

## Introduction

mSAT : a SAT solving library in OCaml. It solves the **satisfiability** of propositional clauses. It is **Modular** : the user provides the theory. And it **produces formal proofs**.

## Conflict Driven Clause Learning

**Propagation** If there exists a clause  $C = C' \vee a$ , where  $C'$  is false in the partial model, then add  $a \mapsto \top$  to the partial model, and record  $C$  as the reason for  $a$ .

**Decision** Take an atom  $a$  that is not yet in the partial model, and add  $a \mapsto \top$  to the model.

**Conflict** A conflict is a clause  $C$  that is false in the current partial model.

**Analyze** Perform resolution between the analyzed clause and the reason behind the propagation of its most recently assigned literal, until the analyzed clause is suitable for backjumping.

**Backjump** A clause is suitable for backjumping if its most recently assigned literal  $a$  is a decision. We can then backtrack to before the decision, and add the analyzed clause to the solver, which will then enable to propagate  $a \mapsto \perp$ .

**SMT** Formulas using first-order theories can be handled using a theory. Each formula propagated or decided is sent to the theory, which then has the duty to check whether the conjunction of all formulas seen so far is satisfiable, if not, it should return a theory tautology (as a clause), that is not satisfied in the current partial model.

## Implementation

- Imperative design
  - ✓ 2-watch literal
  - ✓ Backtrackable theories (less demanding than immutable theories)
- Features
  - ✓ Functorized design, using generative functors
  - ✓ Local assumptions
  - ✓ Model output and proof output (Coq, dot)

## Solver Interface

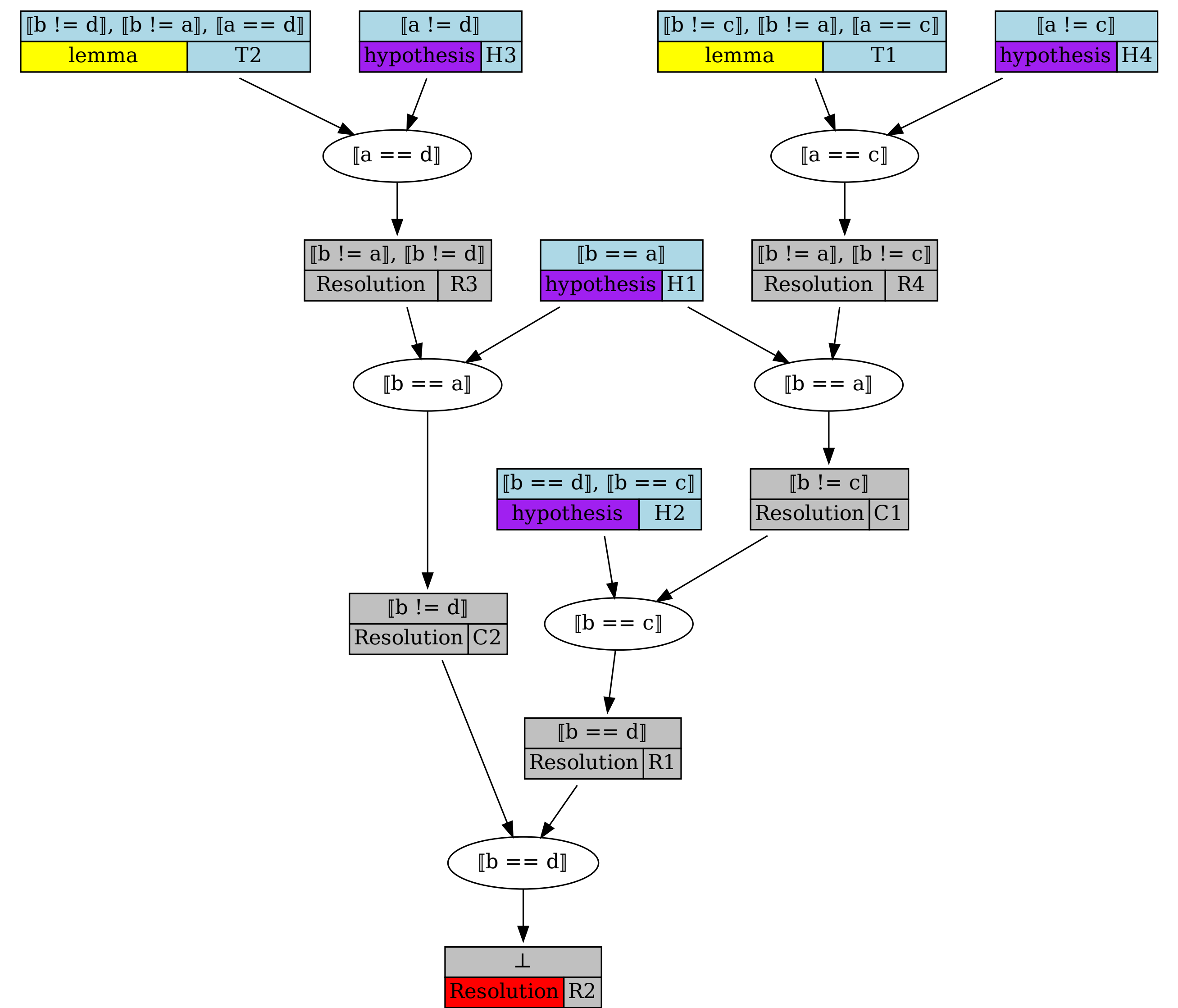
```
module Make(Th: Theory_intf.S)() : sig
  type 'f sat_state = { eval : 'f -> bool; ... }
  type ('c,'p) unsat_state =
    { conflict: unit -> 'c; proof : unit -> 'p }
  type res = Sat of formula sat_state
    | Unsat of (clause, proof) unsat_state
  val assume : ?tag:int -> atom list list -> unit
  val solve : ?assumptions:atom list -> unit -> res
end
```

