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Short communication

Extraction of Cole-Cole model parameters through low-frequency measurements

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ABSTRACT

A novel in-direct method for extracting the four parameters of the single dispersion Cole-Cole model is introduced in this Letter. Compared with the corresponding already introduced methods, the main offered advantage is the relaxed unity-gain bandwidth requirement of the employed active element. This is originated from the fact that only measurements from dc to the cutoff frequency of the realized filter function are required. Experimental results using apples are provided and their comparison with those obtained using a LCR tester confirm the accuracy of the proposed method.

1. Introduction

The measurement of the electrical impedance of biological tissues, also known as bio-impedance, is used to quantify their passive electrical properties and provide details of the electrochemical processes in these tissues which can be used to monitor physiological changes [1]. The equivalent circuits proposed for modeling the bio-impedance are constructed from fractional-order elements because they provide better fit in the behavior of tissues [2]. One of the most widely-used fractional-order models for fitting the measured impedance is the Cole-Cole impedance model and, therefore, the identification of the Cole-Cole impedance model is a powerful tool for studying the biological tissue properties. The Cole-Cole model, shown in Fig. 1, is used for characterizing biological tissues and biochemical materials describing the impedance behavior as a function of frequency [3–10]. It is constructed from two ohmic resistors R_o and R_∞, describing the behavior of the model at very low and high frequencies, respectively, and a fractional-order capacitor (constant phase element-CPE) with order $0 < \alpha < 1$ and pseudo-capacitance C_α in Farad/sec$^{(1-\alpha)}$ [11]. The expression of the impedance is given by (1)

$$Z(s) = R_\infty + \frac{R_o - R_\infty}{(\tau s)^\alpha + 1} \tag{1}$$

where the time-constant (τ) is given by the formula in (2)

$$\tau = [(R_o - R_\infty)C_\alpha]^{\frac{1}{\alpha}} \tag{2}$$

The extraction of the four parameters $\{R_o, R_\infty, C_\alpha, \alpha\}$ that fully describe the Cole-Cole model, can be performed through direct measurements of the impedance described by (1). This is achieved through the stimulation of the tissue by a voltage or current and measuring the magnitude and phase of impedance from dc to very high frequencies or performing measurements. This technique has the obstacle that an expensive impedance analyzer and, also, post processing of the data are required [6]. Another cheaper alternative is the in-direct impedance measurements, where an appropriate filter or integrator setup is established and the extraction of the Cole-Cole model parameters is performed through time-domain [12,13] or frequency-domain [14–16] measurements. A drawback of the frequency based methods is the requirement for collecting data, starting from dc ($\omega \to 0$) and finishing into very high frequencies ($\omega \to \infty$). For example, the frequency range in [14] was (100 Hz, 5 MHz), in [15] was (100 Hz, 1 MHz), while in [16] was (1 Hz, 2 MHz). This results into the requirement for utilizing active elements with reasonable bandwidth in order to derive accurate results. Therefore, the OP27 op-amp, with 5 MHz unity-gain bandwidth, has been utilized in [14] and the AD844 op-amp, with 60 MHz unity-gain bandwidth has been employed in [16]. In addition, the set of critical frequency points for extracting the parameters was $\{0, \infty, \omega_{-3\,\mathrm{dB}}, \omega_{\phi_{max}}\}$ in [14], where $\omega_{-3\,\mathrm{dB}}$ is the half-power frequency where a drop -3 dB from the maximum gain is observed, and $\omega_{\phi_{max}}$ is the frequency where the phase response exhibits a maximum. The corresponding set in [15] was $\{0, \infty, \omega_{-3\,\mathrm{dB}}, \omega_{\phi_{min}}\}$, where $\omega_{\phi_{min}}$ is the frequency where the phase response exhibits a minimum. In [16], the critical points were $\{0, \infty, \omega_{-3\,\mathrm{dB}}, \omega_o\}$, where ω_o is the frequency where the magnitude of the realized filter functions exhibits a minimum.

