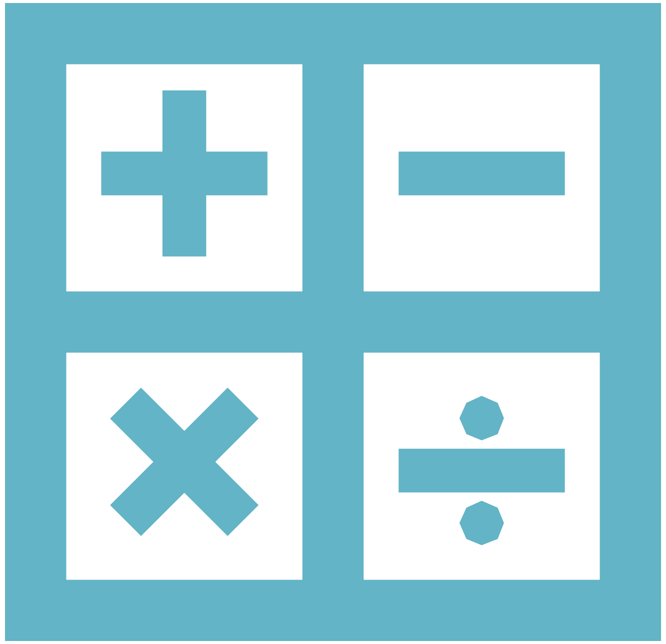
MATHEMATICS FOR COMPUTING

WEEK 3



RELATIONS

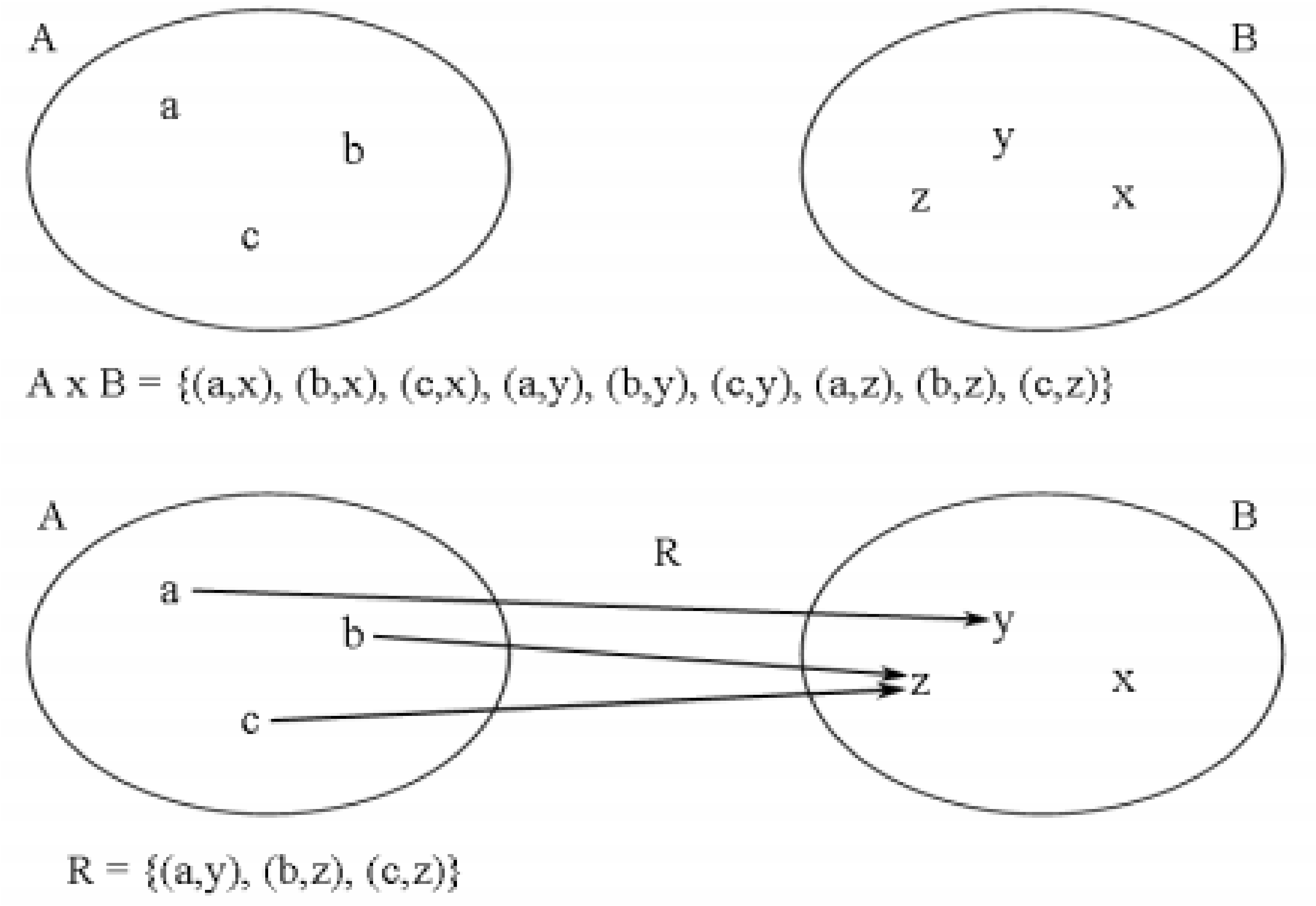
RELATIONS: DEFINITION

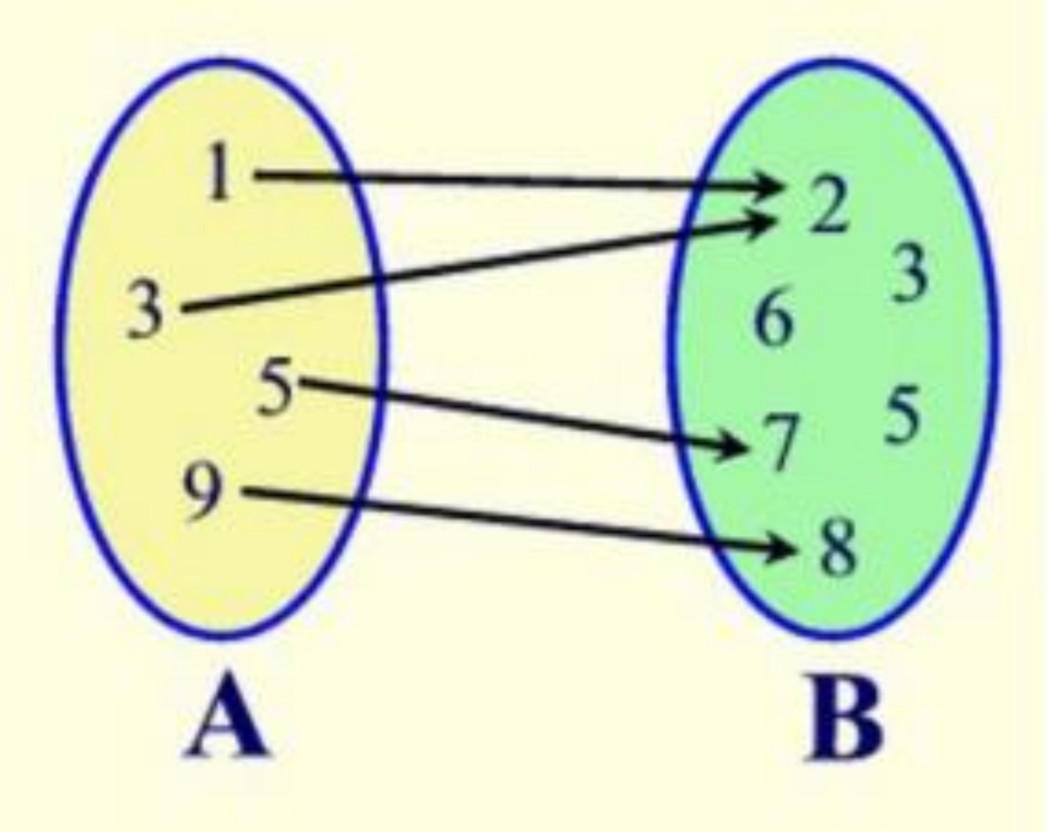
A relation between two sets is a collection of ordered pairs containing one object from each set.

If the object x is from the first set and the object y is from the second set, then the objects are said to be related if the ordered pair (x,y) is in the relation.

EXAMPLE

- Suppose that A and B are sets.
- A relation from A to B is a subset of AxB





BRAIN FOOD

a) What is AxB?

b) What is R?

c) What is the domain?

d) What is the co-domain?

e) What is the range?

The range is the dependent variables.

Can relations work only between two sets?

When a relation is a set of pairs, then it is a BINARY relation

When a relation is a set of triples, then it is a TERNARY relation

It can go on for more!

PROPERTIES OF RELATIONS

- Reflexive: *if for all* $x \in A$, $(x,x) \in R$
- Symmetric: *if for all* $x, y \in A$, *if* $(x,y) \in R$ *then* $(y,x) \in R$
- Transitive *if for all* $x,y,z \in A$, *if* $(x,y) \in R$ *and* $(y,z) \in R$, *then* $(x,z) \in R$

Consider the set {1, 2, 3, 4}

BRAIN FOOD

a) Is R = {(1,1), (2,2), (3,3)} reflexive?

b) Is R = {(1,2), (2,1), (3,1)} symmetric?

c) Is R = {(1,2). (2,3), (1,3), (2,1)} transitive?

d) Is R = {(1,1), (2,2), (3,3)} symmetric? Transitive?

