Data Structures and Algorithms

Lecture 2 : Lists

Outline

- Abstract Data Type (ADT)
- List ADT
- List ADT with Array Implementation
- Linked lists
- Basic operations of linked lists
  - Insert, find, delete, print, etc.
- Variations of linked lists
  - Circular linked lists
  - Doubly linked lists

Abstract Data Type (ADT)

- Data type
  - a set of objects + a set of operations
  - Example: integer
    - set of whole numbers
    - operations: +, -, x, /
- Can this be generalized?
  - (e.g. procedures generalize the notion of an operator)
  - Yes!
- **Abstract** data type
  - high-level abstractions (managing complexity through abstraction)
  - Encapsulation

Encapsulation

- Operation on the ADT can only be done by calling the appropriate function
- no mention of *how* the set of operations is implemented
- The definition of the type and all operations on that type can be localized to one section of the program
- If we wish to change the implementation of an ADT
  - we know where to look
  - by revising one small section we can be sure that there is no subtlety elsewhere that will cause errors
- We can treat the ADT as a primitive type: we have no concern with the underlying implementation
- ADT $\rightarrow$ C++: class
- method $\rightarrow$ C++: member function

ADT…

- Examples
  - the *set* ADT
    - A set of elements
    - Operations: *union, intersection, size* and *complement*
  - the *queue* ADT
    - A set of sequences of elements
    - Operations: c*reate empty queue, insert, examine, delete*, and d*estroy queue*
- Two ADT's are different if they have the same underlying model but different operations
  - E.g. a different set ADT with only the *union* and *find* operations
  - The appropriateness of an implementation depends very much on the operations to be performed

Pros and Cons

- ✓ Implementation of the ADT is separate from its use
- ✓ Modular: one module for one ADT
  - Easier to debug
  - Easier for several people to work simultaneously
- ✓ Code for the ADT can be reused in different applications
- ✓ Information hiding
  - A logical unit to do a specific job
  - implementation details can be changed without affecting user programs
- ✓ Allow rapid prototyping
  - Prototype with simple ADT implementations, then tune them later when necessary
- ✗ Loss of efficiency

The List ADT

- A sequence of zero or more elements

$$A_1, A_2, A_3, \ldots A_N$$

- N: length of the list
- A_1: first element
- A_N: last element
- A_i: position i
- If N=0, then empty list
- Linearly ordered
  - A_i precedes A_{i+1}
  - A_i follows A_{i-1}

Operations

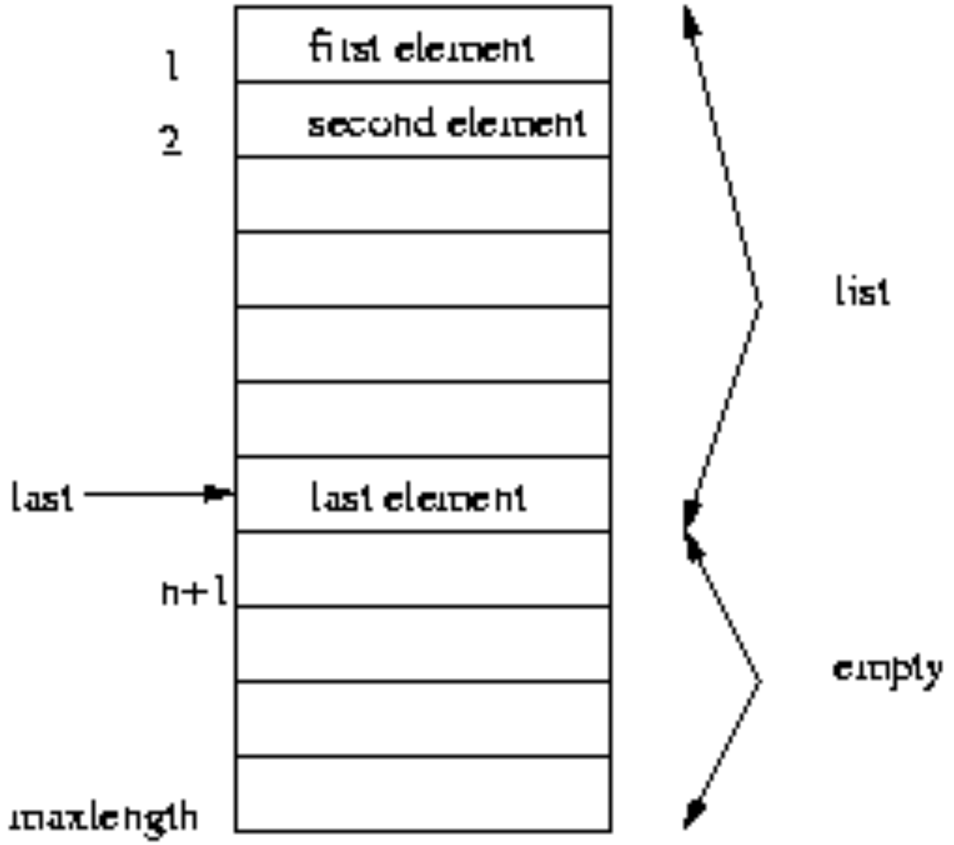
- makeEmpty: create an empty list
- insert: insert an object to a list
  - insert(x,3) → 34, 12, 52, x, 16, 12
- remove: delete an element from the list
  - remove(52) → 34, 12, x, 16, 12
- find: locate the position of an object in a list
  - list: 34,12, 52, 16, 12
  - find(52) → 3
- findKth: retrieve the element at a certain position
- printList: print the list

