Analog Integr Circ Sig Process (2012) 71:107–110
DOI 10.1007/s10470-011-9751-1

MIXED SIGNAL LETTER

Extracting single dispersion Cole–Cole impedance model parameters using an integrator setup

B. Maundy · A. S. Elwakil

Received: 29 July 2010 / Revised: 3 March 2011 / Accepted: 10 August 2011 / Published online: 21 August 2011

Abstract In this letter, we report an integrator-based setup to extract the four parameters that characterize a single-dispersion Cole–Cole impedance model without direct measurement of the complex impedance. Experimental results in the range 100 Hz–1 MHz using apple and plumb fruits are given and results are compared with numerical simulations of the acquired model.

Keywords Cole–Cole model · Bioelectrical impedance · Biomedical instrumentation · Circuit theory · Bioimpedance

1 Introduction

The Cole–Cole impedance model [1, 2] is widely used for characterizing the electrochemical properties of biological tissues [3]. The model comprises three hypothetical circuit elements: a low frequency resistor R_0, a high frequency resistor R_∞ and a Constant Phase Element (CPE), arranged as shown in the feedback path of the op amp in Fig. 1. The CPE is also known as the fractional capacitor [4] and its impedance is $Z_{CPE} = 1/(j\omega C)^\alpha$ where C is the capacitance and α is its order ($0 < \alpha \leq 1$). The Cole–Cole impedance is given by

$$Z = R_\infty + \frac{R_0 - R_\infty}{1 + (j\omega\tau)^\alpha} = Z' + jZ'' \tag{1}$$

and $(j\omega)^\alpha = \omega^\alpha[\cos(\alpha\pi/2) + j\sin(\alpha\pi/2)]$.

To characterize a particular tissue [5], it is required to find the values of the four parameters (R_0, R_∞, τ, α). Note that $1/\tau$ is known as the characteristic frequency of the tissue where $\tau = [(R_0 - R_\infty)C]^{1/\alpha}$. To extract these values, an impedance analyzer is required to measure the impedance of the tissue under consideration and a plot is then constructed relating the imaginary part of the impedance Z'' and the real part Z'. Usually, least squares regression is applied to the acquired data points and a circular arc is obtained, as shown in Fig. 2, from which R_0 and R_∞ can be directly found. The angle φ_{CPE} (see Fig. 2) is equal to $\alpha\pi/2$ and hence measuring φ_{CPE} enables calculating α. Finally, the frequency at which Z'' has its maximum value is measured. This frequency is the characteristic tissue frequency and is therefore equal to $1/\tau$.

The impedance measuring technique thus far has been employed in all tissue characterization applications using the Cole–Cole model [6–8] although it is clear that acquiring Z' and Z'' requires the use of an expensive impedance analyzer and post processing of the data to draw the circular arc. Cheap alternatives include using data acquisition cards and custom software modules which implement the necessary signal processing [9].

In this work we introduce a simple integrator-based measurement technique which enables extracting the values of the parameter set (R_0, R_∞, τ, α) without having to measure the impedance. We apply the developed theory to extract the tissue properties of two apples and a plumb fruit. The simplicity of the proposed method is evident

B. Maundy
Department of Electrical and Computer Engineering,
University of Calgary, Calgary, AB, Canada
e-mail: bmaundy@ucalgary.ca

A. S. Elwakil (✉)
Department of Electrical and Computer Engineering,
University of Sharjah, Sharjah, United Arab Emirates
e-mail: elwakil@ieee.org

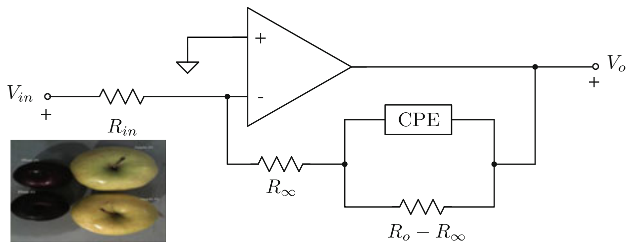

Fig. 1 Integrator setup for extracting the Cole–Cole model parameters for the fruits shown in the *inset*

