Data Structures and Algorithms

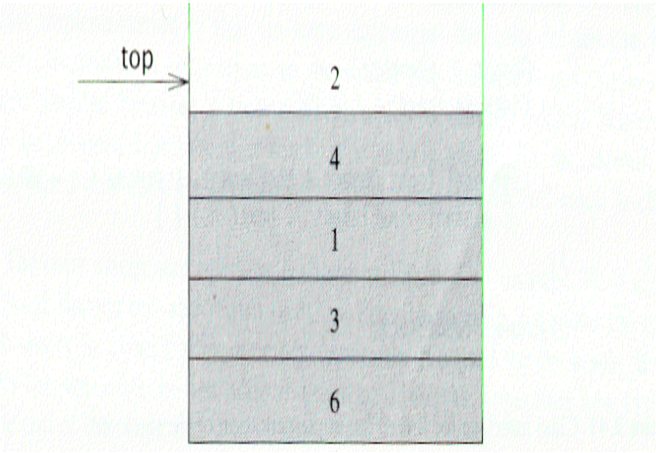
Lecture 3 : Stacks and Queues

Stack Overview

- Stack ADT
- Basic operations of stack
  - Pushing, popping etc.
- Implementations of stacks using
  - array
  - linked list

Stack ADT

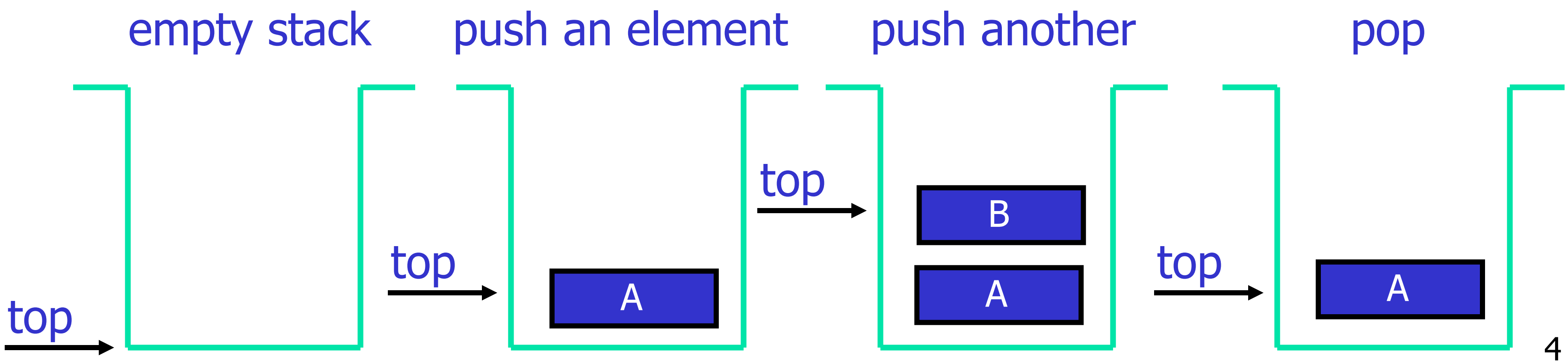
- A *stack* is a list in which insertion and deletion take place at the same end
  - This end is called *top*
  - The other end is called *bottom*



- Stacks are known as LIFO (Last In, First Out) lists.
  - The last element inserted will be the first to be retrieved

Push and Pop

- Primary operations: Push and Pop
- Push
  - Add an element to the top of the stack
- Pop
  - Remove the element at the top of the stack



Implementation of Stacks

- Any list implementation could be used to implement a stack
  - Arrays (static: the size of stack is given initially)
  - Linked lists (dynamic: never become full)
- We will explore implementations based on array and linked list
- Let's see how to use an array to implement a stack first

Stack class

```
class Stack {
public:
        Stack(int size = 10);                    // constructor
        ~Stack() { delete [] values; }   // destructor
        bool IsEmpty() { return top == -1; }
        bool IsFull() { return top == maxTop; }
        double Top();   // examine, without popping
        void Push(const double x);
        double Pop();
        void DisplayStack();
private:
        int maxTop;;    // max stack size = size - 1
        int top;        // current top of stack
        double* values;;        // element array
};
```

