MATHEMATICS FOR COMPUTING

WEEK 9

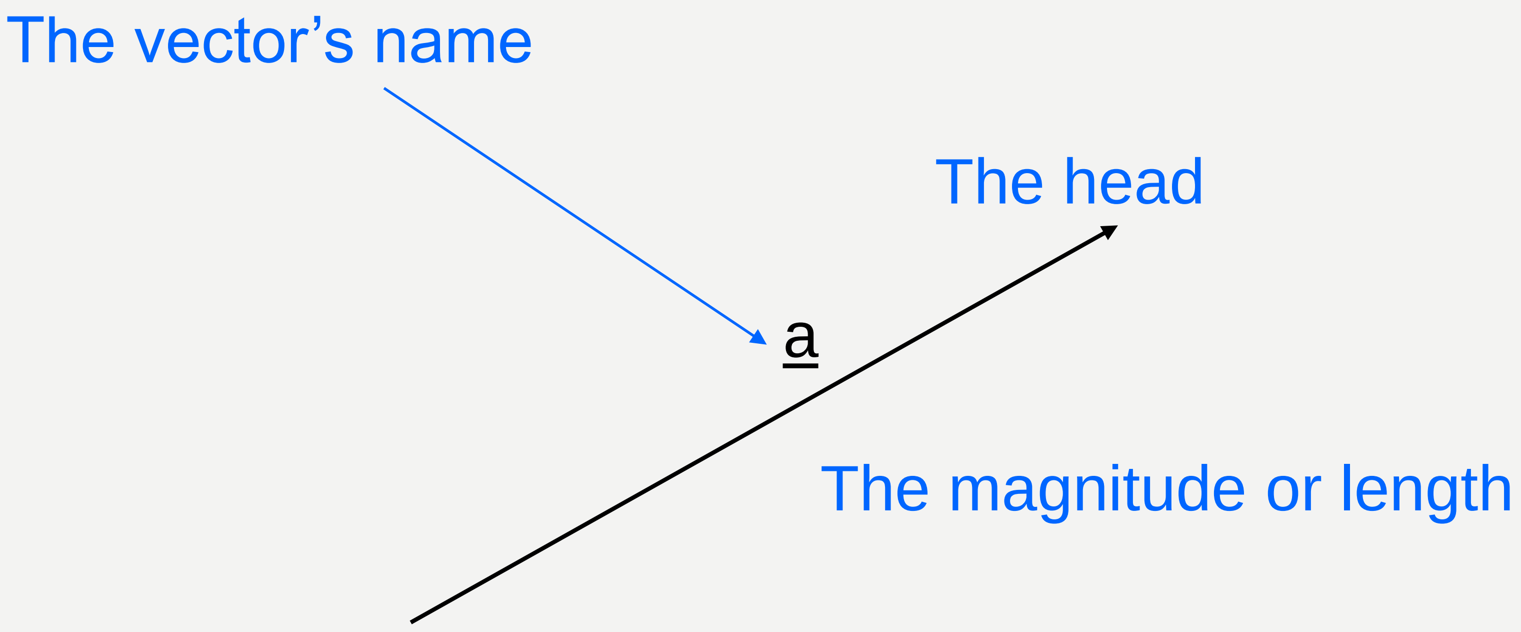
VECTORS

A **vector** is a quantity that is determined by its **magnitude** and **direction**.

A **scalar** is a quantity that is determined by its **magnitude**.

What are some of the vectors you know of?