Implementation of an ADT

- Choose a **data structure** to represent the ADT
  - E.g. arrays, records, etc.
- Each operation associated with the ADT is implemented by one or more subroutines
- Two standard implementations for the list ADT
  - Array-based
  - Linked list

Array Implementation

- Elements are stored in contiguous array positions



Array Implementation...

- Requires an estimate of the maximum size of the list
  - waste space
- printList and find: linear
- findKth: constant
- insert and delete: slow
  - e.g. insert at position 0 (making a new element)
    - requires first pushing the entire array down one spot to make room
  - e.g. delete at position 0
    - requires shifting all the elements in the list up one
  - On average, half of the lists needs to be moved for either operation

Pointer Implementation (Linked List)

- Ensure that the list is not stored contiguously
  - use a linked list
  - a series of structures that are not necessarily adjacent in memory

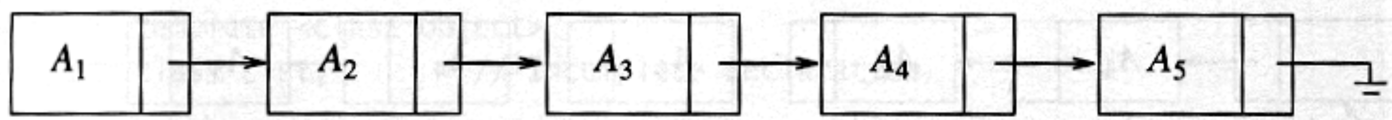
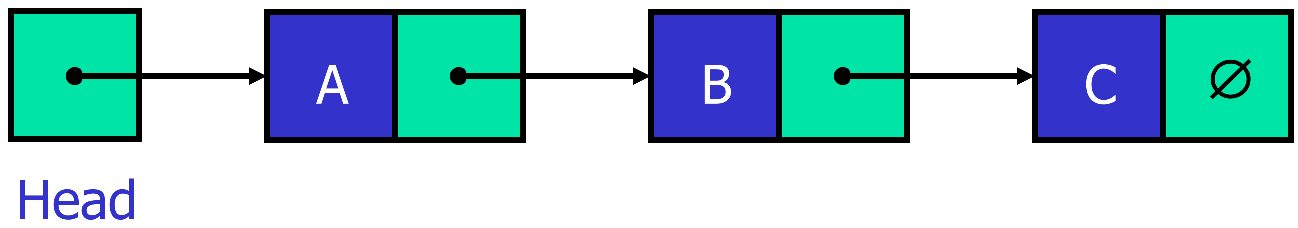