A relation which is reflexive, transitive and symmetrical is called EQUIVALENCE

FUNCTIONS

$f : A \to B$

BRAIN FOOD

$f(x) = 2x + 3$ $g(x) = 7$

Given the two formulas find the below:

$f(5)$

$f(8)$

$f(-4)$

$g(3)$

$g(7)$

$g(-10)$

FUNCTIONS

A relation is a function if and only if you can take EVERY value in the domain, put the value in a formula, and get a SINGLE value in the co-domain

BRAIN FOOD

Suppose I define my Domain to be $\{1, 2, 3\}$
And I define my Codomain to be $\{5, 6, 7, 8\}$
And my formula is $f(x) = x + 5$

Is this a function?

What would happen if the co-domain was changed to $\{5,6,7\}$?

Is x^2 a function?

Is $\sqrt{x}$ a function?

- R is reflexive, symmetric and transitive
- R is reflexive and symmetric but not transitive
- R is reflexive and transitive but not symmetric
- R is reflexive but not transitive nor symmetric
- R is not reflexive, nor transitive, nor symmetric
- R is not reflexive, nor symmetric, but is transitive
- R is not reflexive, nor transitive but symmetric
- R is not reflexive, but is transitive and symmetric

BRAIN FOOD

Find the answers to the given scenarios given that A is a set with 3 elements (i.e. A = {1,2,3}) and R is a relation on A.

Out of your answers, which ones are functions?

GRAPHS

GRAPHS

A graph is a finite set of nodes, with edges between the nodes

Formally, a graph G is a structure (V,E) where

- V is a finite set of nodes, and
- E is a set of pairs of the form (x,y) where x and y are nodes in V

GRAPHS

Consider $V = \{1,2,3,4,5,6\}$ and $E = \{(1,2), (2,4), (3,6)\}$

This is a graph where only several vertices are connected.

There is some pattern represented here – E is a set of all pairs (x , y) such that

$$y = 2x$$

BRAIN FOOD

cheese	£2.99	juice	£4.99
beans	£3.99	cake	£3.99
beer	£2.99	coke	£3.59

V is a set of 6 discounted products

E is a set of all pairs (x,y) such that x is cheaper than y

Identify the sets V and E. Draw the graph representing the relationship between V and E

WHY GRAPHS?

- The nodes represent entities (objects such as products, cars, numbers, words, etc.)
- Edges (x,y) represent relationships between entities x and y, such as:
    - “x < y”
    - “x is cheaper than y”
    - “x is bigger than y”
    - “x larger than y”
    - “x is a longer than y”
    - “x is faster than y”
    - And anything more!

DIRECTED GRAPHS

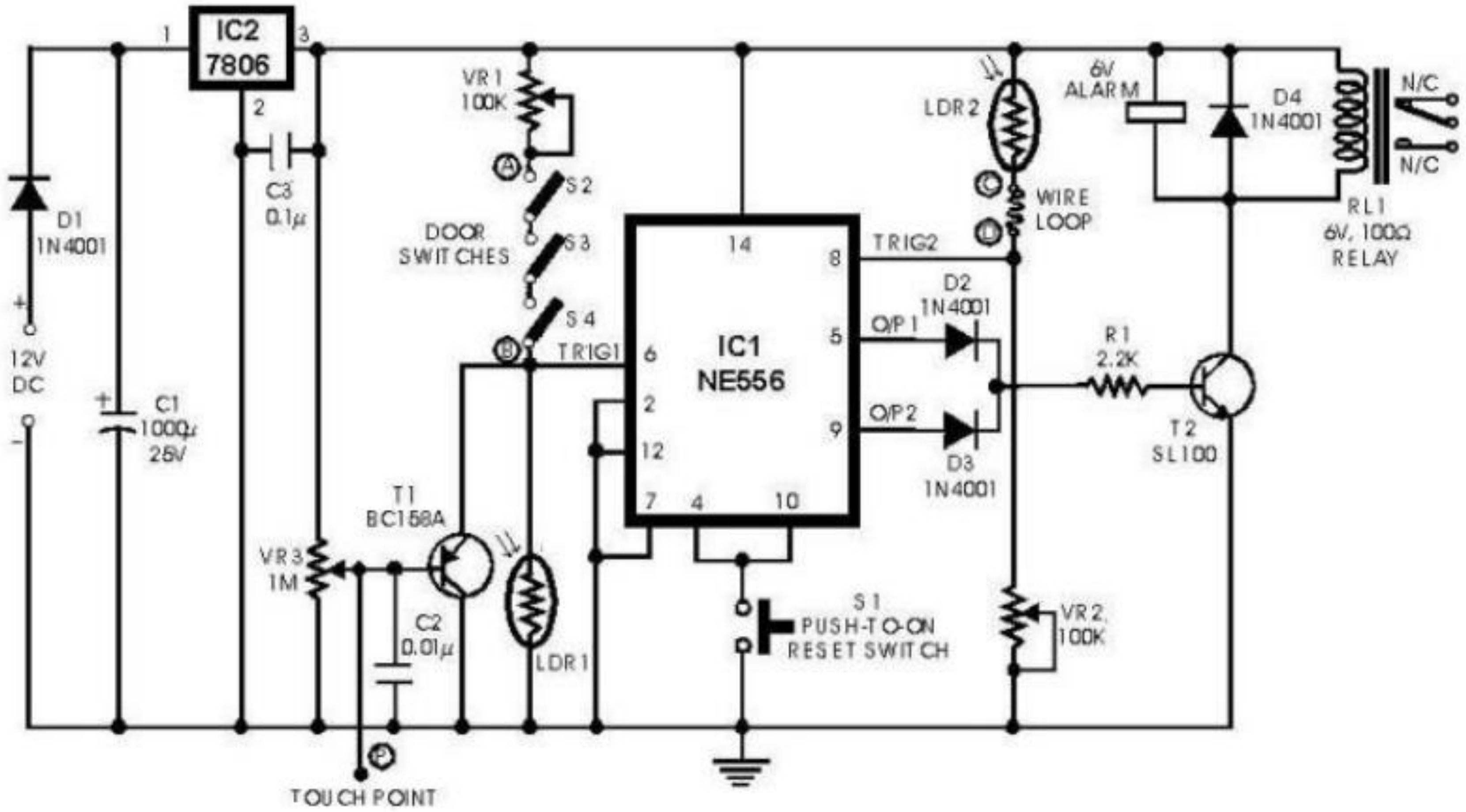
If the directions of the edges matter, then we show the edge directions, and the graph is called a directed graph (or a digraph)

