UNIVERSITY OF DHAKA

Department of Applied Mathematics

Fourth Year B.S. in Applied Mathematics, Academic Session: 2023 – 2024

Course No: **AMTH 450**, Course Title: **MATH LAB IV (Application Software)**

Assignment – 2: Solving Problems on Multivariate and Vector Calculus with its Real-life Applications

Instruction: Write programming code using **Python** to get the outputs and visualize the obtained results of the following problems.

Name: Roll: Group:

1. (a) Find the parametric equations of the tangent line to the following curves:

$$i.\vec{r}(t) = \ln t\,\hat{i} + e^{-t}\,\hat{j} + t^3\hat{k}; t_0 = 2$$

$$ii.\vec{r}(t) = 2\cos\pi t\,\hat{i} + 2\sin\pi t\,\hat{j} + 3t\hat{k}; t_0 = \frac{1}{3}.$$

(b) Find the vector parallel to the line of intersection of the two planes $3x-6y-2z=15$ and $2x+y-2z=5$.

(c) Find the velocity and acceleration of $\vec{r}(t)=3t\,\hat{i}+\sin t\,\hat{j}+t^2\hat{k}$ as a function of $\theta(t)$. Also, plot the graph of $\theta(t)$ versus t.

2. (a) Find the tangent vectors to the plane curve C defined by the vector function $\vec{r}(t)=5\cos t\,\hat{i}+4\sin t\,\hat{j}$ at the points where $t=\frac{\pi}{4}$ and $t=\pi$. Make a sketch of C, and display the position vectors $\vec{r}\left(\frac{\pi}{4}\right)$ and $\vec{r}(\pi)$, and the tangent vectors $\vec{r}'\left(\frac{\pi}{4}\right)$ and $\vec{r}'(\pi)$.

(b) A bug walks along the trunk of a tree following a path modeled by the circular helix $\vec{r}(t)=\langle \cos t, \sin t, t\rangle$. Find the arc length parameterization of the circular helix while starting the bug at the reference point $(1,0,0)$. Also, compute the bug's final coordinates when the bug walks up the helix for a distance of 10 units and graphically show the obtained results.

3. (a) Compute $\vec{T}, \vec{N}, \vec{B}, \kappa,$ and τ for the curves:

$$i.\vec{r}(t) = e^t\,\hat{i} + e^t\cos t\,\hat{j} + e^t\sin t\,\hat{k}; t=0$$

$$ii.\vec{r}(t) = 2\cos t\,\hat{i} + 3\sin t\,\hat{j}; 0\le t\le 2\pi$$

Also, plot the graphs of $\kappa(t)$ and hence comment on the obtained results.

(b) Justify whether or not the function $f(x,y)=y^2\cos(x-y)$ satisfies the *Laplace's equation* and *Cauchy-Riemann equations*. Also, establish the identity $f_{xy}=f_{yx}$ if possible.

(c) Suppose that $w=\sqrt{x^2+y^2+z^2}, x=cos\theta, y=\sin\theta, z=\tan\theta$. Use chain rule to find $\frac{dw}{d\theta}$ when $\theta=\frac{\pi}{4}$.

(d) The temperature (in degrees Celsius) at a point (x,y) on a metal plate in the *xy*-plane is stated as $T(x,y)=3x^2y$. Compute the gradient of $T(x,y)$ at the point $\left(-1,\frac{3}{2}\right)$, and the directional derivative of $T(x,y)$ at the point $\left(-1,\frac{3}{2}\right)$ in the direction $\left(-1,-\frac{1}{2}\right)$. Also, plot of the directional derivative with $-2\le x\le 0, 0\le y\le 2$, and visualize directional derivative over a surface.

4. (a) Sketch the contour plots of $(i)\ f(x,y)=4x^2+y^2, (ii)\ f(x,y,z)=z^2-x^2-y^2$ using level curves of height $k=1,4,9,16,26,36$.

(b) Consider the functions: $(i)\ f(x,y)=y^2-2y\cos x, 1\le x\le 7$
$(ii)\ f(x,y)=|\sin x \sin y|, 0\le x\le 2\pi, 0\le y\le 2\pi$

Plot the three dimensional figures with *python and Matplotlib* to get a better visualization.