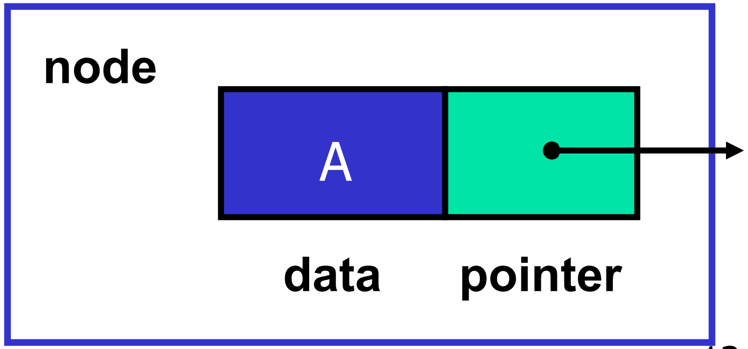
Figure 3.1 A linked list

- Each node contains the element and a pointer to a structure containing its successor
  - the last cell's next link points to NULL
- Compared to the array implementation,
  - ✓the pointer implementation uses only as much space as is needed for the elements currently on the list
  - but requires space for the pointers in each cell

Linked Lists



- A *linked list* is a series of connected *nodes*
- Each node contains at least
  - A piece of data (any type)
  - Pointer to the next node in the list
- *Head*: pointer to the first node
- The last node points to `NULL`



A Simple Linked List Class

- We use two classes: **`Node`** and **`List`**
- Declare `Node` class for the nodes
  - `data`: `double`-type data in this example
  - `next`: a pointer to the next node in the list

```
class Node {
public:
        double  data;           // data
        Node*   next;           // pointer to next
};
```

A Simple Linked List Class

- Declare `List`, which contains
  - `head`: a pointer to the first node in the list.
    Since the list is empty initially, `head` is set to `NULL`
  - Operations on `List`

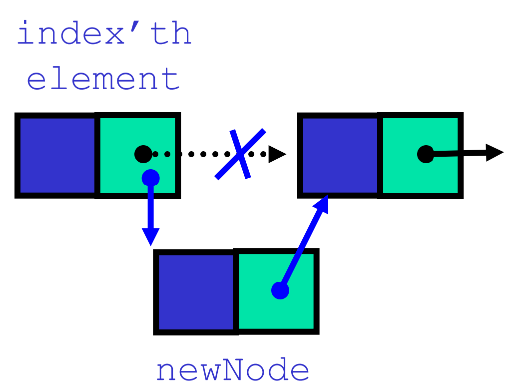
```
class List {
public:
        List(void) { head = NULL; }          // constructor
        ~List(void);                          // destructor

        bool IsEmpty() { return head == NULL; }
        Node* InsertNode(int index, double x);
        int FindNode(double x);
        int DeleteNode(double x);
        void DisplayList(void);
private:
        Node* head;
};
```

A Simple Linked List Class

- Operations of `List`
  - `IsEmpty`: determine whether or not the list is empty
  - `InsertNode`: insert a new node at a particular position
  - `FindNode`: find a node with a given value
  - `DeleteNode`: delete a node with a given value
  - `DisplayList`: print all the nodes in the list

Inserting a new node

- `Node* InsertNode(int index, double x)`
  - Insert a node with data equal to `x` after the `index'`th elements. (i.e., when `index = 0`, insert the node as the first element;
    when `index = 1`, insert the node after the first element, and so on)
  - If the insertion is successful, return the inserted node.
    Otherwise, return `NULL`.
    (If `index` is < 0 or > length of the list, the insertion will fail.)
- Steps
  1. Locate `index`'th element
  2. Allocate memory for the new node
  3. Point the new node to its successor
  4. Point the new node's predecessor to the new node



Inserting a new node

- Possible cases of `InsertNode`
  1. Insert into an empty list
  2. Insert in front
  3. Insert at back
  4. Insert in middle
- But, in fact, only need to handle two cases
  - Insert as the first node (Case 1 and Case 2)
  - Insert in the middle or at the end of the list (Case 3 and Case 4)

Inserting a new node

```
* List::InsertNode(int index, double x) {
	if (index < 0) return NULL;

	int currIndex	=	1;
	Node* currNode	=	head;
	while (currNode && index > currIndex) {
		currNode	=	currNode->next;
		currIndex++;
	}
	if (index > 0 && currNode == NULL) return NULL;

	Node* newNode	=	new	Node;
	newNode->data	=	x;
	if (index == 0) {
		newNode->next	=	head;
		head		=	newNode;
	}
	else {
		newNode->next	=	currNode->next;
		currNode->next	=	newNode;
	}
	return newNode;
```

Try to locate `index`'th node. If it doesn't exist, return `NULL`.

