4COSC007C Mathematics for Computing

Tutorial 8

1. What is the inverse of matrix **A**?

$$\mathbf{A} = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}$$

2. Solve the following system of linear equations using matrices only. You will get a system of $\mathbf{AX} = \mathbf{B}$ and you will need to find the inverse $\mathbf{A}^{-1}$.

$$\begin{aligned} y + 2z &= 17 \\ -2x + 3y - z &= 6 \\ 4x \quad + z &= -1 \end{aligned}$$

3. Let $\mathbf{A} = \begin{pmatrix} 2 & 2 \\ 4 & 5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 4 \\ -1 & -2 \end{pmatrix}$

(a) Find $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$

(b) Verify that $(\mathbf{A} + \mathbf{B})^{-1} \neq \mathbf{A}^{-1} + \mathbf{B}^{-1}$

4. Find the determinant using the diagonal method and find the inverse of the matrix A using the determinant.

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 3 \\ -1 & 4 & -2 \\ 1 & -3 & 5 \end{bmatrix}$$

5 Show that if **A** and **B** are any two invertible and square matrices of the same size, then $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

6 Show that if **A** is invertible and **AB** = **O** then **B** = **O**.

7 Use matrices to solve the following pair of simultaneous linear equations.

$$(3/5)x - (4/5)y = 18$$
$$(4/5)x + (3/5)y = -1$$

Challenge:

1. Solve the following equation for the variable x.

$$\begin{vmatrix} x & x+1 \\ -1 & x-2 \end{vmatrix} = 7$$

2. In general, matrix multiplication is not commutative (i.e., $\mathbf{AB} \neq \mathbf{BA}$). However, in certain special cases the commutative property does hold. Show that:

If **A** and **B** are $n \times n$ diagonal matrices, then **AB** = **BA**.

3. Suppose that $\mathbf{A} = \mathbf{BDB}^{-1}$ where **B** is an invertible matrix and **D** is a diagonal matrix. Find $\mathbf{A}^{100}$.

4. Let **A** be an $n \times n$ matrix and let x and y be vectors in R^n. Show that if $\mathbf{A}x = \mathbf{A}y$ and $x \neq y$, then the matrix $\boldsymbol{A}$ must be singular.