`Stack` class

- Attributes of `Stack`
  - `maxTop`: the max size of stack
  - `top`: the index of the top element of stack
  - `values`: point to an array which stores elements of stack
- Operations of `Stack`
  - `IsEmpty`: return true if stack is empty, return false otherwise
  - `IsFull`: return true if stack is full, return false otherwise
  - **`Top`:** return the element at the top of stack
  - `Push`: add an element to the top of stack
  - `Pop`: delete the element at the top of stack
  - `DisplayStack`: print all the data in the stack

Array impln: Create Stack

- The constructor of `Stack`
  - Allocate a stack array of `size`. By default, `size = 10`.
  - Initially `top` is set to -1. It means the stack is empty.
  - When the stack is full, `top` will have its maximum value, i.e. `size - 1`.

```
Stack::Stack(int size /*= 10*/) {
        values  =       new double[size];
        top     =       -1;
    maxTop      =       size - 1;
}
```

Although the constructor dynamically allocates the stack array, the stack is still static. The size is fixed after the initialization.

Array Impln: Push Stack

- `void Push(const double`
  - Push an element onto the stack
  - Note `top` always represents the index of the top element. After incrementing `top`, push the element.

```
void Stack::Push(const double x) {
        if (IsFull())  // if stack is full, print error
                cout << "Error: the stack is full." << endl;
        else
                values[++top] = x;
}
```

Array Impln: Pop Stack

- `double Pop()`
  - Pop and return the element at the top of the stack
  - Don't forgot to decrement `top`

```
double Stack::Pop() {
        if (IsEmpty()) { //if stack is empty, print error
                cout << "Error: the stack is empty." << endl;
                return -1;
        }
        else {
                return values[top--];
        }
}
```

Array Impln: Stack Top

- `double Top()`
  - Return the top element of the stack
  - Unlike `Pop`, this function does not remove the top element

```
double Stack::Top() {
        if (IsEmpty()) {
                cout << "Error: the stack is empty." << endl;
                return -1;
        }
        else
                return values[top];
}
```

Array Impln: Printing all the elements

- `void DisplayStack()`
  - Print all the elements

```
void Stack::DisplayStack() {
        cout << "top -->";
        for (int i = top; i >= 0; i--)
                cout << “\t|\t" << values[i] << "\t|" << endl;
        cout << "\t|---------------|" << endl;
}
```

```
top -->  |        -8         |
         |        -3         |
         |        6.5        |
         |        5          |
         -------------------
```

Using `Stack`

```
int main(void) {
        Stack stack(5);
        stack.Push(5.0);
        stack.Push(6.5);
        stack.Push(-3.0);
        stack.Push(-8.0);
        stack.DisplayStack();
        cout << "Top: " << stack.Top() << endl;

        stack.Pop();
        cout << "Top: " << stack.Top() << endl;
        while (!stack.IsEmpty()) stack.Pop();
        stack.DisplayStack();
        return 0;
}
```

result

```
top --> |         -8        |
        |         -3        |
        |         6.5       |
        |         5         |
        |-------------------|
Top: -8
Top: -3
top --> |-------------------|
```

Implementation based on Linked List

- Now let's implement a stack based on a linked list
- To make the best out of the code of `List`, we implement `Stack` by inheriting `List`
  - To let `Stack` access private member `head`, we make `Stack` as a friend of `List`

```
class List {
public:
        List(void) { head = NULL; }              // constructor
        ~List(void);                              // destructor
        bool IsEmpty() { return head == NULL; }
        Node* InsertNode(int index, double x);
        int FindNode(double x);
        int DeleteNode(double x);
        void DisplayList(void);
private:
        Node* head;
        friend class Stack;
};
```

Implementation based on Linked List

```
class Stack : public List {
public:
        Stack() {}   // constructor
        ~Stack() {} // destructor
        double Top() {
                if (head == NULL) {
                        cout << "Error: the stack is empty." << endl;
                        return -1;
                }
                else
                        return head->data;
        }
        void Push(const double x) { InsertNode(0, x); }
        double Pop() {
                if (head == NULL) {
                        cout << "Error: the stack is empty." << endl;
                        return -1;
                }
                else {
                        double val = head->data;
                        DeleteNode(val);
                        return val;
                }
        }
        void DisplayStack() { DisplayList(); }
};
```

```
-8
-3
6.5
5
Number of nodes in the list: 4
Top: -8
Top: -3
Number of nodes in the list: 0
```

Note: the stack implementation based on a linked list will never be full.