Formally, a directed graph G is a structure (V,E) where

- V is a finite set of nodes, and
- E is a set of ordered pairs $\{(x,y): x \in V, y \in V\}$

Electrical Circuits.

Vertices represent diodes, transistors, capacitors, switches, etc., and edges represent wires connecting them.



UNDIRECTED GRAPHS

If the directions of the edges does not matter the graph is called an undirected graph

Formally, an undirected graph G is a structure (V,E) where

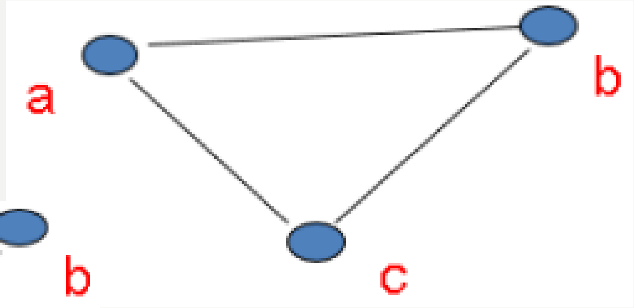
- V is a finite set of nodes, and
- E is a set of unordered pairs $\{\{x,y\}: x \in V, y \in V\}$

Road Maps.

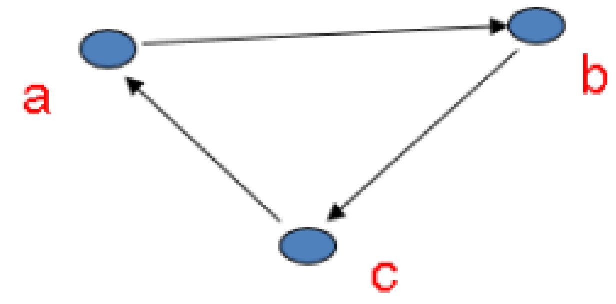
Edges represent streets and vertices represent crossings.

PATHS AND CYCLES

- A path in a graph G is a sequence of nodes $x_1, x_{2,\ldots,} x_k$, such that for any node x_i $(1 \leq \mathrm{i} \leq \mathrm{k})$ there is an edge from it to the next one in the sequence
- A cycle in a graph G is a path where the last node is the same as the first node.

