Section A

Answer ALL questions from this section.

Question 1

(a) For each of the following functions, decide which complexity class it belongs to:

1. $n * (n + \log n)$ **[2 marks]**

2. $2^n + n^4 + 7$ **[2 marks]**

Answer:

1. $n * (n + \log n)$ is in $O(n^2)$
2. $2^n + n^4 + 7$ is in $O(2^n)$

(b) Given the following table of runtimes (in seconds) of two algorithms, determine their complexity. Give a short explanation of how you determined them. **[6 marks]**

Input size	Algorithm 1	Algorithm 2
1000	4	21
2000	15	41
4000	62	85
8000	250	169

Answer: Algorithm 1 grows by about a factor $4 = 2^2$ each time the input size doubles, we would expect it to be in $O(n^2)$.

Algorithm 2 grows by about a factor 2 each time the input size doubles, we would expect it to be in $O(n)$.

[TOTAL 10]

Question 2

Consider this array representation of a heap:

index	0	1	2	3	4	5	6
value	3	4	5	7	6	9	8

(a) Draw the tree representation of this heap. **[2 marks]**

(b) Perform the following sequence of operations, drawing the updated tree representation after each of them:

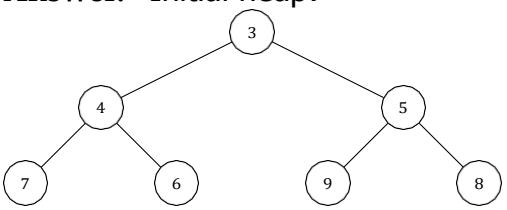
1. Pop the minimal element off the heap **[2 marks]**

2. Insert the value 3 **[2 marks]**

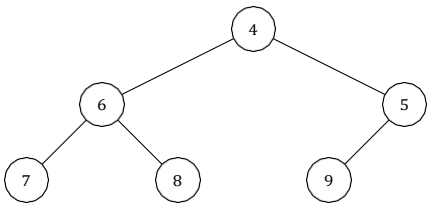
3. Insert the value 1 **[2 marks]**

4. Insert the value 2 **[2 marks]**

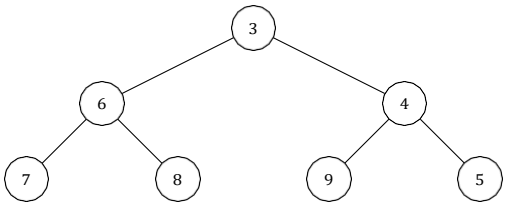
Answer: Initial heap:



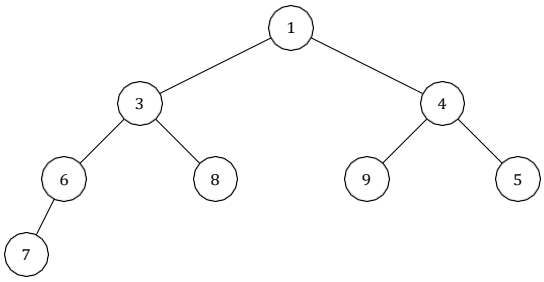
1. Pop the minimum:



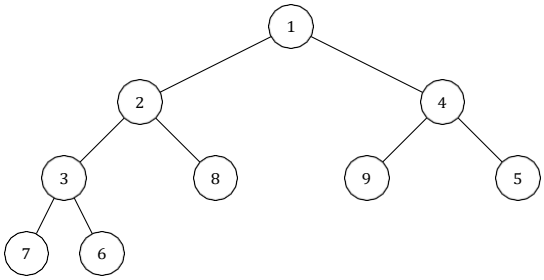
2. Insert 3:



3. Insert 1:



4. Insert 2:



[TOTAL 10]

Question 3

Write a method that removes duplicates in a **sorted** doubly linked list. For example, if the list originally contains the data
1,1,3,4,4,4,7,8
after calling the method it should contain
1,3,4,7,8
Complete either the Java or C++ version below. **[10 marks]**

- Java version:

```
public class DList{
 public class ListNode{
 public int data ;
  public ListNode next, previous;
 }

 public ListNode first, last;

 public void removeDuplicates(){
  ListNode n = first;
  while(n != null){
   while(n.next != null && n.next.data == n.data){
    // remove the next node from the list
    n.next = n.next.next;
    if ( n. next == null ) // n is now the last node last
     = n ;
    else
     n.next.previous = n ;
   }
   n = n.next;
  }
 }
}
```