Application: Balancing Symbols

- To check that every right brace, bracket, and parentheses must correspond to its left counterpart
  - e.g. [( )] is legal, but [( ]) is illegal
- Algorithm

  (1) Make an empty stack.

  (2) Read characters until end of file

    i. If the character is an opening symbol, push it onto the stack

    ii. If it is a closing symbol, then if the stack is empty, report an error

    iii. Otherwise, pop the stack. If the symbol popped is not the corresponding opening symbol, then report an error

  (3) At end of file, if the stack is not empty, report an error

Array implementation versus linked list implementations

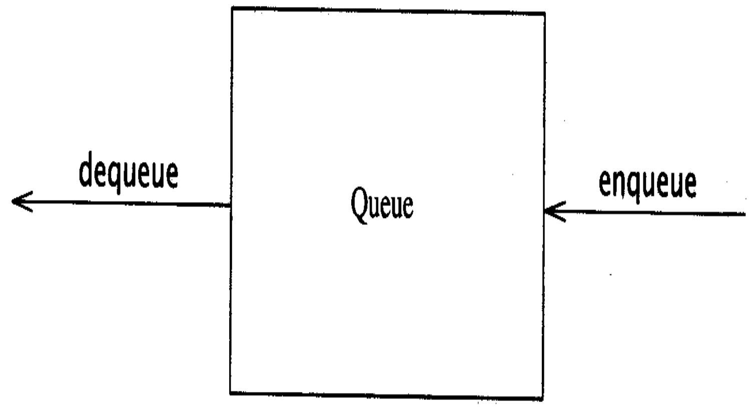
- push, pop, top are all constant-time operations in both array implementation and linked list implementation
  - For array implementation, the operations are performed in very fast constant time

Queue Overview

- Queue ADT
- Basic operations of queue
  - Enqueuing, dequeuing etc.
- Implementation of queue
  - Array
  - Linked list

Queue ADT

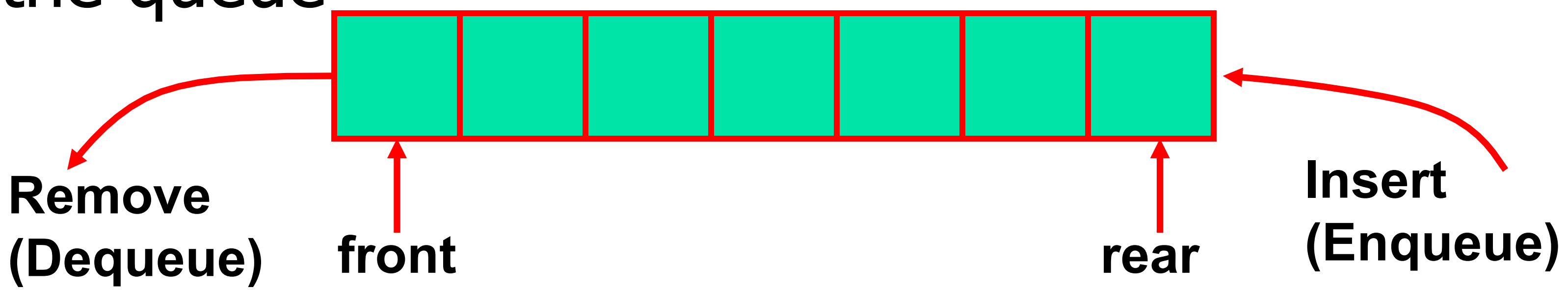
- Like a stack, a *queue* is also a list. However, with a queue, insertion is done at one end, while deletion is performed at the other end.



- Accessing the elements of queues follows a First In, First Out (FIFO) order.
  - Like customers standing in a check-out line in a store, the first customer in is the first customer served.

