sigma_l} \cdot \frac{1}{A_l} dl$$

R_c

MENTRE PER IL MEMBRO A DESTRA DELL'UGUALE:

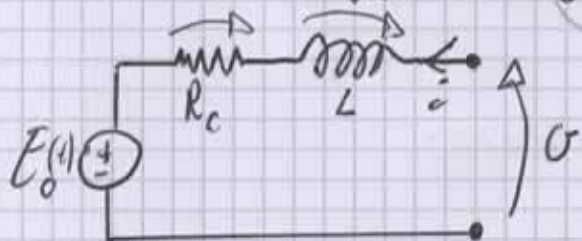
$$\iint_{S_\gamma} \underline{B} \cdot \underline{\hat{n}} dS = \iint_{S_\gamma} \underline{B}_c \cdot \underline{\hat{n}} dS + \iint_{S_\gamma} \underline{B}_0 \cdot \underline{\hat{n}} dS = L i + \phi_0$$

QUINDI:

$$R_c \cdot i - U(t) = -\frac{d}{dt} (L i) - \frac{d}{dt} \phi_0$$

OVVERO:

$$U(t) = R_c \cdot i + \frac{d}{dt} (L i) + \frac{d}{dt} \phi_0 \rightarrow \text{CHE POSSO TRASURRE A LIVELLO CIRCUITALE:}$$



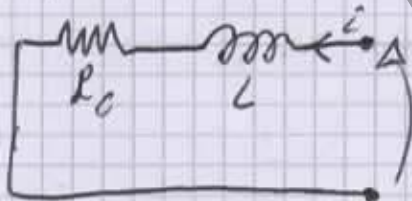
PER AVERE UN INDUTTORE IDEALE, IL CIRCUITO DEVE RIDURSI AD UNA SOLA INDUTTANZA, OVVERO QUANDO:

$$- R_c = 0 \Leftrightarrow \sigma \rightarrow \infty \text{ (COND. PERF.)}$$

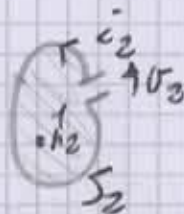
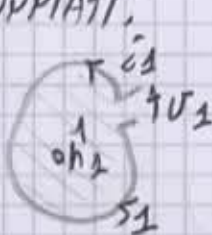
$$- E_0 = 0 \Leftrightarrow \underline{B}_0 \text{ COST.} \Rightarrow \underline{B}_0 = 0$$

$$U(t) = L i'$$

DATO CHE FISICAMENTE NON POSSA AVERE $\sigma \rightarrow \infty$:



STUDIAMO ORA DUE INDUTTORI MUTUAMENTE ACCOPPIATI:



$$U_1(t) = R_1 i_1(t) + L_1 i_1'(t) + \frac{d}{dt} \phi_1^E$$

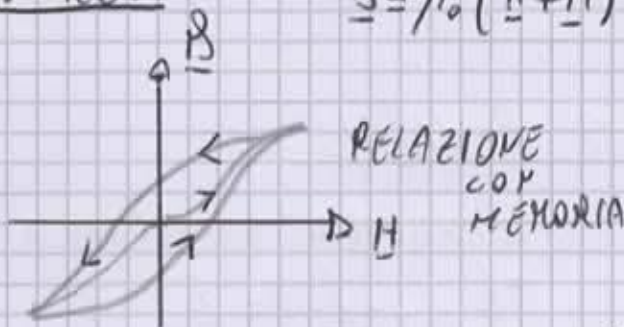
$$U_2(t) = R_2 i_2(t) + L_2 i_2'(t) + \frac{d}{dt} \phi_2^E$$

$$\phi_1^E = M_{12} i_2, \quad \phi_2^E = M_{21} i_1 \Rightarrow \begin{cases} U_1 = R_1 i_1(t) + L_1 i_1' + M_{12} i_2' \\ U_2 = R_2 i_2(t) + L_2 i_2' + M_{21} i_1' \end{cases}$$

ISTERESI

$$\underline{B} = \mu_0 (\underline{H} + \underline{M})$$

(MATERIALE FERROMAGNETICO)



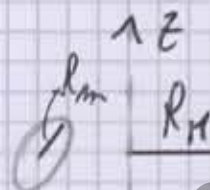
IN GENERALE UN FERROMAGNETE È SUDDIVISO IN DOMINI DI MAGNETIZZAZIONE UNIFORME:



DOVE $\langle \underline{M} \rangle = 0$

MA QUANDO ACCENDIAMO \underline{H} , I DOMINI CHE SONO DIREZIONATI VERSO \underline{H} SI ESPANDONO RIMPICCIANDO GLI ALTRI.