## Other Solvers

regstab	SAT	binary only	only pure SAT
<b>minisat</b>			
<b>sattools</b>	SAT	C bindings	only pure SAT
ocaml-sat-solvers			
Alt-ergo	SMT	binary only	Fixed theory
<b>Alt-ergo-zero</b>	SMT	OCaml lib	Fixed theory
ocamllyices			
yices2	SMT	C bindings	Fixed theory

## Problem Example

Are the following hypotheses satisfiable?  $H1 : a = b$   $H2 : b = c \vee b = d$   
 $H3 : a <> d$   $H4 : a <> c$



## Theory Interface

```
type ('f, 'p) res = Sat | Unsat of 'f list * 'p
type 'f slice = { start:int; length:int; get:int -> 'f }
module type S = sig
  val backtrack : level -> unit
  val current_level : unit -> level
  val assume : formula slice -> (formula, proof) res
end
```

## Proof generation

- ✓ Each clause records its "history" which is the clauses used during analyzing
- ✓ Minimal impact on proof search (already done to compute unsat-core)
- ✓ Sufficient to rebuild the whole resolution tree
- ✓ A proof is a clause and proof nodes are expanded on demand  
→ no memory issue
- ✓ Enables various proof outputs :
  - Dot/Graphviz (see example above)
  - Coq (and soon dedukti) formal proofs

## Performances

solver (package)	Alt-ergo-zero aez	mSAT msat	minisat (minisat/sattools)	cryptominisat (sattools)
uuf100 (1000 pbs)	0.125	0.012	0.004	0.006
uuf125 (100 pbs)	2.217	0.030	0.006	0.013
uuf150 (100 pbs)	67.563	0.087	0.017	0.045
pigeon/hole6	0.120	0.018	0.006	0.006
pigeon/hole7	4.257	0.213	0.015	0.073
pigeon/hole8	31.450	0.941	0.096	2.488
pigeon/hole9	timeout (600)	8.886	0.634	4.075
pigeon/hole10	timeout (600)	161.478	9.579 (minisat) 160.376 (sattools)	72.050