Enqueue and Dequeue

- Primary queue operations: Enqueue and Dequeue
- Like check-out lines in a store, a queue has a front and a rear.
- Enqueue – insert an element at the rear of the queue
- Dequeue – remove an element from the front of the queue

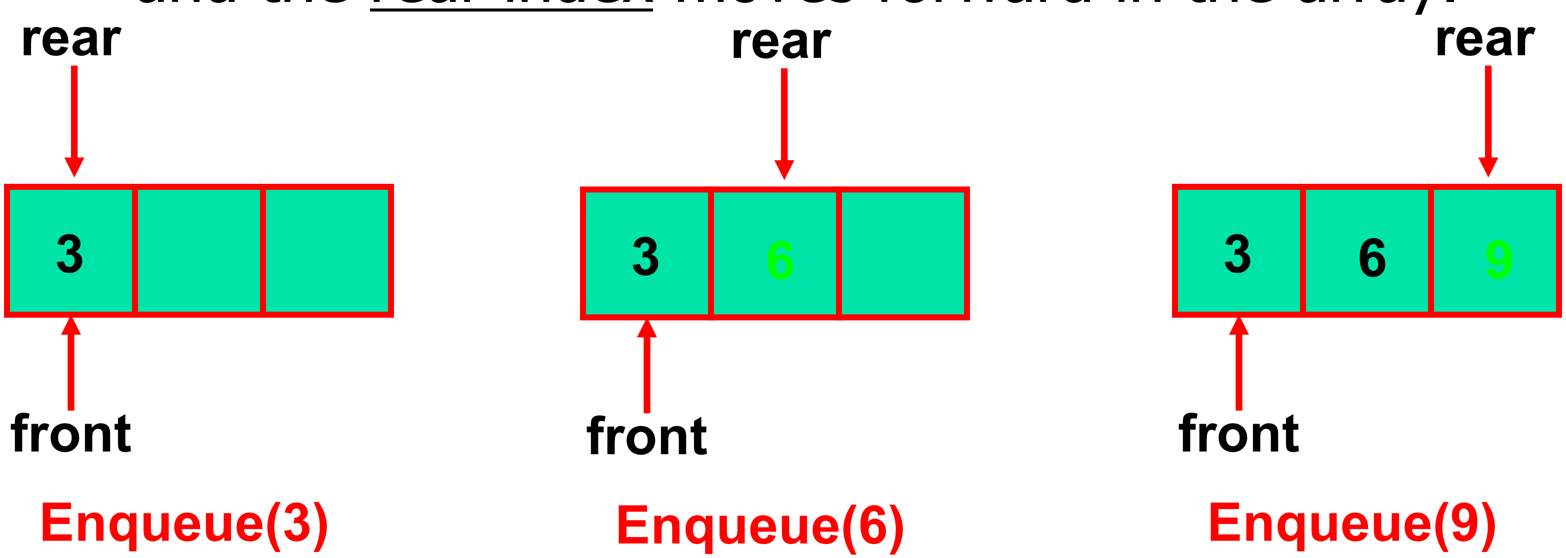


Implementation of Queue

- Just as stacks can be implemented as arrays or linked lists, so with queues.
- Dynamic queues have the same advantages over static queues as dynamic stacks have over static stacks

