PRIMME Documentation

Release 2.0

Andreas Stathopoulos Eloy Romero Alcalde Lingfei Wu James R. McCombs

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PRIMME: PRECONDITIONED ITERATIVE MULTIMETHOD EIGENSOLVER

PRIMME, pronounced as *prime*, finds a number of eigenvalues and their corresponding eigenvectors of a real symmetric, or Hermitian matrix. Also singular values and vectors can be computed. Largest, smallest and interior eigenvalues and singular values are supported. Preconditioning can be used to accelerate convergence. PRIMME is written in C99, but complete interfaces are provided for Fortran 77, MATLAB and Python.

Incompatibilities

From PRIMME 1.x to 2.0:

- Prototype of callbacks has changed: matrixMatvec, applyPreconditioner, massMatrixMatvec and globalSumReal.
- The next parameters are PRIMME_INT: n, nLocal, maxMatvecs, iseed, numOuterIterations, numRestarts, numMatvecs and numMatvecs; use the macro PRIMME INT P to print the values.
- Rename the values of the enum primme_preset_method.
- Rename primme_Free to primme_free ().
- Integer parameters in Fortran functions are of the same size as PRIMME_INT, which is integer *8 by default.
- Extra parameter in many Fortran functions to return the error code.
- Removed primme_display_stats_f77.

Changelog

Changes in PRIMME 2.0 (released on XXX):

- Added interface for singular value problems; see dprimme_svds ().
- Support for float and complex float arithmetic.
- Support for problem dimensions larger than 2^31, without requiring BLAS and LAPACK compiled with 64-bits integers.
- Improve robustness and performance for interior problems; implemented advanced refined and harmonic-Ritz extractions.
- Python interface compatible with NumPy and SciPy Library.
- Added parameter to indicate the leading dimension of the input/output matrices and to return an error code in callbacks matrixMatvec, applyPreconditioner, massMatrixMatvec and globalSumReal.

- Changed to type PRIMME_NUM the options n, nLocal, maxMatvecs and iseed, and the stats counters numOuterIterations, numRestarts, numMatvecs, numPreconds. Also changed realWorkSize to $size_t$. Fortran interface functions will expect an interger of size compatible with PRIMME_NUM for all parameters with integer type: int, PRIMME_NUM and $size_t$; see also parameter value in functions $primmetop_set_member_f77()$, $primmetop_get_member_f77()$, $primme_set_member_f77()$ and $primme_get_member_f77()$.
- Added parameter to return an error code in Fortran interface functions: primmetop_set_member_f77(), primmetop_get_member_f77(), primme_set_member_f77() and primme_get_member_f77().
- Added leading dimension for evecs *ldevecs* and preferred leading dimension for the operators *ldOPs*, such as *matrixMatvec*.
- Optional user-defined convergence function, convTestFun.
- Prefixed methods with PRIMME_. Rename Fortran constants from PRIMMEF77_ to PRIMME_.
- Removed primme_display_stats_f77.

Changes in PRIMME 1.2.2 (released on October 13, 2015):

- Fixed wrong symbols in libdprimme.a and libzprimme.a.
- primme_set_method() sets PRIMME_JDQMR instead of PRIMME_JDQMR_ETol for preset methods PRIMME_DEFAULT_MIN_TIME and PRIMME_DYNAMIC when seeking interior values.
- Fixed compilation of driver with a PETSc installation without HYPRE.
- Included the content of the environment variable INCLUDE for compiling the driver.

Changes in PRIMME 1.2.1 (released on September 7, 2015):

- · Added MATLAB interface to full PRIMME functionality.
- Support for BLAS/LAPACK with 64bits integers (-DPRIMME_BLASINT_SIZE=64).
- Simplified configuration of Make_flags and Make_links (removed TOP variable and replaced defines NUM_SUM and NUM_IBM by F77UNDERSCORE).
- Replaced directories DTEST and ZTEST by TEST, that has:
 - driver.c: read matrices in MatrixMarket format and PETSc binary and call PRIMME with the parameters specified in a file; support complex arithmetic and MPI and can use PETSc preconditioners.
 - ex*.c and ex*.f: small, didactic examples of usage in C and Fortran and in parallel (with PETSc).
- Fixed a few minor bugs and improved documentation (especially the F77 interface).
- Using Sphinx to manage documentation.

Changes in PRIMME 1.2 (released on December 21, 2014):

- A Fortran compiler is no longer required for building the PRIMME library. Fortran programs can still be linked to PRIMME's F77 interface.
- Fixed some uncommon issues with the F77 interface.
- PRIMME can be called now multiple times from the same program.
- Performance improvements in the QMR inner solver, especially for complex arithmetic.
- Fixed a couple of bugs with the locking functionality.
 - In certain extreme cases where all eigenvalues of a matrix were needed.
 - The order of selecting interior eigenvalues.

The above fixes have improved robustness and performance.

- PRIMME now assigns unique random seeds per parallel process for up to 4096^3 (140 trillion) processes.
- For the PRIMME_DYNAMIC method, fixed issues with initialization and synchronization decisions across multiple processes.
- Fixed uncommon library interface bugs, coordinated better setting the method and the user setting of parameters, and improved the interface in the sample programs and makefiles.
- Other performance and documentation improvements.

License Information

PRIMME is licensed under the 3-clause license BSD. Python and Matlab interfaces have BSD-compatible licenses. Source code under file: *tests* is compatible with LGPLv3. Details can be taken from COPYING.txt.

Citing the code

Please cite [r1] and [r6].

More information on the algorithms and research that led to this software can be found in the rest of the papers [r2], [r3], [r4], [r5], [r7]. The work has been supported by a number of grants from the National Science Foundation.

Contact Information

For reporting bugs or questions about functionality contact Andreas Stathopoulos by email, *andreas* at *cs.wm.edu*. See further information in the webpage http://www.cs.wm.edu/~andreas/software and on github.

Directory Structure

The next directories and files should be available:

- COPYING.txt, license;
- Make_flags, flags to be used by makefiles to compile library and tests;
- Link_flags, flags needed in making and linking the test programs;
- include/, directory with headers files;
- src/, directory with the source code for libprimme:
 - include/, common headers;
 - eigs/, eigenvalue interface and implementation;
 - svds/, singular value interface and implementation;
 - tools/, tools used to generated some headers;
- Matlab/, Matlab interface;
- PYTHON/, Python interface;
- examples/, sample programs in C, C++ and F77, both sequential and parallel;

- tests/, drivers for testing purpose and test cases;
- lib/libprimme.a, the PRIMME library (to be made);
- makefile main make file;
- readme.txt text version of the documentation;
- doc/ directory with the HTML and PDF versions of the documentation.

Making and Linking

Make_flags has the flags and compilers used to make libprimme.a:

- CC, compiler program such as gcc, clang or icc.
- *CFLAGS*, compiler options such as -g or -03. Also include some of the following options if required for the BLAS and LAPACK libraries to be linked:
 - -DF77UNDERSCORE, if Fortran appends an underscore to function names (usually they does).
 - DPRIMME_BLASINT_SIZE=64, if the library integers are 64-bit integer (kind=8) type (usually they are not).

By default PRIMME sets the integer type for matrix dimensions and counters (PRIMME_INT) to 64 bits integer int 64_t. This can be changed by setting the macro PRIMME_INT_SIZE to one of the following values:

- 0: use the regular int.
- 32: use C99 int32_t.
- 64: use C99 int64 t.

Note: When <code>-DPRIMME_BLASINT_SIZE=64</code> is set the code uses the type <code>int64_t</code> supported by the C99 standard. In case the compiler doesn't honor the standard, you can set the corresponding type name supported, for <code>instance-DPRIMME_BLASINT_SIZE=__int64</code>.

After customizing Make_flags, type this to generate libprimme.a:

```
make lib
```

Making can be also done at the command line:

```
make lib CC=clang CFLAGS='-03'
```

Link_flags has the flags for linking with external libraries and making the executables located in examples and tests:

- LDFLAGS, linker flags such as -framework Accelerate.
- *LIBS*, flags to link with libraries (BLAS and LAPACK are required), such as -lprimme -llapack -lblas -lqfortran -lm.

After that, type this to compile and execute a simple test:

```
$ make test
...
Test passed!
...
Test passed!
```

In case of linking problems check flags in *LDFLAGS* and *LIBS* and consider to add/remove -DF77UNDERSCORE from *CFLAGS*. If the execution fails consider to add/remove -DPRIMME_BLASINT_SIZE=64 from *CFLAGS*.

Full description of actions that make can take:

- make lib, builds the static library libprimme.a.
- *make solib*, builds the shared library libprimme.so.
- make test, build and execute simple examples.
- *make clean*, removes all *.o, a.out, and core files from src.

Considerations using an IDE

PRIMME can be built in other environments such as Anjuta, Eclipse, KDevelop, Qt Creator, Visual Studio and XCode. To build the PRIMME library do the following:

- 1. Create a new project and include the source files under the directory src.
- 2. Add the directories include and src/include as include directories.

To build an example code using PRIMME make sure:

- to add a reference for PRIMME, BLAS and LAPACK libraries;
- to add the directory include as an include directory.

Tested Systems

PRIMME is primary developed with GNU gcc, g++ and gfortran (versions 4.8 and later). Many users have reported builds on several other platforms/compilers:

- SUSE 13.1 & 13.2
- CentOS 6.6
- Ubuntu 14.04
- MacOS X 10.9 & 10.10
- Cygwin & MinGW
- Cray XC30
- SunOS 5.9, quad processor Sun-Fire-280R, and several other UltraSparcs
- AIX 5.2 IBM SP POWER 3+, 16-way SMP, 375 MHz nodes (seaborg at nersc.gov)

Main Contributors

- · James R. McCombs
- Eloy Romero Alcalde
- Andreas Stathopoulos
- Lingfei Wu

CHAPTER

TWO

EIGENVALUE PROBLEMS

C Library Interface

The PRIMME interface is composed of the following functions. To solve real symmetric and Hermitian standard eigenproblems call respectively:

```
sprimme (float *evals, float *evecs, float *resNorms,
                        primme_params *primme)
     cprimme (float *evals, PRIMME_COMPLEX_FLOAT *evecs,
int
                 float *resNorms, primme_params *primme)
int
     dprimme (double *evals, double *evecs, double *resNorms,
                        primme_params *primme)
     zprimme (double *evals, PRIMME_COMPLEX_DOUBLE *evecs,
                 double *resNorms, primme_params *primme)
Other useful functions:
void primme_initialize (primme_params *primme)
int primme_set_method (primme_preset_method method,
                                                      primme_params *params)
void primme display params (primme params primme)
void primme_free
 (primme_params *primme)
```

PRIMME stores its data on the structure primme_params. See Parameters Guide for an introduction about its fields.

Running

To use PRIMME, follow this basic steps.

1. Include:

```
#include "primme.h" /* header file is required to run primme */
```

2. Initialize a PRIMME parameters structure for default settings:

```
primme_params primme;
primme_initialize (&primme);
```

3. Set problem parameters (see also Parameters Guide), and, optionally, set one of the preset methods:

```
ret = primme_set_method (method, &primme);
...
```

4. Then to solve a real symmetric standard eigenproblems call:

```
ret = dprimme (evals, evecs, resNorms, &primme);
```

To solve Hermitian standard eigenproblems call:

```
ret = zprimme (evals, evecs, resNorms, &primme);
```

The call arguments are:

- evals, array to return the found eigenvalues;
- evecs, array to return the found eigenvectors;
- resNorms, array to return the residual norms of the found eigenpairs; and
- ret, returned error code.
- 5. Before exiting, free the work arrays in PRIMME:

```
primme_free
(&primme);
```

Parameters Guide

PRIMME stores the data on the structure <code>primme_params</code>, which has the next fields:

```
Basic
```

```
int n, matrix dimension.
void (* matrixMatvec) (...), matrix-vector product.
int numEvals, how many eigenpairs to find.
primme_target target, which eigenvalues to find.
int numTargetShifts, for targeting interior eigenpairs.
double * targetShifts
double eps, tolerance of the residual norm of converged eigenpairs.
```

For parallel programs

```
int numProcs
int procID
int nLocal
void (* globalSumReal)(...)
```

Accelerate the convergence

```
void (* applyPreconditioner)(...), preconditioner-vector product.
int initSize, initial vectors as approximate solutions.
int maxBasisSize
int minRestartSize
int maxBlockSize
```

User data

```
void * commInfo
```

```
void * matrix
void * preconditioner
Advanced options
PRIMME_INT ldevecs, leading dimension of the evecs.
int numOrthoConst, orthogonal constrains to the eigenvectors.
int dynamicMethodSwitch
int locking
int maxMatvecs
int maxOuterIterations
int intWorkSize
long int realWorkSize
int iseed [4]
int * intWork
void * realWork
double aNorm
int printLevel
FILE * outputFile
double * ShiftsForPreconditioner
primme_init initBasisMode
struct projection params projectionParams
struct restarting_params restartingParams
struct correction_params correctionParams
struct primme_stats stats
void (* convTestFun)(...)
PRIMME_INT 1dOPs, leading dimension to use in matrixMatvec...
```

PRIMME requires the user to set at least the dimension of the matrix (n) and the matrix-vector product (matrixMatvec), as they define the problem to be solved. For parallel programs, nLocal, procID and qlobalSumReal are also required.