A novel method for extracting the Cole-Cole model parameters is introduced in this Letter, where the unity-gain bandwidth requirement of the employed active elements is reduced in comparison with that of the corresponding already introduced in-direct frequency-domain

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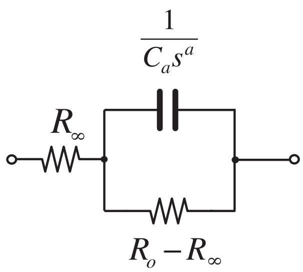


Fig. 1. Single dispersion Cole-Cole model.

setups. This is originated from the fact that the required frequency sweep is limited into the half-power frequency of the filter, avoiding measurements at very high frequencies. Therefore, the extraction of the parameters can be performed using cheap commercially available discrete-component ICs and, also, infrastructure with relatively low maximum frequency of operation. The required frequency points are $\{0, \omega_{-3\,\mathrm{dB}}\}$, where both the magnitude and phase of the transfer function must be recorded. The Letter is organized as follows: the proposed procedure is introduced in Section 2, while its experimental verification is presented in Section 3.

2. Proposed extraction procedure

The proposed setup for extracting the model parameters is given in Fig. 2, where R_{ex} and r are external resistors. The realized transfer function is given by (3)

$$H(s) = \frac{\left(\frac{R_\infty - R_{ex}}{R_{ex}}\right)\cdot(\tau s)^\alpha + \left(\frac{R_o - R_{ex}}{R_{ex}}\right)}{(\tau s)^\alpha + 1} \tag{3}$$

At very low frequencies ($\omega \to 0$), the gain ($G_{\omega\to 0}$) is given by (4)

$$G_{\omega\to 0} = \frac{R_o - R_{ex}}{R_{ex}} \tag{4}$$

Considering now that $R_{ex} = R_o$, the transfer function in (3) becomes

$$H(s)\Bigg|_{R_{ex}=R_o} = -\frac{\left(\frac{R_o - R_\infty}{R_o}\right)\cdot(\tau s)^\alpha}{(\tau s)^\alpha + 1} \tag{5}$$

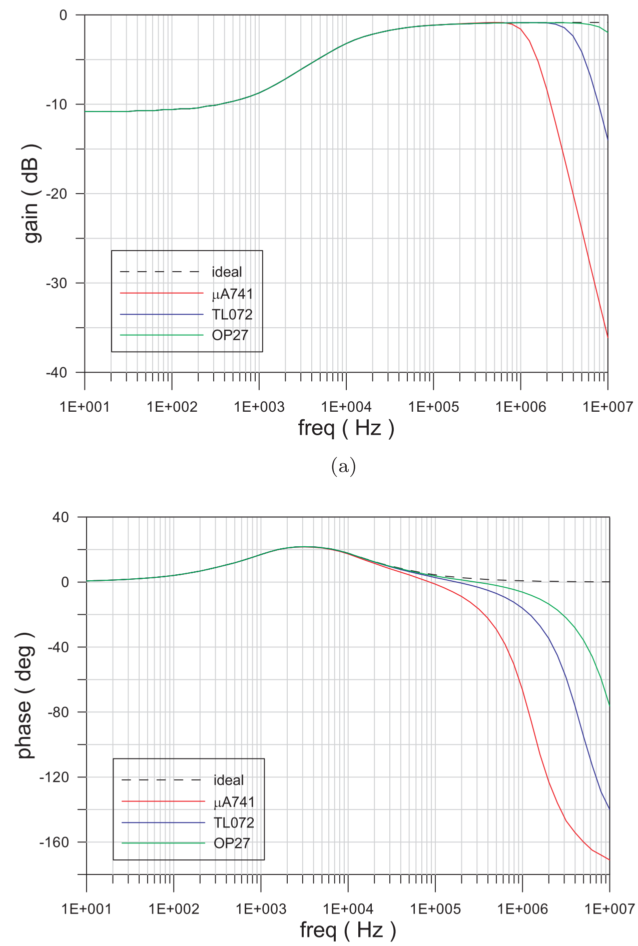
which is a high-pass filter function. The phase at very low frequencies is given by the expression