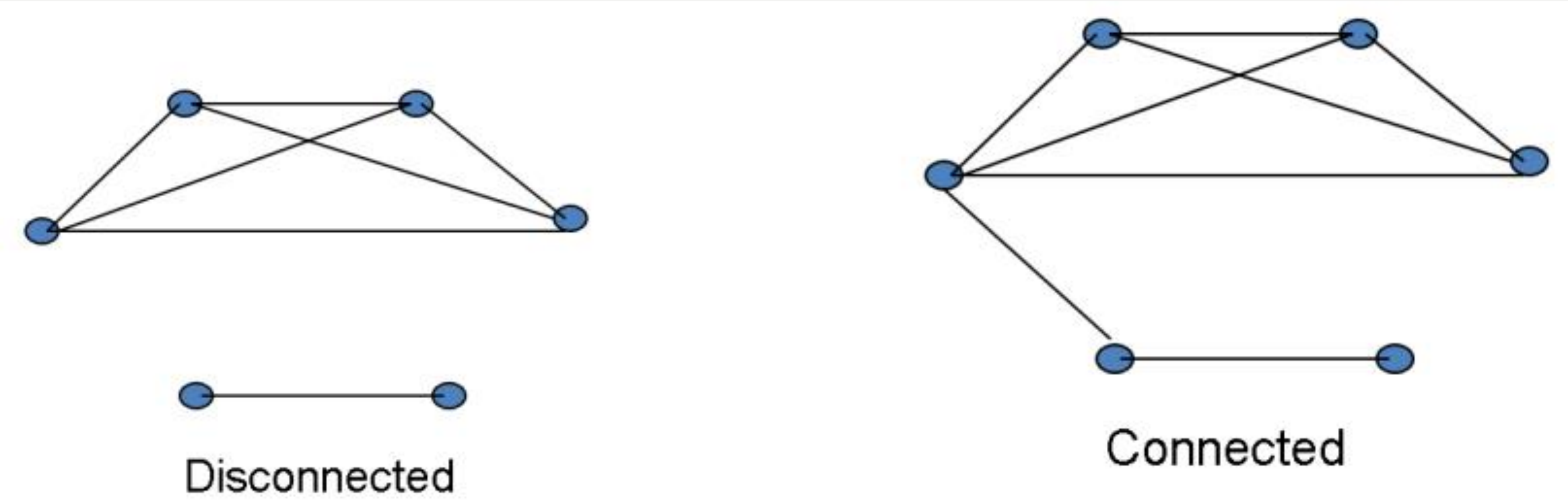


What are the paths and cycles in the given graphs?



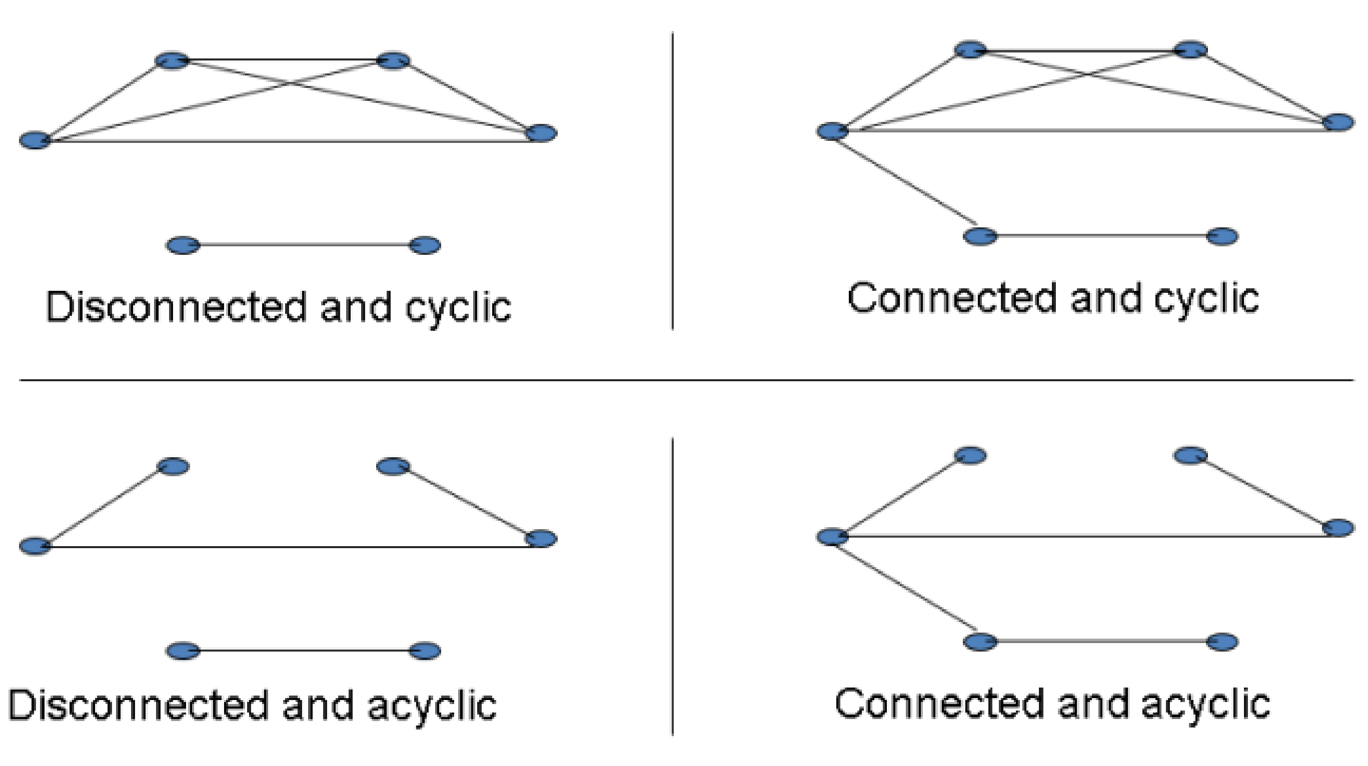
CONNECTIVITY

A graph is connected if there is a path between every pair of nodes. Otherwise, the graph is disconnected