- C++ version:

```
class DList{
 public:
 class ListNode{
  public:
  int data ;
  ListNode *next, *previous;
 };
```

```
ListNode *first, *last;

void removeDuplicates(){
 ListNode *n = first;
 while(n != null){
  while(n->next != null && n->next->data == n->data){
   // remove the next node from the list
   ListNode *temp = n->next;
   n->next = n->next->next;
   if (n -> next == null ) // n is now the last node last
    = n ;
   else
    n->next->previous = n ;
   delete temp; // avoid memory leak
  }
  n = n->next;
 }
}
};
```

[TOTAL 10]

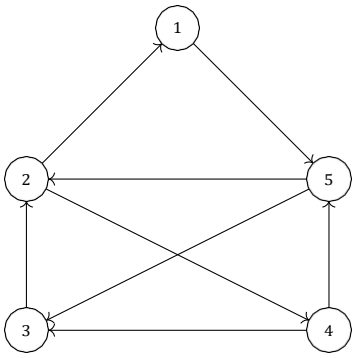
Question 4

A *directed graph* is represented by the following *adjacency matrix* (where `T` represents True and `-` represents False):

	1	2	3	4	5
1	-	-	-	-	T
2	T	-	-	T	-
3	-	T	-	-	-
4	-	-	T	-	T
5	-	T	T	-	-

(a) Draw the above graph. **[5 marks]**

Answer: The graph looks like this.



(b) Does this graph have an *Euler path*? If yes, give one. If not, explain why not. **[5 marks]**

Answer: The graph does have an Euler path, for example $4 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 2 \rightarrow 1 \rightarrow 5 \rightarrow 3$.

[TOTAL 10]

Section B

Answer THREE questions from this section.

Question 5

(a) Consider the following unsorted array:

index	0	1	2	3	4	5	6
value	8	5	34	13	55	3	89

Suppose you try to find the value 8 in this array using binary search. For each iteration of the algorithm, give the values of the `first`, `middle`, and `last` indices, and a (short) explanation of what happens.

[10 marks]

Answer: The variable values per iteration are

1. `first=0, middle=3, last=6`
2. `first=0, middle=1, last=2`
3. `first=2, middle=2, last=2`
4. `first=2, middle=1, last=1`

At this point the algorithm terminates because `first > last`. Even though the value is in the array, the search fails because the array is not sorted, and binary search expects a sorted array.

(b) Instead of finding a specific value, in this problem we are searching an array for a value within a given **range**.

Write a search function which gets a **sorted** array `A` and two numbers `low, high` as its input and returns an index `i` such that `low <= A[i] <= high` (or -1 if there is no such value in `A`).

For example, if the values in the array are [2, 3, 5, 7, 11, 13, 17, 19, 23, 29] and `low=11, high=15` then the function should return either 4 or 5 (because `A[4]=11, A[5]=13` are both in the given range).

Complete either the Java or C++ version below.

[10 marks]

Answer: The algorithm is a variation of binary search. Just like in binary search, we keep track of three positions `first`, `middle`, and `last` in the array. In each iteration we have three possible cases:

- The middle value is less than `low`. Restrict to the first half.
- The middle value is more than `high`. Restrict to the second half.
- Otherwise the middle value in in the range; return it as our result.

- Java version:

```
public class RangedSearch{
 public static int findInRange(int[] A, int low, int high){
  int first =0, last = A.length , middle = A.length /2;
  while(first<=last){
   if(A[middle]<low)
    last=middle-1;
   else if(A[middle]>high)
    first=middle+1;
   else
    return   m iddle  ;
   middle=(first+last)/2;
  }
  return -1;
 }
}
```

- C++ version:

```
int findInRange(vector<int> &A, int low, int high){
 int first=0, last=A.size(), middle=A.size()/2;
 while(first<=last){
  if(A[middle]<low)
   last=middle-1;
  else if(A[middle]>high)
   first=middle+1;
  else
   return   m iddle  ;
  middle=(first+last)/2;
 }
 return -1;
}
```

[TOTAL 20]

Question 6

(a) Consider the following array:

index	0	1	2	3	4	5	6	7	8
value	9	7	5	3	1	2	4	6	8

We want to sort this array using Insertion Sort. Write down the contents of the array after each iteration of the main loop. **[10 marks]**