Inserting a new node

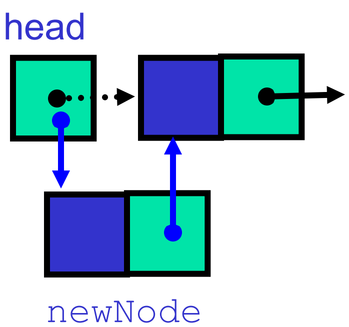
```
List::InsertNode(int index, double x) {
    if (index < 0) return NULL;

    int currIndex       =       1;
    Node* currNode      =       head;
    while (currNode && index > currIndex) {
            currNode  =         currNode->next;
            currIndex++;
    }
    if (index > 0 && currNode == NULL) return NULL;

    Node* newNode       =       new     Node;
    newNode->data       =       x;
    if (index == 0) {
            newNode->next   =       head;
            head            =       newNode;
    }
    else {
            newNode->next   =       currNode->next;
            currNode->next  =       newNode;
    }
    return newNode;
```

Create a new node

Inserting a new node

```
Node* List::InsertNode(int index, double x) {
        if (index < 0) return NULL;

        int currIndex       =       1;
        Node* currNode      =       head;
        while (currNode && index > currIndex) {
                currNode  =         currNode->next;
                currIndex++;
        }
        if (index > 0 && currNode == NULL) return NULL;

        Node* newNode       =       new      Node;
        newNode->data       =       x;
        if (index == 0) {
                newNode->next     =      head;
                head              =      newNode;
        }
        else {
                newNode->next     =      currNode->next;
                currNode->next    =      newNode;
        }
        return newNode;
```

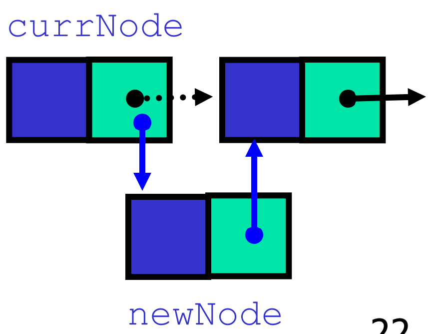
Insert as first element



Inserting a new node

```
Node* List::InsertNode(int index, double x) {
        if (index < 0) return NULL;

        int currIndex       =       1;
        Node* currNode      =       head;
        while (currNode && index > currIndex) {
                currNode  =         currNode->next;
                currIndex++;
        }
        if (index > 0 && currNode == NULL) return NULL;

        Node* newNode       =       new       Node;
        newNode->data       =       x;
        if (index == 0) {
                newNode->next       =       head;
                head                =       newNode;
        }
        else {
                newNode->next       =       currNode->next;
                currNode->next      =       newNode;
        }
        return newNode;
}
```

Insert after `currNode`



Finding a node

- `int FindNode(double x)`
  - Search for a node with the value equal to `x` in the list.
  - If such a node is found, return its position. Otherwise, return 0.

```
int List::FindNode(double x) {
        Node* currNode=        head;
        int currIndex    =        1;
        while (currNode && currNode->data != x) {
                currNode        =        currNode->next;
                currIndex++;
        }
        if (currNode) return currIndex;
        return 0;
}
```

Deleting a node

- `int DeleteNode(double x)`
  - Delete a node with the value equal to `x` from the list.
  - If such a node is found, return its position. Otherwise, return 0.
- Steps
  - Find the desirable node (similar to `FindNode`)
  - Release the memory occupied by the found node
  - Set the pointer of the predecessor of the found node to the successor of the found node
- Like `InsertNode`, there are two special cases
  - Delete first node
  - Delete the node in middle or at the end of the list