CONSIDERO UN TORO = N AVVOLGIMENTI



$$\oint \underline{H} \cdot d\underline{l} = I_{\gamma}$$

$$\underline{H} = H_{\theta}(\rho, z)$$

$$\oint \underline{B} \cdot d\underline{s} = 0$$

$$H_{\theta} = \begin{cases} \frac{\mu_0 I}{2\pi \rho} & \text{NEL SOLENOIDE} \end{cases}$$

$$\underline{B} = \underline{B}(\underline{H})$$

0, ALL'ESTERNO DEL SOLENOIDE

C.R. 10

COSTRUISCO:

$$\text{SE } R_H \gg R_m \rightarrow \frac{1}{\rho} \approx \frac{1}{R_H} \text{ NEL SOLENOIDE}$$

$$H_{\theta} \approx \frac{\mu_0 I}{2\pi R_H}$$

$$\tilde{U}(t) = \frac{d}{dt} \phi_0 = \frac{d}{dt} \left(\mu' \iint_S \underline{B} \cdot d\underline{s} \right) =$$

SINGOLA SPIRA

$$\mu' \iint_S \underline{B}_{\theta} \cdot d\underline{s}$$

$$\tilde{U}(t) = N' A \frac{d}{dt} B_{\theta} \Leftarrow$$



$$\sigma(t) = \int \tilde{\sigma}(\vec{r}) d\vec{r} = N' A B_0$$

PARALLELISMO TRA COND. STAT. E MAGN.

MAGNETOSTATICA

$$\oint \underline{H} \cdot \underline{\hat{e}} dl = I_x$$

$$\oint \underline{B} \cdot \underline{\hat{n}} dS = 0$$

$$\underline{B} = \mu \underline{H}$$

$$C.R. \infty$$

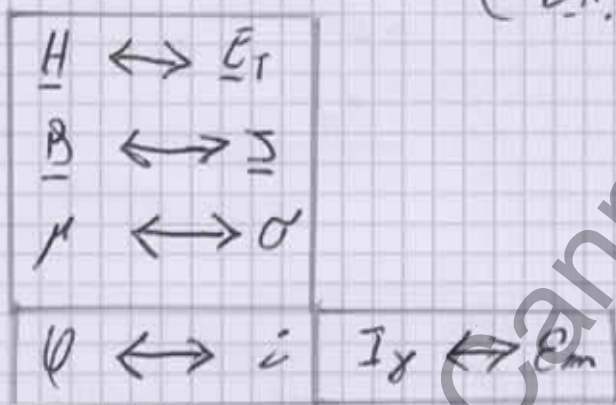
$(\underline{E}_T = \underline{E} - \underline{E}_H)$ CONDUZIONE STAZIONARIA

$$\oint \underline{E}_T \cdot \underline{\hat{e}} dl = E_m$$

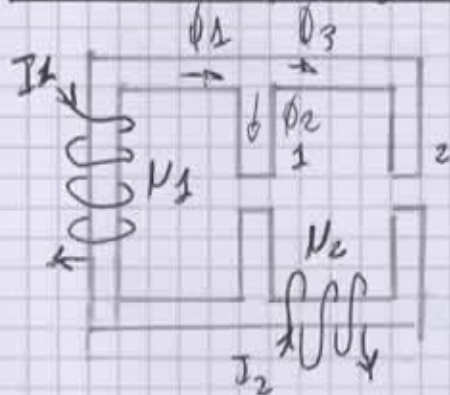
$$\oint \underline{S} \cdot \underline{\hat{n}} dS = 0$$