$$\phi_{\omega\to 0} = \pi + \frac{\alpha\pi}{2} \tag{6}$$

The half-power frequency ($\omega_{-3\,\mathrm{dB}}$) is calculated from the formula

$$\omega_{-3\,\mathrm{dB}} = \frac{1}{\tau}\cdot\left[\sqrt{1 + \cos^2\left(\frac{\alpha\pi}{2}\right)} + \cos\left(\frac{\alpha\pi}{2}\right)\right]^{\frac{1}{\alpha}} \tag{7}$$

The phase and gain at this frequency are calculated by the expressions in (8) and (9), respectively



(b)

Fig. 3. Effect of the finite bandwidth of the op-amp in (a) gain and (b) phase response of the filter function in [14].

$$\phi_{-3\,\mathrm{dB}} = \pi + \frac{\alpha\pi}{2} - \tan^{-1}\frac{\sin\left(\frac{\alpha\pi}{2}\right)}{\sqrt{1 + \cos^2\left(\frac{\alpha\pi}{2}\right)}} \tag{8}$$

$$G_{-3\,\mathrm{dB}} = \frac{R_o - R_\infty}{\sqrt{2}R_o} \tag{9}$$

Therefore, the proposed procedure for extracting the Cole-Cole model parameters is the following:

- **step #1**: Choosing an arbitrary value of R_{ex}, the value of R_o is calculated from (4) by measuring the gain at $\omega \to 0$.
- **step #2**: Setting $R_{ex} = R_o$, the value of the order (α) is calculated from (6).
- **step #3**: Having available the value of order, the half-power frequency can be identified by the phase response, because at this frequency the value of phase is already known through (8). In other

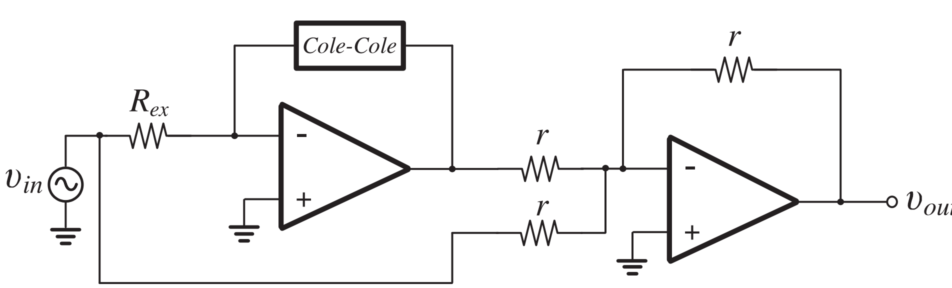


Fig. 2. Proposed setup for extracting the Cole-Cole model parameters.

