5SENG001W - Algorithms, Week

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RECAP

In the first two weeks…

- We studied several problems
- We introduced and compared several solutions
- We saw how performance of algorithms can be compared
  - Big-O notation
  - Some important complexity classes (logarithmic, linear, quadratic, exponential, …)
  - How to determine them empirically (doubling hypothesis)

Algorithms and data structures

- We have already seen how the **size** of the data affects the running time of algorithms
- The data **structure** can be just as important!
- Different data structures offer different performance for elementary operations like insertion or lookup
- Let's have a look at some examples
  - For simplicity, the data contained in the structures will be integers

Sequential data structures: Arrays and linked lists

- Arrays are one of the simplest data structures
- More versatile: Java ArrayLists and C++ vectors
  - These have flexible size, i.e. can grow to accomodate additional data
- Array entries are laid out sequentially in memory:

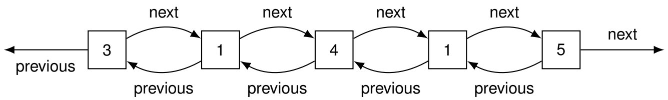
i	0	1	2	3	4
a[i]	3	1	4	1	5

- Some important basic operations on arrays:
  - Reading/writing at any given index: e.g. **`a[i] = a[i+1];`**
    Cost: constant – independent of array size
  - Adding/removing at the end (in ArrayLists or vectors)
    Cost: constant – independent of array size
  - Adding/removing in the middle
    Cost: **linear** – must move the rest of the contents!

Sequential data structures: Arrays and linked lists

- Linked lists store data in **nodes**:

```
public class ListNode{
    private int data;           // contents of this node
    private ListNode next;
    private ListNode previous;
    // ...
}
```



- Some important basic operations on linked lists:
  - Reading/writing at any given index:
    Cost: **linear** – must traverse the list to find the position!
  - Adding/removing at the end
    Cost: constant – independent of array size
  - Adding/removing in the middle
    Cost: constant – assuming we have an iterator

Algorithms and data structures

- Choice of data structure can influence what algorithms are suitable
- Typically there is a trade-off between complexities of operations
- We will be seeing other (non-sequential) structures later
  - Various kinds of trees
  - Graphs

Searching

- Common problem:
  - Given: a data structure, a piece of data
  - Question: Is that piece of data in the structure? If yes, where?
- Case 1: array

```
public static int findLocation(int[] values, int findMe){
    /* What goes here? */
}
```

Searching

- Common problem:
  - Given: a data structure, a piece of data
  - Question: Is that piece of data in the structure? If yes, where?
- Case 1: array – simplest solution:

```
public static int findLocation(int[] values, int findMe){
    for(int i=0; i<values.length; i++)
        if(values[i] == findMe)
            return i;
    return -1;  // indicates failure
}
```

Searching

- Given: a data structure, a piece of data
- Question: Is that piece of data in the structure?
  If yes, where?
- Case 2: Linked list

```
public class ListNode{
    private int data;
    private ListNode next;
    private ListNode previous;

    public ListNode findLocation(int findMe){
    /* What goes here? */
    }
}
```

Searching

- Given: a data structure, a piece of data
- Question: Is that piece of data in the structure?
  If yes, where?
- Case 2: Linked list – simplest solution

```
public class ListNode{
    private int data;
    private ListNode next;
    private ListNode previous;

    public ListNode findLocation(int findMe){
        ListNode current = this;
        while((current != null) && (current.data != findMe))
            current = current.next;
        return current;    //  null result indicates failure
    }
}
```

Brute force algorithms

- Linear search is an example of a **Brute Force** algorithm:
  - In order to find a solution, look at all possible candidates
- On the one hand, this works for many different data structures
- On the other hand, it ignores the benefits that suitable data structures offer
- Often the **easiest** algorithm to implement
- Usually not optimal

Linear vs binary search

- We had to potentially look at each value in the array/list.
- Can we do better? **Yes**, if the container is:
  - indexable
  - sorted
- Idea:
  - Check the middle element
    (This is what we need indexability for)
  - If it is...
    - what we are looking for: success
    - less: repeat with just the right half
    - greater: repeat with just the left half
  - Repeat until no candidates left

Binary search: example

- Searching for 9 in this array:

0	1	2	3	4	5	6	7	8	9	10	11
first					mid						last

- 5 is less than 9, so focus on the right half:

0	1	2	3	4	5	6	7	8	9	10	11
						first		mid			last

- 8 is less than 9, so focus on the right half:

0	1	2	3	4	5	6	7	8	9	10	11
									first	mid	last

- 10 is greater than 9, so focus on the left half:

0	1	2	3	4	5	6	7	8	9	10	11
									first mid last		

Binary search: implementation

```
public int binarySearch(int[] values, int findMe){
    // Everything between first and last is a candidate
    // Initially, that is the whole array
    int first = 0, last = values.length - 1;
    // As long as there are candidates...
    while(first <= last){
        // ...check the middle of the range.
        int middle = (first + last) / 2;
        // If it is what we are looking for: success
        if(values[middle] == findMe)
            return middle;
        // If it is less: repeat with the right half
        else if(values[middle] < findMe)
            first = middle + 1;
        // Else (i.e. it is greater): repeat with the left half
        else
            last = middle - 1;
    }
    return -1;  // indicates failure
}
```

Binary search

- Each iteration cuts the search range in half
- If the array has size n, and n is less than 2^k:
  - After 1 iteration there are less than 2^{k-1} candidates left
  - After 2 iterations there are less than 2^{k-2} candidates left
  - ...
  - After $k-1$ iterations there are less than 2 candidates left
- The search finishes after at most $k = \lceil \log n \rceil$ iterations
- So complexity of binary search is $O(\log n)$
- It relies on the data being indexable and sorted
  - Think about what happens if we try this with a linked list
  - We will be looking at sorting soon

Divide and conquer

- Recall that linear search is an example of a Brute Force algorithm
- Binary search instead follows a strategy called **Divide and Conquer**:
  In order to solve the problem on a data structure,
  1. Split the structure into smaller parts
  2. Solve the problem on each part
  3. Get an overall solution from the partial solutions
- Specifically, in binary search, these parts boil down to
  1. Split the array into the low and high half
  2. Ignore the half where the value cannot be, use recursion on the other
  3. Overall solution is the solution from the relevant half