Task 1

Establish if the following expressions are in the class wff filling the table. For those formulae that are in wff list all their subformulae

		ANSWERS
$\neg q$	YES/NO	
$w \Rightarrow p$	YES/NO	
$w \neg \Rightarrow p$	YES/NO	
$p\ q$	YES/NO	
$p \wedge (q \vee r)$	YES/NO	
$p \neg$	YES/NO	
$p \wedge \neg q$	YES/NO	
$(\neg p) \vee q$	YES/NO	
$(\neg p) \vee (q)$	YES/NO	
$\neg(\neg p \wedge \neg q)$	YES/NO	
$(p \Rightarrow q) \wedge (q \Rightarrow p)$	YES/NO	
$(p \Rightarrow (q \wedge r)) \Rightarrow ((p \Rightarrow q) \wedge (p \Rightarrow r))$	YES/NO	

Task 2

We will learn next the axiomatic construction of propositional logic where the following formulae will be accepted as its axioms (i.e. statement that do need proofs)

Ax.1 $(p \Rightarrow (q \Rightarrow p))$

Ax.2 $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$

Ax.3 $(\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p)$

Prove using the truth tables that the negation of each axiom above is an unsatisfiable formula.

Task 3. Some of the following sequences of formulae which are written in the style of axiomatic proof with annotation, are axiomatic proof and some are not. For the latter, there is a formula in the sequence which appears in the proof but no rule of inference can result in such a formula. Establish which of these sequences are axiomatic proofs with the rule substitution applied and which are not. Explain what went wrong in obtaining the sequences that are not proofs.

Example 1.

1. $(p \Rightarrow (q \Rightarrow p)$	Ax.1
2. $\leftarrow r \Rightarrow (q \Rightarrow \leftarrow r)$	from 1, Substitution $p/\leftarrow r$
3. $\leftarrow r \Rightarrow (s \Rightarrow \leftarrow r)$	from 2, Substitution q/s

Example 2.

1. $(p \Rightarrow (q \Rightarrow p)$	Ax.1
2. $t \Rightarrow (q \Rightarrow p)$	from 1, Substitution p/t

Example 3.

1. $(\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p)$	Ax.3
2. $(\neg s \Rightarrow \neg q) \Rightarrow (q \Rightarrow s)$	from 1, Substitution p/s

Example 4.

1. $(\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p)$	Ax.3
2. $(\neg \neg s \Rightarrow \neg q) \Rightarrow (q \Rightarrow s)$	from 1, Substitution $p/\neg s$ and p/s

Example 5.

1. $(\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p)$	Ax.3
2. $(\neg p \Rightarrow \neg r) \Rightarrow (r \Rightarrow p)$	from 1, Substitution q/r

Example 6.

1. $(\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p)$	Ax.3
2. $(\neg p \Rightarrow \neg \neg r) \Rightarrow (\neg r \Rightarrow p)$	from 1, Substitution $q/\neg r$

Example 7.

1. $(\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p)$	Ax.3
2. $(\neg p \Rightarrow \neg \neg q) \Rightarrow (\neg q \Rightarrow p)$	from 1, Substitution $q/\neg q$

Example 8

1. $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q)) \Rightarrow (p \Rightarrow r))$ Ax.2

2. $(s \Rightarrow (q \Rightarrow r)) \Rightarrow ((s \Rightarrow q)) \Rightarrow (s \Rightarrow r))$ from 1, Substitution p/s,

Example 9

1. $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q)) \Rightarrow (p \Rightarrow r))$ Ax.2

2. $(s \Rightarrow (s \Rightarrow r)) \Rightarrow ((s \Rightarrow s)) \Rightarrow (s \Rightarrow r))$ from 1, Substitution p/s, q/s

Task 4. You are given the rule set and a set of facts. Your task is to apply forward chaining reasoning to obtain the desired goals. Hint – match the left hand sides of the implication with the initial or new facts and apply modus ponens rule.

