MATHEMATICS FOR COMPUTING

WEEK 6

WHAT IS A MATRIX?

- A matrix is a two-dimensional array of elements arranged in rows and columns

$$\mathbf{A} = \begin{bmatrix} a_{11}, a_{12} \ldots, a_{1n} \\ a_{21}, a_{21} \ldots, a_{2n} \\ \ldots\ldots\ldots\ldots \\ a_{m1}, a_{m2} \ldots, a_{mn} \end{bmatrix}_{m\times n}$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 1 & 2 \\ -3 & 4 & 5 \\ \frac{1}{2} & 2 & 1 \end{pmatrix}$$

- Any element

$a_{ij}; i - row\ number$ and $j - column\ number$ is a real number

SPECIAL TYPES OF MATRICES

Row matrix

$$A = \begin{bmatrix} a_1, a_2, \ldots, a_n \end{bmatrix}_{1\times n}$$

Column matrix

$$B = \begin{bmatrix} a_1 \\ a_2 \\ \cdots \\ a_m \end{bmatrix}_{m\times 1}$$

Square matrices

$$\mathbf{A} = \begin{bmatrix} a_{11}, a_{12} \ldots, a_{1n} \\ a_{21}, a_{21} \ldots, a_{2n} \\ \ldots\ldots\ldots\ldots \\ a_{n1}, a_{n2} \ldots, a_{nn} \end{bmatrix}_{n\times n}$$

$$where\ m = n$$

EQUAL MATRICES

- Same dimensions and same entries at corresponding positions of A and B, then A equals B
- Denoted as A=B

E.g. $A = \begin{bmatrix} 1 & 0 \\ 7 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Given that A=B

a=? , b=? , c=? , d=?

SPECIAL MATRICES: NULL

- The null matrix, written **0**, is the matrix all of whose components are zero.
- E.g. The null matrix of order 2×3 is

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

SPECIAL MATRICES: DIAGONAL

- A square matrix
- All off diagonal entries are zero(All entries are zero except the main diagonal)

$$D = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & a_{nn} \end{bmatrix} \quad \text{or} \quad D = \mathrm{diag}[a_{11}, a_{22}, \ldots, a_{nn}]$$

SPECIAL MATRICES: IDENTITY

- The identity matrix, written $\mathbf{I}$, is a square matrix all of which entries are zero except those on the main diagonal, which are ones.

Examples of identity matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

MATRIX TRANSPOSE

- The transpose of a matrix is a new matrix that simply has the rows and columns exchanged
- We denote the transpose of matrix **A** as **A**$^{\mathbf{T}}$

$$If\ \ A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\ then\ \ A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

SYMMETRIC MATRICES

- A square matrix **A** is said to be symmetric if $\mathbf{A}^T = \mathbf{A}$
- Equivalently, a matrix is symmetric if it is symmetric about its main diagonal.

Example: Which of the following matrices is symmetric?

$$\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

SPECIAL MATRICES: UPPER AND LOWER TRIANGULAR

Upper triangular matrices

All lower diagonal entries are zero

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & & a_{2n} \\ \vdots & & \ddots & \\ 0 & 0 & & a_{nn} \end{bmatrix}$$

E.g. $\begin{pmatrix} -2 & 0 & -1 \\ 0 & -4 & 3 \\ 0 & 0 & 5 \end{pmatrix}$

Lower triangular matrices

All upper diagonal entries are zero

$$\begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & & 0 \\ \vdots & & \ddots & \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix}$$

E.g. $\begin{pmatrix} 4 & 0 & 0 \\ -3 & -3 & 0 \\ 0 & -4 & -1 \end{pmatrix}$

BRAIN FOOD

Give the size of each of the following matrices:

$$A = \begin{pmatrix} 1 & 0 & 0 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 3 & 7 \\ 9 & 8 \\ 9 & -9 \end{pmatrix} \qquad C = \begin{pmatrix} 1 \end{pmatrix} \qquad D = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 9 \end{pmatrix}$$

How many elements are there in a matrix whose size is

(a) 3×1 (b) 1×3 (c) $m \times n$ (d) $n \times n$?