CYCLICITY

A graph is cyclic if it has at least one cycle. Otherwise, it is acyclic

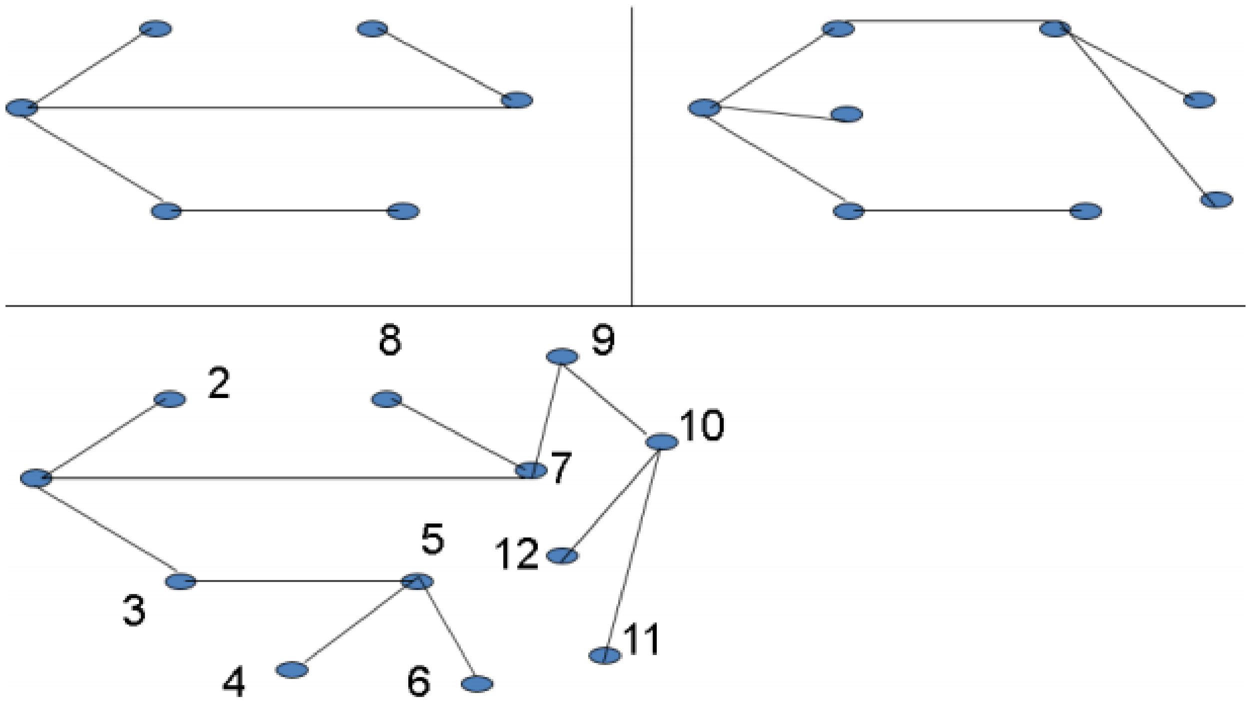


TREES

TREES

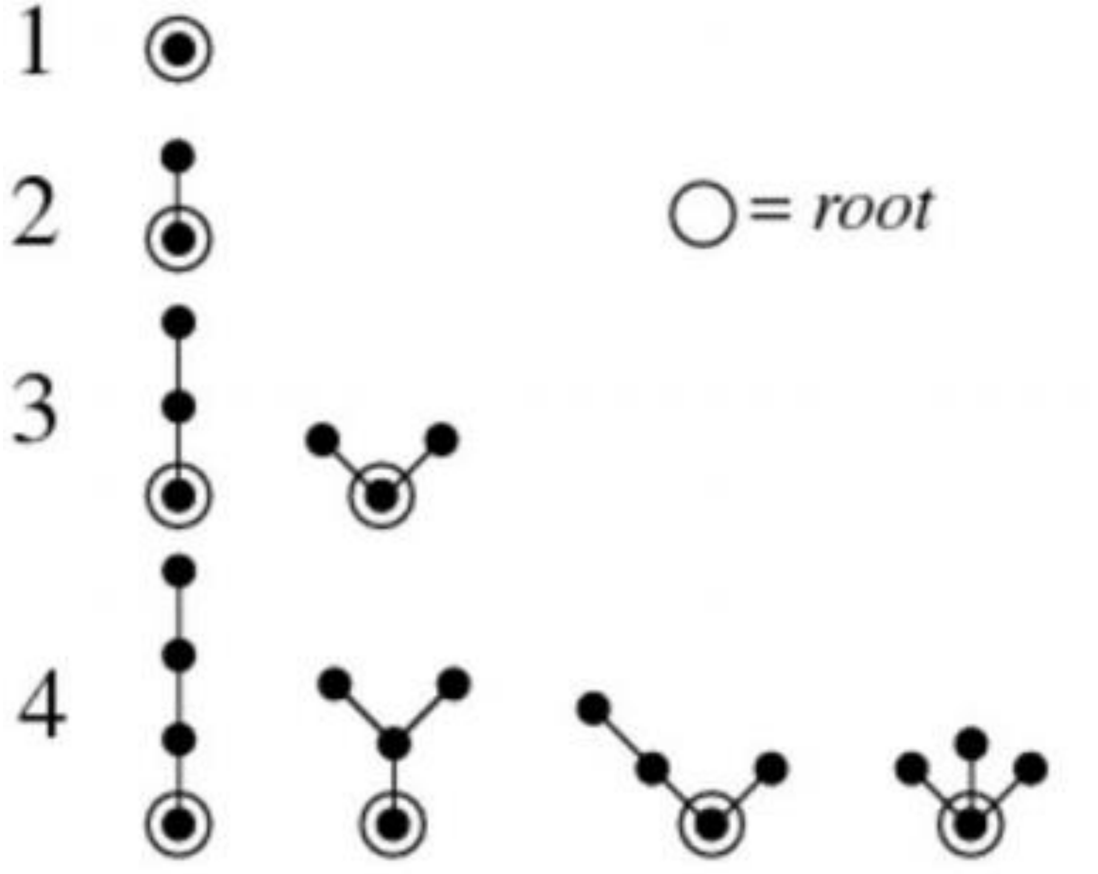
A tree is a connected, acyclic, undirected graph.