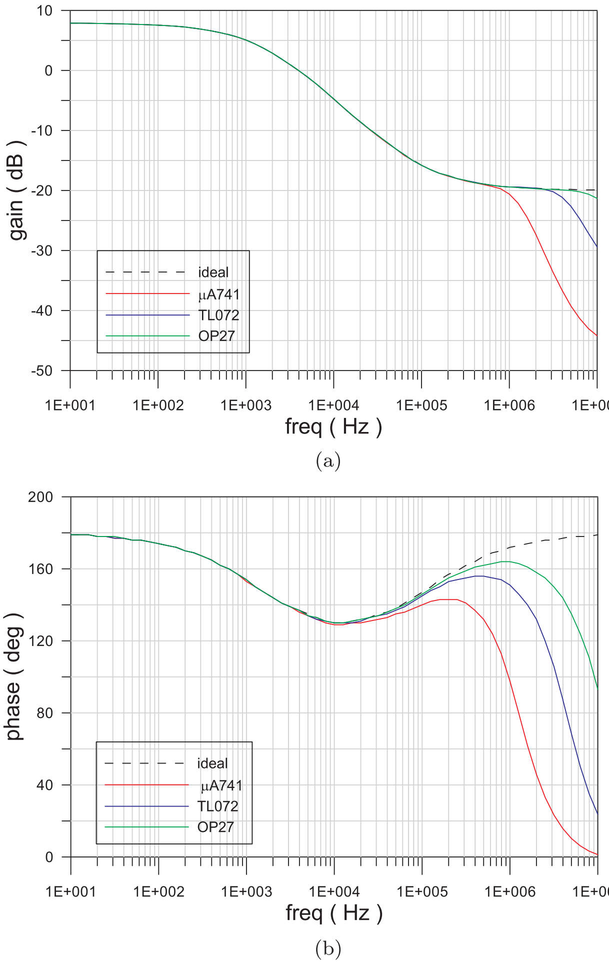


Fig. 4. Effect of the finite bandwidth of the op-amp in (a) gain and (b) phase response of the filter function in [15].

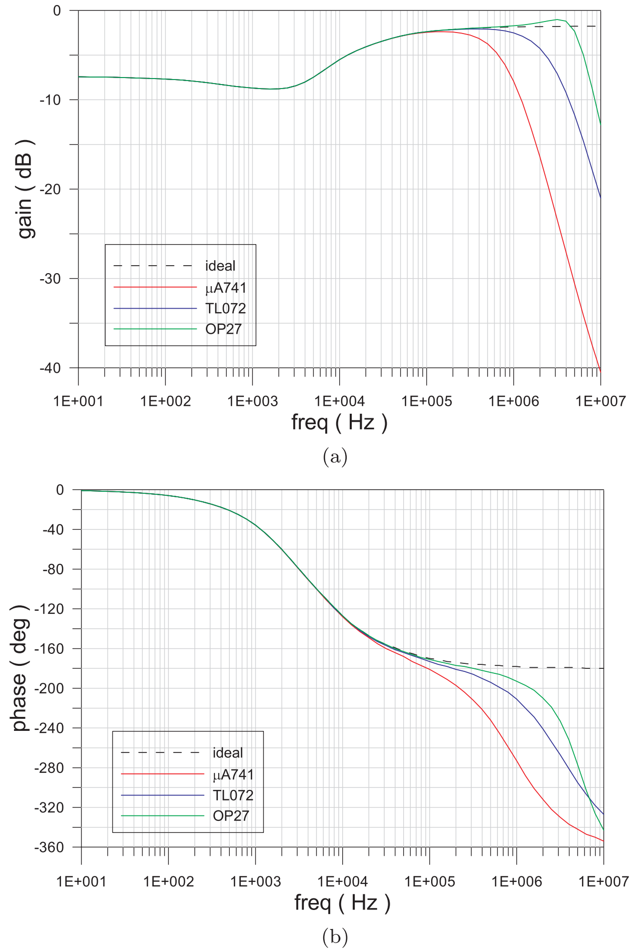


Fig. 5. Effect of the finite bandwidth of the op-amp in (a) gain and (b) phase response of the filter function in [16].

words, without plotting the gain response of the filter, it is possible to locate $\omega_{-3\,\text{dB}}$ from the phase response. Using the expressions in (7) and (9), the remaining parameters R_∞ and C_α can be calculated.

