A simple theoretical and practical approach to measuring dielectric properties with an open-ended coaxial probe

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A simple theoretical and practical approach to measuring dielectric properties with an open-ended coaxial probe

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Abstract. The application of the open-ended coaxial line to the measurements of dielectric constant and conductivity over the 0.01–100 MHz frequency range is presented. A mathematical model that accounts for the electrical cell constant is also proposed. The calculated value of cell constant agreed well with the experimentally determined one. The calibration procedure and its merits with regard to the experimental precision and handling simplicity are discussed. Finally, the results obtained in measurements on yeast cell suspension and human skin, together with the main problems that have appeared in practice, are emphasized.

1. Introduction

Measurements of permittivity and conductivity of heterogeneous systems in the radiofrequency range are of special interest for many fields of scientific research. There are two major types of measurement methods: frequency domain measurements and time domain measurements. Both techniques yield the same quantity, namely the complex permittivity of the sample as a function of frequency. The choice of one or the other must take into account the intrinsic properties of the material under test, the frequency range to be covered and, finally, the type of measuring apparatus available. Some consistent reviews on this subject have been published by Schwan [1], Kaatze and Geise [2] and, more recently, by Kell and Davey [3].

The most often used frequency domain methods are based on the parallel plate capacitor [1, 4–6] and four-electrode system [3, 7]. However, the decay of accuracy at high frequency, mainly due to the artefactual inductance, and the difficulties in manipulation still remain unresolved. Furthermore, in some experiments, such as *in vivo* tissue measurements, the techniques are impractical. The most promising method is the one based on the open-ended coaxial line, previously proposed by Athey *et al* [8] for measurements in the high radiofrequency to microwave frequency range.

The approach presented here has been proven to be very useful in radiofrequency measurements on a wide variety of systems (suspensions of solid particles [9], biological tissues, cell suspensions and others) and has overcome the above-mentioned problems. The method was based on a cell of open-ended coaxial line type connected to an impedance analyser. The measuring probe was modelled as a loss-free transmission line terminated by an infinite lossy transmission line, that is the sample material. The test of the model was carried out by comparing the calculated and experimental values of cell constant and the agreement was found to be good.

The major distinction that has to be made between the present method and the one proposed by Athey *et al* lies in the different basic assumptions used to model the sample material and different frequency ranges covered. Furthermore, an impedance analyser is used here instead of a network analyser.

2. The description of the cell and theoretical analysis

As schematically depicted in figure 1(*a*), the RF probe consists of a system of two coaxial electrodes separated by a Teflon ring. The inner electrode is a platinum wire whilst the outer electrode consists of a cylindrical jacket machined from stainless steel or, alternatively, platinum-coated copper. The platinum-coated jacket is required when the electrode polarization effect is present. The input of the probe is connected, through a BNC plug, to the measuring apparatus. A very well polished surface

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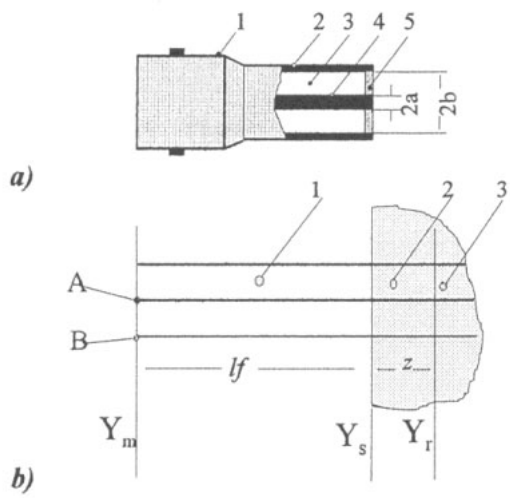

Figure 1. (*a*) A schematic picture of the open-ended coaxial line type probe: 1, BNC plug; 2, metal jacket; 3, air separation; 4, platinum wire; and 5, Teflon ring. (*b*) A schematic diagram of the line model: 1, the loss-free line section; 2, the lossy line section; and 3, the 'remainder sample'. See the text for explanation of the symbols.

of the end of the probe was achieved in order to avoid imperfect contact with the test sample.

The analysis of the cell must provide an expression for the unknown input admittance, Y_m, as measured between A and B (figure 1(*b*)). It is well known that the input admittance, Y_i, of a coaxial line of length l, characteristic admittance Y_0 and propagation constant γ terminated by an admittance Y_t is given by

$$Y_i = Y_0 \frac{Y_t + Y_0 \tanh(\gamma l)}{Y_0 + Y_t \tanh(\gamma l)}. \tag{1}$$

The transmission characteristics are given by

$$Y_0^2 = \frac{G + j\omega C}{R + j\omega L} \tag{2}$$

$$\gamma^2 = (R + j\omega L)(G + j\omega C) \tag{3}$$

where R and L are the resistance and inductance of the line per unit length, G and C are the conductance and capacitance between the inner and outer line per unit length, ω is the angular frequency of the electromagnetic field and $j = (-1)^{1/2}$.

