4COSCOO2W Mathematics for Computing

Tutorial 9 Tasks and Answers

Aim: A few exercises on **induction-recursion**. Apply induction proof to establish equality of algebraic expressions (alternative recursive and function forms), calculate the first terms of an arithmetic sequence defined recursively.

Pre-tutorial work (independent study):

- **Revise:** Lecture 9
- **Reading:** Chapter 7 *Peter Grossman (2009) Discrete Mathematics for computing*

TASK 1

Let ***t*(*n*)** be a sequence defined for all non–negative integers by the following recursive definition:

$$t(0) = 1$$

$$t(1) = 3$$

$$t(n) = t(n\text{-}2) + t(n\text{-}1),\ n \geq 2$$

Find out the value of **t(7)**.

TASK 2

Prove by induction that the following statement is true for whichever value of natural number ***n***:

P(*n*) statement: $\sum_{i=0}^{n} 2^{(i-1)} = 2^n - 1$

TASK 3

Prove by induction that the following statement is true for whichever value of natural number ***n***:

P(*n*) statement: $2^n > n$

TASK 4 (Challenge)

Find a formula in function form to use as an alternative to the sum (which can be seen as the successive term of a sequence expressed in recursive form):

$$\sum_{i=1}^{n} \frac{1}{2^i}$$

Hint: evaluate the sum for the cases n=1,2,3,4. You will see a pattern emerge from the calculation results. Guess the function form formula. Make a statement (in the fashion of task 2 and 3) and prove it by induction.