In a tree any two nodes are connected by exactly one path.

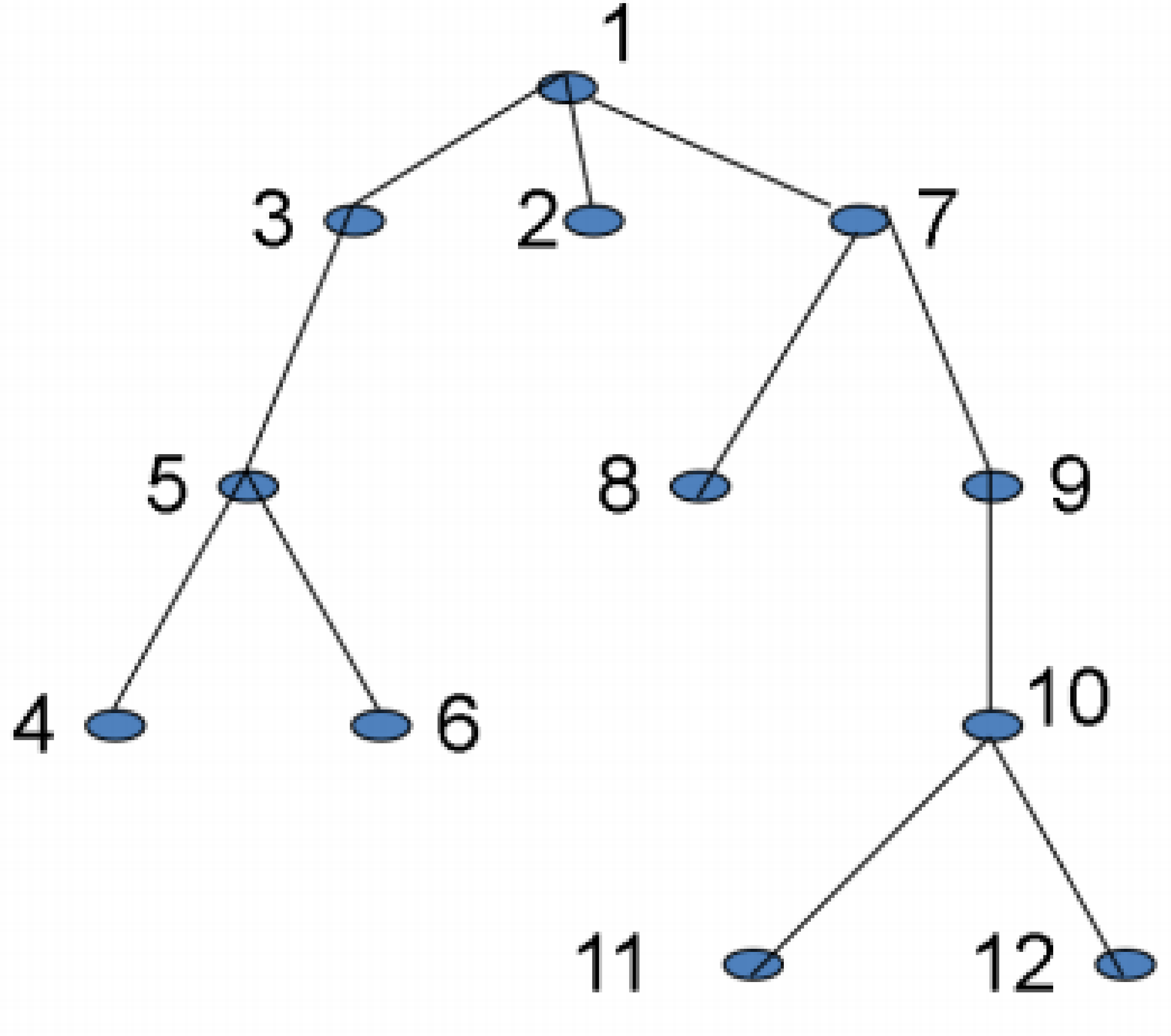


ROOTED TREES

A rooted tree is a tree where one of the nodes is designated as the root node. (We cannot have two roots of a tree)