BRAIN FOOD

Given that the following matrices are equal (A = B), find the values of *x*, *y*, and *z*.

$$A = \begin{bmatrix} 2 & 1 \\ \frac{2}{3} & -5 \\ 6 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} x & 1 \\ \frac{2}{3} & y-10 \\ \frac{z}{2} & 4 \end{bmatrix}$$

MATRIX ADDITION

- To add matrices we simply add numbers in corresponding positions. However, to add matrices they must be of the same order.

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 6 \\ 5 & 2 \end{pmatrix}$$

$$C = A + B = \begin{pmatrix} 2+0 & 3+6 \\ 4+5 & 5+2 \end{pmatrix} = \begin{pmatrix} 2 & 9 \\ 9 & 7 \end{pmatrix}$$

MATRIX SUBTRACTION

- Works a lot like addition

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 9 & 8 & 7 \end{pmatrix} \qquad B = \begin{pmatrix} 5 & 1 & 1 \\ 5 & 7 & 9 \end{pmatrix}$$

$$C = A - B = ??$$

MATRIX MULTIPLICATION BY A SCALAR

$$A = \begin{pmatrix} 1 & 3 \\ 5 & 0 \end{pmatrix}$$

$$2A = 2 \times \begin{pmatrix} 1 & 3 \\ 5 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 10 & 0 \end{pmatrix}$$

MATRIX PROPERTIES

1. $A + B = B + A$
2. $A + (B + C) = (A + B) + C$
3. $b\,(A + B) = bA + bB$
4. $(b+d)A = bA + dA$
5. $b(dA) = (bd)A = d(bA)$

BRAIN FOOD

Find the resulting matrices

i) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix}$

ii) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 1 & 7 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 3 & 9 \\ 7 & 0 & 0 & 1 \end{pmatrix}$

iii) A= $\begin{pmatrix} 1 & 0 & 1 \\ -2 & 1 & 3 \\ 4 & 1 & 0 \end{pmatrix}$ B= $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 4 & 1 & 1 \end{pmatrix}$

Given A and B find

2A + 3B

3A – B

MATRIX MULTIPLICATION

- This is a very useful operation because it enables us to combine a series of operations into one matrix – more of which later
- If we have two matrices **A** and **B** then we have two products **AB and BA**
- Be careful, matrix multiplication is NOT commutative. That is, usually **AB** $\neq$ **BA**

MATRIX MULTIPLICATION

- Consider two matrices A and B

[m x n] and [p x q]

- The multiplication AB exists if the dimensions

[m x n] and [p x q]

n = p

And resulting matrix have

[m x n] and [p x q]