In addition, most users would want to specify how many eigenpairs to find, and provide a preconditioner (if available).

It is useful to have set all these before calling <code>primme_set_method()</code>. Also, if users have a preference on <code>maxBasisSize</code>, <code>maxBlockSize</code>, etc, they should also provide them into <code>primme_params</code> prior to the <code>primme_set_method()</code> call. This helps <code>primme_set_method()</code> make the right choice on other parameters. It is sometimes useful to check the actual parameters that PRIMME is going to use (before calling it) or used (on return) by printing them with <code>primme_display_params()</code>.

Interface Description

The next enumerations and functions are declared in primme.h.

sprimme

```
int sprimme (float *evals, float *evecs, float *resNorms, primme_params *primme) Solve a real symmetric standard eigenproblem.
```

Parameters

- **evals** array at least of size *numEvals* to store the computed eigenvalues; all processes in a parallel run return this local array with the same values.
- **resNorms** array at least of size *numEvals* to store the residual norms of the computed eigenpairs; all processes in parallel run return this local array with the same values.
- **evecs** array at least of size *nLocal* times *numEvals* to store columnwise the (local part of the) computed eigenvectors.
- **primme** parameters structure.

Returns error indicator; see Error Codes.

dprimme

```
int dprimme (double *evals, double *evecs, double *resNorms, primme_params *primme) Solve a real symmetric standard eigenproblem.
```

Parameters

- **evals** array at least of size *numEvals* to store the computed eigenvalues; all processes in a parallel run return this local array with the same values.
- **resNorms** array at least of size *numEvals* to store the residual norms of the computed eigenpairs; all processes in parallel run return this local array with the same values.
- **evecs** array at least of size *nLocal* times *numEvals* to store columnwise the (local part of the) computed eigenvectors.
- **primme** parameters structure.

Returns error indicator; see Error Codes.

cprimme

```
int cprimme (float *evals, PRIMME_COMPLEX_FLOAT *evecs, float *resNorms, primme_params *primme)

Solve a Hermitian standard eigenproblem; see function sprimme().
```

zprimme

```
int zprimme (double *evals, PRIMME_COMPLEX_DOUBLE *evecs, double *resNorms, primme_params *primme)

Solve a Hermitian standard eigenproblem; see function dprimme().
```

primme initialize

Parameters

• primme – parameters structure.

primme_set_method

Parameters

• method – preset configuration; one of

```
PRIMME_DYNAMIC

PRIMME_DEFAULT_MIN_TIME

PRIMME_DEFAULT_MIN_MATVECS

PRIMME_Arnoldi

PRIMME_GD

PRIMME_GD_plusK

PRIMME_GD_Olsen_plusK

PRIMME_JD_Olsen_plusK

PRIMME_JDQR

PRIMME_JDQR

PRIMME_JDQMR

PRIMME_JDQMR_ETol

PRIMME_SUBSPACE_ITERATION

PRIMME_LOBPCG_OrthoBasis_Window
```

• **primme** – parameters structure.

See also Preset Methods.

primme_display_params

Parameters

• **primme** – parameters structure.

primme_free

```
void primme_free (

primme_params *primme)

Free memory allocated by PRIMME.
```

Parameters

• **primme** – parameters structure.

FORTRAN Library Interface

The next enumerations and functions are declared in primme_f77.h.

ptr

Fortran datatype with the same size as a pointer. Use integer *4 when compiling in 32 bits and integer *8 in 64 bits.

primme_initialize_f77

```
primme_initialize_f77 (primme)
```

Set PRIMME parameters structure to the default values.

Parameters

• **primme** (ptr) – (output) parameters structure.

primme_set_method_f77

```
primme_set_method_f77 (method, primme, ierr)
```

Set PRIMME parameters to one of the preset configurations.

Parameters

• method (integer) - (input) preset configuration. One of:

```
PRIMME_DYNAMIC

PRIMME_DEFAULT_MIN_TIME

PRIMME_DEFAULT_MIN_MATVECS

PRIMME_Arnoldi

PRIMME_GD

PRIMME_GD_plusK

PRIMME_GD_Olsen_plusK

PRIMME_JD_Olsen_plusK

PRIMME_JDQR

PRIMME_JDQR

PRIMME_JDQMR

PRIMME_JDQMR_ETO1

PRIMME_SUBSPACE_ITERATION

PRIMME_LOBPCG_OrthoBasis_Window
```

See primme preset method.

- **primme** (ptr) (input) parameters structure.
- ierr (integer) (output) if 0, successful; if negative, something went wrong.

primme_free_f77

primme_free_f77 (primme)

Free memory allocated by PRIMME and delete all values set.

Parameters

• **primme** (ptr) – (input/output) parameters structure.

dprimme_f77

dprimme_f77 (evals, evecs, resNorms, primme, ierr)

Solve a real symmetric standard eigenproblem.

Parameters

- evals (*) (double precision) (output) array at least of size numEvals to store the computed eigenvalues; all parallel calls return the same value in this array.
- **resNorms** (*) (double precision) (output) array at least of size numEvals to store the residual norms of the computed eigenpairs; all parallel calls return the same value in this array.
- evecs (*) (double precision) (input/output) array at least of size nLocal times numEvals to store columnwise the (local part of the) computed eigenvectors.
- **primme** (ptr) parameters structure.
- ierr (integer) (output) error indicator; see Error Codes.

zprimme f77

zprimme_f77 (evals, evecs, resNorms, primme, ierr)

Solve a Hermitian standard eigenproblem. The arguments have the same meaning as in function dprimme f77().

Parameters

- evals(*) (double precision) (output)
- resNorms(*) (double precision) (output)
- evecs(*) (complex double precision) (input/output)
- **primme** (ptr) (input) parameters structure.
- **ierr** (*integer*) (output) error indicator; see Error Codes.

primmetop_set_member_f77

primmetop_set_member_f77 (primme, label, value)

Set a value in some field of the parameter structure.

Parameters

- **primme** (ptr) (input) parameters structure.
- **label** (*integer*) field where to set value. One of:

```
PRIMME n
PRIMME_matrixMatvec
PRIMME_applyPreconditioner
PRIMME_numProcs
PRIMME_procID
PRIMME comminfo
PRIMME nLocal
PRIMME_globalSumDouble
PRIMME numEvals
PRIMME_target
PRIMME numTargetShifts
PRIMME_targetShifts
PRIMME_locking
PRIMME_initSize
PRIMME numOrthoConst
PRIMME maxBasisSize
PRIMME minRestartSize
PRIMME_maxBlockSize
PRIMME maxMatvecs
PRIMME_maxOuterIterations
PRIMME intWorkSize
PRIMME realWorkSize
PRIMME_iseed
PRIMME intWork
PRIMME realWork
PRIMME_aNorm
PRIMME_eps
PRIMME_printLevel
PRIMME_outputFile
PRIMME matrix
PRIMME preconditioner
PRIMME_restartingParams_scheme.
PRIMME restartingParams maxPrevRetain
PRIMME_correctionParams_precondition
PRIMME correctionParams robustShifts
PRIMME correctionParams maxInnerIterations
PRIMME_correctionParams_projectors_LeftQ
PRIMME_correctionParams_projectors_LeftX
PRIMME correctionParams projectors RightQ
PRIMME_correctionParams_projectors_RightX
PRIMME_correctionParams_projectors_SkewQ
PRIMME_correctionParams_projectors_SkewX
PRIMME correctionParams convTest
PRIMME_correctionParams_relTolBase
PRIMME stats numOuterIterations
PRIMME_stats_numRestarts
PRIMME_stats_numMatvecs
```

```
PRIMME_stats_numPreconds
PRIMME_stats_elapsedTime
PRIMME_dynamicMethodSwitch
PRIMME_massMatrixMatvec
```

• **value** – (input) value to set.

If the type of the option is integer (int, PRIMME_INT, size_t), the type of value should be as long as PRIMME_INT, which is integer *8 by default.

Note: Don't use this function inside PRIMME's callback functions, e.g., matrixMatvec or applyPreconditioner, or in functions called by these functions. In those cases use primme_set_member_f77().

primmetop get member f77

primmetop_get_member_f77 (primme, label, value)

Get the value in some field of the parameter structure.

Parameters

- **primme** (ptr) (input) parameters structure.
- label (integer) (input) field where to get value. One of the detailed in function primmetop set member f77().
- **value** (output) value of the field.

If the type of the option is integer (int, PRIMME_INT, size_t), the type of value should be as long as PRIMME_INT, which is integer *8 by default.

Note: Don't use this function inside PRIMME's callback functions, e.g., matrixMatvec or applyPreconditioner, or in functions called by these functions. In those cases use primme_get_member_f77().

Note: When label is one of PRIMME_matrixMatvec, PRIMME_applyPreconditioner, PRIMME_commInfo, PRIMME_intWork, PRIMME_realWork, PRIMME_matrix and PRIMME_preconditioner, the returned value is a C pointer (void*). Use Fortran pointer or other extensions to deal with it. For instance:

```
use iso_c_binding
MPI_Comm comm

comm = MPI_COMM_WORLD
call primme_set_member_f77(primme, PRIMME_commInfo, comm)
...
subroutine par_GlobalSumDouble(x,y,k,primme)
use iso_c_binding
implicit none
...
MPI_Comm, pointer :: comm
type(c_ptr) :: pcomm
```

```
call primme_get_member_f77(primme, PRIMME_commInfo, pcomm)
call c_f_pointer(pcomm, comm)
call MPI_Allreduce(x,y,k,MPI_DOUBLE,MPI_SUM,comm,ierr)
```

Most users would not need to retrieve these pointers in their programs.

primmetop get prec shift f77

```
primmetop_get_prec_shift_f77 (primme, index, value)
```

Get the value in some position of the array ShiftsForPreconditioner.

Parameters

- **primme** (ptr) (input) parameters structure.
- index (integer) (input) position of the array; the first position is 1.
- **value** (output) value of the array at that position.

primme_set_member_f77

```
primme_set_member_f77 (primme, label, value)
```

Set a value in some field of the parameter structure.

Parameters

- primme (ptr) (input) parameters structure.
- label (integer) field where to set value. One of the vales defined in primmetop_set_member_f77().
- value (input) value to set.

If the type of the option is integer (int, PRIMME_INT, size_t), the type of value should be as long as PRIMME_INT, which is integer *8 by default.

Note: Use this function exclusively inside PRIMME's callback functions, e.g., matrixMatvec or applyPreconditioner, or in functions called by these functions. Otherwise, e.g., from the main program, use the function primmetop_set_member_f77().

primme_get_member_f77

primme_get_member_f77 (primme, label, value)

Get the value in some field of the parameter structure.

Parameters

- primme (ptr) (input) parameters structure.
- label (integer) (input) field where to get value. One of the detailed in function primmetop set member f77().

• **value** – (output) value of the field.

If the type of the option is integer (int, PRIMME_INT, size_t), the type of value should be as long as PRIMME_INT, which is integer *8 by default.

Note: Use this function exclusively inside PRIMME's callback functions, e.g., matrixMatvec or applyPreconditioner, or in functions called by these functions. Otherwise, e.g., from the main program, use the function primmetop_get_member_f77().

Note: When label is one of PRIMME_matrixMatvec, PRIMME_applyPreconditioner, PRIMME_commInfo, PRIMME_intWork, PRIMME_realWork, PRIMME_matrix and PRIMME_preconditioner, the returned value is a C pointer (void*). Use Fortran pointer or other extensions to deal with it. For instance:

```
use iso_c_binding
MPI_Comm comm

comm = MPI_COMM_WORLD
call primme_set_member_f77(primme, PRIMME_commInfo, comm)
...
subroutine par_GlobalSumDouble(x,y,k,primme)
use iso_c_binding
implicit none
...
MPI_Comm, pointer :: comm
type(c_ptr) :: pcomm

call primme_get_member_f77(primme, PRIMME_commInfo, pcomm)
call c_f_pointer(pcomm, comm)
call MPI_Allreduce(x,y,k,MPI_DOUBLE,MPI_SUM,comm,ierr)
```

Most users would not need to retrieve these pointers in their programs.

primme_get_prec_shift_f77

primme_get_prec_shift_f77 (primme, index, value)

Get the value in some position of the array ShiftsForPreconditioner.