rooted trees



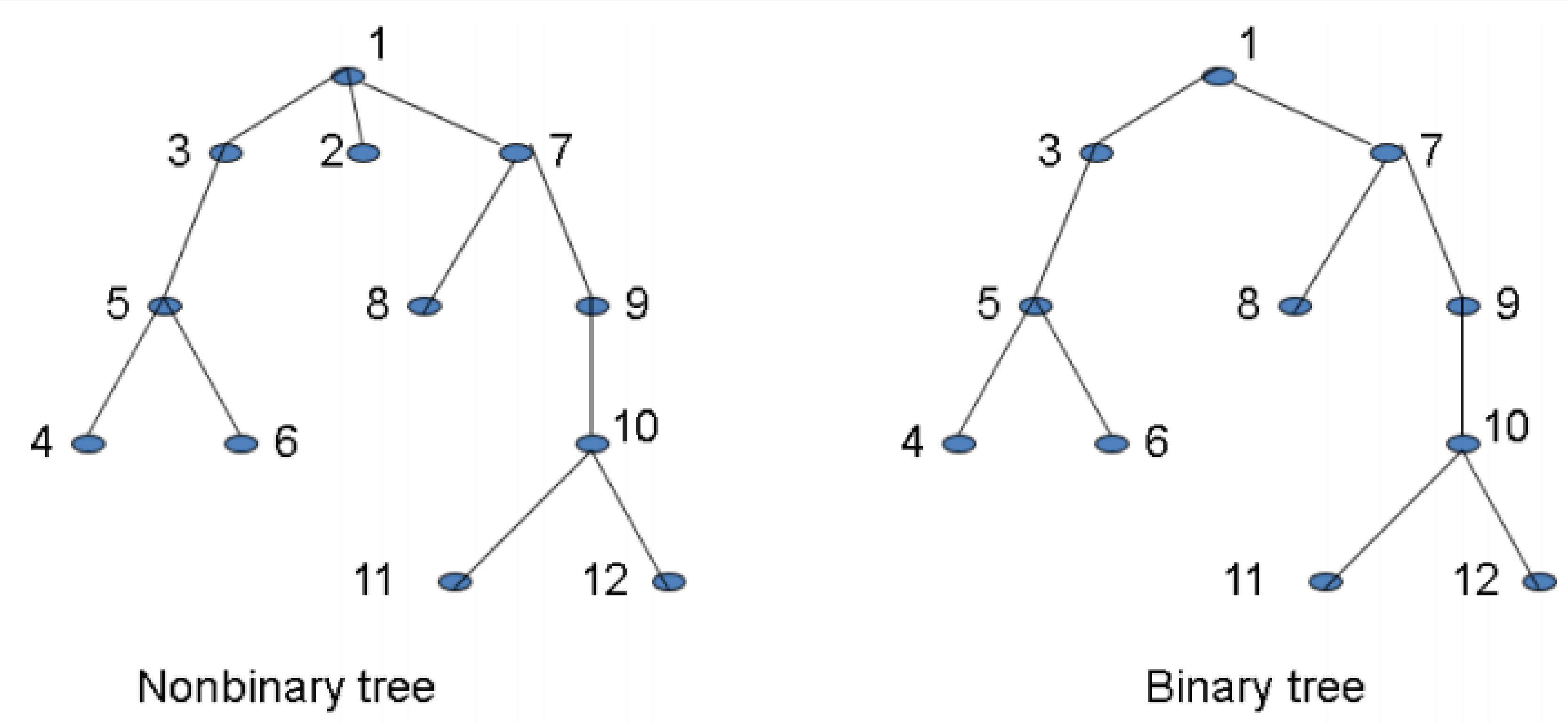
BRAIN FOOD

In relation to the given tree,

Explain and give an example of what children of a node, parent of a node, descendants of a node, and ancestors of a node are. Find the leaf nodes, and the depth of the tree.

BINARY TREES

A tree is a binary tree if every node has at most two children



BINARY TREES – DEFINITIONS

- The children of any node in a binary tree are ordered into a left child and a right child
- A node can have a left and a right child, a left child only, a right child only, or no children
- The tree made up of a left child (of a node x) and all its descendants is called the left subtree of x
- Right subtrees are defined similarly

WHERE ARE GRAPHS AND TREES USED?

SELF STUDY!

QUESTIONS?

SAPNA.K@IIT.AC.LK