Printout

Monday, October 7, 2019 12:40 PM

10-7 Exam Review
CSCI 3022 Fall 19

$X \sim bin(n,p)$

$E[X] = np.$

$E[X^2] = \sum x^2 P(X=x)$

$E[X] = \sum x \cdot P(X=x)$

$= 0 \cdot P(X=0)$

$+ 1 \cdot P(X=1)$

$+ 2 \cdot P(X=2)$

You are taking a 2-question quiz and believe there to be about a 75% chance that any one of your answers is correct. What is the pmf of your grade on the quiz? Your expected grade on the quiz? What is the variance in your quiz grade?

$P(X=0) = 1/4 \cdot 1/4 = 1/16$

$P(X=1) = 6/16$

$P(X=2) = 3/4 \cdot 3/4 = 9/16$

$= \frac{1 \cdot (6) + 2 \cdot (9)}{16} = \frac{24}{16}$

$= 1.5$

$Var[X] = E[(X - E[X])^2]$

$= \sum (x - 1.5)^2 f(x)$

$= (0-1.5)^2 P(X=0)$ $\quad 9/4 \cdot 1/16$

$(1-1.5)^2 P(X=1)$ $\quad + 1/4 \cdot 6/16$

$(2-1.5)^2 P(X=2)$ $\quad + 1/4 \cdot 9/16$

$= np(1-p).$

Opening Sol... and a variance shortcut

$E[(X - E[X])^2]$

$= E[X^2 - 2X E[X] + (E[X])^2]$

$$= E[X^2 - 2XE[X] + (E[X])^2]$$

$$= E[X^2] - E[2X E[X]] + E[(E[X])^2]$$

random not not random

$$= E[X^2] - 2E[X] \cdot E[X] + (E[X])^2$$

$\int x^2 f(x)dx$ $\left(\int x f(x) dx\right)^2$

$$Var[X] = E[X^2] - (E[X])^2$$

Announcements and Reminders:

- Practicum 1 due a week from ~~today~~. Friday.
- Exam Review Monday in class.
- Exam Tuesday: 6:30-8:00 PM on Tuesday 8 October, this section in HUMN 1B50: Inform me by 5pm today.
- You are allowed to use a calculator. No smartphones or other devices that can store large amounts of data or access the internet.
- You are allowed one 3x5-inch notecard as a cheat sheet. You can write whatever you want on it and can use both sides. No magnifying glasses.
- Quizzes, notebooks, homework, past exams are all good references!

On your exam page...

Bayes' theorem	$p(A \mid B) = \dfrac{p(B \mid A)p(A)}{p(B)}$	
Law of total probability	$p(E) = \sum_{i=1}^{N} p(E \mid F_i)p(F_i)$	
Union of sets	$p(A \cup B) = p(A) + p(B) - p(A \cap B)$	Inclusion Exclusion
Conditional probability	$p(A \mid B) = \dfrac{p(A \cap B)}{p(B)}$	
Expectation & variance	$Var(X) = E[X^2] - E[X]^2$	
You can do it!		

Sample Stats

A List

Sample mean

" variance

" std

" median

" IQR

Computation

1 2 3 4 5

Q_1

Q_3

Q_3: median of the upper half of data

n even $n/2$ | $n/2$

n odd. Q_1 Q_3

median is "in" lower half

median is "in" upper half

$.25 = F(X)$

Data Viz

Histograms

frequency

density

Start with freq...
then make area = 1
by dividing by current
area.

Boxplots

make & read them

Probability Theory

Sets Conditions

Independence: $P(B|A)=P(B)$ or $P(A|B)=P(A)$ or

$P(A\cap B)=P(A)\cdot P(B)$

multiplication rule

De Morgan's Laws

$P((A\cup B)^c)=P(A^c\cap B^c)$

Disjoint/Mutually Exclusive events

if A & B don't overlap: $P(A\cup B)=P(A)+P(B)$ $\cancel{-P(A\cap B)}$

Prob:

① $P(\Omega)=1$

add up pmf

integrate all of pdf

② Ensure $P(A)\geq 0$

Discrete RVs

Tools	$f(x)$	Properties μ	σ^2
Bern(p) coin flip	$p^x(1-p)^{1-x}$	p	$p(1-p)$
Binomial (n,p) many coin flip fixed #	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$
Geometric ? coinflips until a "1" or "H"	$p(1-p)^{x-1}$	$1/p$	$\frac{1-p}{p^2}$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