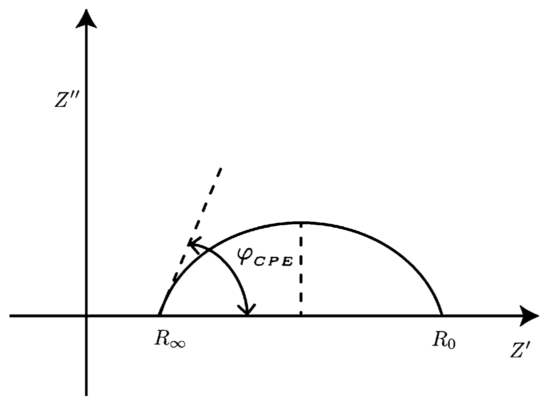

Fig. 2 Impedance loci used to extract the parameter set (R_0, R_∞, τ, α) in classical impedance-based analysis

when compared to the impedance measuring technique used in [10] to model several fruits and vegetables.

2 Integrator-based setup

Figure 1 shows a simple op amp inverting integrator with an input resistance R_{in}. The Cole impedance is assumed in the feedback path and hence it can be shown that the transfer function of this filter is

$$T(s) = \frac{V_o}{V_i} = -\frac{G_1 + G_2(\tau s)^\alpha}{1 + (\tau s)^\alpha} \tag{2}$$

where $G_1 = R_0/R_{in}, G_2 = R_\infty/R_{in}$ and $s = j\omega$. We then note the following:

(a) at $\omega = 0 \rightarrow |T(j\omega)| = G_1$. Hence, measuring the DC gain of the transfer function and knowing R_{in} yields R_0.

(b) at $\omega = \infty \rightarrow |T(j\omega)| = G_2$. Hence, measuring the high frequency gain and knowing R_{in} yields R_∞.

(c) the -3 dB point at which $|T(j\omega_{3\mathrm{dB}})| = G_1/\sqrt{2}$ is given by

$$\omega_{-3\mathrm{dB}} = \frac{1}{\tau}\left[a\left(\sqrt{\frac{b+\cos(\alpha\pi)}{2}} - \cos(\frac{\alpha\pi}{2})\right)\right]^{1/\alpha} = \frac{f_1(\alpha)}{\tau} \tag{3a}$$

$$\text{with } a = \frac{G_1(G_1 - 2G_2)}{G_1^2 - 2G_2^2} \quad \text{and} \quad b = \frac{G_1(3G_1 - 4G_2)}{(G_1 - 2G_2)^2} \tag{3b}$$

(d) the phase angle $\angle T(j\omega)$ always exhibits a minimum value at a frequency $\omega_{\phi_{min}}$ given by

$$\omega_{\phi_{min}} = \frac{1}{\tau}\left(\sqrt{\frac{G_1}{G_2}}\right)^{1/\alpha} = \frac{f_2(\alpha)}{\tau} \tag{4}$$

and at this frequency, the magnitude $|T(j\omega_{\phi_{min}})|$ is uniquely equal to $\sqrt{G_1 G_2}$. Therefore, without plotting the phase of the transfer function, it is possible to locate $\omega_{\phi_{min}}$ from the magnitude response.

(e) from (3a, 3b) and (4), it is seen that the ratio $\omega_{-3\mathrm{dB}}/\omega_{\phi_{min}} = f_1(\alpha)/f_2(\alpha)$ is independent of τ and a function only of α. Therefore assuming $\omega_{-3\mathrm{dB}}/\omega_{\phi_{min}} = p$, α can be found by numerically solving the equation

$$\left(\frac{\sqrt{G_1 G_2}(G_1 - 2G_2)}{(G_1^2 - 2G_2^2)}\right) \times \left(\sqrt{\frac{b+\cos(\alpha\pi)}{2}} - \cos\left(\frac{\alpha\pi}{2}\right)\right) = p^\alpha \tag{5}$$

Knowing α, τ could then be found from (4).