GEOMETRIC REPRESENTATION



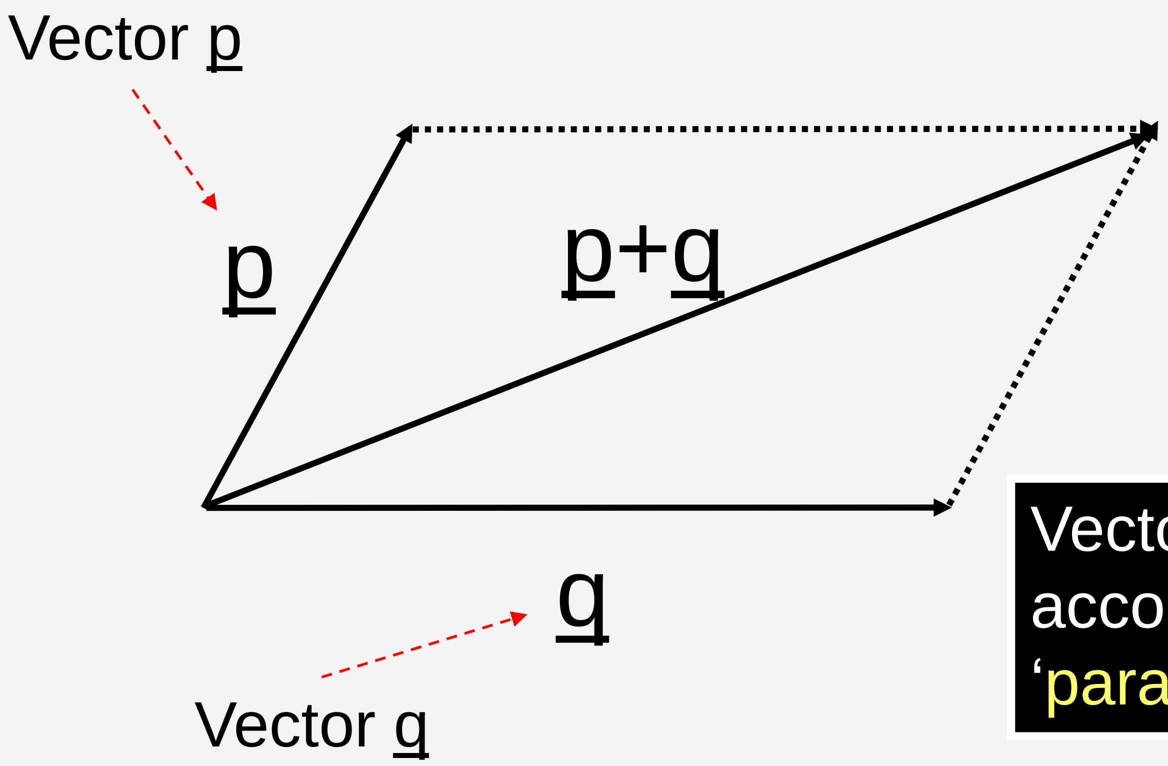
ZERO VECTOR

The zero vector is the vector whose magnitude is zero, and whose direction is arbitrary. It is denoted by the symbol **0**

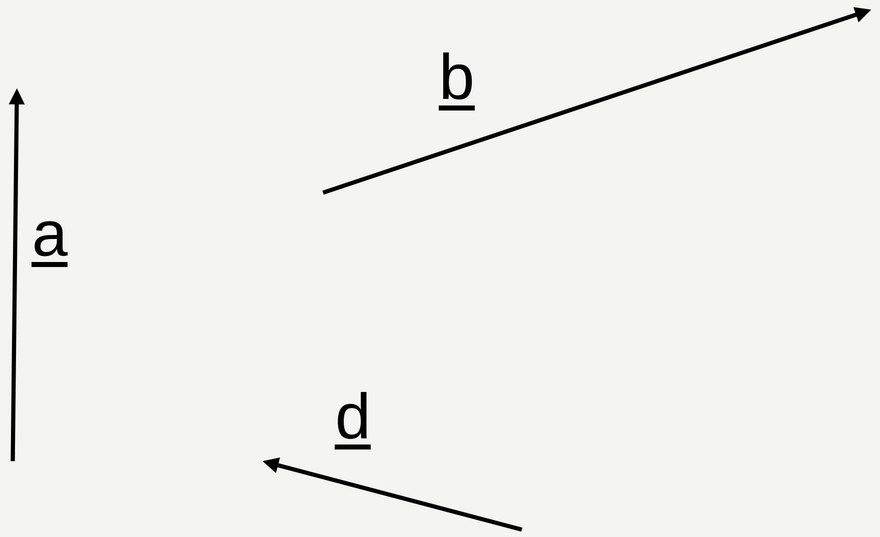
What is the displacement vector of a point from itself?

OPERATIONS

ADDITION



Vectors can be added according to the 'parallelogram law'