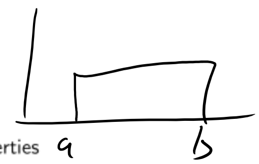
Discrete RVs

Tools	Properties
Neg Binom: P fail always until you succeed "r" times. also known as: do the geometric r times.	$\binom{x-1}{r-1} p^r (1-p)^{x-r}$ $x \in [r, r+1, \ldots]$ $\mu = r/p$
Poisson:	$\mu = \lambda$
Discrete Uniform: all outcomes equal	

Continuous RVs

Tools

Properties



uniform $f(x)=\frac{1}{b-a}$ $\mu=\frac{a+b}{2}$

exponential

$f(x)=\lambda e^{-\lambda x}$ $x>0$

$\mu = 1/\lambda$

kinda like Poisson...

Expected Value

Example 1

Let X = the outcome when a fair die is rolled once. Suppose that, before the die is rolled, you are offered a choice: Option #1: a guarantee of $\frac{1}{4}$ dollars (whatever the outcome of the roll); Option #2: $h(X) = 1/X$ dollars. Which option would you prefer? Justify your answer.

What is the variance of each option?

$E[\text{option } 1] = 1/4$ $\quad Var[\text{option } 1] = 0$

$X = 1$ w/ Prob $1/6$
$\quad 2$ " "
$\quad \vdots$

$E[1/X] = \sum_x \frac{1}{x} \cdot P(X=x)$

$= \sum_x \frac{1}{x} \cdot 1/6$

$= 1/6(1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6)$

$> 1/4.$

OR find PMF for $1/X$.

$h(X)$ pmf:
$h(x) = 1$ w/ Prob $1/6$
$\quad 1/2$ " $1/6$
$\quad 1/3$ " " "

$E[(1/x)^2]$ vs. $Var(1/x)$.
$\hookrightarrow = \sum_x (\frac{1}{x})^2 P(X=x) = 1/6(1 + 1/4 + 1/9 + \cdots)$

Solution Slide

Examples

A company offers cable, phone, and internet services. The probability a customer will purchase cable is 0.35, purchase phone services is 0.20, and purchase internet services is 0.40. A customer will purchase both cable and phone services with probability 0.18, both cable and internet is 0.14, and both phone and internet is 0.26. A customer will purchase all three items with probability 0.08. What is the probability that a customer purchases:

$P(A^c \cap B^c \cap C^c)$

$P(C) = .35$

$P(P) = .2$

$P(I) = .4$

$P(C \cap P) = .18$

$P(C \cap I) = .14$

$P(P \cap I) = .26$

$P(\text{all}) = .08$

AND

1. None of these items?
2. Two or more of these items?
3. Exactly one of these items? *Hint: This is easier if you solve using parts (a) and (b).*
4. Are these events independent? Why or why not?

for all?

is: $P(C) \cdot P(I) = P(C \cap I)$

OR

$= P((C \cup P \cup I)^c)$

$1 - P(C \cup P \cup I)$

inclusion-exclusion

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Note: prob won't ask w/ 3 events...

Solution Slide

Examples

Suppose that you have 10 light bulbs in a room, and that the lifetime of each bulb is independent of the other bulbs. Assume that each light bulb's lifetime has an exponential distribution with a mean equal to 3.

1. You pick one of the lightbulbs at random. What is the probability that it fails before 2 years?
2. What is the probability that exactly five of the 10 bulbs fail before 2 years?
3. After two years, you find that 5 of the lightbulbs are still burning. What is the probability that all 5 lightbulbs will burn an additional 2 years?

$= 1-e^{-2/3}$

1. $\int_0^2 \lambda e^{-\lambda t} dt$ for $\lambda = 1/3$ $= A$.

2. $Bin(10, p=A)$ at $x=5$: $\binom{10}{5} p^5 (1-p)^5$

$p = A$

3. each: $1-A$

all 5: $(1-A)^5$.

Solution Slide

Examples

You have 6 coins in front of you. Five of the coins are unbiased (i.e., the probability of tossing a head is 50%). The sixth coin is biased, and the probability of tossing a head is 70%. It is not possible to tell which is the biased coin just by looking. You plan to pick a coin at random and then will flip it three times.

1. If an unbiased coin was picked, what is the probability that (exactly) two of three tosses will be heads?
2. If the biased coin was picked, what is the probability that (exactly) two of three tosses will be heads?
3. What is the total probability of tossing (exactly) two heads?
4. You pick your coin and toss it three times and get two heads. What is the probability you selected the biased coin?

Solution Slide