2048 GAME ON PYTHON

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Abstract — **The 2048 game involves tiles labelled with powers of two that can be merged to form bigger powers of two; variants of the same puzzle involve similar merges of other tile values. We analyse the Maximum score achievable in these games by proving a min-max theorem equating this maximum Score (in an abstract generalized variation of 2048 that allows all the moves of the original game) With the minimum value that causes a greedy change-making algorithm to use a given number Of coins. A widely-followed strategy in 2048 maintains tiles that represent the move number in binary notation, and a similar strategy in the Fibonacci number variant of the game (987) Maintains the Zeckendorf representation of the move number as a sum of the fewest possible Fibonacci numbers; our analysis shows that the ability to follow these strategies is intimately Connected with the fact that greedy change-making is optimal for binary and Fibonacci coinage. For variants of 2048 using tile values for which greedy change-making is suboptimal, it is the Greedy strategy, not the optimal representation as sums of tile values that controls the length of the game. In particular, the game will always terminate whenever the sequence of allowable tile Values has arbitrarily large gaps between consecutive values.**

Introduction **-The solitaire game 2048 was But other variants of 2048 use different tile values than powers of two. Threes uses the sequence of numbers that are either powers of two or three times a power of two:**
1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, . . .
(It also restricts tile merges to pairs of tiles whose values are equal or differ by a factor of two.) Fives uses 2, 3, and powers of two times 5, giving the sequence of allowable values [12]
1, 2, 3, 5, 10, 20, 40, 80, 160, 320, 640, . . .
Another variant, called 987, uses as its tile values the Fibonacci numbers,
1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, . . .
We can find analogous ad-hoc arguments for why these games must terminate, but can we generalize them to arbitrary systems of tile values? If we define a 2048-like game with a set as its tile values, is the length of the game and the maximum value that can be achieved controlled, as it is for binary numbers, by the lengths of the shortest representations of arbitrary numbers as sums of members of S ?
For instance, suppose that we allow any practical number as a tile value, and any merge of two tiles that produces another practical number. The practical numbers are defined by the property that, for a practical number
n
, every integer

$m < n$
can be expressed as a sum of distinct divisors of n. Their sequence begins
1, 2, 4, 6, 8, 12, 16, 18, 20, 24, 28, 30, 32, . . .
There are many more practical numbers than powers of two, and the practical numbers behave in many ways like the prime numbers. In particular, analogously to Goldbach's conjecture for the prime numbers, every even integer can be expressed as a sum of two practical numbers [17], and therefore every integer can be expressed as a sum of three practical numbers. Because we can express every tile value using a bounded number of practical-number tiles, does the practical-number variant of 2048 go on forever?
Alternatively, suppose we use 3-smooth tile values, the numbers whose only prime factors are two or three:
1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 32, 36, . . .
developed in 2014 by Gabriele Cirulli, based on another game called Threes developed earlier in 2014 by Asher Vollmer [28]. It is played on a 16-cell square grid, each cell of which can either be empty or contain a tile labelled with a power of two. In each turn, a tile of value 2 or 4 is placed by the game software on a randomly chosen empty cell. The player then must tilt the board in one of the four cardinal directions, causing its tiles to slide until reaching the edge of the board or another tile. When two tiles of equal value slide

into each other, they merge into a new tile of twice the value. The game stops when the whole board fills with tiles, and the goal is to achieve the highest single tile value possible. Figure 1 shows the state of the game after approximately 4000 moves, when a tile with value 8192 has been reached. As most players of the game quickly learn, it is not possible to keep playing a single game of 2048 forever. At any step of the game, there must be at least one tile for each nonzero bit in the binary representation of the total tile value. For total tile values just below a large power of two, the number of ones in the binary representation is similarly large, eventually exceeding the number of cells in the board.