$$\underline{S} = \sigma \underline{E}_T$$

$$C.R. \infty$$

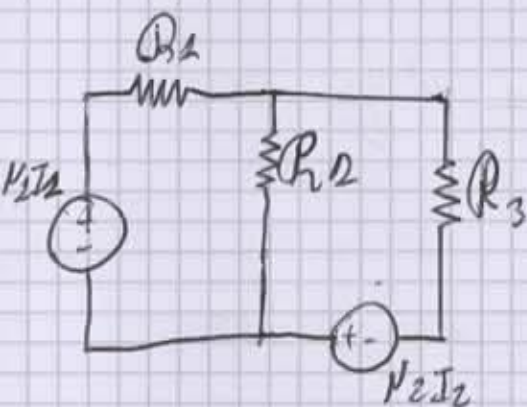


CIRCUITI MAGNETICI



- VALORE L.E. CKE PER I FLUSSI

- VALORE L.E. CKT



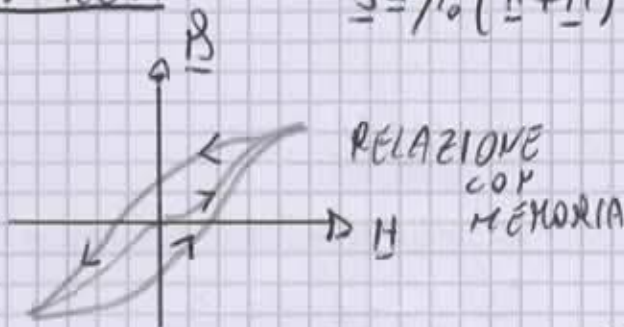
$$R_1 = \int \frac{dl}{\mu_1 A} = 0$$

$$R_2 = \int \frac{dl}{\mu_2 A} = \int \frac{dl}{\mu_2^m A} + \int \frac{dl}{\mu_2^0 A} = \frac{l_A}{\mu_0 S_A}$$

ISTERESI

$$\underline{B} = \mu_0 (\underline{H} + \underline{M})$$

(MATERIALE FERROMAGNETICO)



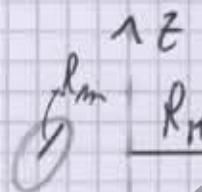
IN GENERALE UN FERROMAGNETE È SUDDIVISO IN DOMINI DI MAGNETIZZAZIONE UNIFORME:



DOVE $\langle \underline{M} \rangle = 0$

MA QUANDO ACCENDIAMO \underline{H} , I DOMINI CHE SONO DIREZIONATI VERSO \underline{H} SI ESPANDONO RIMPICCIANDO GLI ALTRI.

CONSIDERO UN TORO = N AVVOLGIMENTI



$$\oint \underline{H} \cdot d\underline{l} = I_{\gamma}$$

$$\underline{H} = H_{\theta}(\rho, z)$$

$$\oint \underline{B} \cdot d\underline{s} = 0$$

$$H_{\theta} = \begin{cases} \frac{\mu_0 I}{2\pi \rho} & \text{NEL SOLENOIDE} \\ 0 & \text{ALL'ESTERNO DEL SOLENOIDE} \end{cases}$$

$$\underline{B} = \underline{B}(\underline{H})$$

C.R. A

COSTRUISCO:

$$\text{SE } R_H \gg R_m \rightarrow \frac{1}{\rho} \approx \frac{1}{R_H} \text{ NEL SOLENOIDE}$$

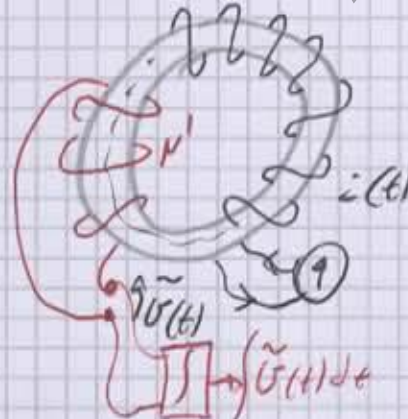
$$H_{\theta} \approx \frac{\mu_0 I}{2\pi R_H}$$

$$\tilde{U}(t) = \frac{d}{dt} \phi_0 = \frac{d}{dt} \left(\mu' \iint_S \underline{B} \cdot d\underline{s} \right) =$$

