Lab Week 5 - Calibration of the Heston model

In the lecture we studied the technique of parameterisation to calibrate the local volatility model using market data. In the lab session, we use the technique of regularisation to calibrate the Heston stochastic volatility model. Recall that the Heston model under a risk-neutral measure is given as

$$\begin{aligned} \frac{\mathrm{dS}(t)}{\mathrm{S}(t)} &= r\mathrm{d}t + \sqrt{v(t)}\mathrm{dW}(t), \\ \mathrm{d}v(t) &= \kappa(\theta - v(t))\mathrm{d}t + \sigma\sqrt{v(t)}\mathrm{dW}'(t), \quad v(0) = v_0, \end{aligned}$$

where $2\kappa\theta > \sigma^2$ and ρ is the instantaneous correlation of W and W′. The formula for European call and put under the Heston model can be used to accurately compute option prices by different numerical techniques. This was covered in the lab session of Week 4.

The same formula can also be used to calibrate the model parameters $\Theta = (\kappa, \theta, \sigma, \rho, v_0)$ from the market option prices by formulating the following inverse problem

$$\min_{\Theta} \mathrm{S}(\Theta) = \min_{\Theta} \sum_{i,j} w_{ij} \left(\mathrm{P}^{\mathrm{model}(\Theta)}(\mathrm{T}_i, \mathrm{K}_j) - \mathrm{P}^{\mathrm{market}}(\mathrm{T}_i, \mathrm{K}_j)\right)^2.$$

The above problem can be solved using MATLAB's `lsqnonlin` function. For a given maturity T_i and strike K_j we choose $w_{ij} = \frac{1}{|\mathrm{P}^{\mathrm{market,bid}}(\mathrm{T}_i,\mathrm{K}_j) - \mathrm{P}^{\mathrm{market,ask}}(\mathrm{T}_i,\mathrm{K}_j)|}$ and $\mathrm{P}^{\mathrm{market}}(\mathrm{T}_i, \mathrm{K}_j) = \frac{\mathrm{P}^{\mathrm{market,bid}}(\mathrm{T}_i,\mathrm{K}_j) + \mathrm{P}^{\mathrm{market,ask}}(\mathrm{T}_i,\mathrm{K}_j)}{2}$ where $\mathrm{P}^{\mathrm{market,bid}}(\mathrm{T}_i, \mathrm{K}_j)$ and $\mathrm{P}^{\mathrm{market,ask}}(\mathrm{T}_i, \mathrm{K}_j)$ are bid and ask option prices, respectively. Use the provided code from Moodley (2005) using MATLAB's `lsqnonlin` function to calibrate the Heston model. Notice the sensitivity of the method with respect to the initial parameters. Using the calibrated parameters, compare the model option prices with the market given option prices.

Course coordinator: Dr. Ankush Agarwal
ankush.agarwal@glasgow.ac.uk