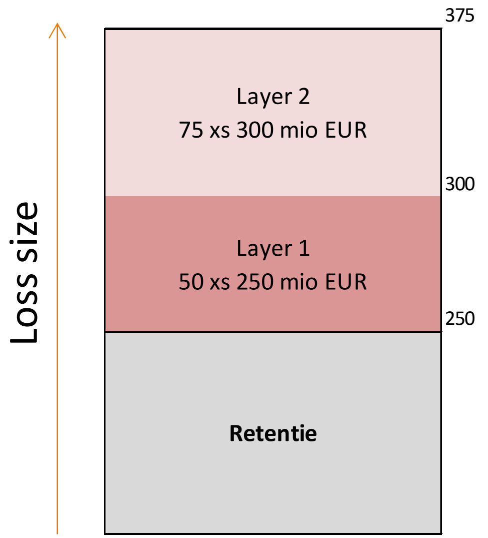
Case: price this structure

Non-proportional

- Excess of Loss (XoL)
- 1 paid reinstatement



Information available

- Loss history
- Output vendor models
- Explanation of methods
- Internet
- Note: Economic growth / indexation!
- Note: Earned Premium Income is 150 mln. EUR

Please prepare:

- Premium for layer 1 & 2
- Explanation of your pricing

Excess of Loss – XoL – Appendix Calculations

<u>Generic formula</u> - for each layer, the recovery would be:

$$Y^l = \min\{ \max(X - Deductible^l), Limit^l \} \qquad Eq.1$$

$\sum_{l=1}^{n} Y^l$ - total recovery from the reinsurance contract ; sum of layer recoveries

<u>Example</u>: 20mln xs 10mln; A loss *X* of 15 mln would generate a 5 milion recovery. Therefore 10 mln is **Retained**; 5 is **Reinsured**

Premium can be often given in Euro's, or in a **<u>% - called RoL – Rate on Line</u>** – a percentage of the Limit
e.g. Expected Reinsurance Premium = 20% * 20mln = 4 mln.

If Reinstatements are present, a **Reinstatement Premium** is calculated after the first loss. This could be free **1@0%** or fully paid **1@100%**.

$$Expected\ Reinsurance\ Premium * \frac{Loss}{Limit} * Free|Paid\%$$

<u>Example</u>: In our case $4mln * \frac{5}{20} * 100\% = 1\ mln$.

(1)Burning Cost methodology

Step 1 - Consider the reinsurance structure

Reinsurance Program				
Layer	Deductable	Limit	Reinstatement	
1	500	500	1	100%
2	1000	500	1	100%

Step 2 - Premium Information

Year	Premium
2012	5000
2013	5696
2014	6371
2015	7125
2016	7844
2017	8916
2018	10000

Step 3 - analize the losses given the XoL formula

Year	Loss	Liability L1	Liability L2
2012	740	240	0
2012	840	340	0
2012	980	480	0
2012	1110	500	110
2013	710	210	0
2013	900	400	0
2013	960	460	0
2013	1130	500	130
2014	730	230	0
2014	810	310	0
2014	920	420	0
2014	1090	500	90
2014	1200	500	200
2015	750	250	0
2015	890	390	0
2015	980	480	0
2015	1090	500	90
2015	1110	500	110
2016	720	220	0
2016	830	330	0
2016	940	440	0
2016	1010	500	10
2016	1080	500	80
2016	1340	500	340
2017	750	250	0
2017	880	380	0
2017	950	450	0
2018	1020	500	20
2018	1250	500	250
2018	1480	500	480

Step 4 - Yearly Premiums and Losses

Year	Premium	Liability L1	Liability L2
2012	5.000	1.000	110
2013	5.696	1.000	233
2014	6.371	1.000	449
2015	7.125	1.000	200
2016	7.844	1.000	430
2017	8.916	1.000	0
2018	10.000	1.000	750

$$Y^l = \min\{\max(X - Deductible^l), Limit^l\}$$

(1)Burning Cost methodology

Step 5 - determine the loss ratios

Year	Layer1	Layer2
2012	20,00%	2,20%
2013	17,56%	4,09%
2014	15,70%	7,05%
2015	14,04%	2,81%
2016	12,75%	5,48%
2017	11,22%	0,00%
2018	10,00%	7,50%

Step 6 - Determine the final BC ratio

	L1	L2
Aritmethical	14,46%	4,16%
Weighted average	13,74%	4,26%

Final rate ?