a.) Rule

set R1: $p \Rightarrow$

q R2: $q \Rightarrow r$

R3: $r \Rightarrow s$

The set of initial facts: p

Goal: s

b.) Rule

set

R1: $r \Rightarrow (t \Rightarrow s)$

The set of initial facts t, r

Goal s

c) Rule

set

R1: $r \Rightarrow (w \Rightarrow t)$

R2: $r \Rightarrow w$

The set of initial facts r

Goal t

d) **CHALLENGE**

Rule set

R1: $p \Rightarrow q$

R2: $q \wedge s \Rightarrow r$

R3: $r \Rightarrow (\neg t \vee u)$

The set of initial facts: p, s, t

Goal: $u \vee w$

Task 5. You are given the following rule set.

```
( RULE mammal_1
     IF ( ANIMAL HAS HAIR )
   THEN ( ANIMAL IS A MAMMAL ))

( RULE mammal_2
     IF ( ANIMAL IS A MILK GIVER )
   THEN ( ANIMAL IS A MAMMAL ))

( RULE bird_1
     IF ( ANIMAL HAS FEATHERS )
   THEN ( ANIMAL IS A BIRD ))

( RULE bird_2
     IF ( ANIMAL IS A FLIER ) AND ( ANIMAL IS AN EGG LAYER )
   THEN ( ANIMAL IS A BIRD ))

( RULE carnivore_1
     IF ( ANIMAL IS A MEAT EATER )
   THEN ( ANIMAL IS A CARNIVORE ))

( RULE carnivore_2
     IF ( ANIMAL HAS POINTED TEETH ) AND
       ( ANIMAL HAS CLAWS ) AND
       ( ANIMAL HAS FORWARD EYES )
   THEN ( ANIMAL IS A CARNIVORE ))

( RULE ungulate_1
     IF ( ANIMAL IS A MAMMAL ) AND
       ( ANIMAL HAS HOOVES )
   THEN ( ANIMAL IS UNGULATE ))

( RULE ungulate_2
     IF ( ANIMAL IS A MAMMAL ) AND
       ( ANIMAL IS A CUD CHEWER )
   THEN ( ANIMAL IS UNGULATE ))

( RULE even_toed
     IF ( ANIMAL IS A MAMMAL ) AND
       ( ANIMAL IS A CUD CHEWER )
   THEN ( ANIMAL IS EVEN TOED ))
```

<u>Find which rules are fired given the following facts. What can you conclude about an animal in these cases?</u>

a) The only fact is “Animal gives milk.”
b) The only fact is “Animal has hair but does not give milk.”
c) The only fact is “Animal has hooves.”
d) Facts are “Animal gives milk” and “Animal has hooves.”

e) Facts are “animal is a mammal” and “animal is a cud chewer”.

Task 6. Establish applying reasoning by refutation on the semantical conditions of the following formulae that they are valid. (Hint – by refutation assume that a formula
is not valid and reason form this trying to derive contradiction, use T[p] to write that p
is assigned value true and F[p] that p is assigned false.

a. $(p \Rightarrow p)$
b. $(p \vee \leftarrow p)$
c. $p \Rightarrow (p \vee q)$
d. **CHALLENGE** $(p \vee \neg q) \Rightarrow (q \Rightarrow p)$

TASK 7 Build tableau for the formulae below and establish if they are valid.

1. $p \Rightarrow p$
2. $\neg (p \Rightarrow p)$
3. $p \vee \neg p$
4. $\neg (p \vee \neg p)$
5. $(p \wedge q) \Rightarrow p$
6. $p \Rightarrow (p \vee q)$
7. $((p \Rightarrow q) \wedge \neg q) \Rightarrow \neg p$
8. $((p \Rightarrow q) \wedge \neg p) \Rightarrow \neg q$
9. $((p \Rightarrow q) \wedge p) \Rightarrow \neg q$
10. $(\neg p \vee \neg q) \Rightarrow \neg (p \wedge q)$
11. $\neg (p \wedge q) \Rightarrow (\neg p \vee \neg q)$
12. $(p \Rightarrow q) \Rightarrow (\neg p \vee \neg q)$

TASK 8. Prove using the tableau technique that each of the axioms for CPL has a tableau refutation.

Ax.1 $(p \Rightarrow (q \Rightarrow p))$
Ax.2 $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$ (Challenge)
Ax.3 $(\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p)$

Challenge. TASK 9.

a) Find out how to use the tableau technique to show that a formula is not satisfiable.
b) Show using the tableau that the negations of all axioms are not satisfiable formulae.

TASK 10 Build tableau for the formulae below and establish if they are valid.

a) $(p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p)$

b) $(\neg p \vee \neg q) \Rightarrow (p \vee q)$

c) $(p \vee q) \Rightarrow (p \vee (q \vee r))$

d) $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$

TASK 11. Challenge

Justify that if during the construction of a tableau a node *n* becomes labelled by an inconsistent configuration then you can stop expanding this node. This technique is often called "on-the-fly" checking. Show applying when possible, the "on-the-fly" technique, that the following formulae are valid

a) $((p \Rightarrow q) \wedge (p \Rightarrow r)) \Rightarrow (p \Rightarrow (q \wedge r))$

b) $((p \Rightarrow q) \wedge (r \Rightarrow q)) \Rightarrow ((p \vee r) \Rightarrow q)$