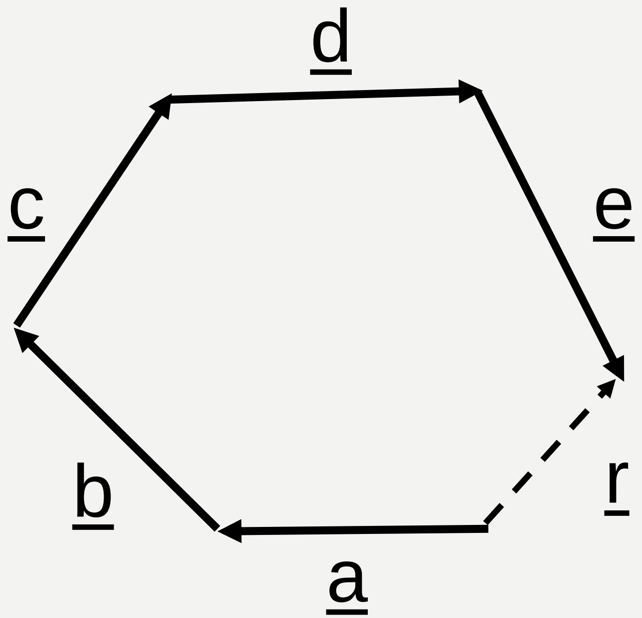


BRAIN FOOD

Add the following vectors a + b, a + d, b + d

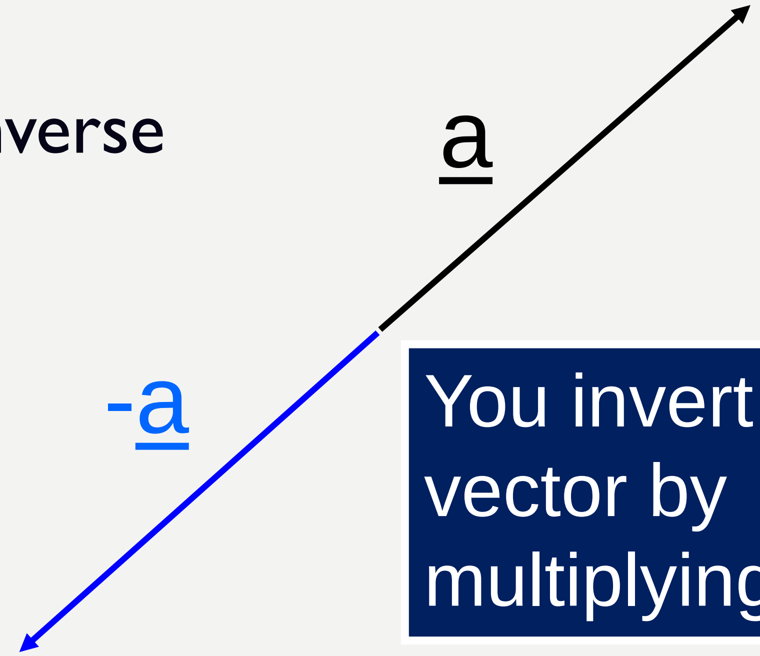
ADDING MULTIPLE VECTORS

r is the vector resulting from the addition of the other vectors



SUBTRACTION

- Pictorially, this is very similar to adding them except that you add the inverse of one of the vectors
- If we have a vector $\underline{\mathbf{a}}$ then $-\underline{\mathbf{a}}$ is its inverse
- Consider the following:
  - $\underline{\mathbf{c}} = \underline{\mathbf{a}} - \underline{\mathbf{b}}$
  - or $\underline{\mathbf{c}} = \underline{\mathbf{a}} + (-\underline{\mathbf{b}})$



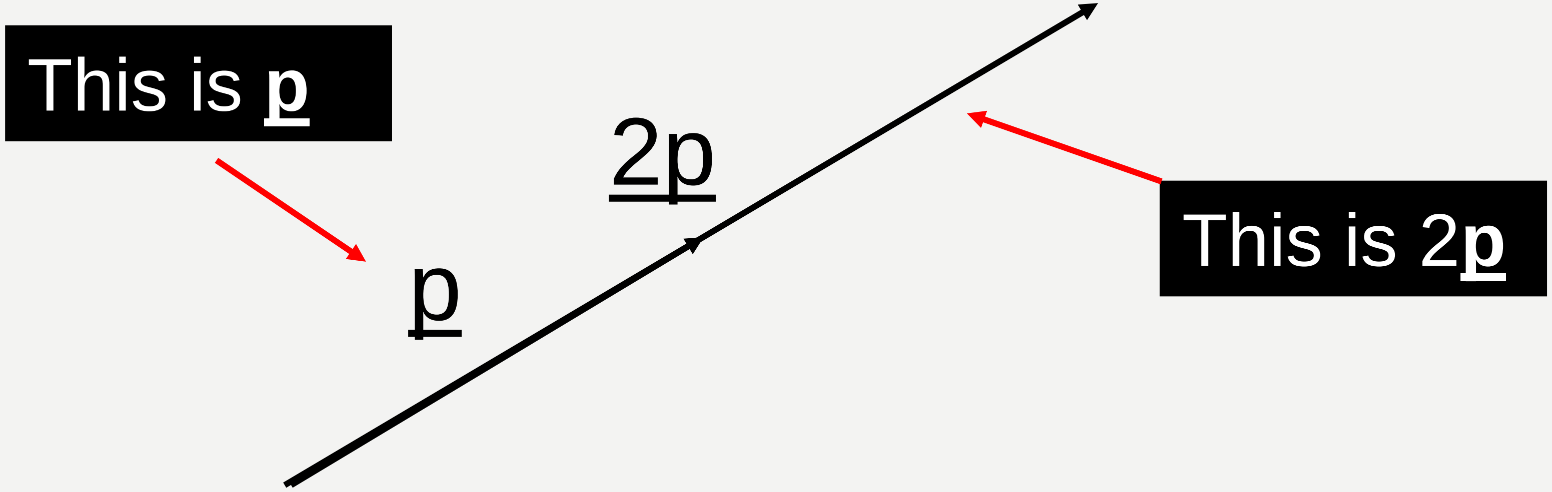
You invert a vector by multiplying it by -1