General mathematical formulas:

Defining the Burning Cost of each year as the biggest liability of the layer devided by the yearly earned premium income (EPI):

$$BC_t = \frac{L_t}{EPI_t}$$

The so called (maximum) Annual Aggregate Limit woud be:

$$MaxAAL = \left(Reinst^l + 1\right) * Limit^l$$

The pure yearly AAL for the XoL contract is then:

$$AAL = \min\{MaxAAL, Y^l\}$$

(2)Deterministic probability sampling

- Curve is already modelled by a third party – vendor model
- The probability distribution is more or less given – often Poisson or Negative Binomial

Step 1

- An occurrence exceedence probability would be:

Event	Pr(Xn=z)	X
1	0,5338	2.088.614
2	0,2896	5.714.001
3	0,0918	15.714.066
4	0,04	31.169.935
5	0,0162	60.727.029
6	0,006	75.661.148
7	0,0044	89.427.523
8	0,009	123.968.407
9	0,0046	274.067.520
10	0,002	466.991.413
11	0,002	619.323.623
12	0,0002	1.024.447.925
13	0,0004	1.306.015.728

- This is, in fact, the probability density function (**pdf**)

Step 2

- Having the **Pdf** we can determine the **Cdf:**

Event	F=Pr(Xn≤z)	X
1	0,5338	2.088.614
2	0,8234	5.714.001
3	0,9152	15.714.066
4	0,9552	31.169.935
5	0,9714	60.727.029
6	0,9774	75.661.148
7	0,9818	89.427.523
8	0,9908	123.968.407
9	0,9954	274.067.520
10	0,9974	466.991.413
11	0,9994	619.323.623
12	0,9996	1.024.447.925
13	1	1.306.015.728

$$F_i = F_{i-1} + P_i \; for \; i = \{2, \dots, n\} \; and \; with \; F_1 = P_1$$

- *F is also* known under the term of *Exceedance Frequency*

(2)Deterministic probability sampling

- Following the Cumulative Distribution Function, we can know proceed to our simulation framework

- Herein, for each event sampled in a given year/scenario, a uniform distribution can be sampled betweem 0 and 1 – ***Uniform (0,1)***

- The realisation of a draw is used to look up for an event entry thgough the ***cdf****, or the Exceedance Frequency*

Step 3

- Draw a random number $u \sim Uniform\ (0,1)$

Year/ Scenario	*u*
1	0,846490613
2	0,501508536
3	0,103457697
4	0,953091016
5	0,999612179
6	0,134665844
7	0,077069823
8	0,490500106
9	0,85312786
10	0,630134672
11	0,781986351
12	0,144245606
13	0,615513371
14	0,862617672
15	0,895532451
16	0,096197391
17	0,598806112
18	0,050791757
19	0,30769088
20	0,326647545

(2)Deterministic probability sampling

- Once we have the random sample from the Uniform distribution, we can look up the event with the first exceedance frequency greater then our random number $\boldsymbol{u}$

- If we look at our first sampled year value in the table below, $u_1 = 0{,}8464$, this would be $0{,}8464 < 0{,}9152$, which corresponds in our initial constructed cdf table to **Event 3**, therefore its Loss of *15.714.066* will be picked up.

Step 3

Year/ Scenario	*u*
1	0,846490613
2	0,501508536
3	0,103457697
4	0,953091016
5	0,999612179
6	0,134665844
7	0,077069823
8	0,490500106
9	0,85312786
10	0,630134672
11	0,781986351
12	0,144245606
13	0,615513371
14	0,862617672
15	0,895532451
16	0,096197391
17	0,598806112
18	0,050791757
19	0,30769088
20	0,326647545

Event	*F=Pr(Xn≤z)*	*X*
1	0,5338	2.088.614
2	0,8234	5.714.001
3	0,9152	15.714.066
4	0,9552	31.169.935
5	0,9714	60.727.029
6	0,9774	75.661.148
7	0,9818	89.427.523
8	0,9908	123.968.407
9	0,9954	274.067.520
10	0,9974	466.991.413
11	0,9994	619.323.623
12	0,9996	1.024.447.925
13	1	1.306.015.728