SINGOLA SPIRA

$$\mu' \iint_S \underline{B}_{\theta} \cdot d\underline{s}$$

$$\tilde{U}(t) = N' A \frac{d}{dt} B_{\theta} \Leftarrow$$



$$\sigma(t) = \int \tilde{\sigma}(\vec{r}) d\vec{r} = N' A B_0$$

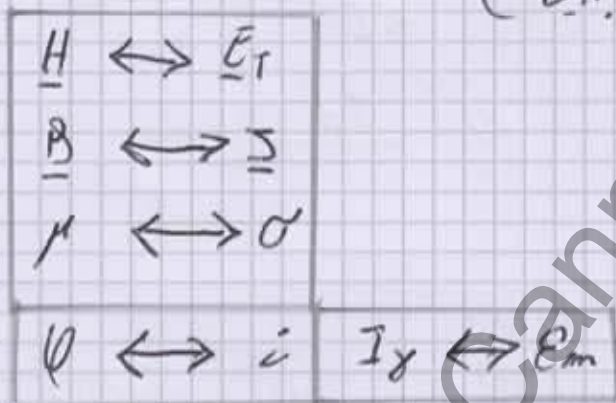
PARALLELISMO TRA COND. STAZ. E MAGN.

MAGNETOSTATICA

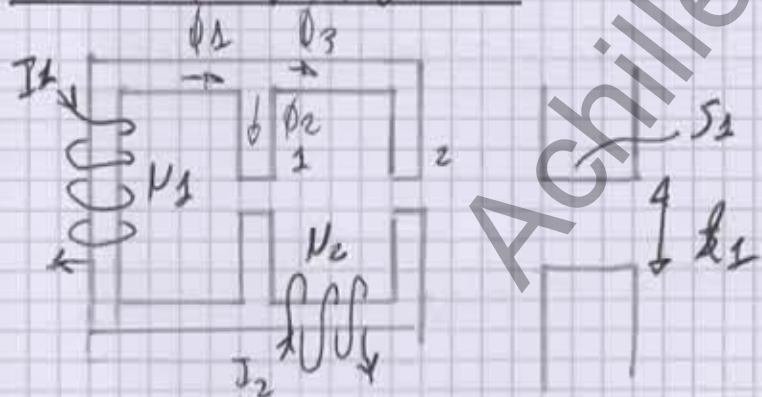
$$\begin{cases} \oint_{\gamma} \underline{H} \cdot \underline{\hat{e}} d\ell = I_{\gamma} \\ \oint_S \underline{B} \cdot \underline{\hat{n}} dS = 0 \\ \underline{B} = \mu \underline{H} \\ C.R. \infty \end{cases}$$

($\underline{E}_T = \underline{E} - \underline{E}_N$) CONDUZIONE STAZIONARIA

$$\begin{cases} \oint_{\gamma} \underline{E}_T \cdot \underline{\hat{e}} d\ell = E_m \\ \oint_S \underline{\Sigma} \cdot \underline{\hat{n}} dS = 0 \\ \underline{\Sigma} = \sigma \underline{E}_T \\ C.R. \infty \end{cases}$$