Parameters

- **primme** (ptr) (input) parameters structure.
- index (integer) (input) position of the array; the first position is 1.
- **value** (output) value of the array at that position.

Note: Use this function exclusively inside the function matrixMatvec, massMatrixMatvec, or applyPreconditioner. Otherwise use the function primmetop_get_prec_shift_f77().

Python Interface

Primme.eigsh (A, k=6, M=None, sigma=None, which='LM', v0=None, ncv=None, maxiter=None, tol=0, return_eigenvectors=True, Minv=None, OPinv=None, mode='normal', lock=None, return_stats=False, maxBlockSize=0, minRestartSize=0, maxPrevRetain=0, method=None) Find k eigenvalues and eigenvectors of the real symmetric square matrix or complex Hermitian matrix A.

Solves $A \star x[i] = w[i] \star x[i]$, the standard eigenvalue problem for w[i] eigenvalues with corresponding eigenvectors x[i].

If M is specified, solves A * x[i] = w[i] * M * x[i], the generalized eigenvalue problem for w[i] eigenvalues with corresponding eigenvectors x[i]

Parameters

- A (An N x N matrix, array, sparse matrix, or LinearOperator representing) the operation A * x, where A is a real symmetric matrix or complex Hermitian.
- **k** (*int*, *optional*) The number of eigenvalues and eigenvectors desired. *k* must be smaller than N. It is not possible to compute all eigenvectors of a matrix.
- M (An N x N matrix, array, sparse matrix, or LinearOperator representing) the operation M * x for the generalized eigenvalue problem

$$A * x = w * M * x$$
.

M must represent a real, symmetric matrix if A is real, and must represent a complex, hermitian matrix if A is complex. For best results, the data type of M should be the same as that of A.

- sigma (real, optional) Find eigenvalues near sigma.
- v0 (N x i, ndarray, optional) Starting vectors for iteration.
- ncv (int, optional) The maximum size of the basis
- which (str ['LM' | 'SM' | 'LA' | 'SA' | 'BE']) If A is a complex hermitian matrix, 'BE' is invalid. Which k eigenvectors and eigenvalues to find:
 - 'LM': Largest (in magnitude) eigenvalues
 - 'SM': Smallest (in magnitude) eigenvalues
 - 'LA': Largest (algebraic) eigenvalues
 - 'SA': Smallest (algebraic) eigenvalues
 - 'BE': Half (k/2) from each end of the spectrum (not supported)

When sigma != None, 'which' refers to the shifted eigenvalues w'[i]

- maxiter (int, optional) Maximum number of iterations.
- **tol** (*float*) Accuracy for eigenvalues (stopping criterion). The default value is sqrt of machine precision.
- Minv((not supported))-
- OPinv (N x N matrix, array, sparse matrix, or LinearOperator) –
 Preconditioner to accelerate the convergence. Usually it is an approximation of the inverse
 of (A sigma*M).
- return_eigenvectors (bool) Return eigenvectors (True) in addition to eigenvalues

- mode (string ['normal' | 'buckling' | 'cayley']) Only 'normal' mode is supported.
- **lock** (*N x i, ndarray, optional*) Seek the eigenvectors orthogonal to these ones. The provided vectors *should* be orthonormal. Useful to not converge some already computed solutions.
- maxBlockSize (int, optional) Maximum number of vectors added at every iteration.
- minRestartSize (int, optional) Number of approximate eigenvectors kept from last iteration in restart.
- maxPrevRetain (int, optional) Number of approximate eigenvectors kept from previous iteration in restart. Also referred as +k vectors in GD+k.
- method (int, optional) Preset method, one of:
 - DEFAULT_MIN_TIME: a variant of JDQMR,
 - DEFAULT_MIN_MATVECS: GD+k
 - DYNAMIC : choose dynamically between both previous methods.

See a detailed description of the methods and other possible values in 2 .

• report_stats(bool, optional)-If True, it is also returned extra information from PRIMME.

Returns

- w (array) Array of k eigenvalues
- v (array) An array representing the k eigenvectors. The column v [:, i] is the eigenvector corresponding to the eigenvalue w [i].
- stats (dict, optional (if return_stats)) Extra information reported by PRIMME:
 - "numOuterIterations": number of outer iterations
 - "numRestarts": number of restarts
 - "numMatvecs": number of A*v
 - "numPreconds": number of OPinv*v
 - "elapsedTime": time that took
 - "estimateMinEVal": the leftmost Ritz value seen
 - "estimateMaxEVal": the rightmost Ritz value seen
 - "estimateLargestSVal": the largest singular value seen

Raises PrimmeError – When the requested convergence is not obtained.

The PRIMME error code can be found as err attribute of the exception object.

See also:

scipy.sparse.linalg.eigs() eigenvalues and eigenvectors for a general (nonsymmetric) matrix A Primme.svds() singular value decomposition for a matrix A

² Preset Methods, http://www.cs.wm.edu/~andreas/software/doc/readme.html#preset-methods

Notes

This function is a wrapper to PRIMME functions to find the eigenvalues and eigenvectors ¹.

References

Examples

```
>>> import Primme, scipy.sparse
>>> A = scipy.sparse.spdiags(range(100), [0], 100, 100) # sparse diag. matrix
>>> evals, evecs = Primme.eigsh(A, 3, tol=1e-6, which='LA')
>>> evals # the three largest eigenvalues of A
array([ 99., 98., 97.])
>>> evals, evecs = Primme.eigsh(A, 3, tol=1e-6, which='LA', lock=evecs)
>>> evals # the next three largest eigenvalues
array([ 96., 95., 94.])
```

MATLAB Interface

function [varargout] = primme_eigs(varargin)

primme_eigs() finds a few eigenvalues and eigenvectors of a real symmetric or Hermitian matrix, A, by calling the function PRIMME_mex (flag,dim,...). This in turn calls PRIMME. Full PRIMME functionality is supported.

Input: [A, numEvals, target, opts, eigsMethod, P]

Output: [evals, evecs, norms, primmeout]

We provide different levels of function calls, similarly to MATLAB eigs():

```
primme_eigs(A)
primme_eigs(A, numEvals)
primme_eigs(A, numEvals, target)
primme_eigs(A, numEvals, target, opts)
primme_eigs(A, numEvals, target, opts, eigsMethod)
primme_eigs(A, numEvals, target, opts, eigsMethod, P)
primme_eigs(A, numEvals, target, opts, eigsMethod, P1,P2)
primme_eigs(A, numEvals, target, opts, eigsMethod, Pfun)
primme_eigs(Afun, dim,...)
```

primme_eigs (A) returns a vector of A's 6 largest algebraic eigenvalues. A must be real symmetric or complex Hermitian and should be large and sparse.

primme_eigs (Afun, dim) accepts a function AFUN instead of a matrix. AFUN is a function handle and y = Afun(x) returns the matrix-vector product A*x. primme_eigs (A,...) could be replaced by primme_eigs (Afun, dim,...) in any of above levels of function calls. Examples are given in PRIMME_MEX_Readme.txt in the root directory of PRIMME_MEX folder.

[V,D] = primme_eigs (A) returns a diagonal matrix D, of A's 6 largest algebraic eigenvalues and a matrix V whose columns are the corresponding eigenvectors.

¹ PRIMME Software, https://github.com/primme/primme

[V,D,norms,primmeout] = primme_eigs(A) also returns an array of the residual norms of the computed eigenpairs, and a struct to report statistical information about numOuterIterations, numRestarts, numMatvecs and numPreconds.

primme_eigs (A, numEvals) finds the *numEvals* largest algebraic eigenvalues. numEvals must be less than the dimension of the matrix A.

primme_eigs (A, numEvals, target) returns numEvals target eigenvalues. target could be a string like below:

- 'LA': primme_largest (default)
- •'SA': primme_smallest
- 'CGT': primme_closest_geq
- 'CLT': primme_closest_leq
- 'CT': primme_closest_abs

primme_eigs (A, numEvals, target, opts, eigsMethod) specifies any of a set of possible options as explained below in the opts structure.

eigsMethod is an integer specifying one of the preset methods in PRIMME:

- •0: PRIMME_DYNAMIC, (default) Switches dynamically to the best method
- •1: PRIMME_DEFAULT_MIN_TIME, Currently set at JDQMR_ETol
- •2: PRIMME DEFAULT MIN MATVECS, Currently set at GD+block
- •3: PRIMME_Arnoldi, obviously not an efficient choice
- •4: PRIMME_GD, classical block Generalized Davidson
- •5: PRIMME_GD_plusK, GD+k block GD with recurrence restarting
- •6: PRIMME_GD_Olsen_plusK, GD+k with approximate Olsen precond.
- •7: PRIMME_JD_Olsen_plusK, GD+k, exact Olsen (two precond per step)
- •8: PRIMME_RQI, Rayleigh Quotient Iteration. Also INVIT, but for INVIT provide targetShifts
- •9: PRIMME_JDQR, Original block, Jacobi Davidson
- •10: PRIMME_JDQMR, Our block JDQMR method (similar to JDCG)
- •11: PRIMME JDQMR ETo1, Slight, but efficient JDQMR modification
- •12: PRIMME SUBSPACE ITERATION, equiv. to GD(block, 2*block)
- •13: PRIMME LOBPCG OrthoBasis, equiv. to GD(nev,3*nev)+nev
- •14: PRIMME_LOBPCG_OrthoBasis_Window equiv. to GD(block,3*block)+block nev>block

primme_eigs(A, numEvals, target, opts, eigsMethod, P)

primme_eigs (A, numEvals, target, opts, eigsMethod, P1, P2) uses preconditioner P or P = P1*P2 to accelerate convergence of the methods. If P is [] then a preconditioner is not applied. P may be a function handle Pfun such that Pfun(x) returns Px.

opts is an option structure which contain following parameters:

- •aNorm: the estimate norm value of matrix A [{0.0}|scaler]
- •eps: desired computing accuracy [{1e-12}|scaler]
- •maxBlockSize: maximum block size the PRIMME uses [{1}|scaler]

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- •printLevel: different level reporting(0-5) [{1}|scaler]
- •outputFile: output file name where user wants to save results
- •precondition: set to 1 if use preconditioner [{0}|1]
- •isreal: the complexity of A represented by AFUN [{ture}|false]
- •numTargetShifts: number of shifts for interior eigenvalues [{0}|scaler]
- •targetShifts: shifts for interior eigenvalues [{}|vector]
- initSize: On INPUT, the number of initial guesses provided in evecs array. ON OUTPUT, the number of converged eigenpairs [{0}|scaler]
- •numOrthoConst: Number of external orthogonalization constraints provided in the first numOrtho-Const vectors of evecs [{0}|scaler]
- •locking: If set to 1, hard locking will be used, otherwise the code will try to use soft locking [{0}|1]
- •maxBasisSize: maximum basis size allowed in the main iteration
- •minRestartSize: minimum Ritz vectors to restart
- •maxMatvecs: maximum number of matrix vector multiplications [{INT_MAX}|scaler]
- $\bullet \textit{maxOuterIterations}: maximum number of outer iterations \ [\{INT_MAX\}| scaler]$
- •restartingParams. scheme: the restart scheme [{primme_thick}| primme_dtr]
- •restartingParams. maxPrevRetain: number of approximations from previous iteration to be retained after restart [{1}|scaler]
- robust Shifts: set to 1 if use robust Shifting to help avoid stagnation and misconverge [{0}|1]
- •maxInnerIterations: number of inner QMR iterations [{0}|scaler]
- •LeftQ: a projector with Q must be applied on the left [$\{0\}$ |1]
- •LeftX: a projector with X must be applied on the left [$\{0\}$ |1]
- •Right Q: a projector with Q must be applied on the right [{0}|1]
- Right X: a projector with X must be applied on the right [{0}|1]
- SkewQ: the Q right projector must be skew [{0}|1]
- SkewX: the X right projector must be skew [{0}|1]
- •relTolBase: a legacy from calssical JDQR (recommend not use)
- •convTest: how to stop the inner QMR Method
- •iseed: set iseed value for initialization
- •intWorkSize: memory size for integer workspace
- •realWorkSize: memory size for real or complex workspace

See also Matlab/readme.txt.

Appendix

primme_params

PRIMME_INT

Integer type used in matrix dimensions (such as n and nLocal) and counters (such as numMatvecs).

The integer size is controlled by the compilation flag PRIMME_INT_SIZE, see Making and Linking.

