4COSC007C Mathematics for Computing

Tutorial 4

1. Given the set A = {2,3,4,5,6,7,8,9} form a new set B which consists of all elements of set A that are:
    a. Prime numbers.
    b. Even numbers.
    c. Odd numbers that are greater or equal to 3.
    d. Those numbers that being multiplied by 2 give a number that is also an element of A.
    e. Those numbers that being multiplied by 2 give a number that is not in the given set A.
    f. Those numbers that being squared resulting in a number which also belongs to A.

2. Let N be a set of natural numbers {1,2,3…}. For each of the cases below we define a new set B using set builder notation. Your task is to list all elements of B and to establish the cardinality of B.
    a) $B = \{x: x \in N \text{ and } x^2 = x\}$
    b) $B = \{x: x \in N \text{ and } x^2 = 2x\}$
    c) $B = \{(x,y): x \in N \text{ and } y \in N \text{ and } x < y \text{ and } y \leq 3\}$
    d) $B = \{(x,y): x \in N \text{ and } y \in N \text{ and } x = y \text{ and } y < 5\}$
    e) $B = \{(x,y,z): (y \in N \text{ and } z \in N \text{ and } z = x + 2^y \text{ and } x = 10 \text{ and } 1<y < 5\}$

3. Identify if the following statements are true or false.
    a) $3 \in (3, 5]$
    b) $5 \in (3, 5]$
    c) $\{1,2,3\} \subseteq \{3,2,1\}$
    d) $\{1,2,3\} \subset \{1,2,3\}$
    e) $[1,2] \subset \{1,2,3\}$
    f) $\{1,2,3\} \subset [1,2]$
    g) $\{-1,0,1\} \subseteq [-1,1]$

4. Let R abbreviate a set of real numbers. Define the following intervals of real numbers using set builder notation:
    a) [2, 9]
    b) [2, 9)
    c) (2,9]
    d) (2, 9)

5. Let A = [2,9] be a closed interval of all real numbers 2 to 9. Which new interval is introduced by the following set builder notation: $\{ x \in A: \sqrt{x} \in A\}$

6. Define, using the set builder notation, the set C which is obtained via the following operations to sets A and B:

   a) $C = A \cap B$
   b) $C = A \cup B$
   c) $C = A \backslash B$
   d) A'

7. Let A and B be the only sets in U and A = {5, 6, 10, 12} and B = {5, 7, 11} Apply the following operations to sets A and B

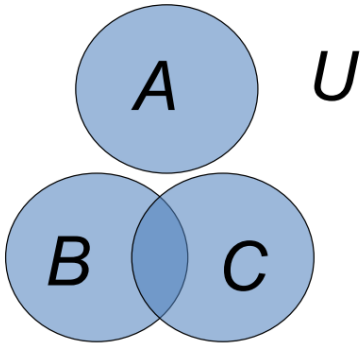
   a) $A \cup B$
   b) $A \cap B$
   c) $A \backslash B$
   d) $B \backslash A$
   e) A'
   f) B'
   g) $A' \cap B'$

8. Below U is the universal set, {} is the empty set and A is an arbitrary set. Based on the definition of the empty and universal sets establish what should be the resulting set of the following operations:

   a) $A \cap U$
   b) $A \cup U$
   c) $A \backslash U$
   d) $(U \backslash A) \cup A$
   e) $\{\} \cup A$
   f) $\{\} \cap A$

9. Consider sets A = {a}, B = {g, h, i, j} and C = {i, j, k, l}. In the Venn diagram below place the elements of the following sets and establish what are the sets resulting in the following operations:

   a) $B \cap C$
   b) $A \cup B$
   c) $A \cap B$
   d) $B \cup (B \cap C)$
   e) $A \cap \{B \setminus U\}$
   f) $U \cap (A \cup B)$



10. Let A = {a,b,c} and B = {1,0}.
    a) Write down all elements of the Power Set of A and Power Set of B
    b) List all elements of AxB
    c) List all elements of BxA
    d) Calculate |AxB| and |BxA|
    e) What is AxB \ BxA

11. CHALLENGE

a. Show that if A ⊆ B and B ⊆ A then A = B, i.e. that A and B are equivalent, i.e. they have the same elements.
b. Let A = [2,9] be a closed interval of all real numbers 2 to 9 Which interval is introduced by the following set builder notation: { $x \in$ A: $\sqrt{x} \in$ A & x < $\sqrt{20}$ }

12. CHALLENGE

Let A = {m | m is an integer satisfying 0 < m < 13} and B = {n | n is an integer satisfying 7 < n < 23}. Calculate |(A x B) \ (B x A)|.

Note that the task is to only calculate the number of elements but not to list the elements of the resulting set!