m x q

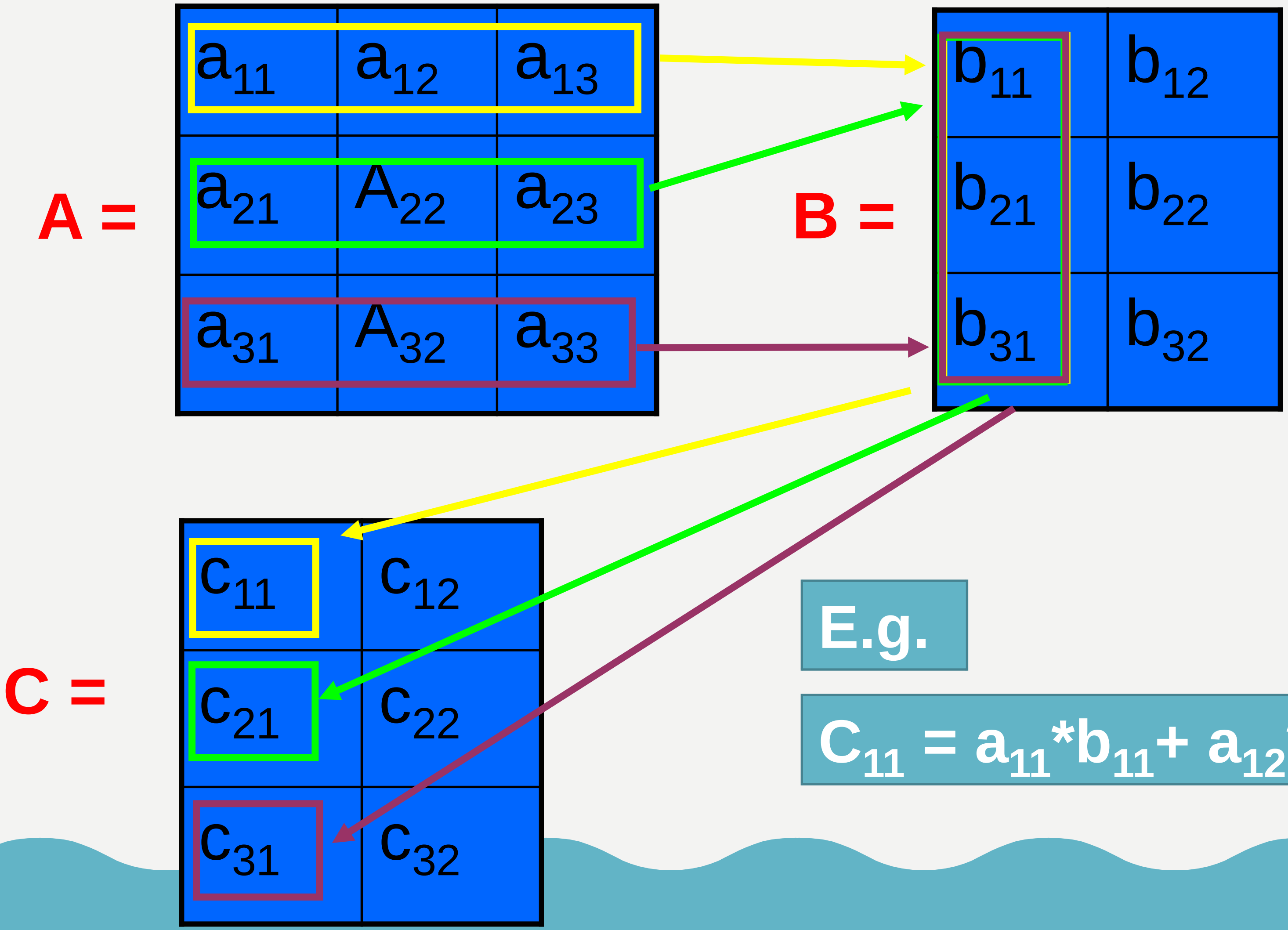
- To find BA?

BRAIN FOOD

Determine if the following matrices can be multiplied, and the order of the resulting matrix

Matrix A	Matrix B	Can find AB?	Can find BA?	Resulting Matrix
4x3	3x3			
2x1	2x1			
2x4	4x1			
3x3	3x3			
4x4	3x4			

HOW TO MULTIPLY C = AB



E.g.

$C_{11} = a_{11}*b_{11}+ a_{12}*b_{21}+a_{13}*b_{31}$

MATRIX PROPERTIES

1. $(AB)C = A(BC)$
2. $A(B+C) = AB + AC$
3. $(A+B)C = AC + BC$
4. $A(kB) = k(AB) = (kA)B$

MATRIX TRANSPOSE PROPERTIES

(1) $(A^T)^T = A$

(2) $(A + B)^T = A^T + B^T$

(3) For a scalar c, $(cA)^T = cA^T$

(4) $(AB)^T = B^T A^T$

BRAIN FOOD

- Let

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 1 & 3 \\ 4 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 4 & 1 & 1 \end{pmatrix}$$

Show that these matrices satisfy

1. $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
2. $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$

QUESTIONS?

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