PRIMME_COMPLEX_FLOAT

Macro that is complex float in C and std::complex<float> in C++.

PRIMME_COMPLEX_DOUBLE

Macro that is complex double in C and std::complex<double> in C++.

primme_params

Structure to set the problem matrices and eigensolver options.

```
PRIMME INT n
```

Dimension of the matrix.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read by dprimme().
```

```
void (*matrixMatvec) (void *x, PRIMME_INT *ldx, void *y, PRIMME_INT *ldy, int *blockSize, primme params *primme, int *ierr)
```

Block matrix-multivector multiplication, y = Ax in solving $Ax = \lambda x$ or $Ax = \lambda Bx$.

Parameters

- x matrix of size nLocal x blockSize in column-major order with leading dimension ldx.
- 1dx the leading dimension of the array x.
- **y** matrix of size *nLocal* **x** blockSize in column-major order with leading dimension ldy.
- **ldy** the leading dimension of the array y.
- blockSize number of columns in x and y.
- **primme** parameters structure.
- ierr output error code; if it is set to non-zero, the current call to PRIMME will stop.

The actual type of x and y depends on which function is being calling. For <code>dprimme()</code>, it is double, for <code>zprimme()</code> it is <code>PRIMME_COMPLEX_DOUBLE</code>, for <code>sprimme()</code> it is <code>float</code> and for for <code>cprimme()</code> it is <code>PRIMME_COMPLEX_FLOAT</code>.

Input/output:

```
primme_initialize() sets this field to NULL;
this field is read by dprimme().
```

Note: If you will have performance issues with leading dimension different than nLocal, set 1dOPs to nLocal.

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Block preconditioner-multivector application, $y = M^{-1}x$ where M is usually an approximation of $A - \sigma I$ or $A - \sigma B$ for finding eigenvalues close to σ . The function follows the convention of matrixMatvec.

Input/output:

```
primme_initialize() sets this field to NULL;
this field is read by dprimme().
```

```
void (*massMatrixMatvec) (void *x, PRIMME_INT *ldx, void *y, PRIMME_INT *ldy, int *block-
Size, primme params *primme, int *ierr)
```

Block matrix-multivector multiplication, y = Bx in solving $Ax = \lambda Bx$. The function follows the convention of matrixMatvec.

Input/output:

```
primme_initialize() sets this field to NULL;
this field is read by dprimme().
```

Warning: Generalized eigenproblems not implemented in current version. This member is included for future compatibility.

int numProcs

Number of processes calling dprimme () or zprimme () in parallel.

Input/output:

```
primme_initialize() sets this field to 1;
this field is read by dprimme().
```

int procID

The identity of the local process within a parallel execution calling <code>dprimme()</code> or <code>zprimme()</code>. Only the process with id 0 prints information.

Input/output:

```
primme_initialize() sets this field to 0;
dprimme() sets this field to 0 if numProcs is 1;
this field is read by dprimme().
```

int nLocal

Number of local rows on this process.

Input/output:

```
primme_initialize() sets this field to 0;
dprimme() sets this field to n if numProcs is 1;
this field is read by dprimme().
```

void *commInfo

A pointer to whatever parallel environment structures needed. For example, with MPI, it could be a pointer to the MPI communicator. PRIMME does not use this. It is available for possible use in user functions defined in matrixMatvec, applyPreconditioner, massMatrixMatvec and globalSumReal.

Input/output:

```
primme_initialize() sets this field to NULL;
```

Global sum reduction function. No need to set for sequential programs.

Parameters

- sendBuf array of size count with the local input values.
- recvBuf array of size count with the global output values so that the i-th element of recvBuf is the sum over all processes of the i-th element of sendBuf.
- count array size of sendBuf and recvBuf.
- primme parameters structure.
- ierr output error code; if it is set to non-zero, the current call to PRIMME will stop.

The actual type of sendBuf and recvBuf depends on which function is being calling. For dprimme() and zprimme() it is double, and for sprimme() and cprimme() it is float. Note that count is the number of values of the actual type.

Input/output:

```
primme_initialize() sets this field to an internal function;
dprimme() sets this field to an internal function if numProcs is 1 and globalSumReal is
NULL;
this field is read by dprimme().
```

When MPI is used this can be a simply wrapper to MPI_Allreduce().

When calling sprimme () and cprimme () replace MPI_DOUBLE by `MPI_FLOAT.

int numEvals

Number of eigenvalues wanted.

Input/output:

```
\label{eq:primme_initialize} \textit{primme_initialize()} \ \ \textit{sets this field to 1;} \\ \textit{this field is read by } \textit{primme\_set\_method()} \ \ \textit{(see Preset Methods)} \ \ \textit{and } \textit{dprimme()}.
```

primme_target target

Which eigenpairs to find:

```
primme_smallest Smallest algebraic eigenvalues; targetShifts is ignored.
primme_largest Largest algebraic eigenvalues; targetShifts is ignored.
primme_closest_geq Closest to, but greater or equal than the shifts in targetShifts.
primme_closest_leq Closest to, but less or equal than the shifts in targetShifts.
primme_closest_abs Closest in absolute value to the shifts in targetShifts.
```

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primme_largest_abs Furthest in absolute value to the shifts in targetShifts.

Input/output:

```
primme_initialize() sets this field to primme_smallest;
this field is read by dprimme().
```

int numTargetShifts

Size of the array targetShifts. Used only when target is primme_closest_geq, primme_closest_leq or primme_closest_abs. The default values is 0.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read by dprimme().
```

double *targetShifts

Array of shifts, at least of size numTargetShifts. Used only when target is primme_closest_geq, primme_closest_leq or primme_closest_abs.

Input/output:

```
primme_initialize() sets this field to NULL;
this field is read by dprimme().
```

The i-th shift (or the last one, if it is not given) is taken into account in finding the i-th eigenvalue.

Note: Considerations for interior problems:

- •PRIMME will try to compute the eigenvalues in the order given in the <code>targetShifts</code>. However, for code efficiency and robustness, the shifts should be ordered. Order them in ascending (descending) order for shifts closer to the lower (higher) end of the spectrum.
- •If some shift is close to the lower (higher) end of the spectrum, use either <code>primme_closest_geq</code> (<code>primme_closest_leq</code>) or <code>primme_closest_abs</code>.
- •primme_closest_leq and primme_closest_geq are more efficient than primme_closest_abs.
- •For interior eigenvalues larger maxBasisSize is usually more robust.

int printLevel

The level of message reporting from the code. One of:

- •0: silent
- •1: print some error messages when these occur.
- •2: as 1, and info about targeted eigenpairs when they are marked as converged:

```
#Converged $1 eval[ $2 ]= $3 norm $4 Mvecs $5 Time $7
```

or locked:

```
#Lock epair[ $1 ]= $3 norm $4 Mvecs $5 Time $7
```

•3: as 2, and info about targeted eigenpairs every outer iteration:

```
OUT $6 conv $1 blk $8 MV $5 Sec $7 EV $3 |r| $4
```

Also, if it is used the dynamic method, show JDQMR/GDk performance ratio and the current method in use.

•4: as 3, and info about targeted eigenpairs every inner iteration:

```
INN MV $5 Sec $7 Eval $3 Lin|r| $9 EV|r| $4
```

•5: as 4, and verbose info about certain choices of the algorithm.

Output key:

- \$1: Number of converged pairs up to now.
- \$2: The index of the pair currently converged.
- \$3: The eigenvalue.
- \$4: Its residual norm.
- \$5: The current number of matrix-vector products.
- \$6: The current number of outer iterations.
- \$7: The current elapsed time.
- \$8: Index within the block of the targeted pair .
- \$9: QMR norm of the linear system residual.

In parallel programs, output is produced in call with *procID* 0 when *printLevel* is from 0 to 4. If *printLevel* is 5 output can be produced in any of the parallel calls.

Input/output:

```
primme_initialize() sets this field to 1;
this field is read by dprimme().
```

Note: Convergence history for plotting may be produced simply by:

```
grep OUT outpufile | awk '{print $8" "$14}' > out
grep INN outpufile | awk '{print $3" "$11}' > inn
```

Then in Matlab:

```
plot(out(:,1),out(:,2),'bo');hold; plot(inn(:,1),inn(:,2),'r');
```

Or in gnuplot:

```
plot 'out' w lp, 'inn' w lp
```

double aNorm

An estimate of the norm of A, which is used in the default convergence criterion (see eps).

If aNorm is less than or equal to 0, the code uses the largest absolute Ritz value seen. On return, aNorm is then replaced with that value.

Input/output:

```
primme_initialize() sets this field to 0.0;
this field is read and written by dprimme().
```

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double eps

If convTestFun is NULL, an eigenpairs is marked as converged when the 2-norm of the residual vector is less than eps*aNorm. The residual vector is $Ax - \lambda x$ or $Ax - \lambda Bx$.

Input/output:

```
primme_initialize() sets this field to 10^{-12}; this field is read by dprimme().
```

FILE *outputFile

Opened file to write down the output.

Input/output:

```
primme_initialize() sets this field to the standard output;
this field is read by dprimme() and primme_display_params().
```

int dynamicMethodSwitch

If this value is 1, it alternates dynamically between PRIMME_DEFAULT_MIN_TIME and PRIMME_DEFAULT_MIN_MATVECS, trying to identify the fastest method.

On exit, it holds a recommended method for future runs on this problem:

```
-1: use PRIMME_DEFAULT_MIN_MATVECS next time.
```

- -2: use PRIMME_DEFAULT_MIN_TIME next time.
- -3: close call, use PRIMME_DYNAMIC next time again.

Input/output:

```
primme_initialize() sets this field to 0;
written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

Note: Even for expert users we do not recommend setting *dynamicMethodSwitch* directly, but through *primme_set_method()*.

Note: The code obtains timings by the <code>gettimeofday</code> Unix utility. If a cheaper, more accurate timer is available, modify the <code>PRIMMESRC/COMMONSRC/wtime.c</code>

int locking

If set to 1, hard locking will be used (locking converged eigenvectors out of the search basis). If set to 0, the code will try to use soft locking (à la ARPACK), when large enough minRestartSize is available.

Input/output:

```
primme_initialize() sets this field to -1;
written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

int initSize

On input, the number of initial vector guesses provided in evecs argument in dprimme() or zprimme().

On output, <code>initSize</code> holds the number of converged eigenpairs. Without <code>locking</code> all <code>numEvals</code> approximations are in <code>evecs</code> but only the <code>initSize</code> ones are converged.

During execution, it holds the current number of converged eigenpairs. In addition, if locking is used, these are accessible in evals and evecs.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read and written by dprimme().
```

PRIMME INT ldevecs

The leading dimension of evecs. The default is nLocal.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read by dprimme().
```

int numOrthoConst

Number of vectors to be used as external orthogonalization constraints. These vectors are provided in the first <code>numOrthoConst</code> positions of the <code>evecs</code> argument in <code>dprimme()</code> or <code>zprimme()</code> and must be orthonormal.

PRIMME finds new eigenvectors orthogonal to these constraints (equivalent to solving the problem with $(I-YY^*)A(I-YY^*)$ and $(I-YY^*)B(I-YY^*)$ matrices where Y are the given constraint vectors). This is a handy feature if some eigenvectors are already known, or for finding more eigenvalues after a call to dprimme() or dprimme() or dprimme(), possibly with different parameters (see an example in dprimme()).

Input/output:

```
primme_initialize() sets this field to 0;
this field is read by dprimme().
```

int maxBasisSize

The maximum basis size allowed in the main iteration. This has memory implications.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read and written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

int minRestartSize

Maximum Ritz vectors kept after restarting the basis.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read and written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

int maxBlockSize

The maximum block size the code will try to use.

The user should set this based on the architecture specifics of the target computer, as well as any a priori knowledge of multiplicities. The code does *not* require that maxBlockSize > 1 to find multiple eigenvalues. For some methods, keeping to 1 yields the best overall performance.

Input/output:

```
primme_initialize() sets this field to 1;
this field is read and written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

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Note: Inner iterations of QMR are not performed in a block fashion. Every correction equation from a block is solved independently.

PRIMME INT maxMatvecs

Maximum number of matrix vector multiplications (approximately equal to the number of preconditioning operations) that the code is allowed to perform before it exits.

Input/output:

```
primme_initialize() sets this field to INT_MAX;
this field is read by dprimme().
```

PRIMME_INT maxOuterIterations

Maximum number of outer iterations that the code is allowed to perform before it exits.

Input/output:

```
primme_initialize() sets this field to INT_MAX;
this field is read by dprimme().
```

int intWorkSize

If dprimme() or zprimme() is called with all arguments as NULL except for primme_params then PRIMME returns immediately with intWorkSize containing the size in bytes of the integer workspace that will be required by the parameters set in PRIMME.

Otherwise if <code>intWorkSize</code> is not 0, it should be the size of the integer work array *in bytes* that the user provides in <code>intWorkSize</code> is 0, the code will allocate the required space, which can be freed later by calling <code>primme_free()</code>.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read and written by dprimme().
```

size t realWorkSize

If dprimme () or zprimme () is called with all arguments as NULL except for primme_params then PRIMME returns immediately with realWorkSize containing the size in bytes of the real workspace that will be required by the parameters set in PRIMME.