3 Experimental validation

Figure 1 was constructed using an OP27 op amp and $R_{in} = 10$ kΩ to extract the Cole model parameters for two apples and two plumb fruits in the range 100 Hz–1 MHz. Figure 3(a) shows the measured magnitude and phase responses of the integrator transfer function when an apple is used while Fig. 3(b) shows the same responses for the plumb. We have plotted the phase response in order to show the existence of a minimum phase point, as theoretically predicted. However, the phase response is *not needed* to extract any of the model parameters.

From Fig. 3(a) we measure a DC gain of 8.711 dB which yields $R_0 = 27.261$ kΩ and a high frequency gain of -14.964 dB yielding $R_\infty = 1.786$ kΩ. The $\omega_{-3\mathrm{dB}}$ frequency was located at 1.674 kHz while $\omega_{\phi\min}$ was located at 19.189 kHz corresponding to a magnitude of -3.127 dB. Thus $\omega_{-3\mathrm{dB}}/\omega_{\phi\ \min} = 0.087$ and numerically solving (5) then yields $\alpha = 0.71$ and hence $\tau = 56.47$ µs. The Matlab magnitude and phase response simulated using these extracted parameters are also plotted in Fig. 3(a) (dashed lines) showing very good agreement with the experimental measurements within the limits of accuracy of the Cole model itself [2]. Measurements were repeated on another apple, and are given in Table 1 where the extracted capacitance C is also given.