The cell is essentially a loss-free transmission line of length l_f, characteristic admittance Y_{0f} and propagation constant γ_f terminated by the admittance of the test sample Y_S. Consequently, considering $\omega L/R \gg 1$ and $\omega C/G \gg 1$ over the whole radiofrequency range, the characteristics of the loss-free transmission line become $Y_{0f} = (C/L)^{1/2}$ and $\gamma_f = j\omega(LC)^{1/2}$ and the measured admittance is expressed as

$$Y_m = \frac{Y_S + j\omega C_e}{1 + Y_S j\omega L_e} \tag{4}$$

where by C_e and L_e were denoted the quantities Cl_f and Ll_f, respectively and $\gamma_f l_f$ was considered to be small. The capacitance C_e and inductance L_e of the loss-free section (including the probe connector) are compensated as explained in section 3.

There remains the determination of the sample admittance, Y_S. In this respect, we tentatively use the assumption that the material being measured can be modelled as a lossy transmission line of an arbitrary length z, characteristic admittance Y_{0l} and propagation constant γ_l, terminated by the admittance Y_r of the 'remainder sample' (see figure 1(*b*)). It is assumed that the inner and outer lines of the lossy section as well as the separation between them have the complex conductivity $\sigma + j\omega\varepsilon$. As with Rosen *et al* [4], the transverse admittance is given by

$$G + j\omega C = F(\sigma + j\omega\varepsilon) \tag{5}$$

and the resistance by

$$R = \frac{1}{E(\sigma + j\omega\varepsilon)} \tag{6}$$

where E is approximately equal to the area of the inner line and F is a form factor:

$$F = 8\pi \Big/ \left[1 + 4\ln\left(\frac{b}{a}\right)\right]. \tag{7}$$

By considering $\omega L \ll |R|$, the characteristics of the lossy section become

$$Y_{0l} = (EF)^{1/2}(\sigma + j\omega\varepsilon) \tag{8}$$

$$\gamma_l = (F/E)^{1/2}. \tag{9}$$

Thus, the sample admittance

$$Y_S = (EF)^{1/2}(\sigma + j\omega\varepsilon) \times \frac{Y_r + (EF)^{1/2}(\sigma + j\omega\varepsilon)\tanh[(F/E)^{1/2}z]}{(EF)^{1/2}(\sigma + j\omega\varepsilon) + Y_r \tanh[(F/E)^{1/2}z]} \tag{10}$$

takes, for an 'infinite' sample ($z \to \infty$), the form

$$Y_S = Y_{0l} = k(\sigma + j\omega\varepsilon) \tag{11}$$

where k is called the cell constant and defined as the quantity $(EF)^{1/2}$. In fact, the lossy line is finite, but large enough to allow all the field lines to be confined within the sample. By substituting equation (11) into equation (4) one obtains

$$Y_m = \frac{k(\sigma + j\omega\varepsilon) + j\omega C_e}{1 + j\omega L_e k(\sigma + j\omega\varepsilon)}. \tag{12}$$

Because L_e is usually very small (about 10^{-9} H) for frequency below 100 MHz, equation (12) takes the form

$$Y_m = k\sigma + 2\omega^2 L_e k^2 \sigma\varepsilon + j\omega[k\varepsilon + C_e - L_e k^2(\sigma^2 - \omega^2\varepsilon^2)]. \tag{13}$$

3. The method of calibration

The measurement system was based on the RF probe connected to a HP 4194A impedance analyser. Measurements of capacity and conductance were carried out over the 0.01–100 MHz frequency range.

The calibration principle was essentially based on the Hewlett–Packard standard procedure [10]. In this method, three known terminations, a short line, an open line and a matched load (50 Ω HP standard impedance), are consecutively placed at the probe connector's end. Thus the residual admittances and impedances of connecting cable are internal balanced by the apparatus. However, this procedure leaves uncompensated the residual capacitance and inductance of the probe, which leads to poor accuracy of experimental data. This problem was eliminated by performing the zero–open/short off-set measurements, as for the HP 16099A test fixture [10]. Under such circumstances, the artefactual parameters C_e and L_e can be internally balanced by the measuring apparatus and equation (13) reduces to

$$Y_m = Y_{0l} = k(\sigma + j\omega\varepsilon). \tag{14}$$

By separating into the real and imaginary parts one may readily obtain

$$G_m = k\sigma \tag{15}$$

$$C_m = k\varepsilon. \tag{16}$$

The zero–open off-set measurements were made in air, whilst for zero–short ones the short termination was obtained by pressing a flexible metal foil on the end plane of the cell. Having performed the off-set measurements, no further corrections are necessary. This feature confers simplicity on the measurement procedure and provides relatively highly accurate data over the whole frequency range.