Making Change in 2048 significantly different than two consecutive tiles of value 2 that then became merged, but it can interact with the board geometry to cause tiles to become out of position, making continued play more difficult. And in many of these games, the location of each newly placed tile could be any previously-open cell. These unknowns make the game nondeterministic, and complicate the definition of the longest play or highest achievable tile value: do we mean the worst case (the best that a player could achieve against a malicious adversary), best case (the best one could hope to achieve against repeated play with a random adversary), or some kind of probabilistic analysis that determines the distribution or expected value of scores? To avoid these complications, we define a class of variants of 2048 in which they are eliminated.

(Abstract generalized games).

Given a set A of allowable tile values, an initial element a $\in$ A (usually a = 1), and a number n of cells, we define the abstract generalized 2048 game for A and a to be a solitaire game in which there are n indistinguishable cells, each of which can either be empty or contain a tile with a value in A. We define a position of the game to be an assignment of either a tile with a value in A or no tile to each cell of the game. The initial position of the game is a position in which all cells are empty. Starting from the initial position, each step of the game consists of the following actions: The player chooses any empty cell, and a tile of value a is placed into that cell. The player may choose to merge any sets of non-empty cells whose total value belongs to A into a single tile, which is placed on a single cell from its set. The remaining cells in each chosen set become empty. The game ends when, after one of these steps, all cells are nonempty. When this happens,

there would be nowhere to place the new tile of value a in the next step. We denote the abstract generalized 2048 game on n cells with tile value set A and initial tile value a by AGG(n, A, a) or (when a = 1) by AGG(n, A). I

Observation 2

(simulation by abstract games).

With the possible exception of the value of each newly placed tile, each action in 2048, Threes, Fives, or 987 can be simulated by a corresponding action in the abstract generalized 2048 game with the same set of tile values and the same number of cells. Therefore, any upper bound on the number of moves or maximum tile value in the abstract generalized 2048 game provides a valid upper bound for the number of moves or maximum tile value in the corresponding sliding-tile game.

Observation 3 Optimal strategy in the abstract game The abstract generalized 2048 game eliminates the complications of board geometry, tile position, and sliding mechanics from the game, making its analysis much simpler. As a consequence, we can characterize the optimal strategies in this game. We begin by describing some helpful move-ordering principles.

(eager sequences) .

We say that a sequence of steps in AGG (n, A) is eager if each merge of tiles is performed in the first step at which all of the tiles to be merged have their merged values, rather than delaying the merge until some later step. I

Observation 4 (all sequences can be made eager). If a position in AGG (n, A) can be reached by a sequence of steps, it can be reached by an eager sequence of steps

Discussion - We have described an abstract version of the game 2048 that eliminates the geometry and other complicating factors of the game, allowing us to provide a complete analysis of our abstract game for any set of allowable tile values and any number of cells. We proved a min-max theorem equating the maximum total tile value that can be achieved in this game with the minimum value that would cause a greedy change-making algorithm, using coins of the same value as the tiles, to use the same number of coins as the number of cells in the game. Finally, we showed how to compute the values from this theorem by a streaming algorithm that uses only a constant number of integer variables beyond the requirements of generating the tile value sequence itself, and used our implementation to compute the sequences of maximum game values for several choices of allowable tile value sets. It would be of interest to understand in more detail for which non-abstract 2048-like games this analysis is tight or nearly tight, and for which it fails to capture the game dynamics and produces a bound on the total game value that is large compared to the actual achievable value. For instance, experience with 2048 and 987 suggests that, in those games, a strategy close to that of the abstract game can usually be followed, leading to total game values similar to what could be achieved in the abstract game. On the other hand, in Threes, the inability to add some pairs of game tiles such as 1 + 3 , even when the sum would be another allowable tile value, may cause this game's maximum achievable value to be closer to 2n than to the (4n+11) / 3 formula for the total score achievable on the corresponding abstract game. We leave such questions open for future research. Additionally, some variants of 2048 are not amenable to our analysis. These include 2048 Circle of Fifths, a game based on the circle of fifths in music theory whose tile values involve modular arithmetic [9], and 2048 Numberwang, in which the tile combinations that are allowed on each move vary randomly [8]. Developing a theoretical analysis of these games could be fun.

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