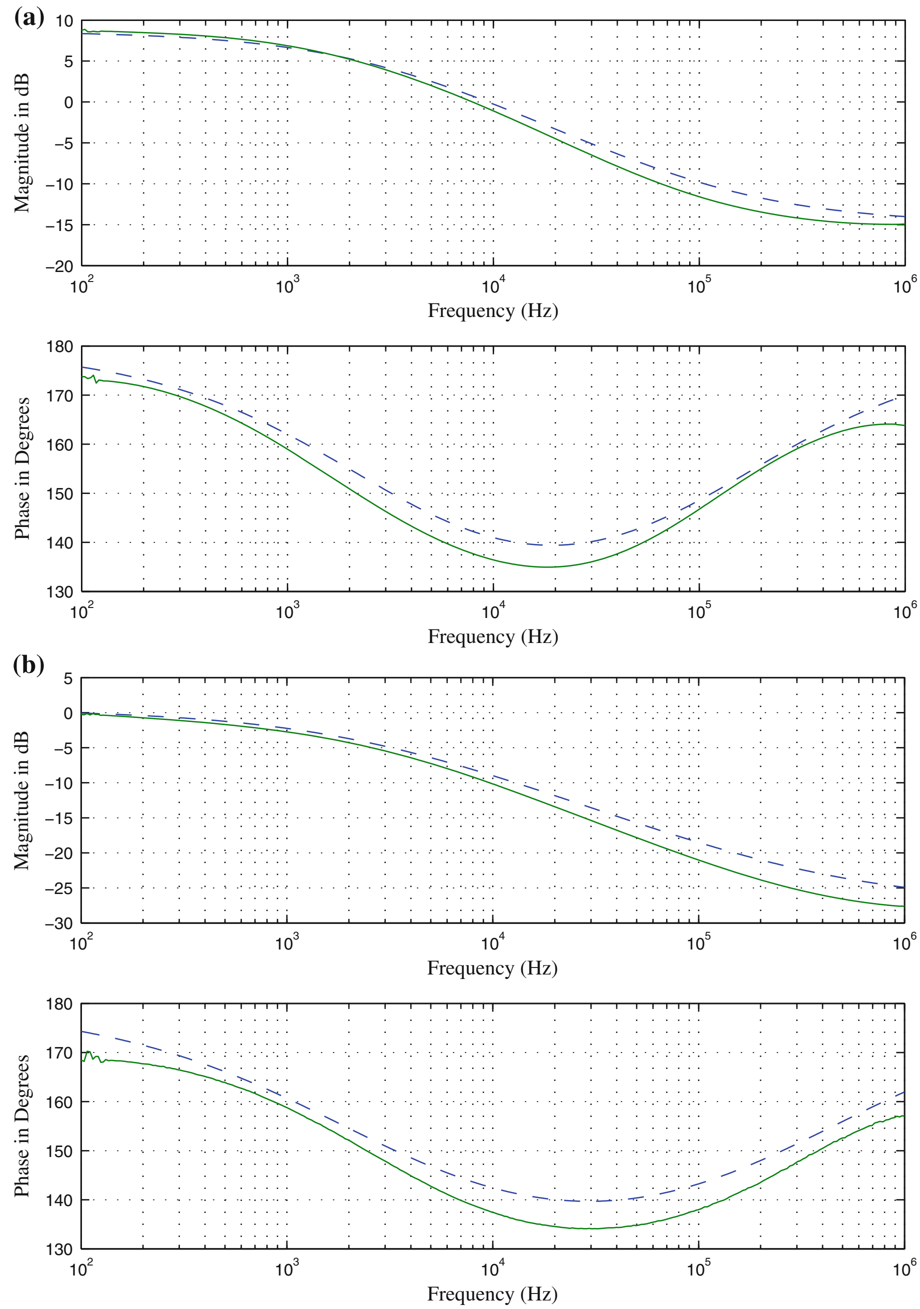

Fig. 3 Experimental (*solid*) and simulated (*dashed*) transfer function magnitude and phase responses using **a** an apple fruit and **b** a plumb fruit in the integrator of Fig. 1

Table 1 Experimentally extracted Cole model parameters (resistors in kΩ)

	R_0	R_∞	α	τ (µs)	C (nF)
Apple 1	24.141	1.95	0.696	58.56	51.02
Apple 2	27.261	1.786	0.71	56.47	37.63
Plumb 1	10.655	0.424	0.64	67.34	208.5
Plumb 2	15.795	0.408	0.698	69.78	81.12

From Fig. 3(b), obtained using a plumb, the DC gain, high frequency gain, $\omega_{-3\mathrm{dB}}$ and $\omega_{\phi\min}$ were found to be 0.551 dB, −27.445 dB, 1.127 kHz and 29.274 kHz respectively leading to a parameter set (R_0, R_∞, α, τ) = 10.655 kΩ, 424.38 Ω, 0.64, 67.37 µs. The experimental data is also compared with the numerical simulation using the extracted parameter values.

4 Conclusion

In an integrator setup that incorporates a tissue, and by plotting only the magnitude response, all parameters of the Cole–Cole model which characterize this tissue can be extracted without need for an impedance analyzer. The only computational overhead is the need to numerically solve (5) for α. To the best of the authors' knowledge, this is the simplest possible method yet reported. It can also be automated using a microcontroller based embedded system.