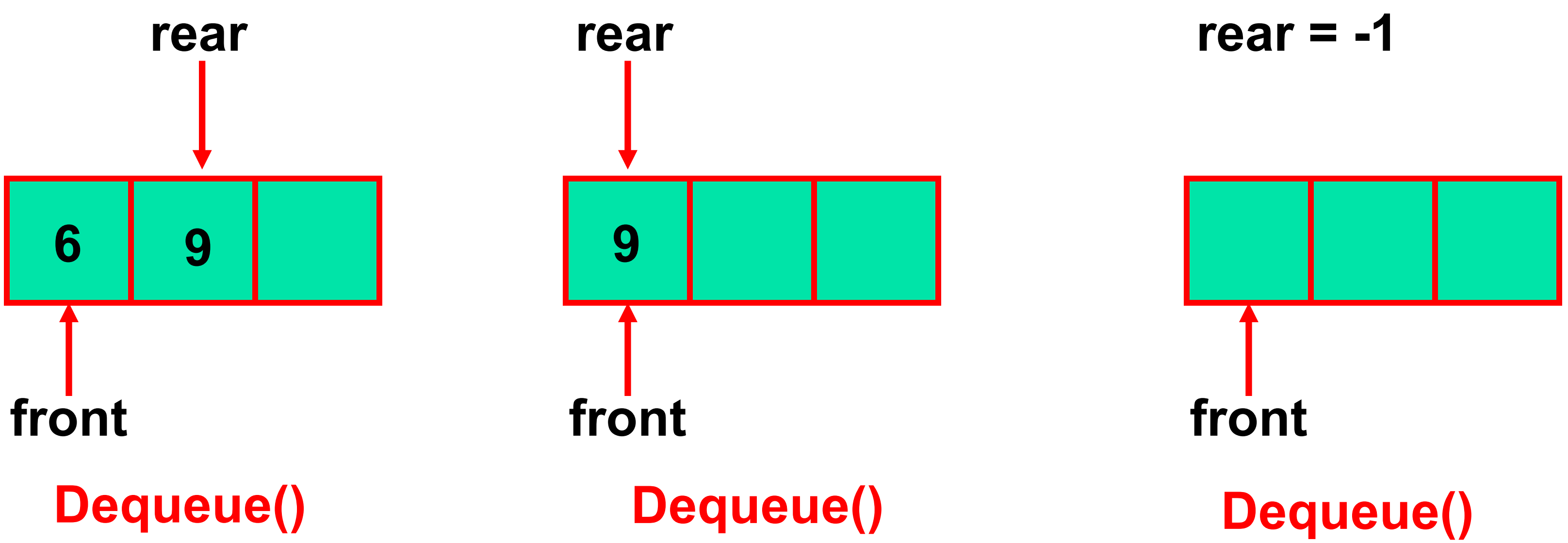
Queue Implementation Using Array

- There are several different algorithms to implement Enqueue and Dequeue
- Naïve way
  - When enqueuing, the <u>front index</u> is always fixed and the <u>rear index</u> moves forward in the array.



Queue Implementation Using Array

- Naïve way (cont'd)
  - When dequeuing, the front index is fixed, and the element at the front of the queue is removed. Move all the elements after it by one position. (Inefficient!!!)



Queue Implementation Using Array

- A better way
  - When an item is enqueued, the rear index moves forward.
  - When an item is dequeued, the front index also moves forward by one element

(front) XXXXOOOOO (rear)
OXXXXOOOO (after 1 dequeue, and 1 enqueue)
OOXXXXXOO (after another dequeue, and 2 enqueues)
OOOOXXXXX (after 2 more dequeues, and 2 enqueues)

The problem here is that the rear index cannot move beyond the last element in the array.

Implementation using Circular Array

- Using a circular array
- When an element moves past the end of a circular array, it wraps around to the beginning, e.g.
  - OOOOO7963 → 4OOOO7963 (after Enqueue(4))
  - After Enqueue(4), the rear index moves from 3 to 4.
- How to detect an empty or full queue, using a circular array algorithm?
  - Use a counter of the number of elements in the queue.

Queue Implementation Using Array

```
class Queue {
public:
        Queue(int size = 10);                          // constructor
        ~Queue() { delete [] values; }                 // destructor
        bool IsEmpty(void);
        bool IsFull(void);
        bool Enqueue(double x);
        bool Dequeue(double & x);
        void DisplayQueue(void);
private:
        int front;                  // front index
        int rear;                   // rear index
        int counter;                // number of elements
        int maxSize;                // size of array queue
        double* values;             // element array
};
```

Queue Class

- Attributes of `Queue`
  - `front/rear`: front/rear index
  - `counter`: number of elements in the queue
  - `maxSize`: capacity of the queue
  - `values`: point to an array which stores elements of the queue
- Operations of `Queue`
  - `IsEmpty`: return true if queue is empty, return false otherwise
  - `IsFull`: return true if queue is full, return false otherwise
  - `Enqueue`: add an element to the rear of queue
  - `Dequeue`: delete the element at the front of queue
  - `DisplayQueue`: print all the data

Create Queue

- `Queue(int size = 10)`
  - Allocate a queue array of `size`. By default, `size = 10`.
  - `front` is set to `0`, pointing to the first element of the array
  - `rear` is set to `-1`. The queue is empty initially.

```
Queue::Queue(int size /* = 10 */) {
        values      =       new double[size];
        maxSize     =       size;
        front       =       0;
        rear        =       -1;
        counter     =       0;
}
```