Otherwise if realWorkSize is not 0, it should be the size of the real work array in bytes that the user provides in realWork. If realWorkSize is 0, the code will allocate the required space, which can be freed later by calling primme_free().

Input/output:

```
primme_initialize() sets this field to 0;
this field is read and written by dprimme().
```

int *intWork

Integer work array.

If NULL, the code will allocate its own workspace. If the provided space is not enough, the code will return the error code -37.

On exit, the first element shows if a locking problem has occurred. Using locking for large numEvals may, in some rare cases, cause some pairs to be practically converged, in the sense that their components are in the basis of evecs. If this is the case, a Rayleigh Ritz on returned evecs would provide the accurate eigenvectors (see [r4]).

Input/output:

```
primme_initialize() sets this field to NULL;
this field is read and written by dprimme().
```

void *realWork

Real work array.

If NULL, the code will allocate its own workspace. If the provided space is not enough, the code will return the error code -36.

Input/output:

```
primme_initialize() sets this field to NULL;
this field is read and written by dprimme().
```

PRIMME_INT iseed

The PRIMME_INT iseed[4] is an array with the seeds needed by the LAPACK dlarnv and zlarnv.

The default value is an array with values -1, -1, -1 and -1. In that case, iseed is set based on the value of procID to avoid every parallel process generating the same sequence of pseudorandom numbers.

Input/output:

```
primme_initialize() sets this field to [-1, -1, -1, -1]; this field is read and written by dprimme().
```

void *matrix

This field may be used to pass any required information in the matrix-vector product matrixMatvec.

Input/output:

```
primme_initialize() sets this field to NULL;
```

void *preconditioner

This field may be used to pass any required information in the preconditioner function applyPreconditioner.

Input/output:

```
primme_initialize() sets this field to NULL;
```

double *ShiftsForPreconditioner

Array of size blockSize provided during execution of dprimme () and zprimme () holding the shifts to be used (if needed) in the preconditioning operation.

For example if the block size is 3, there will be an array of three shifts in ShiftsForPreconditioner. Then the user can invert a shifted preconditioner for each of the block vectors $(M - ShiftsForPreconditioner_i)^{-1}x_i$. Classical Davidson (diagonal) preconditioning is an example of this.

this field is read and written by dprimme ().

primme_init initBasisMode

Select how the search subspace basis are initialized up to minRestartSize vectors if not enough initial vectors are provided (see initSize):

•primme_init_krylov, with a Krylov subspace generated by the matrix problem and the last initial vector if given or a random vector otherwise.

•primme_init_random, with random vectors.

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primme_init_user, the initial basis will have only initial vectors if given, or a single random vector.

Input/output:

```
primme_initialize() sets this field to primme_init_krylov;
this field is read by dprimme().
```

primme_projection projectionParams.projection

Select the extraction technique, i.e., how the approximate eigenvectors x_i and eigenvalues λ_i are computed from the search subspace \mathcal{V} :

```
•primme proj RR, Rayleigh-Ritz, Ax_i - Bx_i\lambda_i \perp \mathcal{V}.
```

- •primme_proj_Harm, Harmonic Rayleigh-Ritz, $Ax_i Bx_i\lambda_i \perp (A \tau B)V$, where τ is the current target shift (see targetShifts).
- •primme_proj_ref, refined extraction, compute $||x_i||=1$ so that minimizes $||(A-\tau B)x_i||$; the eigenvalues are computes as the Rayleigh quotients, $\lambda_i=\frac{x_i^*Ax_i}{x_i^*Bx_i}$.

Input/output:

```
primme_initialize() sets this field to primme_proj_default;
primme_set_method() and dprimme() sets it to primme_proj_RR if it is
primme_proj_default.
```

primme_restartscheme restartingParams.scheme

Select a restarting strategy:

- •primme_thick, Thick restarting. This is the most efficient and robust in the general case.
- •primme_dtr, Dynamic thick restarting. Helpful without preconditioning but it is expensive to implement.

Input/output:

```
primme_initialize() sets this field to primme_thick;
written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

int restartingParams.maxPrevRetain

Number of approximations from previous iteration to be retained after restart (this is the locally optimal restarting, see [r2]). The restart size is minRestartSize plus maxPrevRetain.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read and written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

int correctionParams.precondition

Set to 1 to use preconditioning. Make sure applyPreconditioner is not NULL then!

Input/output:

```
primme_initialize() sets this field to 0;
this field is read and written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

int correctionParams.robustShifts

Set to 1 to use robust shifting. It tries to avoid stagnation and misconvergence by providing as shifts in *ShiftsForPreconditioner* the Ritz values displaced by an approximation of the eigenvalue error.

```
Input/output:
```

```
primme_initialize() sets this field to 0;
written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

int correctionParams.maxInnerIterations

Control the maximum number of inner QMR iterations:

- •0: no inner iterations;
- •>0: perform at most that number of inner iterations per outer step;
- •<0: perform at most the rest of the remaining matrix-vector products up to reach maxMatvecs.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read and written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

See also convTest.

double correctionParams.relTolBase

Parameter used when convTest is primme_decreasing_LTolerance.

Input/output:

```
primme_initialize() sets this field to 0;
written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

primme_convergencetest correctionParams.convTest

Set how to stop the inner QMR method:

- •primme full LTolerance: stop by iterations only;
- •primme_decreasing_LTolerance, stop when relTolBase outlts where outlts is the number of outer iterations and retTolBase is set in relTolBase; This is a legacy option from classical JDQR and we recommend **strongly** against its use.
- •primme_adaptive, stop when the estimated eigenvalue residual has reached the required tolerance (based on Notay's JDCG).
- •primme_adaptive_ETolerance, as primme_adaptive but also stopping when the estimated eigenvalue residual has reduced 10 times.

Input/output:

```
primme_initialize() sets this field to primme_adaptive_ETolerance;
written by primme_set_method() (see Preset Methods);
this field is read by dprimme().
```

Note: Avoid to set maxInnerIterations to -1 and convTest to primme_full_LTolerance.

See also maxInnerIterations.

```
int correctionParams.projectors.LeftQ
int correctionParams.projectors.LeftX
int correctionParams.projectors.RightQ
```

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int correctionParams.projectors.RightX

int correctionParams.projectors.SkewQ

int correctionParams.projectors.SkewX

Control the projectors involved in the computation of the correction appended to the basis every (outer) iteration.

Consider the current selected Ritz value Λ and vectors X, the residual associated vectors $R = AX - X\Lambda$, the previous locked vectors Q, and the preconditioner M^{-1} .

When maxInnerIterations is 0, the correction D appended to the basis in GD is:

RightX	SkewX	D
0	0	$M^{-1}R$ (Classic GD)
1	0	$M^{-1}(R - \Delta X)$ (cheap Olsen's Method)
1	1	$(I - M^{-1}X(X^*M^{-1}X)^{-1}X^*)M^{-1}R$ (Olsen's Method)
0	1	error

Where Δ is a diagonal matrix that $\Delta_{i,i}$ holds an estimation of the error of the approximate eigenvalue $\Lambda_{i,i}$.

The values of RightQ, SkewQ, LeftX and LeftQ are ignored.

When maxInnerIterations is not 0, the correction D in Jacobi-Davidson results from solving:

$$P_O^l P_X^l (A - \sigma I) P_X^r P_O^r M^{-1} D' = -R, \quad D = P_X^r P_O^l M^{-1} D'.$$

For LeftQ:

$$\begin{aligned} 0 \colon P_Q^l &= I; \\ 1 \colon P_Q^l &= I - QQ^*. \end{aligned}$$

For LeftX:

0:
$$P_X^l = I$$
;
1: $P_X^l = I - XX^*$.

For RightQ and SkewQ:

RightQ	SkewQ	P_Q^r
0	0	I
1	0	$I - QQ^*$
1	1	$I - KQ(Q^*KQ)^{-1}Q^*$
0	1	error

For RightX and SkewX:

RightX	SkewX	P_X^r
0	0	I
1	0	$I - XX^*$
1	1	$I - KX(X^*KX)^{-1}X^*$
0	1	error

Input/output:

primme_initialize() sets all of them to 0;
this field is written by primme_set_method() (see Preset Methods);
this field is read by dprimme().

See [r3] for a study about different projector configurations in JD.

PRIMME INT ldOPs

Recommended leading dimension to be used in matrixMatvec, applyPreconditioner and massMatrixMatvec. The default value is zero, which means no recommendation.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read by dprimme().
```

PRIMME INT stats.numOuterIterations

Hold the number of outer iterations. The value is available during execution and at the end.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

PRIMME INT stats.numRestarts

Hold the number of restarts during execution and at the end.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

PRIMME_INT stats.numMatvecs

Hold how many vectors the operator in *matrixMatvec* has been applied on. The value is available during execution and at the end.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

PRIMME_INT stats.numPreconds

Hold how many vectors the operator in *applyPreconditioner* has been applied on. The value is available during execution and at the end.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

PRIMME INT stats.numGlobalSum

Hold how many times <code>globalSumReal</code> has been called. The value is available during execution and at the end.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

double stats.volumeGlobalSum

Hold how many REAL have been reduced by *globalSumReal*. The value is available during execution and at the end.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

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double stats.elapsedTime

Hold the wall clock time spent by the call to <code>dprimme()</code> or <code>zprimme()</code>. The value is available at the end of the execution.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

double stats.timeMatvec

Hold the wall clock time spent by matrixMatvec. The value is available at the end of the execution.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

double stats.timePrecond

Hold the wall clock time spent by applyPreconditioner. The value is available at the end of the execution.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

double stats.timeOrtho

Hold the wall clock time spent by orthogonalization. The value is available at the end of the execution.

Input/output:

```
primme_initialize() sets this field to 0; written by dprimme().
```

double stats.timeGlobalSum

Hold the wall clock time spent by globalSumReal. The value is available at the end of the execution.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

double stats.estimateMinEVal

Hold the estimation of the smallest eigenvalue for the current eigenproblem. The value is available during execution and at the end.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

double stats.estimateMaxEVal

Hold the estimation of the largest eigenvalue for the current eigenproblem. The value is available during execution and at the end.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

double stats.estimateLargestSVal

Hold the estimation of the largest singular value (i.e., the absolute value of the eigenvalue with largest absolute value) for the current eigenproblem. The value is available during execution and at the end.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

double stats.maxConvTol

Hold the maximum residual norm of the locked eigenvectors. The value is available during execution and at the end.

Input/output:

```
primme_initialize() sets this field to 0;
written by dprimme().
```

```
void (*convTestFun) (double *eval, void *evecs, double *resNorm, int *isconv, primme_params *primme, int *ierr)
```

Function that evaluates if the approximate eigenpair has converged. If NULL, it is used the default convergence criteria (see eps).

Parameters

- **eval** the approximate value to evaluate.
- **x** one dimensional array of size <code>nLocal</code> containing the approximate vector; it can be NULL. The actual type depends on which function is being calling. For <code>dprimme()</code>, it is <code>double</code>, for <code>zprimme()</code> it is <code>PRIMME_COMPLEX_DOUBLE</code>, for <code>sprimme()</code> it is <code>float</code> and for for <code>cprimme()</code> it is <code>PRIMME_COMPLEX_FLOAT</code>.
- resNorm the norm of residual vector.
- isconv (output) the function sets zero if the pair is not converged and non zero otherwise.
- **primme** parameters structure.
- ierr output error code; if it is set to non-zero, the current call to PRIMME will stop.

Input/output:

```
primme_initialize() sets this field to NULL;
this field is read by dprimme().
```

Error Codes

The functions *dprimme()* and *zprimme()* return one of the next values:

- 0: success.
- 1: reported only amount of required memory.
- -1: failed in allocating int or real workspace.
- -2: malloc failed in allocating a permutation integer array.
- -3: main_iter() encountered problem; the calling stack of the functions where the error occurred was printed in stderr.
- -4: if argument primme is NULL.
- -5: if $n \le 0$ or $nLocal \le 0$.
- -6: if numProcs < 1.
- -7: if matrixMatvec is NULL.

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- -8: if applyPreconditioner is NULL and precondition is not NULL.
- -10: if numEvals > n.
- -11: if numEvals < 0.
- -12: if eps > 0 and eps < machine precision.
- -13: if target is not properly defined.
- -14: if target is one of primme_closest_geq, primme_closest_leq or primme_closest_abs but numTargetShifts <= 0 (no shifts).
- -15: if target is one of primme_closest_geq, primme_closest_leq or primme_closest_abs but targetShifts is NULL (no shifts array).
- -16: if numOrthoConst < 0 or numOrthoConst >= n. (no free dimensions left).
- -17: if *maxBasisSize* < 2.
- -18: if minRestartSize <= 0.
- -19: if $maxBlockSize \le 0$.
- -20: if maxPrevRetain < 0.
- -21: if scheme is not one of primme_thick or primme_dtr.
- -22: if initSize < 0.
- -23: if not locking and initSize > maxBasisSize.
- -24: if locking and initSize > numEvals.
- -25: if maxPrevRetain + minRestartSize >= maxBasisSize.
- -26: if minRestartSize >= n.
- -27: if printLevel < 0 or printLevel > 5.
- -28: if convTest is not one of primme_full_LTolerance, primme_decreasing_LTolerance, primme_adaptive_ETolerance or primme_adaptive.
- -29: if convTest == primme_decreasing_LTolerance and relTolBase <= 1.
- -30: if evals is NULL, but not evecs and resNorms.
- -31: if evecs is NULL, but not evals and resNorms.
- $ext{-}32$: if resNorms is NULL, but not evecs and evals.
- -33: if not locking and minRestartSize < numEvals.
- -34: if ldevecs < nLocal
- -35: if 1dOPs is not zero and less than nLocal
- -36: not enough memory for realWork
- -37: not enough memory for intWork

Preset Methods

primme_preset_method

PRIMME DEFAULT MIN TIME

Set as PRIMME_JDQMR_ETol when target is either primme_smallest or primme_largest, and as PRIMME_JDQMR otherwise. This method is usually the fastest if the cost of the matrix vector product is inexpensive.

PRIMME_DEFAULT_MIN_MATVECS

Currently set as PRIMME_GD_Olsen_plusK; this method usually performs fewer matrix vector products than other methods, so it's a good choice when this operation is expensive.

PRIMME DYNAMIC

Switches to the best method dynamically; currently, between methods PRIMME_DEFAULT_MIN_TIME and PRIMME_DEFAULT_MIN_MATVECS.

With PRIMME_DYNAMIC primme_set_method() sets dynamicMethodSwitch = 1 and makes the same changes as for method PRIMME_DEFAULT_MIN_TIME.

PRIMME Arnoldi

Arnoldi implemented à la Generalized Davidson.

With PRIMME_Arnoldi primme_set_method() sets:

- •locking = 0;
- •maxPrevRetain = 0;
- •precondition = 0;
- •maxInnerIterations = 0.

PRIMME GD

Generalized Davidson.

With PRIMME_GD primme_set_method() sets:

- •locking = 0;
- •maxPrevRetain = 0;
- robustShifts = 1;
- •maxInnerIterations = 0;
- •RightX = 0;
- •SkewX = 0.

PRIMME_GD_plusK

GD with locally optimal restarting.

With PRIMME_GD_plusK primme_set_method() sets maxPrevRetain = 2 if maxBlockSize is 1 and numEvals > 1; otherwise it sets maxPrevRetain to maxBlockSize. Also:

- •locking = 0;
- •maxInnerIterations = 0;
- •RightX = 0;
- •SkewX = 0.

PRIMME_GD_Olsen_plusK

GD+k and the cheap Olsen's Method.

With $PRIMME_GD_Olsen_plusK$ $primme_set_method()$ makes the same changes as for method $PRIMME_GD_plusK$ and sets RightX = 1.

PRIMME_JD_Olsen_plusK

GD+k and Olsen's Method.

With $PRIMME_JD_Olsen_plusK$ $primme_set_method()$ makes the same changes as for method $PRIMME_GD_plusK$ and also sets robustShifts = 1, RightX to 1, and SkewX to 1.

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 $\bullet SkewX = 0;$

PRIMME ROI

(Accelerated) Rayleigh Quotient Iteration.

```
•locking = 1;
•maxPrevRetain = 0;
•robustShifts = 1;
•maxInnerIterations = -1;
•LeftQ = 1;
•LeftX = 1;
•RightQ = 0;
•RightX = 1;
•SkewQ = 0;
```

•convTest = primme_full_LTolerance.

With PRIMME_RQI primme_set_method() sets:

Note: If numTargetShifts > 0 and targetShifts are provided, the interior problem solved uses these shifts in the correction equation. Therefore RQI becomes INVIT (inverse iteration) in that case.

PRIMME_JDQR

Jacobi-Davidson with fixed number of inner steps.

With PRIMME_JDQR primme_set_method() sets:

```
•locking = 1;
•maxPrevRetain = 1;
•robustShifts = 0;
•maxInnerIterations = 10 if it is 0;
•LeftQ = 0;
•LeftX = 1;
•RightQ = 1;
•RightX = 1;
•SkewQ = 1;
•SkewX = 1;
•relTolBase = 1.5;
•convTest = primme_full_LTolerance.
```

PRIMME_JDQMR

Jacobi-Davidson with adaptive stopping criterion for inner Quasi Minimum Residual (QMR).

With PRIMME_JDQMR primme_set_method() sets:

```
•locking = 0;
•maxPrevRetain = 1 if it is 0
•maxInnerIterations = -1;
•LeftQ = precondition;
•LeftX = 1;
•RightQ = 0;
•RightX = 0;
•SkewQ = 0;
•SkewX = 1;
•convTest = primme_adaptive.
```

PRIMME_JDQMR_ETol

JDQMR but QMR stops after residual norm reduces by a 0.1 factor.

With PRIMME_JDQMR_ETol primme_set_method() makes the same changes as for the method PRIMME_JDQMR and sets convTest = primme_adaptive_ETolerance.

PRIMME SUBSPACE ITERATION

Subspace iteration.

```
With PRIMME_SUBSPACE_ITERATION primme_set_method() sets:
```

- •locking = 1;
- •maxBasisSize = numEvals * 2;
- •minRestartSize = numEvals;
- •maxBlockSize = numEvals;
- •scheme = primme_thick;
- •maxPrevRetain = 0;
- •robustShifts = 0;
- •maxInnerIterations = 0;
- •RightX = 1;
- •SkewX = 0.

PRIMME_LOBPCG_OrthoBasis

LOBPCG with orthogonal basis.

With PRIMME_LOBPCG_OrthoBasis primme_set_method() sets:

- •locking = 0;
- •maxBasisSize = numEvals * 3;
- •minRestartSize = numEvals;
- •maxBlockSize = numEvals;
- •scheme = primme thick;
- •maxPrevRetain = numEvals;
- robustShifts = 0;
- •maxInnerIterations = 0;
- •RightX = 1;
- •SkewX = 0.

PRIMME_LOBPCG_OrthoBasis_Window

LOBPCG with sliding window of maxBlockSize < 3 * numEvals.

With PRIMME_LOBPCG_OrthoBasis_Window primme_set_method() sets:

- •locking = 0;
- •maxBasisSize = maxBlockSize * 3;
- $\bullet minRestartSize = maxBlockSize;$
- •maxBlockSize = numEvals;
- •scheme = primme thick;
- •maxPrevRetain = maxBlockSize;
- robustShifts = 0;
- •maxInnerIterations = 0;
- •RightX = 1;
- •SkewX = 0.

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CHAPTER

THREE

SINGULAR VALUE PROBLEMS

C Library Interface

The PRIMME SVDS interface is composed of the following functions. To solve real and complex singular value problems call respectively:

Other useful functions:

```
void primme_svds_initialize (primme_svds_params *primme_svds)
int primme_svds_set_method (primme_svds_preset_method method,
    primme_preset_method methodStage1,
    primme_preset_method methodStage2, primme_svds_params *primme_svds)
void primme_svds_display_params (primme_svds_params primme_svds)
void primme_svds_free
    (primme_svds_params *primme_svds)
```

PRIMME SVDS stores its data on the structure <code>primme_svds_params</code>. See Parameters Guide for an introduction about its fields.

Running

To use PRIMME SVDS, follow this basic steps.

1. Include:

```
#include "primme.h" /* header file is required to run primme */
```

2. Initialize a PRIMME SVDS parameters structure for default settings:

```
primme_svds_params primme_svds;
primme_svds_initialize (&primme_svds);
```

3. Set problem parameters (see also Parameters Guide), and, optionally, set one of the preset methods:

4. Then to solve a real singular value problem call:

```
ret = dprimme_svds (svals, svecs, resNorms, &primme_svds);
To solve complex singular value problems call:
ret = zprimme_svds (svals, svecs, resNorms, &primme_svds);
```

The call arguments are:

- svals, array to return the found singular values;
- svecs, array to return the found left and right singular vectors;
- resNorms, array to return the residual norms of the found triplets; and
- ret, returned error code.
- 5. Before exiting, free the work arrays in PRIMME SVDS:

```
primme_svds_free
(&primme_svds);
```

Parameters Guide

PRIMME SVDS stores the data on the structure <code>primme_svds_params</code>, which has the next fields:

Basic

```
int m, number of rows of the matrix.
int n, number of columns of the matrix.
void (* matrixMatvec)(...), matrix-vector product.
int numSvals, how many singular triplets to find.
primme_svds_target target, which singular values to find.
double eps, tolerance of the residual norm of converged triplets.
```

For parallel programs

```
int numProcs
int procID
int mLocal
int nLocal
void (* globalSumReal) (...)
```

Accelerate the convergence

```
void (* applyPreconditioner) (...), preconditioner-vector product.
int initSize, initial vectors as approximate solutions.
int maxBasisSize
int minRestartSize
```

int maxBlockSize

User data

```
void * commInfo
void * matrix
void * preconditioner
```

Advanced options

```
int numTargetShifts, for targeting interior singular values.
double * targetShifts
int numOrthoConst, orthogonal constrains to the singular vectors.
int locking
int maxMatvecs
int intWorkSize
long int realWorkSize
int iseed [4]
int * intWork
void * realWork
double aNorm
int printLevel
FILE * outputFile
primme_svds_operator method
primme_svds_operator methodStage2
primme_params primme
primme params primmeStage2
```

PRIMME SVDS requires the user to set at least the matrix dimensions (m x n) and the matrix-vector product (matrixMatvec), as they define the problem to be solved. For parallel programs, mLocal, nLocal, procID and globalSumReal are also required.

In addition, most users would want to specify how many singular triplets to find, and provide a preconditioner (if available).

It is useful to have set all these before calling <code>primme_svds_set_method()</code>. Also, if users have a preference on <code>maxBasisSize</code>, <code>maxBlockSize</code>, etc, they should also provide them into <code>primme_svds_params</code> prior to the <code>primme_svds_set_method()</code> call. This helps <code>primme_svds_set_method()</code> make the right choice on other parameters. It is sometimes useful to check the actual parameters that PRIMME SVDS is going to use (before calling it) or used (on return) by printing them with <code>primme_svds_display_params()</code>.

Interface Description

The next enumerations and functions are declared in primme.h.

sprimme_svds

int **sprimme_svds** (float *svals, float *svecs, float *resNorms, primme_svds_params *primme_svds)

Solve a real singular value problem.

Parameters

- **svals** array at least of size *numSvals* to store the computed singular values; all processes in a parallel run return this local array with the same values.
- **resNorms** array at least of size *numSvals* to store the residual norms of the computed triplets; all processes in parallel run return this local array with the same values.
- **svecs** array at least of size (*mLocal* + *nLocal*) times *numSvals* to store columnwise the (local part of the) computed left singular vectors and the right singular vectors.
- primme svds parameters structure.

Returns error indicator; see Error Codes.

dprimme_svds

int dprimme_svds (double *svals, double *svecs, double *resNorms, primme_svds_params *primme_svds)

Solve a real singular value problem.

Parameters

- **svals** array at least of size *numSvals* to store the computed singular values; all processes in a parallel run return this local array with the same values.
- **resNorms** array at least of size *numSvals* to store the residual norms of the computed triplets; all processes in parallel run return this local array with the same values.
- **svecs** array at least of size (*mLocal* + *nLocal*) times *numSvals* to store columnwise the (local part of the) computed left singular vectors and the right singular vectors.
- primme_svds parameters structure.

Returns error indicator; see Error Codes.

cprimme svds

```
int cprimme_svds (float *svals, PRIMME_COMPLEX_FLOAT *svecs, float *resNorms, primme_svds_params *primme_svds)

Solve a complex singular value problem; see function dprimme_svds().
```

zprimme svds