References

1. Cole, K., & Cole, R. (1941). Dispersion an absorption in dielectrics. *Journal of Chemical Physics, 9*, 341–351.
2. Grimnesand, S., & Martinsen, O. (2005). Cole electrical impedance model—a critique and an alternative. *IEEE Transactions on Biomedical Engineering, 52*, 132–135.
3. Grimnesand, S., & Martinsen, O. (2000). Bioimpedance and bioelectricity basics. London: Academic Press.
4. Maundy, B., Elwakil, A., & Gift, S. (2010). On a multivibrator that employs a fractional capacitor. *Journal of Analog Integrated Circuits and Signal Processing, 62*, 99–103.
5. Rigaudt, B., Hamzaouit, L., Frikhaf, M., Chauveaut, N., & Moruccit, J. (1995). In vitro tissue characterization and modelling using electrical impedance measurements in the 100 Hz–10 MHz frequency range. *Physiological Measurement, 16*, 15–28.
6. Tang, C., You, F., Cheng, G., Gao, D., Fu, F., & Dong, X. (2009). Modeling the frequency dependence of the electrical properties of the live human skull. *Physiological Measurement, 30*, 1293–1301.
7. Gupta, D., Lammersfeld, C., Vashi, P., King, J., Sahlk, S., Grutsch, J., & Lis, C. (2008). Bioelectrical impedance phase angle as a prognostic indicator in breast cancer. *BMC Cancer, 8*, 249–256.
8. Ionescu, C., & De Keyser, R. (2009). Relations between fractional-order model parameters and lung pathology in chronic obstructive pulmonary disease. *IEEE Transactions on Biomedical Engineering, 56*, 978–987.
9. Sloten, J., Verdonck, P., Nyssen, M., & Haueisen, V. (2009). A virtual instrument for bio-impedance measurement in oral cavity. In *Proceedings of 4th European conference of the international federation for medical and biological engineering* (Vol. 9, pp. 1133–1136). Berlin: Springer.
10. Jesus, I., Machado, J., & Cunha, J. (2008). Fractional electrical impedances in botanical elements. *Journal of Vibration and Control, 14*, 1389–1402.

B. Maundy received the B.Sc. degree in Electrical Engineering and a M.Sc. degree in Electronics and Instrumentation in 1983 and 1986, respectively from the University of the West Indies, Trinidad. In 1992 he received the Ph.D. in Electrical from Dalhousie University, Halifax, Nova Scotia. He completed a one-year postdoctoral fellow at Dalhousie University where he was actively involved in its analog microelectronics group. Subsequent to that he taught at the University of the West Indies, and was a visiting Professor at the University of Louisville for 7 months. He later worked in the defense industry for 2 years on mixed signal projects. In 1997 Dr. Maundy joined the Department of Electrical and Computer Engineering at the University of Calgary where he is currently a Professor. Dr. Maundy's current research is in the design of linear circuit elements, high-speed amplifier design, active analog filters, and CMOS circuits for signal processing and communication applications. Dr. Maundy is a member of the IEEE and is a past Associate Editor for the IEEE Transactions on Circuits and Systems I.

A. S. Elwakil was born in Cairo, Egypt. He received the B.Sc. and M.Sc. degrees from the Department of Electronics & Communications at Cairo University and his Ph.D. from the Department of Electrical & Electronic Engineering at the National University of Ireland, University College Dublin. Since September 2000, He has been with the Department of Electrical & Computer Engineering at Sharjah University, United Arab Emirates where he is currently an Associate Professor. His research interests are primarily in the areas of Circuit theory, Nonlinear dynamics, Chaos theory and Fractional-order circuits where he has authored and coauthored over 90 publications in these areas. Dr. Elwakil is a member of the IEEE Technical Committee on Nonlinear Circuits and has served as a reviewer and review committee member for many journals and numerous international conferences. He has also served as an instructor for a number of courses on basic VLSI design organized by the United Nations University (UNU) and the International Centre for Theoretical Physics (ICTP), where he is also an Associate member. Dr. Elwakil is an Associate Member of the Centre for Chaos Control and Synchronization at the City University of Hong Kong and serves on the Editorial Boards of the Int. J. Circuit Theory & Applications and the J. of Electrical & Computer Engineering. He currently also serves as an Associate Editor for the J. Dynamics of Continuous & Discrete Impulsive Systems: Series-B and the newly launched IEICE J. Nonlinear Theory and its Applications. Dr. Elwakil is a member of the UAE Society of Engineers Advisory Committee and was awarded the Egyptian Government first-class medal for achievements in Engineering Sciences in 2003 and 2009.