(c) Locate all *relative extrema* and *saddle points* of the following functions:

$$(i)\ f(x,y)=4xy-x^4-y^4 \text{ and } (ii)\ f(x,y)=4x^2e^y-2x^4-e^{4y}.$$

Confirm that your obtained results are consistent with graphs.

5. (a) Consider the ellipsoid $x^2+4y^2+z^2=18$

i. Find an equation of the tangent plane to the ellipsoid at the point (1, 2, 1).

ii. Find parametric equations of the line that is normal to the ellipsoid at the point (1, 2, 1).

iii. Find the acute angle that the tangent plane at the point (1, 2, 1) makes with the *xy*-plane.

iv. Visualize the obtained results.

(b) A space probe has the shape of an ellipsoid $4x^2+y^2+4z^2=16$ and after sitting in the sun for an hour, the temperature on its surface is given by $T(x,y)=8x^2+4yz-16z+600$. Apply *Lagrange multipliers* approach to find the hottest point on the surface.

6. (a) Compute the integrals:

$$(i)\int_0^1\int_0^{1-x^2}\int_3^{4-x^2-y^2} xe^{-y}\cos(z)dzdydx \text{ and } (ii)\iint_R \frac{xy}{\sqrt{x^2+y^2+1}}dA;\ R=\{(x,y):0\le x\le 1, 0\le y\le 1\}.$$

(b) Find the surface area of that portion of the surface $z=\sqrt{4-x^2}$ that lies above the rectangle R in the *xy*-plane whose coordinates satisfy $0\le x\le 1$ and $0\le y\le 4$.

(c) Find the volume of the solid that lies below the paraboloid $z=4-x^2-y^2$, above the *xy*-plane, and inside the cylinder $(x-1)^2+y^2=1$.

7. (a) Suppose that the temperature in degrees Celsius at a point (x, y) on a flat metal plate is described as $T(x,y)=10-8x^2-2y^2$, where x and y are in meters. Find the average temperature of the rectangular portion of the plate for which $0\le x\le 1$ and $0\le y\le 2$.

(b) Evaluate the line integral $\int_C (xy+z^3)ds$ from $(1,0,0)$ to $(-1,0,\pi)$ along the helix C that is represented by the parametric equations $x=\cos t, y=\sin t, z=t\ (0\le t\le \pi)$.

(c) Find the mass of a cylinder with radius r and height h centered at origin with density $\rho(x,y)=x^2+y^2$.

(d) Let $\vec{F}(x,y)=e^y\hat{i}+xe^y\hat{j}$ denotes a force field in the *xy*-plane.

i. Verify that the force field $\vec{F}(x,y)$ is conservative on the entire xy-plane.

ii. Find a potential function ϕ.

iii. Find the work done by the field on a particle that moves from $(1,0)$ to $(-1,0)$ along the semicircular path C.

8. (a) Apply Green's Theorem to find the work done by the force field

$$\vec{F}(x,y)=\left(e^{x}-y^{3}\right)\hat{i}+\left(\cos y+x^{3}\right)\hat{j}$$

on a particle that travels once around the unit circle $x^2+y^2=1$ in the counterclockwise direction.

(b) Evaluate the surface integral $\iint_{\sigma} x^2 dS$ over the sphere $x^2+y^2+z^2=1$.

(c) Use the Divergence Theorem to find the outward flux of the vector field

$$\vec{F}(x,y)=x^3\hat{i}+y^3\hat{j}+z^2\hat{k}$$

across the surface of the region that is enclosed by the circular cylinder $x^2+y^2=9$ and the planes $z=0$ and $z=2$.

(d) Verify Stoke's Theorem for the vector field $\vec{F}(x,y,z)=2z\,\hat{i}+3x\,\hat{j}+5y\,\hat{k}$, taking σ to be the portion of the paraboloid $z=4-x^2-y^2$ for which $z\geq 0$ with upward orientation, and C to be the positively oriented circle $x^2+y^2=4$ that forms the boundary of σ in the xy-plane.

The End