From the described steps it is obvious that measurements at $\omega \to \infty$ are avoided and only phase and gain measurements at $\omega \to 0$ and $\omega_{-3\,\text{dB}}$ are required for extracting the parameters of the model. As a result, the limitation imposed by the finite gain-bandwidth product of the employed op-amps is more relaxed in the proposed method. In order to demonstrate this claim, let us consider the case of the Cole-Cole model of the *starking delicious* type of apples, where the parameters have values: $R_o = 25\,\text{k}\Omega$, $R_\infty = 1\,\text{k}\Omega$, $C_\alpha = 35\,\text{nF/sec}^{1-\alpha}$, and $\alpha = 0.77$. Using (7), the calculated value of the half-power frequency $f_{-3\,\text{dB}}$ will be 2.46 kHz and, therefore, this will be also the required bandwidth where measurements will be performed following the proposed method. In order to demonstrate this, the magnitude and phase responses of the transfer function used for extracting the Cole-Cole model parameters using the method in [14–16] are demonstrated in Figs. 3–5, respectively, in the cases of $\mu A741$ (unity-gain bandwidth equal to 1 MHz), *TL*072 (unity-gain bandwidth equal to 3 MHz), and *OP*27 (unity-gain bandwidth equal to 5 MHz) op-amps. As it was expecting, the circuits behave correctly only in the case of the *OP*27 op-amp, due to its relatively higher bandwidth. It should be also mentioned at this point that phase measurements are not required in all the aforementioned methods and, therefore, only the gain responses are critical. The corresponding plots for the proposed setup in Fig. 2 are depicted in Fig. 6.

Taking into account that the maximum frequency of the magnitude and phase measurements is $f_{-3\,\text{dB}} = 2.46\,\text{kHz}$, it is obvious that a general purpose op-amp with unity-gain bandwidth equal to 1 MHz could be capable for employment in order to extract the four unknown parameters of the model. In order to demonstrate the benefits offered by the proposed technique, a comparison has been performed with the corresponding already published methods which are based on the frequency sweep. The derived results are summarized in Table 1, where it is readily obtained that the main benefits of the proposed setup is the relaxed requirement about the unity-bandwidth of the employed op-amp as well as the fact that the number of the frequency points that are required for performing calculations for extracting the Cole-Cole model parameters is halved, in comparison with the corresponding already published schemes. The price paid, is that the active and passive component count is increased.

3. Experimental results

The *starking delicious* type of apple has been utilized for performing the experiments. The experimental setup is given in Fig. 7, where $\mu A741$ op-amps biased at $\pm$ 15 V have been utilized.

The values of the resistors in the summation stage were equal to 10 kΩ. Due to the high-pass frequency response of the established setup, the level of the applied input signal at very low frequencies should be sufficient large (close to the power supply voltage) in order the measured output voltage to be above the noise floor. With regards to the choice of the "very low frequency", according to the results provided in Table 2 the cutoff frequency of the high pass filter is 2–4 kHz. Therefore, the selection of the frequency in the range as 0.1–1 Hz is a reasonable choice in order to guarantee that $\omega \ll \omega_{-3\,\text{dB}}$. In the performed

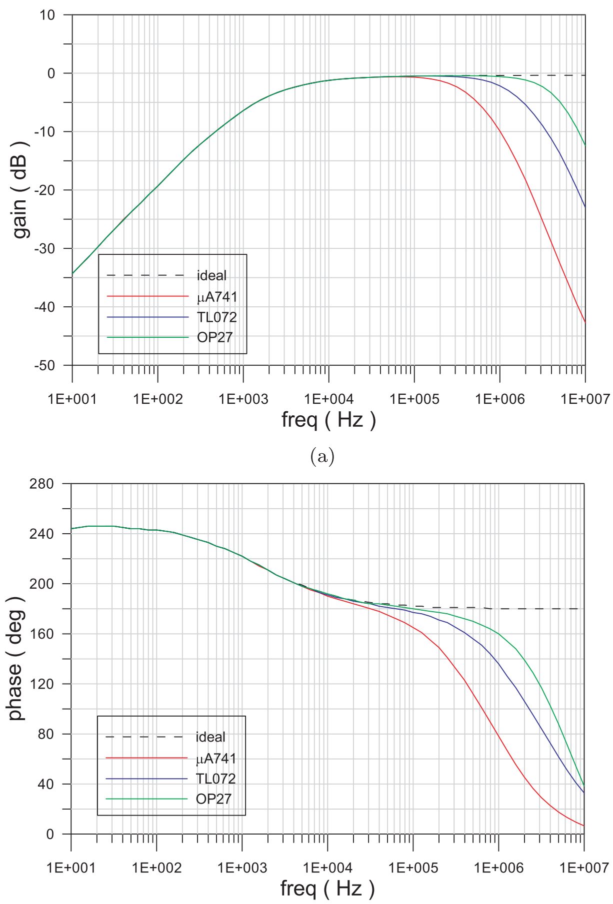


Fig. 6. Effect of the finite bandwidth of the op-amp in (a) gain and (b) phase response of the filter function in Fig. 2.

experiments, the minimum applied frequency was 0.1 Hz. The measured characteristics points for 4 apples of the aforementioned type are summarized in Table 2, while the corresponding extracted values of the Cole-Cole model parameters, are summarized in Table 3.