```
int zprimme_svds (double *svals, PRIMME_COMPLEX_DOUBLE *svecs, double *resNorms, primme_svds_params *primme_svds)

Solve a complex singular value problem; see function dprimme_svds().
```

primme svds initialize

Parameters

• primme_svds - parameters structure.

primme_svds_set_method

Parameters

- method preset method to compute the singular triplets; one of
 - primme_svds_default, currently set as primme_svds_hybrid.
 - primme_svds_normalequations, compute the eigenvectors of A^*A or AA^* .
 - primme_svds_augmented, compute the eigenvectors of the augmented matrix, $\begin{pmatrix} 0 & A^* \\ A & 0 \end{pmatrix}$.
 - primme_svds_hybrid, start with primme_svds_normalequations; use the resulting approximate singular vectors as initial vectors for primme_svds_augmented if the required accuracy was not achieved.
- **methodStage1** preset method to compute the eigenpairs at the first stage; see available values at primme_set_method().
- methodStage2 preset method to compute the eigenpairs with the second stage of primme_svds_hybrid; see available values at primme_set_method().
- primme_svds parameters structure.

See also Preset Methods.

primme_svds_display_params

Parameters

• primme_svds - parameters structure.

primme_svds_free

Parameters

• primme_svds – parameters structure.

FORTRAN Library Interface

The next enumerations and functions are declared in primme_svds_f77.h.

dprimme_svds_f77

dprimme_svds_f77 (svals, svecs, resNorms, primme_svds)
Solve a real singular value problem.

Parameters

- svals(*) (double precision) (output) array at least of size numSvals to store the computed singular values; all processes in a parallel run return this local array with the same values.
- resNorms (*) (double precision) array at least of size numSvals to store the residual norms of the computed triplets; all processes in parallel run return this local array with the same values.
- svecs(*) (double precision) array at least of size (mLocal + nLocal) times numSvals to store columnwise the (local part of the) computed left singular vectors and the right singular vectors.
- primme_svds (ptr) parameters structure.

Returns error indicator; see Error Codes.

zprimme_svds_f77

zprimme_svds_f77 (svals, svecs, resNorms, primme_svds)
Solve a real singular value problem.

Parameters

- **svals**(*) (double precision) (output) array at least of size numSvals to store the computed singular values; all processes in a parallel run return this local array with the same values.
- resNorms (*) (double precision) array at least of size numSvals to store the residual norms of the computed triplets; all processes in parallel run return this local array with the same values.
- **svecs(*)** (complex*16) array at least of size (mLocal + nLocal) times numSvals to store columnwise the (local part of the) computed left singular vectors and the right singular vectors.
- primme_svds (ptr) parameters structure.

Returns error indicator; see Error Codes.

primme svds initialize f77

```
primme_svds_initialize_f77 (primme_svds)
    Set PRIMME SVDS parameters structure to the default values.
```

Parameters

• **primme svds** (ptr) – (output) parameters structure.

primme svds set method f77

primme_svds_set_method_f77 (method, methodStage1, methodStage2, primme_svds, ierr)
Set PRIMME SVDS parameters to one of the preset configurations.

Parameters

- method (integer) (input) preset configuration to compute the singular triplets; one of
 - PRIMME_SVDS_default, currently set as PRIMME_SVDS_hybrid.
 - PRIMME_SVDS_normalequations, compute the eigenvectors of A^*A or AA^* .
 - PRIMME_SVDS_augmented, compute the eigenvectors of the augmented matrix, $\begin{pmatrix} 0 & A^* \\ A & 0 \end{pmatrix}$.
 - PRIMME_SVDS_hybrid, start with PRIMME_SVDS_normalequations;
 use the resulting approximate singular vectors as initial vectors for PRIMME_SVDS_augmented if the required accuracy was not achieved.
- methodStage1 (primme_preset_method) (input) preset method to compute the eigenpairs at the first stage; see available values at primme_set_method_f77().
- methodStage2 (primme_preset_method) (input) preset method to compute the eigenpairs with the second stage of PRIMME_SVDS_hybrid; see available values at primme_set_method_f77().
- primme_svds (ptr) (input/output) parameters structure.
- ierr (integer) (output) if 0, successful; if negative, something went wrong.

primme_svds_display_params_f77

primme_svds_display_params_f77 (primme_svds)

Display all printable settings of primme_svds into the file descriptor outputFile.

Parameters

• primme_svds (ptr) - (input) parameters structure.

primme svds free f77

```
primme_svds_free_f77 (primme_svds)
```

Free memory allocated by PRIMME SVDS and delete all values set.

Parameters

• **primme_svds** (ptr) – (input/output) parameters structure.

primme sydstop set member f77

primme_svdstop_set_member_f77 (primme_svds, label, value)

Set a value in some field of the parameter structure.

Parameters

- **primme_svds** (ptr) (input) parameters structure.
- label (integer) field where to set value. One of:

```
PRIMME SVDS primme
PRIMME_SVDS_primmeStage2
PRIMME SVDS m
PRIMME_SVDS_n
PRIMME_SVDS_matrixMatvec
PRIMME SVDS applyPreconditioner
PRIMME SVDS numProcs
PRIMME_SVDS_procID
PRIMME SVDS mLocal
PRIMME_SVDS_nLocal
PRIMME SVDS comminfo
PRIMME_SVDS_globalSumDouble
PRIMME_SVDS_numSvals
PRIMME_SVDS_target
PRIMME SVDS numTargetShifts
PRIMME_SVDS_targetShifts
PRIMME_SVDS_method
PRIMME_SVDS_methodStage2
PRIMME SVDS intWorkSize
PRIMME_SVDS_realWorkSize
PRIMME SVDS intWork
PRIMME_SVDS_realWork
PRIMME_SVDS_matrix
PRIMME_SVDS_preconditioner
PRIMME SVDS locking
PRIMME_SVDS_numOrthoConst
PRIMME SVDS aNorm
PRIMME_SVDS_eps
PRIMME_SVDS_precondition
PRIMME SVDS initSize
PRIMME SVDS maxBasisSize
PRIMME_SVDS_maxBlockSize
PRIMME SVDS maxMatvecs
PRIMME_SVDS_iseed
PRIMME SVDS printLevel
PRIMME_SVDS_outputFile
PRIMME_SVDS_stats_numOuterIterations
PRIMME_SVDS_stats_numRestarts
PRIMME SVDS stats numMatvecs
PRIMME SVDS stats numPreconds
PRIMME_SVDS_stats_elapsedTime
```

• **value** – (input) value to set.

Note: Don't use this function inside PRIMME SVDS's callback functions, e.g., matrixMatvec or applyPreconditioner, or in functions called by these functions. In those cases use

```
primme svds set member f77().
```

primme_svdstop_get_member_f77

 $\label{primme_svdstop_get_member_f77} \textbf{(primme_svds, label, value)}$

Get the value in some field of the parameter structure.

Parameters

- **primme_svds** (ptr) (input) parameters structure.
- label (integer) (input) field where to get value. One of the detailed in function primmesvds_top_set_member_f77().
- **value** (output) value of the field.

Note: Don't use this function inside PRIMME SVDS's callback functions, e.g., matrixMatvec or applyPreconditioner, or in functions called by these functions. In those cases use primme_svds_get_member_f77().

Note: When label is one of PRIMME_SVDS_matrixMatvec, PRIMME_SVDS_applyPreconditioner, PRIMME_SVDS_commInfo, PRIMME_SVDS_intWork, PRIMME_SVDS_realWork, PRIMME_SVDS_matrix and PRIMME_SVDS_preconditioner, the returned value is a C pointer (void*). Use Fortran pointer or other extensions to deal with it. For instance:

```
use iso_c_binding
MPI_Comm comm

comm = MPI_COMM_WORLD
call primme_svds_set_member_f77(primme_svds, PRIMME_SVDS_commInfo, comm)
...
subroutine par_GlobalSumDouble(x,y,k,primme_svds)
use iso_c_binding
implicit none
...
MPI_Comm, pointer :: comm
type(c_ptr) :: pcomm

call primme_svds_get_member_f77(primme_svds, PRIMME_SVDS_commInfo, pcomm)
call c_f_pointer(pcomm, comm)
call MPI_Allreduce(x,y,k,MPI_DOUBLE,MPI_SUM,comm,ierr)
```

Most users would not need to retrieve these pointers in their programs.

primme_svds_set_member_f77

primme_svds_set_member_f77 (primme_svds, label, value)

Set a value in some field of the parameter structure.

Parameters

• **primme_svds** (ptr) – (input) parameters structure.

- label (integer) field where to set value. One of the vales defined in primme_svdstop_set_member_f77().
- value (input) value to set.

Note: Use this function exclusively inside PRIMME SVDS's callback functions, e.g., matrixMatvec or applyPreconditioner, or in functions called by these functions. Otherwise, e.g., from the main program, use the function primme_svdstop_set_member_f77().

primme_svds_get_member_f77

```
primme_svds_get_member_f77 (primme_svds, label, value)

Get the value in some field of the parameter structure.
```

Parameters

- primme_svds (ptr) (input) parameters structure.
- label (integer) (input) field where to get value. One of the detailed in function primme_svdstop_set_member_f77().
- **value** (output) value of the field.

Note: Use this function exclusively inside PRIMME SVDS's callback functions, e.g., matrixMatvec or applyPreconditioner, or in functions called by these functions. Otherwise, e.g., from the main program, use the function primme_svdstop_get_member_f77().

Note: When label is one of PRIMME_SVDS_matrixMatvec, PRIMME_SVDS_applyPreconditioner, PRIMME_SVDS_commInfo, PRIMME_SVDS_intWork, PRIMME_SVDS_realWork, PRIMME_SVDS_matrix and PRIMME_SVDS_preconditioner, the returned value is a C pointer (void*). Use Fortran pointer or other extensions to deal with it. For instance:

```
use iso_c_binding
MPI_Comm comm

comm = MPI_COMM_WORLD
call primme_svds_set_member_f77(primme_svds, PRIMME_SVDS_commInfo, comm)
...
subroutine par_GlobalSumDouble(x,y,k,primme_svds)
use iso_c_binding
implicit none
...
MPI_Comm, pointer :: comm
type(c_ptr) :: pcomm

call primme_svds_get_member_f77(primme_svds, PRIMME_SVDS_commInfo, pcomm)
call c_f_pointer(pcomm, comm)
call MPI_Allreduce(x,y,k,MPI_DOUBLE,MPI_SUM,comm,ierr)
```

Most users would not need to retrieve these pointers in their programs.

Python Interface

Primme.svds (A, k=6, ncv=None, tol=0, which='LM', v0=None, maxiter=None, return_singular_vectors=True, precAHA=None, precAH=None, precAug=None, u0=None, locku0=None, lockv0=None, return_stats=False, maxBlockSize=0)

Compute k singular values and vectors for a sparse matrix.

Parameters

- A({sparse matrix, LinearOperator}) Array to compute the SVD on, of shape (M, N)
- **k** (*int*, *optional*) Number of singular values and vectors to compute. Must be 1 <= k < min(A.shape).
- ncv (int, optional) The maximum size of the basis
- tol (float, optional) Tolerance for singular values. Zero (default) means machine precision.
- which (str ['LM' | 'SM'] or number, optional) Which k singular values to find:
 - 'LM': largest singular values
 - 'SM': smallest singular values
 - number : closest singular values to (referred as sigma later)
- **u0** (ndarray, optional) Left starting vectors for the iterations.

Should be approximate left singular vectors. If only u0 or v0 is provided, the other is computed.

- v0 (ndarray, optional) Right starting vectors for the iterations.
- maxiter (int, optional) Maximum number of iterations.
- precAHA ({N x N matrix, array, sparse matrix, LinearOperator}, optional) Approximate inverse of (A.H*A sigma**2*I). If provided and M>N, it usually accelerates the convergence.
- **precAAH** ({M x M matrix, array, sparse matrix, LinearOperator}, optional) Approximate inverse of (A*A.H sigma**2*I). If provided and M<N, it usually accelerates the convergence.
- **precAug** ({ (M+N) x (M+N) matrix, array, sparse matrix, LinearOperator}, optional) Approximate inverse of ([zeros() A.H; zeros() A] sigma*I). It usually accelerates the convergence if tol<dtype.eps**.5.
- locku0 (ndarray, optional) Left orthogonal vector constrain.

Seek singular triplets orthogonal to locku0 and lockv0. The provided vectors *should* be orthonormal. If only locku0 or lockv0 is provided, the other is computed. Useful to not converge some already computed solutions.

- lockv0 (ndarray, optional) Right orthogonal vector constrain. See locku0.
- maxBlockSize (int, optional) Maximum number of vectors added at every iteration
- report_stats (bool, optional) If True, it is also returned extra information from PRIMME.

Returns

- **u** (*ndarray*, *shape*=(*M*, *k*), *optional*) Unitary matrix having left singular vectors as columns. Returned if *return_singular_vectors* is True.
- \mathbf{s} (*ndarray*, *shape*=(k,)) The singular values.
- **vt** (*ndarray, shape=(k, N), optional*) Unitary matrix having right singular vectors as rows. Returned if *return_singular_vectors* is True.
- **stats** (*dict*, *optional* (*if return_stats*)) Extra information reported by PRIMME:
 - "numOuterIterations": number of outer iterations
 - "numRestarts": number of restarts
 - "numMatvecs": number of A*v
 - "numPreconds": number of OPinv*v
 - "elapsedTime": time that took

Returned if return stats is True.

See also:

Primme.eigsh() eigenvalue decomposition for a sparse symmetrix/complex Hermitian matrix A
scipy.sparse.linalg.eigs() eigenvalues and eigenvectors for a general (nonsymmetric) matrix A

Examples

Appendix

primme svds params

primme_svds_params

Structure to set the problem matrix and the solver options.

PRIMME INT m

Number of rows of the matrix.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read by dprimme().
```

PRIMME INT n

Number of columns of the matrix.

