# DL Meeting

H&D Group

April 27, 2025

# 1 REMS Model

## 1.1 Constants

$n$ - number of people $d$ - number of shifts ($2 \times$ number of days in the month) $t_j$ - $\{\, 1\ isnightshift$ $-1 isdayshift$ $h_j$ - is shift $j$ a high demand shift (Thursday, Friday, Saturday nights) (binary) $a_{ij}$ - is person $i$ available at shift $j$ $o_i$ - is person $i$ a member of duty crew (non-observer) $c_i$ - is person $i$ off campus

## 1.2 Decision Variables

$x_{ij}$ - is person $i$ assigned to shift $j$ $b_j$ - penalty for if shift $j$ has less than three people $m_i$ - penalty for if person $i$ has less than 2 shifts

## 1.3 Objective Components

$underutilization = \sum_{i=1}^{n} m_i$ $understaffed = \sum_{j=1}^{d} h_j b_j + .2 \sum_{j=1}^{d} (1-h_j) b_j$ $shiftimbalance = \sum_{i=1}^{n} (\sum_{j=1}^{d} t_j x_{ij})^2$

$$\min\ 5 \cdot underutilization + 10 \cdot understaffed + shiftimbalance$$

$x_{ij} \leq a_{ij}$
$b_j + \sum_{i=1}^{n} o_i x_{ij} = 2 \quad \forall j \in [d]$
$b_j + \sum_{i=1}^{n} x_{ij} \leq 3 \quad \forall j \in [d]$
$m_i + \sum_{j=1}^{d} x_{ij} = 2 \quad \forall i \in [n]$
$\sum_{i=1}^{n} c_i x_{ij} \leq 2 \quad \forall j \in [d]$
$x_{ij} \in \{0, 1\}$

# 2 2/9/2025 - Meeting 1

## 2.1 Basic Model

Define:
Skills $k = 1, \ldots, h$
Shift $j = 1, \ldots, m$
Employee $i = 1, \ldots, n$
$s_{ij}$: an employee $i$ has skill to work task $k$
$c_{ij}$: an employee $i$ can work shift $j$
$b_{jk}$: the number of employees needed with skill to work task $k$ at time $j$
$x_{ijk}$: decision variable; employee $i$ working shift $j$ on skill $k$

$$\min 0$$
$$\text{s.t. } \sum_{i=1}^{n} x_{ijk} \geq b_{ij} \forall jk \text{ (all shifts have needed tasks covered)}$$
$$\sum_{k=1}^{h} x_{ijk} \leq 1 \forall i, j \text{ (all employees can do one job per shift)}$$
$$x_{ijk} \leq c_{ij} \forall i, j \text{ (enforcing availability)}$$
$$x_{ijk} \leq s_{ik} \forall i, k \text{ (enforcing skillset) } x_{ijk} \in \{0, 1\}$$

## 2.2 With Penalty

Define:
Skills $k = 1, \ldots, h$
Shift $j = 1, \ldots, m$
Employee $i = 1, \ldots, n$
$s_{ij}$: an employee $i$ has skill to work task $k$
$c_{ij}$: an employee $i$ can work shift $j$
$b_{jk}$: the number of employees needed with skill to work task $k$ at time $j$
$p_{jk}$: penalty associated with no employee working task k at time j
$x_{ijk}$: decision variable; employee $i$ working shift $j$ on skill $k$

$$\min \sum \sum p_{jk}$$
$$\text{s.t. } \sum_{i=1}^{n} x_{ijk} + p_{jk} \geq b_{ij} \forall jk \text{ (all shifts have needed tasks covered)}$$
$$\sum_{k=1}^{h} x_{ijk} \leq 1 \forall i, j \text{ (all employees can do one job per shift)}$$
$$x_{ijk} \leq c_{ij} \forall i, j \text{ (enforcing availability)}$$
$$x_{ijk} \leq s_{ik} \forall i, k \text{ (enforcing skillset) } x_{ijk} \in \{0, 1\} \; p_{jk} \geq 0$$

Note: Consider changing 1st constraint to adding a string $\geq 1$ later on

## 2.3 Split Variables

Consider adding:
$y_{ij}$: is employee i working shift j
$z_{ik}$: is employee i working task k

# 3 2/23/2025 - Meeting 2

## Quadratic Model

Define:
$x_{ik}$: where i is shift i, k is employee k
$a_{ij}$: number of chefs needed for shift i of level j
$E_j$: subset of employees of level j

$$\begin{gathered}\min \textstyle\sum_{k\in K}((\sum_{i\in I} x_{ik} - 40)^2 + \alpha(\sum_{i\in I} x_{ik} - 40)) \\ \text{s.t. } \textstyle\sum_{x_{ik}\in E_j} x_{ik} \geq a_{ij} \quad \forall i, j\end{gathered}$$

## 3.1 Linear Version

Let $z_{ik}$ be the overworked hours for employee k, for shift i
Let $w_k$ be the decision variable that enforces the underworking constraint.

$$\begin{gathered}\max 2\textstyle\sum x_{ik} + \sum z_{ik} - \sum w_k \\ \text{s.t. } \textstyle\sum z_{ik} + \sum x_{ik} \geq a_{ij} \quad \forall i, j \\ \textstyle\sum_i x_{ik} \leq 40 \\ w_k + \textstyle\sum_i x_{ik} = 40\end{gathered}$$

## 3.2 Consecutive Shifts Constraint

Sum over 1 day's shifts and night-morning shift is equal to 1.