4COSC007C Mathematics for Computing

Tutorial 3

1. For each of the compound propositions below list all combinations of input values for their atomic propositions that make it true and those that make it false.
   Hint: build the truth tables, determine which rows have the final value true and which false and determine for each of these rows the input values for atomic propositions on the left of the truth table.

   a. $q \vee \neg p$
   b. $(p \wedge q) \vee \neg p$
   c. $\neg (p \wedge q) \vee \neg p$
   d. $((p \Rightarrow q) \wedge q) \Rightarrow p$
   e. $((p \Rightarrow q) \wedge \neg p) \Rightarrow \neg q$

2. For the compound propositions below prove that there is no input into the truth table which would make them false
   Hint: build the truth tables which explicitly show that for any input the output value is true.

   a. $(p \wedge q) \Rightarrow (q \wedge p)$
   b. $(p \vee q) \Rightarrow (q \vee p)$
   c. $\neg (p \wedge q) \Rightarrow (\neg p \vee \neg q)$
   d. $\neg (p \vee q) \Rightarrow (\neg p \wedge \neg q)$
   e. $\neg (p \Rightarrow q) \Rightarrow (p \wedge \neg q)$
   f. $p \Rightarrow (p \vee (q \wedge r))$
   g. $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$

3. For the compound propositions below prove that there is no input into the truth table which would make them true.
   Hint: build the truth tables which explicitly show that for any input the output value is false.

   a. $\neg (p \Rightarrow (q \Rightarrow p))$
   b. $\neg ((p \wedge \neg p) \Rightarrow q)$
   c. $\neg ((p \wedge (q \vee r)) \Rightarrow p)$
   d. $\neg ((p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r)))$

4. For each of the cases below establish if there is logical consequence between the given knowledge base KB and the proposition A

   Hint: following the definition of logical consequence given in Lecture 4 assemble the required compound proposition with the components of the knowledge base on the left of implication and proposition A on the right-hand side, build a truth table and determine if it shows that the formula you have assembled is valid.

   a. $KB = \{p \Rightarrow q,\ q\};\ A = p$
   b. $KB = \{p \Rightarrow q,\ p\};\ A = q$
   c. $KB = \{p \Rightarrow q,\ \neg q\};\ A = \neg p$
   d. $KB = \{p \Rightarrow q,\ \neg p\};\ A = \neg q$
   e. $KB = \{p \vee q,\ p\};\ A = q$
   f. $KB = \{p \vee q,\ \neg p\};\ A = q$
   g. $KB = \{p \vee q,\ \neg q\};\ A = p$
   h. $KB = \{p,\ \neg p\};\ A = q$

5. **Challenge**

   Establish, only analyzing their structure and not constructing long truth tables, if there are input values that make the following formulae false

   a. $((p \Rightarrow (q \wedge r)) \Rightarrow ((s \Rightarrow q) \wedge (u \Rightarrow v))) \Rightarrow (t \Rightarrow t)$
   b. $((p \vee \neg p) \Rightarrow (q \wedge \neg q)) \Rightarrow (((r \Rightarrow q) \wedge (u \Rightarrow s)) \Rightarrow (t \Rightarrow w))$

6. **Challenge**

   We defined that B is a logical consequence of a knowledge base $A_1, A_2, A_3, \ldots A_n$ if the following formula is valid:

   $$(A_1 \wedge (A_2 \wedge (A_3 \wedge \ldots A_n))) \Rightarrow B$$

   a) Show that the following reasoning is wrong: *if B is a logical consequence of a knowledge base $A_1, A_2, A_3, \ldots A_n$ then B is a logical consequence of each of the components of this knowledge base*

   b) Show that the following reasoning is correct: *if B is a logical consequence of at least of one of the components of the knowledge base $A_1, A_2, A_3, \ldots A_n$ then B is a logical consequence of this knowledge base.*

   Hint: consider the specifics of how conjunction and implication work as a Boolean operation, you can either reason or find a counter- example that fails the reasoning above.