

Thapar Institute of Engineering & Technology, Patiala School of Mathematics	
B.E/ B.Tech. (Semester 2 <sup>nd</sup> )	Time: 02 hrs. M.M. 30
UMA 004: Mathematics - II	Wednesday: March 09, 2023
<b>Name of the Faculty:</b> Ankush Pathania, Anuj Kumar, Harish Garg, Jatinderdeep Kaur, Kavita, Meenu Rani, Md. Hasanuzzaman, Munish Kansal, Pankaj Narula, Pramod Kumar Vaishnav, S.S. Bhatia, Sapna Sharma, Satish Kr. Sharma, Sumit Chandok, Vivek Sangwan	

**Note:** 1) All questions are compulsory to attempt.

2) **Attempt all parts of each question at one place.**

1. (a) If  $y_1 = x$  is one of the solutions of the differential equation

$$(1 - x^2) \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - 2y = 0,$$

then find the general solution of this equation. (3)

- (b) Using the method of undetermined coefficient, find the general solution of the differential equation: (4.5)

$$\frac{d^2 y}{dx^2} + 6 \frac{dy}{dx} + 9y = e^{-3x} + x$$

2. (a) Solve the following differential equation by using method of Variation of Parameters. (3)

$$\frac{d^2 y}{dx^2} + 4y = \sec 2x$$

- (b) Check whether the following differential equation is exact or not. If not, then use the integrating factor to make it exact and hence find the general solution. (4.5)

$$\left(xy^2 - e^{\frac{1}{x^3}}\right) dx - x^2 y dy = 0$$

3. (a) Solve the differential equation: (3)

$$x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = 0$$

- (b) Using definition, find the Laplace transform of the following function. (4.5)

$$f(t) = \begin{cases} 2t & ; 0 \leq t \leq 5 \\ 1 & ; t > 5 \end{cases}$$

4. (a) Using properties, find the Laplace transform (3)

$$e^{2t}(t^{3/2} + 3 \cos 2t)$$

- (b) Evaluate the integral using Laplace transform (4.5)

$$\int_0^\infty \frac{e^{-3t} \sin 2t}{t} dt$$