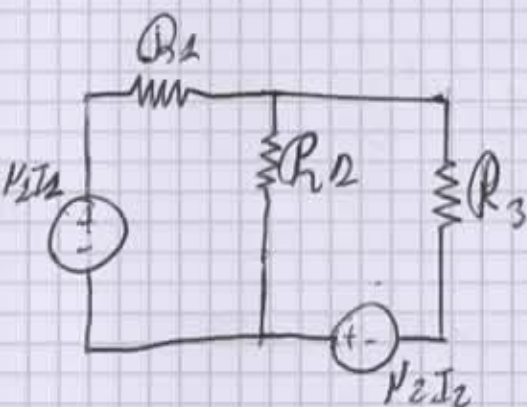


CIRCUITI MAGNETICI



- VALORE L.C. CRE. PER I FLUSSI

- VALORE L.C. CRT



$$R_1 = \int_{\gamma_1} \frac{d\ell}{\mu A} = 0$$

$$R_2 = \int_{\gamma_2} \frac{d\ell}{\mu A} = \int_{\gamma_2^m} \frac{d\ell}{\mu A} + \int_{\gamma_2^0} \frac{d\ell}{\mu_0 A} = \frac{l_A}{\mu_0 S_A}$$

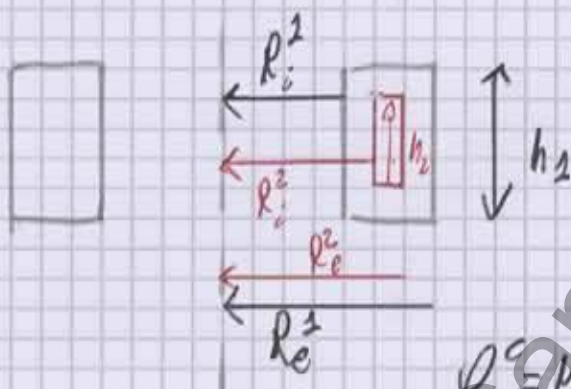
$$L_1 = \left. \frac{\phi_1^c}{I_1} \right|_{I_2=0}, \quad L_2 = \left. \frac{\phi_2^c}{I_2} \right|_{I_1=0}, \quad M = M_{12} = M_{21}$$

$$\phi_1^c = N_1 \phi_1$$

$$\phi_2^c = N_2 \cdot \phi_2$$

$$\Rightarrow L_1 = \frac{N_1^2}{R_2/R_3}, \quad L_2 = \frac{N_2^2}{R_1/R_2}, \quad M = \frac{N_1 N_2}{R_3}$$

PER. VV. SOLENOIDS E TOROIDALI:



$$B_\theta = \int \frac{\mu I}{2\pi r} \frac{1}{r} dr$$

$$B_\theta = \frac{\mu_1 I_1}{2\pi r \mu_0}$$

$$\phi_1^c = N_1 \cdot \phi_1$$

$$\begin{aligned} \phi_1 &= \iint_{S_1} \underline{B}^1 \cdot \underline{n} \, dS = \frac{\mu_1 I_1}{2\pi \mu_0} \iint_{S_1} \frac{1}{r} \, dS = \frac{\mu_1 I_1}{2\pi \mu_0} \int_{z_1^1}^{z_2^1} dz \int_{R_i^1}^{R_o^1} \frac{1}{r} \cdot r \, dr = \\ &= \frac{\mu_1 I_1}{2\pi \mu_0} h_1 \ln\left(\frac{R_o^1}{R_i^1}\right) \end{aligned}$$

$$\Rightarrow L_1 = \frac{\phi_1 N_1}{I_1} = \mu_1^2 \frac{I_1 \cdot N_1}{2\pi \mu_0 I_1} \ln\left(\frac{R_o^1}{R_i^1}\right) = \frac{\mu_1^2 N_1^2 \ln\left(\frac{R_o^1}{R_i^1}\right)}{2\pi \mu_0}$$

$$M_{21} = \left. \frac{\phi_2^c}{I_1} \right|_{I_2=0} = \frac{\mu_0}{I_1} \cdot \frac{\mu_1 \cdot I_1}{2\pi \mu_0} h_2 \ln\left(\frac{R_o^2}{R_i^2}\right) = \frac{\mu_1 \mu_2 h_2 \ln\left(\frac{R_o^2}{R_i^2}\right)}{2\pi \mu_0}$$

$$\phi_2^c = N_2 \phi_2$$

$$\phi_2 = \iint_{S_2} \underline{B}^2 \cdot \underline{n} \, dS = \frac{\mu_2 I_2}{2\pi \mu_0} h_2 \ln\left(\frac{R_o^2}{R_i^2}\right)$$