MULTIPLYING BY A SCALAR

Vectors can be multiplied by any scalar

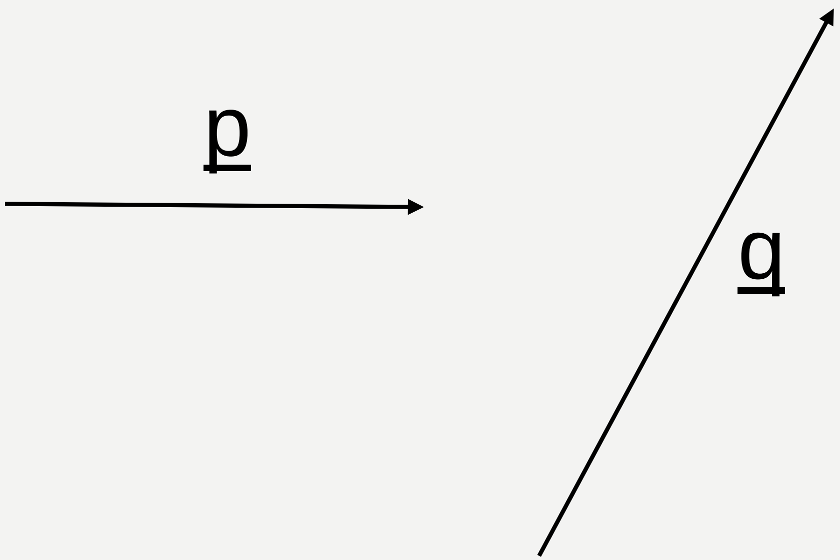
Multiplying by -1 gives us the inverse of a vector

Multiplying by 1 leaves the vector unchanged



BRAIN FOOD

For the vectors **p** and **q** shown below sketch **p** + **q**, **p** – **q** and 2**p** + 1/2**q**



ALGEBRAIC RULES FOR SCALING AND ADDING VECTORS

Let $\underline{\mathbf{a}}$, $\underline{\mathbf{b}}$ and $\underline{\mathbf{c}}$ be vectors, and let m, m_1, and m_2 be scalars.

1. Addition is commutative: $\underline{\mathbf{a}} + \underline{\mathbf{b}} = \underline{\mathbf{b}} + \underline{\mathbf{a}}$
2. Addition is associative: $(\underline{\mathbf{a}} + \underline{\mathbf{b}}) + \underline{\mathbf{c}} = \underline{\mathbf{a}} + (\underline{\mathbf{b}} + \underline{\mathbf{c}})$
3. $m\underline{\mathbf{a}}$ is a vector with magnitude $|m|\ |\underline{\mathbf{a}}|$, in the same direction as $\underline{\mathbf{a}}$ when $m > 0$ and in the opposite direction when $m < 0$
4. Scaling is associative: $m_1(m_2\underline{\mathbf{a}}) = (m_1m_2)\underline{\mathbf{a}}$
5. Scaling is distributive: $(m_1 + m_2)\underline{\mathbf{a}} = m_1\underline{\mathbf{a}} + m_2\underline{\mathbf{a}}$
6. Addition and scaling involving the zero vector are as expected: $\mathbf{0} + \underline{\mathbf{a}} = \underline{\mathbf{a}}$ and $\mathbf{0}\underline{\mathbf{a}} = \mathbf{0}$
7. Subtraction is defined by $\underline{\mathbf{a}} - \underline{\mathbf{b}} = \underline{\mathbf{a}} + (-1)\ \underline{\mathbf{b}}$

TRANSFORMATIONS


COFFEE

TRANSFORMATIONS

Transformation is a general term for four specific ways to manipulate the shape of a point, a line, or shape.