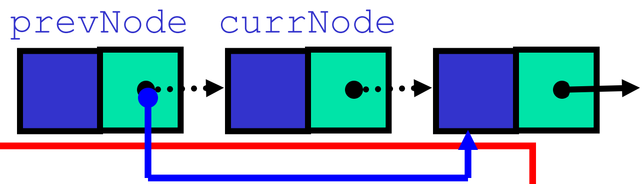
Deleting a node

```
int List::DeleteNode(double x) {
        Node* prevNode    =       NULL;
        Node* currNode    =       head;
        int currIndex     =       1;
        while (currNode && currNode->data != x) {
                prevNode =        currNode;
                currNode  =       currNode->next;
                currIndex++;
        }
        if (currNode) {
                if (prevNode) {
                        prevNode->next    =       currNode->next;
                        delete currNode;
                }
                else {
                        head              =       currNode->next;
                        delete currNode;
                }
                return currIndex;
        }
        return 0;
}
```

Try to find the node with its value equal to x

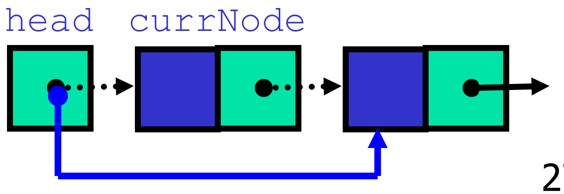
Deleting a node

```
int List::DeleteNode(double x) {
        Node* prevNode      =       NULL;
        Node* currNode      =       head;
        int currIndex       =       1;
        while (currNode && currNode->data != x) {
                prevNode =          currNode;
                currNode  =         currNode->next;
                currIndex++;
        }
        if (currNode) {
                if (prevNode) {
                        prevNode->next      =       currNode->next;
                        delete currNode;
                }
                else {
                        head                =       currNode->next;
                        delete currNode;
                }
                return currIndex;
        }
        return 0;
}
```



Deleting a node

```
int List::DeleteNode(double x) {
        Node* prevNode      =       NULL;
        Node* currNode      =       head;
        int currIndex       =       1;
        while (currNode && currNode->data != x) {
                prevNode =          currNode;
                currNode  =         currNode->next;
                currIndex++;
        }
        if (currNode) {
                if (prevNode) {
                        prevNode->next      =       currNode->next;
                        delete currNode;
                }
                else {
                        head                =       currNode->next;
                        delete currNode;
                }
                return currIndex;
        }
        return 0;
}
```



Printing all the elements

- `void DisplayList(void)`
  - Print the data of all the elements
  - Print the number of the nodes in the list

```
void List::DisplayList()
{
   int num              =       0;
   Node* currNode       =       head;
   while (currNode != NULL){
         cout << currNode->data << endl;
         currNode       =       currNode->next;
         num++;
   }
   cout << "Number of nodes in the list: " << num << endl;
}
```

Destroying the list

- `~List(void)`
  - Use the destructor to release all the memory used by the list.
  - Step through the list and delete each node one by one.

```
List::~List(void) {
   Node* currNode = head, *nextNode = NULL;
   while (currNode != NULL)
   {
         nextNode    =    currNode->next;
         // destroy the current node
         delete currNode;
         currNode    =    nextNode;
   }
}
```