Answer: The contents of the array after each iteration are

index	0	1	2	3	4	5	6	7	8
iteration 1:	7	9	5	3	1	2	4	6	8
iteration 2:	5	7	9	3	1	2	4	6	8
iteration 3:	3	5	7	9	1	2	4	6	8
iteration 4:	1	3	5	7	9	2	4	6	8
iteration 5:	1	2	3	5	7	9	4	6	8
iteration 6:	1	2	3	4	5	7	9	6	8
iteration 7:	1	2	3	4	5	6	7	9	8
iteration 8:	1	2	3	4	5	6	7	8	9

(b) **Cycle Sort** is a sorting algorithm which works as follows: For each element in the array `A`, figure out where it should go (by counting how many smaller elements there are) and swap it with whatever is at that position. We assume that `A` contains **no duplicates** to simplify matters. In more detail:

- Maintain an additional boolean array `B`, initially all false; `B[i]` keeps track of whether `A[i]` is known to be in the right position.
- Starting at position `i=0`, do the following until `i` reaches the end:
  - If `B[i]` is true, increase `i`
  - Otherwise, count how many elements in `A` are less than `A[i]`. Call this number `j`, swap `A[i]` with `A[j]`, and update `B`.

For example, starting with this array:

index	0	1	2	3	4	5
A	5	3	1	2	6	8
B	false	false	false	false	false	false

First we look at `A[0]=5`. There are 3 values which are smaller, so this belongs at position 3. After swapping, we have

index	0	1	2	3	4	5
A	2	3	1	5	6	8
B	false	false	false	true	false	false

Now the 5 is in the right position, and `A[0]=2`. There is 1 value which is smaller, so this belongs at position 1, and we get

index	0	1	2	3	4	5
A	3	2	1	5	6	8
B	false	true	false	true	false	false

and so on. Based on this description, complete either the Java or C++ version below. **[10 marks]**

- Java version:

```
public class CycleSort{

 public static void sort ( int [] values ){
  boolean[] inPlace = new boolean[values.length]; // initially all f
  for(int i=0; i<values.length; )
   if(inPlace[i])
// this value is in the right place , move on i++;
   else {
// determine where this value belongs
    int p=0;
    for ( int j : values)
     if(i < values[i]) p
     ++;
    // swap entries at positions i and p int
    temp = values[i];
    values[i] = values[p];
    values[p] = temp;
    inPlace[p] = true;
   }
 }
}
```

- C++ version:

```
void cycleSort(vector<int> &values){
  vector<boolean> inPlace(values.size, false);
  for(int i=0; i<values.size(); )
  if(inPlace[i])
// this value is in the right place , move on
```

```
        i++;
      else {
// determine where this value belongs
       int p=0;
       for ( int j : values)
        if(i < values[i]) p
        ++;
       // swap entries at positions i and p int
       temp = values[i];
       values[i] = values[p];
       values[p] = temp;
       inPlace[p] = true;
      }
}
```

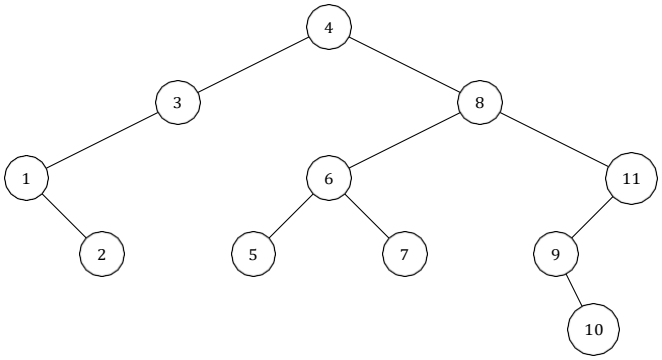
[TOTAL 20]

Question 7

(a) Draw the *Binary Search Tree (BST)* which results from inserting the following values in the sequence given into an initially empty BST:

4,8,6,3,7,11,1,5,9,2,10 **[4 marks]**

Answer: The resulting tree is



(b) Give the pseudo code for the **inorder** tree traversal algorithm. Apply the algorithm to the BST constructed for Part **(a)** and show the order in which nodes will be visited. **[4 marks]**

Answer:

```
inorder(tree):
    IF ( tree not empty ) THEN
        inorder the left subtree
        print the root node
        inorder the rightsubtree
```

The result is: 1 2 3 4 5 6 7 8 9 10 11

(c) Draw the BSTs which you get by removing the following three values, one at a time, from the tree created in Part **(a)**:

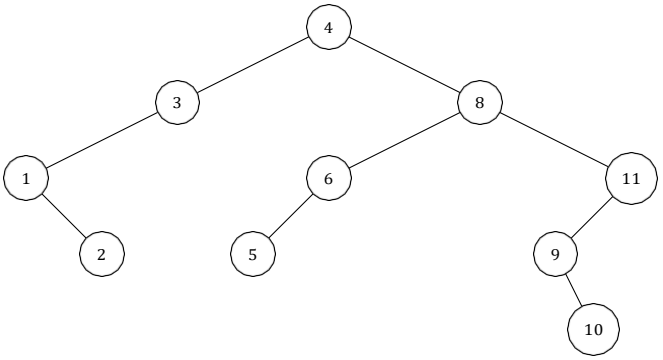
(i) 7 **[4 marks]**

(ii) 1 **[4 marks]**

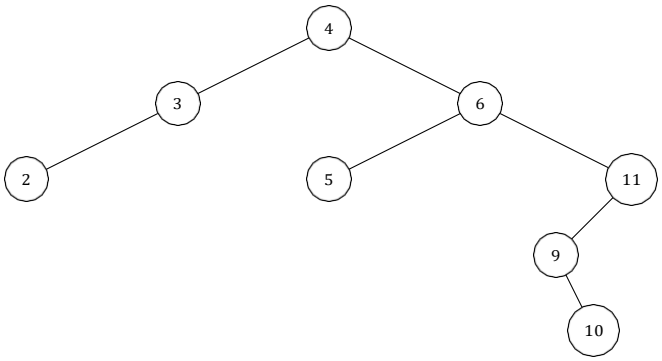
(iii) 8 **[4 marks]**

Answer:

(i) After removing 7:

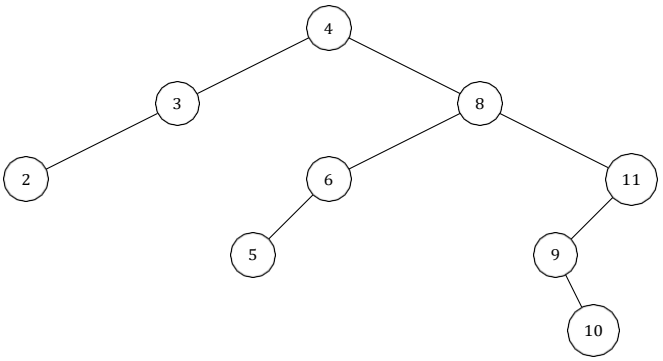


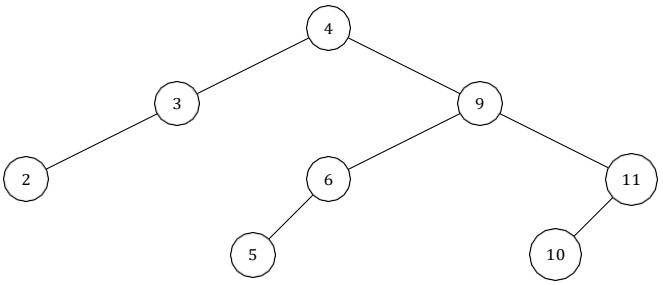
(ii) After removing 1:



(iii) After removing 8, we have **either:**

or:





[TOTAL 20]

Question 8

You are given the following adjacency matrix of a directed graph:

id	1	2	3	4	5	6
1	0	0	1	0	1	0
2	0	0	1	1	0	0
3	0	0	0	1	0	0
4	0	0	0	0	1	1
5	0	1	0	0	0	0
6	0	1	1	0	0	0

(a) Show the steps that are taken by breadth-first search when exploring this graph, starting at vertex 1. Do this by writing for each step which vertex is explored from which predecessor (e.g. "Exploring vertex 7 from vertex 9"). **[10 marks]**

Answer:

```
Exploring vertex 3 from vertex 1
Exploring vertex 5 from vertex 1
Exploring vertex 4 from vertex 3
Exploring vertex 2 from vertex 5
Exploring vertex 6 from vertex 4
```

(b) Show the steps that are taken by iterative depth-first search when exploring this graph, starting at vertex 1. Do this the same way as in Part **(a)**. **[10 marks]**

Answer:

```
Exploring vertex 3 from vertex 1
Exploring vertex 4 from vertex 3
Exploring vertex 5 from vertex 4
Exploring vertex 2 from vertex 5
Exploring vertex 6 from vertex 4
```

[TOTAL 20]