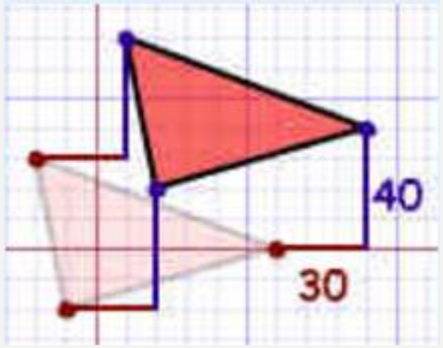
– Translation
– Reflection
– Rotation
– Dilation (resizing)

TRANSLATION

To Translate a shape:

– Every point of the shape must move:

- the same distance
- in the same direction



Example: to say the shape gets moved **30 Units in the "X" direction**, and **40 Units in the "Y" direction**, we can write:

$$(x, y) \rightarrow (x + 30, y + 40)$$

Which says "all the x and y coordinates become x+30 and y+40"

TRANSLATION IN MATRIX FORMAT

The algebraic form of translation is:

$$x' = x + t_x \qquad\qquad y' = y + t_y$$

and the equivalent in matrix form is:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

HOMOGENOUS COORDINATES

- What we do is augment our vertices with a further ‘coordinate’, which is usually called *w*.

w is any arbitrary non-zero constant, called the ‘weight’.

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

- This converts the vertices into *homogeneous coordinates*
- Most of the time we set the value of *w* to 1
- This means that our coordinates are now represented by 3x1 order matrices (2D)

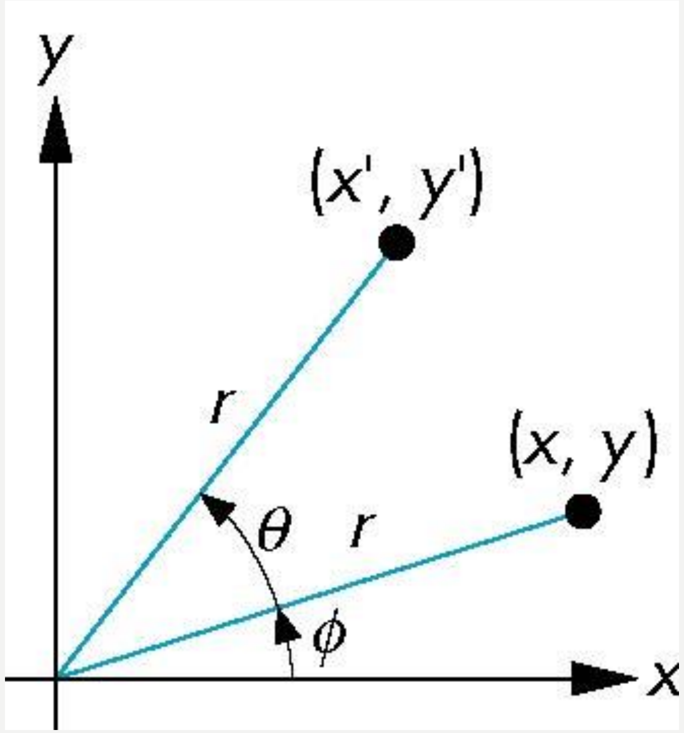
TRANSLATION WITH 'W'

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

ROTATION – 2D

- **Consider rotation about the origin by ϕ degrees :** *radius stays the same, angle increases by ϑ*

$$x' = x \cos\theta - y \sin\theta$$
$$y' = x \sin\theta + y \cos\theta$$



$$x' = r \cos(\phi + \theta)$$
$$y' = r \sin(\phi + \theta)$$

$$x = r \cos\phi$$
$$y = r \sin\phi$$

THE ROTATION MATRIX – 2D

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

CONCATENATING?

WHAT DO WE DO IF WE WANT TO PERFORM MULTIPLE TRANSFORMATIONS TOGETHER?

- A point is given at (2,2) and it goes under the following transformations.
  - Translated by (3,3)
  - Rotated by 90 degrees

BRAIN FOOD

1. Find the translation matrix
2. Find the rotation matrix
3. Find the transformation matrices for TR and RT orders

4, Find and compare new coordinates for both transformation orders

CONCATENATION WRAPPED UP

- The order in which you multiply matrices matters
- Multiplying **R** by **T** gave a different result to multiplying **T** by **R**
- Since we are placing objects in world coordinates, we generally want **TR**, **which is Rotate first and then Translate.** We read it from right to left
- In general if we have to apply several transformation matrices $\mathbf{M_1}$, $\mathbf{M_2}$, $\mathbf{M_3}$,…..to a point X THE composite operation is given by:

$$\mathbf{X' = ...M_3M_2M_1X = MX}$$

QUESTIONS?

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