The cell constant was determined by immersing the probe in a liquid of well-known dielectric constant and conductivity [11]. By introducing the mean values of the measured capacitance and conductance at fixed frequency (10 MHz) into the equations (15) and (16) was obtained $k = 0.002 \pm 2 \times 10^{-5}$ m. The given error represents the standard deviation obtained for ten measurements. On the other hand, as determined from geometrical dimensions ($2a = 0.6 \times 10^{-3}$ m, $2b = 5 \times 10^{-3}$ m) the calculated value of k is found to be 0.0016 m. As can readily be seen, the agreement between the two values of cell constant is good. The existing difference is, most probably, due to the fringe of the field lines inside the sample that gives an apparent increase in the radii of the lossy section.

A simple experiment was performed to determine how much the fringe field effects can influence the precision of measurements. In this respect, the capacitance and conductance of both distilled water and 0.02 M NaCl solution were measured by immersing the end plane of the probe at various distances from the liquid surface. Significant dependence on distance was only observed (see table 1) in the measurements on distilled water. This feature suggested that the cylindrical part of the outer electrode does not contribute to the total measured admittance and hence undesirable interactions with the surrounding do not appear. Consequently, special precautions, related to the vessel dimensions and material, are not necessary. However, when one measures solid samples, which implies simple contact between the probe end and sample surface, a flattened end of the outer electrode is rather preferable.

Table 1. The deviation from the true values of measured permittivity and conductivity of water and 0.02 M NaCl solution as a function of distance between the end plane of the probe and liquid surface. The measuring frequency was 100 kHz and the temperature was 20 ± 0.2 °C. The sample vessel, 3 cm in diameter and 5 cm in height, was made from glass. The values represent the mean obtained on five measurements. Corrections for electrode polarization were made when necessary.

	ε deviation (%)		σ deviation (%)	
Immersion depth (mm)	Water	NaCl	Water	NaCl
1	1.5	0.3	7.6	0.1
3	0.8	0	2	0
5	0	0	0.1	0

Measurements of permittivity and conductivity on saline solutions were carried out to evaluate the minimum thickness for which the concept of an 'infinite sample' is still correct. A deviation of about 2% from the true values of both permittivity and conductivity, at a sample thickness of 1.5 mm, was found. As can be readily calculated, this value is identical to the deviation of $\tanh[(E/F)^{1/2}z]$ (see equation (10)) from unity, at the same sample thickness. These findings provide a simple and useful method to find the minimum thickness of the sample, for given dimensions of the probe, if the accuracy of the measuring apparatus is known.

4. Examples of experimental data and accuracy

Figures 2 and 3 show two examples of applications of the RF probe to measurements of dielectric constant and conductivity. The measurements were made at fixed temperature (30 ± 0.5 °C). In the case of human skin, the probe with flattened outer electrode was simply placed in contact with the sample.

The steep rise of permittivity as frequency decreased was due to the polarization effects that always appear at the interface between an electrolytic medium and electrodes. The problem of electrode polarization was extensively treated in a paper by Schwan [1], which summarized the methods used in correcting for polarization capacitance.

One of the simplest ways to avoid the electrode polarization is to use the platinated copper electrodes that minimize the interface contribution to the measured

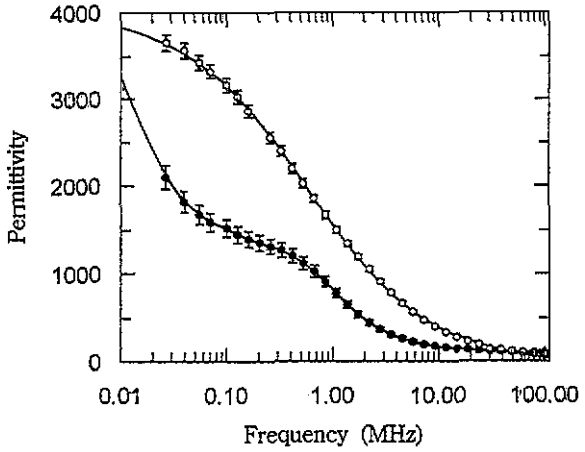

Figure 2. The permittivity of a yeast cell suspension (●) and human skin (○) versus frequency. Error bars represent the standard deviations obtained from five sets of measurements.