Using `List`

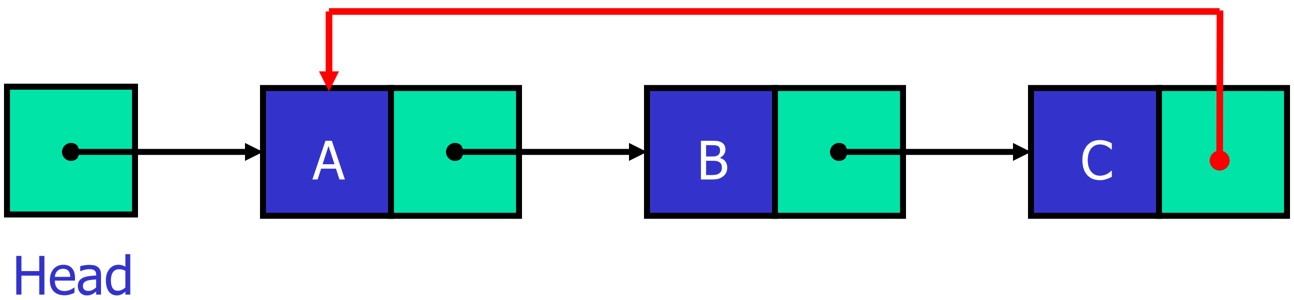
```
int main(void)
{
        List list;
        list.InsertNode(0, 7.0);      // successful
        list.InsertNode(1, 5.0);      // successful
        list.InsertNode(-1, 5.0);     // unsuccessful
        list.InsertNode(0, 6.0);      // successful
        list.InsertNode(8, 4.0);      // unsuccessful
        // print all the elements
        list.DisplayList();
        if(list.FindNode(5.0) > 0)   cout << "5.0 found" << endl;
        else                          cout << "5.0 not found" << endl;
        if(list.FindNode(4.5) > 0) cout << "4.5 found" << endl;
        else                          cout << "4.5 not found" << endl;
        list.DeleteNode(7.0);
        list.DisplayList();
        return 0;
}
```

result

```
6
7
5
Number of nodes in the list: 3
5.0 found
4.5 not found
6
5
Number of nodes in the list: 2
```

Variations of Linked Lists

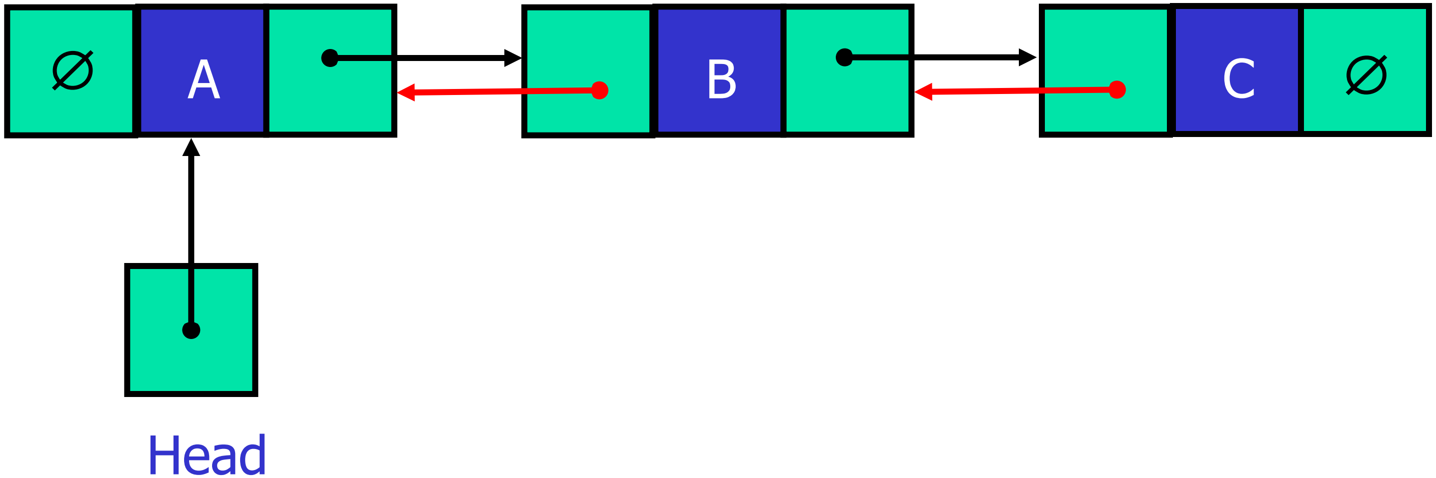
- *Circular linked lists*
  - The last node points to the first node of the list



  - How do we know when we have finished traversing the list? (Tip: check if the pointer of the current node is equal to the head.)