Verification of the extracted parameters was carried out using a precise impedance analyzer (HIOKI 3522-50) where the impedance of the apple was measured in the range (0.1 Hz, 100 kHz) and a total of 100 points were recorded. Fig. 8 shows the Nyquist plot of the measured data for apple#1.

Owing to the fact that the maximum frequency of operation of the specific analyzer is 100 kHz, it is obvious that it is unable to record data at $\omega \to \infty$. In order to overcome this obstacle, the concept presented in Fig. 9 will be followed.

The following equations are redaily derived from this plot:

$$R_o = Re(Z)_{\omega \to 0} \tag{10}$$

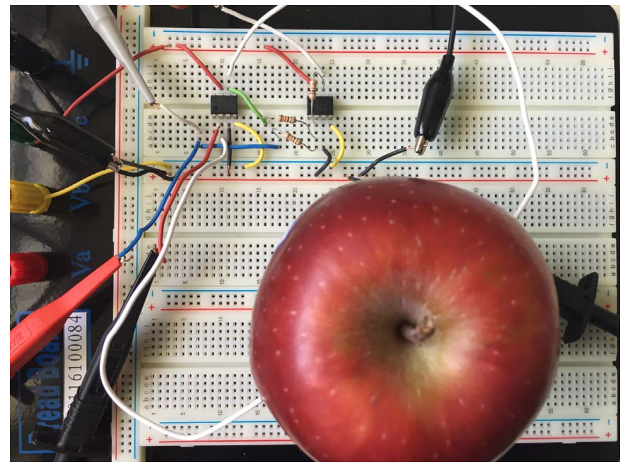


Fig. 7. Experimental setup for extracting the Cole-Cole model parameters.

Table 2
Experimental values of the characteristic points of the frequency response of the setup in Fig. 2 for 4 apples.

Variable	Apple #1	Apple #2	Apple #3	Apple #4
$G_{\omega\to 0}$	0.2775	0.434	0.278	0.4691
$\phi_{\omega\to 0}$	245 deg	248 deg	249 deg	251 deg
$f_{-3\,\text{dB}}$	3.9 kHz	2.5 kHz	2.8 kHz	2.25 kHz
$G_{-3\,\text{dB}}$	0.665	0.6625	0.625	0.6466

Table 3
Extracted values of Cole-Cole model parameters for 4 apples of starking delicious type.

Variable	Apple #1	Apple #2	Apple #3	Apple #4
R_o	25.55 kΩ	28.7 kΩ	25.56 kΩ	29.4 kΩ
R_∞	1.52 kΩ	1.8 kΩ	2.96 kΩ	2.5 kΩ
C_α	42 nF/sec$^{1-\alpha}$	35 nF/sec$^{1-\alpha}$	34.9 nF/sec$^{1-\alpha}$	27 nF/sec$^{1-\alpha}$
α	0.7222	0.7576	0.7667	0.7889

$$Re(Z)\Big|_{\omega=\omega_{peak}} = \frac{R_o + R_\infty}{2} \tag{11}$$

$$Im(Z)\Big|_{\omega=\omega_{peak}} = \frac{R_o - R_\infty}{2}\cdot tan\left(\frac{\alpha\pi}{4}\right) \tag{12}$$

The value of R_o is readily extracted from the measured real part of the impedance at very low-frequency, as it is given by (10). The parameter R_∞ will be calculated solving (11), while the order (α) will be derived using (12). The parameter C_α will be calculated according to the formula in (2). The measured real part of the impedance at 0.1 Hz was 28.2 kΩ and, therefore, $R_o = 28.2$ kΩ. The peak frequency was measured as $f_{peak} = 4$ kHz and the real and imaginary parts of the impedance (at this frequency) were equal to 14.8 kΩ and 7.8 kΩ , respectively. Using (12) and (11) the calculated values of the model

Table 1
Comparison results for the proposed setup and those introduced in [14–16].