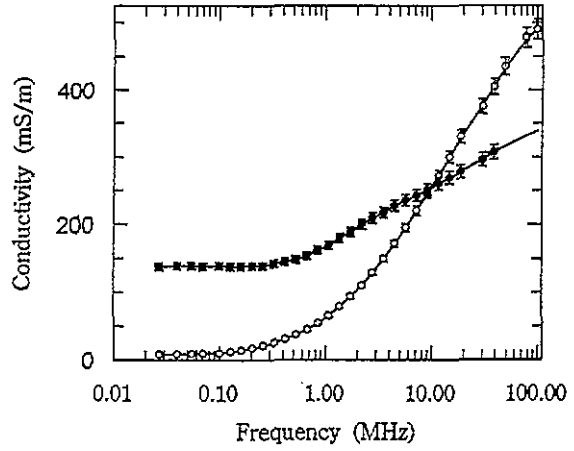

Figure 3. The conductivity of a yeast cell suspension (●) and human skin (○) versus frequency. The errors are as in figure 2.

electrical parameters. Nevertheless, in most cases, such as biological tissues and suspensions of solid particles, the cleaning of the electrode surfaces leads to weakly reproducible results.

In examples presented here, a computational method was used in correcting for electrode polarization. In this respect, the measured permittivity values were considered as a sum between the contributions of sample permittivity and polarization capacitance divided by the cell constant (that is, C_p/k). The former contribution was simulated by using the Cole–Cole dispersion function while the latter was, according to Schwan [1], taken as

$$\varepsilon_p = C_p/k = Af^{-b} \tag{17}$$

where f is the frequency and A and b are experimentally determined constants. For measurements on human skin, no noticeable polarization capacitance was observed.

The most important non-systematic errors that appeared in practice were found to depend on accuracy in the cell constant determination and the precision of the impedance analyser. Other types of errors occasionally encountererd in practice have been analysed in a paper by Stuchly *et al* [12].

To improve the accuracy in determination of the cell constant, the standard solutions were carefully prepared and the apparatus was switched on to the settings integration time = mead and number of averaging = 256 [10].

The precision of the impedance analyser drastically decays at very high loss tangent ($\sigma/\omega\varepsilon$), that is low frequency and/or high conductivity of the sample. Therefore, accurate measurements on biological materials in the low frequency range (below 1 MHz) require an increased dielectric constant, which is achievable by increasing the volume fraction of the sample.

To illustrate, let us evaluate the relative errors in measurements on a cell suspension having a dielectric constant ($\varepsilon/\varepsilon_0$) of 1300 at 100 kHz and about 200 at 10 MHz. Let us assume, for simplicity, a fixed conductivity value of 0.2 S m^{-1}. From equations (15) and (16), one may readily obtain the expressions for the estimated accuracies of conductivity and permittivity:

$$A(\sigma) = A(G_m) + A(k) \tag{18}$$

$$A(\varepsilon) = A(C_m) + A(k) \tag{19}$$

where the symbol $A()$ denotes the accuracy as a percentage. If the loss tangent exceeds a value of 5, then the accuracy of the measured capacity is taken as $A(G_m)\tan\delta$, whilst the accuracy of G_m can be evaluated by means of more complicated formulae [10]. The formulae for accuracy of both capacitance and conductance are also complicated if the loss tangent lies between 0.2 and 5, and they can be found in [10]. Under such circumstances, we limit ourselves to giving only the calculated value.

The uncertainties in G_m were estimated to be about 0.2% at 100 kHz and 1.6% at 10 MHz, leading to 1.2% and 2.6%, respectively for conductivity (see equation (18)). Accordingly, the uncertainty in permittivity was 6.5% at 100 kHz and 4.5% at 10 MHz (see equation (19)). To compare, by taking now the permittivity at a half of its former value ($\varepsilon/\varepsilon_0 = 650$ at 100 kHz), the loss tangent is twofold greater and yields an accuracy of 12% in permittivity at 100 kHz. Obviously, the estimated uncertainties in permittivity take higher values for smaller permittivity values and/or lower frequency (above 40% at 10 kHz). Nevertheless, the errors in conductivity still remain acceptable (less than 3%) over the whole frequency range.

As can readily be seen in figures 2 and 3, the standard deviations obtained on five sets of measurements lie within the estimated uncertainty limits.

5. Conclusions

The measurements with the open-ended coaxial line provide accurate data over the entire radiofrequency range. The method offers all the advantages that *in vivo*

measurements may offer. Also, the method has been proven to be fast enough (30 frequency points in less than 20 s) to be used, for example, in monitoring the fermentation processes in biotechnology. However, the fermentation processes imply low volume fraction of the cells (namely, low dielectric constant of the suspension) and so only conductivity measurements can be made with very high precision.

The most important disadvantage is related to the uncertainties in determining the cell constant, but this can be easily avoided by making repeated measurements on carefully prepared saline solutions.

It is to be noted that the present probe is, in its construction, quite similar to that previously proposed for the microwave frequency range [8, 13]. Therefore, it is valid to say that the open-ended coaxial line is a broad-band dielectric probe for *in vivo* measurements, which is appropriate for a large variety of systems.

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