Year/ Scenario	*Event / Loss Selection*	*Selected/Sampled Event*
1	15.714.066,26	3
2	2.088.613,56	1
3	2.088.613,56	1
4	31.169.934,86	4
5	1.306.015.728,21	13
6	2.088.613,56	1
7	2.088.613,56	1
8	2.088.613,56	1
9	15.714.066,26	3
10	5.714.000,99	2
11	5.714.000,99	2
12	2.088.613,56	1
13	5.714.000,99	2
14	15.714.066,26	3
15	15.714.066,26	3
16	2.088.613,56	1
17	5.714.000,99	2
18	2.088.613,56	1
19	2.088.613,56	1
20	2.088.613,56	1

(2)Deterministic probability sampling

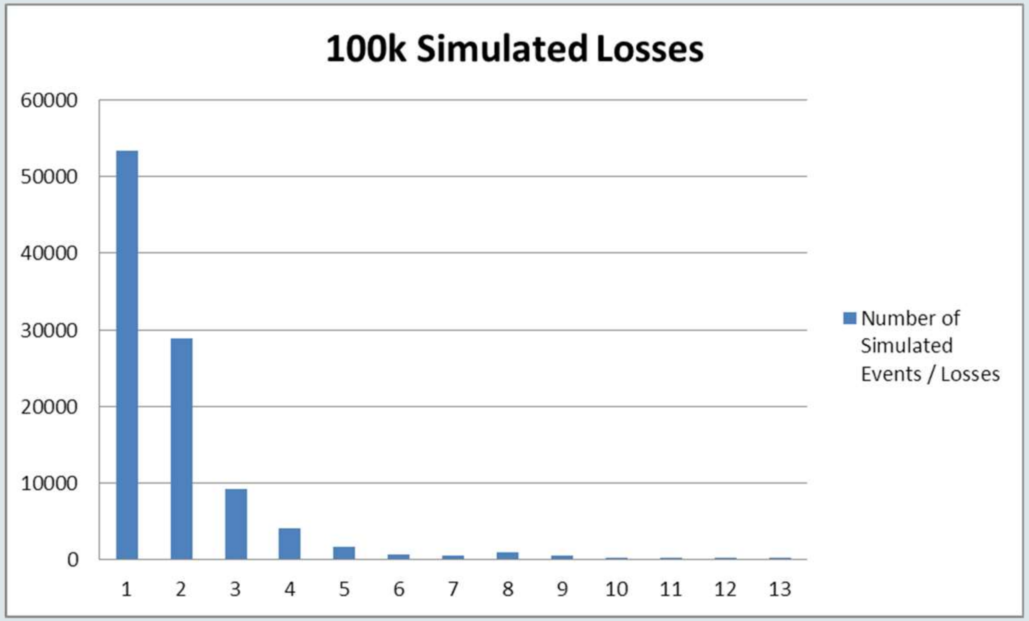
- Note, in the previous slide, our sample contains only 20 years, assuming 1 event a year occurs.

- It is recommended a sample size of at least 50.000 years is drawn. Given the fact that the underlying event frequency is represented by Poisson distribution, on average, how many events would you expect in a year?

- Below, you can see that the distribution of the events follows the initial provided probability density function, when using 100.000 simulated years **=> The average of all sampled years will be your expected <u>Gross Loss $E[X]$</u>**

<u>Final step, increasing the sample</u>

Event	*Loss*	*Event Counts within Sample*
1	2.088.613,56	53401
2	5.714.000,99	28872
3	15.714.066,26	9161
4	31.169.934,86	4088
5	60.727.028,51	1597
6	75.661.147,95	605
7	89.427.523,03	438
8	123.968.407,05	917
9	274.067.520,22	449
10	466.991.413,31	199
11	619.323.623,47	216
12	1.024.447.925,30	21
13	1.306.015.728,21	36
	Total Scenarios	100.000



(2)Deterministic probability sampling

<u>Tips</u>

- Using the XoL formula for each Layer, you can determined the loss within the layer for a given simulated year.

- *The average of all recovered/net losses within the layer would be your $E[Y]$ - so your* **<u>Pure Premium</u>** for that Layer.

- In case you assume there are more events occuring within one year, we would recomend sampling an Event Frequency $N \sim Poisson\ (\lambda)$, where λ could equal 0,6931. (i.e this is also the E[N], expected value of yearly events in the sample)