Parameter	[14]	[15]	[16]	New
Frequency range	$(0, \infty)$	$(0, \infty)$	$(0, \infty)$	$(0, \omega_{-3\,\text{dB}})$
Op-amp with high BW	Yes	Yes	Yes	No
Frequency points	$\{0, \omega_{-3\,\text{dB}}, \omega_{max}, \infty\}$	$\{0, \omega_{-3\,\text{dB}}, \omega_{\phi min}, \infty\}$	$\{0, \omega_{-3\,\text{dB}}, \omega_o, \infty\}$	$\{0, \omega_{-3\,\text{dB}}\}$
Record of gain	Yes	Yes	Yes	Yes
Record of phase	no	No	No	Only at $\omega \to 0$ and $\omega_{-3\,\text{dB}}$
No. of active elements	1	1	1	2
No. of passive elements	1	1	3	4

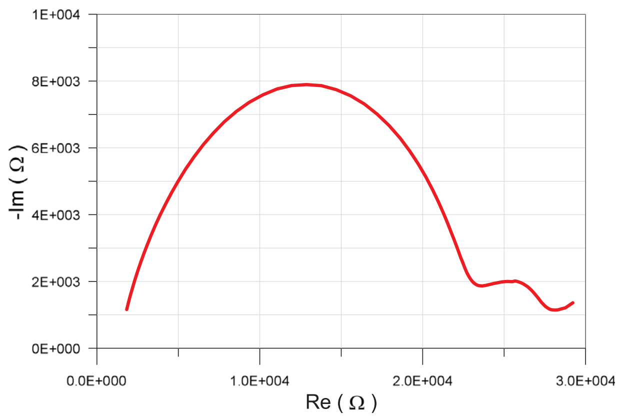


Fig. 8. Nyquist plot for apple #1 obtained using the HIOKI 3522-50 LCR tester.

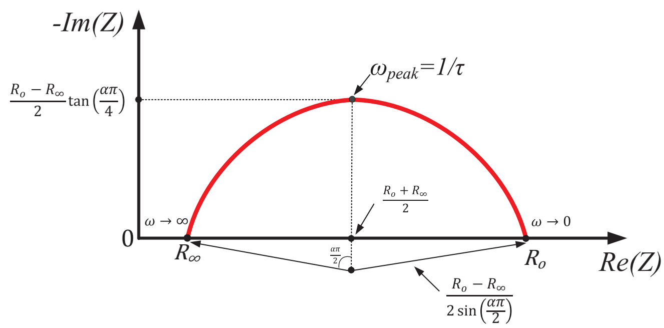


Fig. 9. Nyquist plot of the Cole-Cole model with its characteristics points.

parameters were: $R_\infty = 1.4\,\mathrm{k}\Omega$ and $\alpha = 0.6712$. Using (2), it is derived that $C_a = 41.54\,\mathrm{nF}$. The calculated values are close to those obtained by the proposed setup and the observed differences are mainly caused by the $\mu A741$ op-amp non-idealities.

4. Conclusion

A novel experimental setup for extracting the parameters of the Cole-Cole model has been proposed in this Letter. Simulation results show that it is capable for operation using an op-amp with 1 MHz unity-gain bandwidth and has been also proved through experimental results. The extracted parameters are very close to those obtained using a high-precision LCR meter. Therefore, the proposed scheme could be considered as an attractive cheap solution for extracting the parameters of the Cole-Cole model through in-direct frequency measurements. It should be also mentioned at this point that the values of the extracted parameters only depend on the position of the electrodes [17].

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