IsEmpty & IsFull

- Since we keep track of the number of elements that are actually in the queue: `counter`, it is easy to check if the queue is empty or full.

```
bool Queue::IsEmpty() {
        if (counter)        return false;
        else                return true;
}
bool Queue::IsFull() {
        if (counter < maxSize)          return false;
        else                            return true;
}
```

Enqueue

```
bool Queue::Enqueue(double x) {
	if (IsFull()) {
		cout << "Error: the queue is full." << endl;
		return false;
	}
	else {
		// calculate the new rear position (circular)
		rear		= (rear + 1) % maxSize;
		// insert new item
		values[rear]	= x;
		// update counter
		counter++;
		return true;
	}
}
```

Dequeue

```
bool Queue::Dequeue(double & x) {
        if (IsEmpty()) {
                cout << "Error: the queue is empty." << endl;
                return false;
        }
        else {
                // retrieve the front item
                x       = values[front];
                // move front
                front   = (front + 1) % maxSize;
                // update counter
                counter--;
                return true;
        }
}
```

Printing the elements

```
void Queue::DisplayQueue() {
        cout << "front -->";
        for (int i = 0; i < counter; i++) {
                if (i == 0) cout << "\t";
                else      cout << "\t\t";
                cout << values[(front + i) % maxSize];
                if (i != counter - 1)
                        cout << endl;
                else
                        cout << "\t<-- rear" << endl;
        }
}
```

```
front -->       0
                1
                2
                3
                4       <-- rear
```

Using Queue

```
Enqueue 5 items.
Now attempting to enqueue again...
Error: the queue is full.
front -->       0
                1
                2
                3
                4       <-- rear
Retrieved element = 0
front -->       1
                2
                3
                4       <-- rear
front -->       1
                2
                3
                4
                7       <-- rear
```

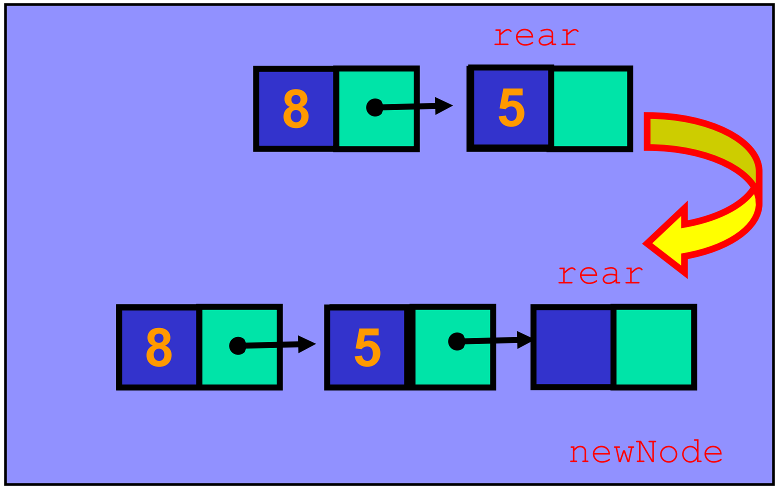
```
int main(void) {
        Queue queue(5);
        cout << "Enqueue 5 items." << endl;
        for (int x = 0; x < 5; x++)
                queue.Enqueue(x);
        cout << "Now attempting to enqueue again..." << endl;
        queue.Enqueue(5);
        queue.DisplayQueue();
        double value;
        queue.Dequeue(value);
        cout << "Retrieved element = " << value << endl;
        queue.DisplayQueue();
        queue.Enqueue(7);
        queue.DisplayQueue();
        return 0;
}
```

Queue Implementation Using Linked List

```
class Queue {
public:
        Queue() {               // constructor
                front = rear = NULL;
                counter   = 0;
        }
        ~Queue() {                              // destructor
                double value;
                while (!IsEmpty()) Dequeue(value);
        }
        bool IsEmpty() {
                if (counter)            return false;
                else                    return true;
        }
        void Enqueue(double x);
        bool Dequeue(double & x);
        void DisplayQueue(void);
private:
        Node* front;            // pointer to front node
        Node* rear;             // pointer to last node
        int counter;            // number of elements
};
```