Variations of Linked Lists

- *Doubly linked lists*
  - Each node points to not only successor but the predecessor
  - There are two `NULL:` at the first and last nodes in the list
  - Advantage: given a node, it is easy to visit its predecessor. Convenient to traverse lists backwards



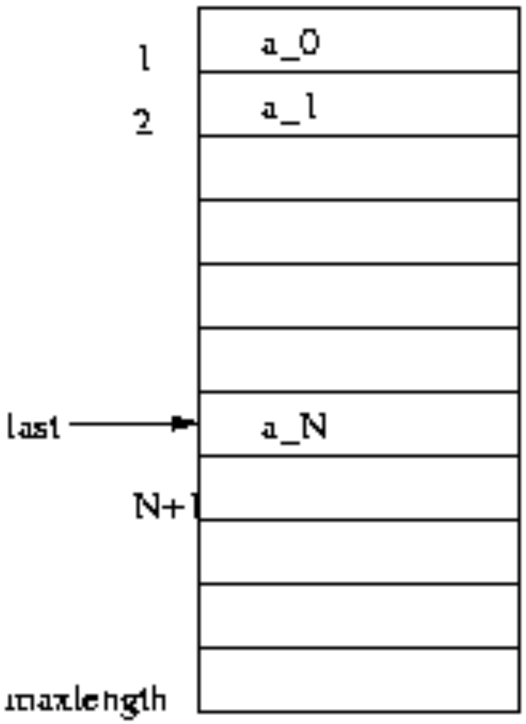
Array versus Linked Lists

- Linked lists are more complex to code and manage than arrays, but they have some distinct advantages.
  - **Dynamic**: a linked list can easily grow and shrink in size.
    - We don’t need to know how many nodes will be in the list. They are created in memory as needed.
    - In contrast, the size of a C++ array is fixed at compilation time.
  - **Easy and fast insertions and deletions**
    - To insert or delete an element in an array, we need to copy to temporary variables to make room for new elements or close the gap caused by deleted elements.
    - With a linked list, no need to move other nodes. Only need to reset some pointers.

Example: The Polynomial ADT

- An ADT for single-variable polynomials $f(x) = \sum_{i=0}^{N} a_i x^i$

- Array implementation



The Polynomial ADT…

- Acceptable if most of the coefficients A_j are nonzero, undesirable if this is not the case
- E.g. multiply

$$P_1(x) = 10x^{1000} + 5x^{14} + 1$$

$$P_2(x) = 3x^{1990} - 2x^{1492} + 11x + 5$$

  - most of the time is spent multiplying zeros and stepping through nonexistent parts of the input polynomials

- Implementation using a singly linked list
  - Each term is contained in one cell, and the cells are sorted in decreasing order of exponents

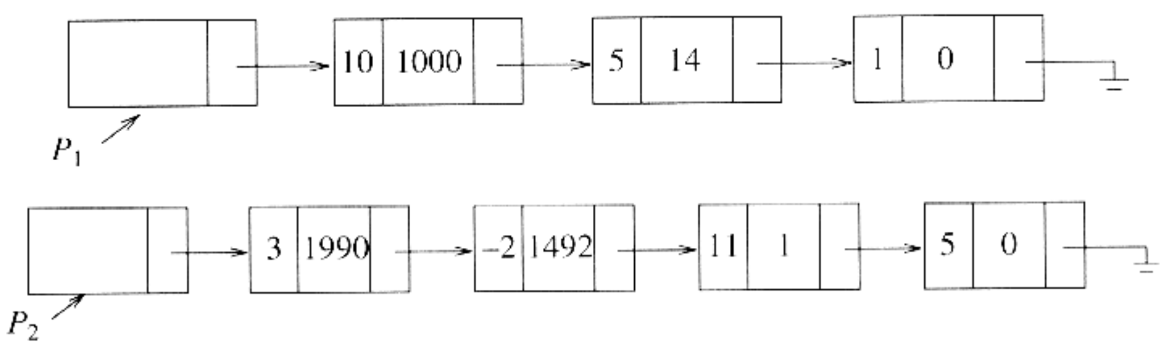


Figure 3.23 Linked list representations of two polynomials