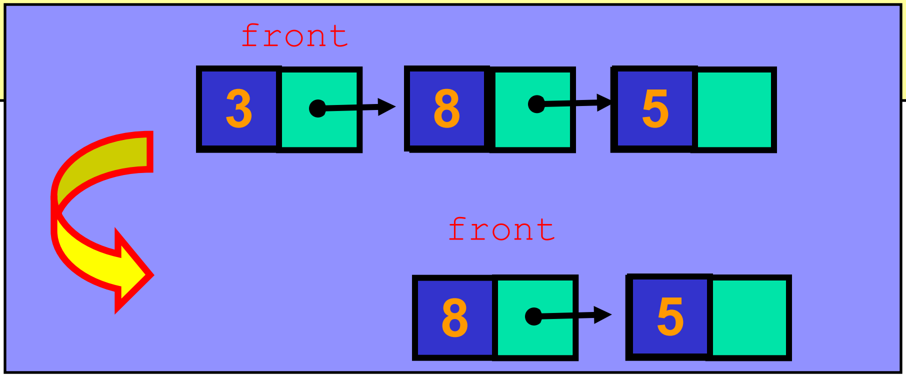
Enqueue

```
void Queue::Enqueue(double x) {
    Node* newNode       =   new Node;
    newNode->data       =   x;
    newNode->next       =   NULL;
    if (IsEmpty()) {
        front           =   newNode;
        rear            =   newNode;
    }
    else {
        rear->next      =   newNode;
        rear            =   newNode;
    }
    counter++;
}
```



Dequeue

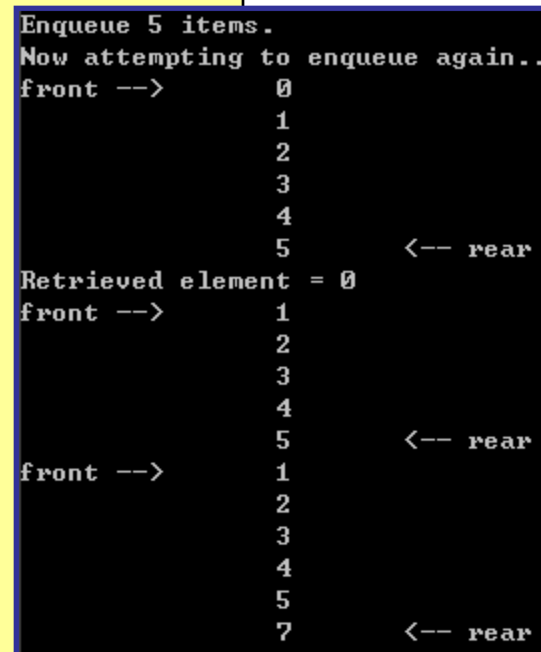
```
bool Queue::Dequeue(double & x) {
        if (IsEmpty()) {
                cout << "Error: the queue is empty." << endl;
                return false;
        }
        else {
                x               =       front->data;
                Node* nextNode  =       front->next;
                delete front;
                front           =       nextNode;
                counter--;
        }
}
```



Printing all the elements

```
void Queue::DisplayQueue() {
        cout << "front -->";
        Node* currNode        =       front;
        for (int i = 0; i < counter; i++) {
                if (i == 0)
                        cout << "\t";
                else
                        cout << "\t\t";
                cout << currNode->data;
                if (i != counter - 1)
                        cout << endl;
                else
                        cout << "\t<-- rear" << endl;
                currNode        =       currNode->next;

        }
}
```



Result

- Queue implemented using linked list will be never full

```
Enqueue 5 items.
Now attempting to enqueue again...
Error: the queue is full.
front -->       0
                1
                2
                3
                4       <-- rear
Retrieved element = 0
front -->       1
                2
                3
                4       <-- rear
front -->       1
                2
                3
                4
                7       <-- rear
```

based on array

```
Enqueue 5 items.
Now attempting to enqueue again..
front -->       0
                1
                2
                3
                4
                5       <-- rear
Retrieved element = 0
front -->       1
                2
                3
                4
                5       <-- rear
front -->       1
                2
                3
                4
                5
                7       <-- rear
```

based on linked list

Queue applications

- When jobs are sent to a printer, in order of arrival, a queue.
- Customers at ticket counters …