Input/output:

```
primme_initialize() sets this field to 0;
this field is read by dprimme().
```

```
void (*matrixMatvec) (void *x, PRIMME_INT ldx, void *y, PRIMME_INT ldy, int *block-
Size, int *transpose, primme_svds_params *primme_svds,
int *ierr)
```

Block matrix-multivector multiplication, y = Ax if transpose is zero and $y = A^*x$ otherwise.

If transpose is zero, then x and y are array of dimensions nLocal x blockSize and mLocal x blockSize respectively. Elsewhere they have dimensions mLocal x blockSize and nLocal x blockSize. Both arrays are column-major (consecutive rows are consecutive in memory) and their real type is double* and double complex* when called from $dprimme_svds()$ $zprimme_svds()$ respectively.

Parameters

- \mathbf{x} input array.
- **ldx** leading dimension of x.
- **y** output array.
- 1dy leading dimension of y.
- blockSize number of columns in x and y.
- **transpose** if non-zero, the transpose A should be applied.
- **primme_svds** parameters structure.
- ierr output error code; if it is set to non-zero, the current call to PRIMME will stop.

Input/output:

```
primme_initialize() sets this field to NULL;
this field is read by dprimme_svds() and zprimme_svds().
```

Note: Integer arguments are passed by reference to make easier the interface to other languages (like Fortran).

Block preconditioner-multivector application. Depending on mode it is expected an approximation of the inverse of

```
 \begin{split} & \texttt{-primme\_svds\_op\_AtA:} \ y = A^*Ax - \sigma^2 I, \\ & \texttt{-primme\_svds\_op\_AAt:} \ y = AA^*x - \sigma^2 I, \\ & \texttt{-primme\_svds\_op\_augmented:} \left( \begin{array}{c} 0 & A^* \\ A & 0 \end{array} \right) - \sigma I. \end{split}
```

Where σ is the current target (see $target \S hifts$) (for finding the smallest σ is zero). If mode is primme_svds_op_AtA, then x and y are array of dimensions nLocal x blockSize; if mode is primme_svds_op_AAt, they are mLocal x blockSize; and otherwise they are (mLocal + nLocal) x blockSize. Both arrays are column-major (consecutive rows are consecutive in memory) and their real type is double* and double complex* when called

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```
from dprimme_svds() zprimme_svds() respectively.
       Parameters
          • \mathbf{x} – input array.
          • 1dx – leading dimension of x.
          • y – output array.
          • ldy – leading dimension of y.
          • blockSize – number of columns in x and y.
          • mode - one of primme_svds_op_AtA, primme_svds_op_AAt or
            primme_svds_op_augmented.
          • primme_svds – parameters structure.
          • ierr – output error code; if it is set to non-zero, the current call to PRIMME will
            stop.
    Input/output:
       primme_initialize() sets this field to NULL;
       this field is read by dprimme_svds() and zprimme_svds().
int numProcs
    Number of processes calling dprimme_svds() or zprimme_svds() in parallel.
    Input/output:
       primme_initialize() sets this field to 1;
       this field is read by dprimme () and zprimme svds ().
int procID
    The identity of the local process within a parallel execution calling dprimme_svds() or
    zprimme_svds(). Only the process with id 0 prints information.
    Input/output:
       primme_svds_initialize() sets this field to 0;
        dprimme_svds() sets this field to 0 if numProcs is 1;
       this field is read by dprimme_svds() and zprimme_svds().
PRIMME INT mLocal
    Number of local rows on this process. The value depends on how the matrix and preconditioner
    is distributed along the processes.
    Input/output:
       primme_svds_initialize() sets this field to 0;
        dprimme_svds() sets this field to m if numProcs is 1;
       this field is read by dprimme_svds() and zprimme_svds().
    See also: matrixMatvec and applyPreconditioner.
PRIMME_INT nLocal
    Number of local columns on this process. The value depends on how the matrix and precondi-
    tioner is distributed along the processes.
    Input/output:
       primme_svds_initialize() sets this field to 0;
        dprimme svds() sets this field to to n if numProcs is 1;
       this field is read by dprimme_svds() and zprimme_svds().
void *commInfo
```

A pointer to whatever parallel environment structures needed. For example, with MPI, it could be a pointer to the MPI communicator. PRIMME does not use this. It is available for possible use in user functions defined in matrixMatvec, applyPreconditioner and globalSumReal.

Input/output:

```
primme_svds_initialize() sets this field to NULL;
void (*globalSumReal) (double
                                   *sendBuf,
                                                double
                                                         *recvBuf,
                                                                     int
                                                                            *count,
                          primme svds params *primme svds, int *ierr)
   Global sum reduction function. No need to set for sequential programs.
       Parameters
          • sendBuf – array of size count with the local input values.
          • recvBuf - array of size count with the global output values so that the i-th
           element of recvBuf is the sum over all processes of the i-th element of sendBuf.
          • count - array size of sendBuf and recvBuf.
          • primme svds – parameters structure.
          • ierr – output error code; if it is set to non-zero, the current call to PRIMME will
            stop.
   The actual type of sendBuf and recvBuf depends on which function is being calling. For
    dprimme_svds() and zprimme_svds() it is double, and for sprimme_svds() and
    cprimme_svds() it is float. Note that count is the number of values of the actual type.
   Input/output:
       primme_svds_initialize() sets this field to an internal function;
       dprimme svds () sets this field to an internal function if numProcs is 1 and
       globalSumReal is NULL;
       this field is read by dprimme_svds() and zprimme_svds().
   When MPI is used this can be a simply wrapper to MPI Allreduce().
    void par_GlobalSumForDouble(void *sendBuf, void *recvBuf, int *count,
                                primme_svds_params *primme_svds, int *ierr) {
       MPI_Comm communicator = *(MPI_Comm *) primme_svds->commInfo;
       if (MPI_Allreduce(sendBuf, recvBuf, *count, MPI_DOUBLE, MPI_SUM,
                       communicator) == MPI_SUCCESS) {
           *ierr = 0;
       } else {
           *ierr = 1;
    }
    When calling :c:func:`sprimme_svds` and :c:func:`cprimme_svds`_
    →replace ``MPI_DOUBLE`` by ```MPI_FLOAT``.
int numSvals
   Number of singular triplets wanted.
   Input/output:
       primme_svds_initialize() sets this field to 1;
       this field is read by primme_svds_set_method() (see Preset Methods) and
       dprimme svds().
primme_svds_target target
   Which eigenpairs to find:
   primme_svds_smallest Smallest singular values; targetShifts is ignored.
   primme_svds_largest Largest singular values; targetShifts is ignored.
   primme_svds_closest_abs Closest in absolute value to the shifts in targetShifts.
   Input/output:
       primme_svds_initialize() sets this field to primme_svds_smallest;
       this field is read by dprimme svds () and zprimme svds ().
```

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targetShifts.

Used

only

when

target

int numTargetShifts
Size of the a

array

```
primme_svds_closest_abs. The default values is 0.
    Input/output:
        primme_svds_initialize() sets this field to 0;
        this field is read by dprimme_svds() and zprimme_svds().
double *targetShifts
    Array of shifts, at least of size numTargetShifts. Used only when target is
    primme_closest_abs.
    Input/output:
        primme_svds_initialize() sets this field to NULL;
        this field is read by dprimme_svds() and zprimme_svds().
    The i-th shift (or the last one, if it is not given) is taken into account in finding the i-th singular
    value.
    Note: Eventually this is used by dprimme () and zprimme (). Please see considerations of
    targetShifts.
int printLevel
    The level of message reporting from the code. For now it controls the reporting level of the
    underneath eigensolvers. See printLevel.
    Input/output:
        primme_svds_initialize() sets this field to 1;
        this field is read by dprimme_svds() and zprimme_svds().
double aNorm
    An estimate of the 2-norm of A, which is used in the default convergence criterion (see eps).
    If a Norm is less than or equal to 0, the code uses the largest absolute Ritz value seen. On return,
    aNorm is then replaced with that value.
    Input/output:
        primme_svds_initialize() sets this field to 0.0;
        this field is read and written by dprimme svds () and zprimme svds ().
double eps
    A triplet is marked as converged when the 2-norm of the residual vectors is less than eps *
    aNorm. The residual vectors are Av - \sigma u and A^*u - \sigma v for the triplet (u, \sigma, v).
    Input/output:
       primme_svds_initialize() sets this field to 10^{-12};
        this field is read by dprimme_svds() and zprimme_svds().
FILE *outputFile
    Opened file to write down the output.
    Input/output:
       primme_svds_initialize() sets this field to the standard output;
        this field is read by dprimme svds (), zprimme svds () and
        primme_svds_display_params()
int locking
    If set to 1, the underneath eigensolvers will use hard locking. See locking.
    Input/output:
```

primme_svds_initialize() sets this field to -1;

```
written by primme_svds_set_method() (see Preset Methods); this field is read by dprimme_svds() and zprimme_svds().
```

int initSize

On input, the number of initial vector guesses provided in svecs argument in dprimme_svds() and zprimme_svds().

On output, <code>initSize</code> holds the number of converged triplets. Without <code>locking</code> all <code>numSvals</code> approximations are in <code>svecs</code> but only the first <code>initSize</code> are converged.

During execution, it holds the current number of converged triplets.

Input/output:

```
primme_svds_initialize() sets this field to 0;
this field is read and written by dprimme_svds() and zprimme_svds().
```

int numOrthoConst

Number of vectors to be used as external orthogonalization constraints. The left and the right vector constrains are provided in the first 2xlSnumOrthoConstl vectors of the svecs argument in <code>dprimme_svds()</code> and <code>zprimme_svds()</code>, and must be orthonormal.

PRIMME SVDS finds new triplets orthogonal to these constraints (equivalent to solving the problem $(I-UU^*)A(I-VV^*)$ where U and V are the given left and right constraint vectors). This is a handy feature if some singular triplets are already known, or for finding more triplets after a call to $dprimme_svds()$ or $zprimme_svds()$, possibly with different parameters (see an example in TEST/exsvd_zseq.c).

Input/output:

```
primme_svds_initialize() sets this field to 0;
this field is read by dprimme_svds() and zprimme_svds().
```

int maxBasisSize

The maximum basis size allowed in the main iteration. This has memory implications.

Input/output:

```
primme_svds_initialize() sets this field to 0;
this field is read and written by primme_svds_set_method() (see Preset
Methods);
this field is read by dprimme_svds() and zprimme_svds().
```

int maxBlockSize

The maximum block size the code will try to use.

The user should set this based on the architecture specifics of the target computer, as well as any a priori knowledge of multiplicities. The code does *not* require that maxBlockSize > 1 to find multiple triplets. For some methods, keeping to 1 yields the best overall performance.

Input/output:

```
primme_svds_initialize() sets this field to 1;
this field is read and written by primme_svds_set_method() (see Preset
Methods);
this field is read by dprimme_svds() and zprimme_svds().
```

PRIMME INT maxMatvecs

Maximum number of matrix vector multiplications (approximately half the number of preconditioning operations) that the code is allowed to perform before it exits.

Input/output:

```
primme_svds_initialize() sets this field to INT_MAX;
this field is read by dprimme_svds() and zprimme_svds().
```

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int intWorkSize

If dprimme_svds() or zprimme_svds() is called with all arguments as NULL except for primme_svds_params then it returns immediately with intWorkSize containing the size in bytes of the integer workspace that will be required by the parameters set.

Otherwise if <code>intWorkSize</code> is not 0, it should be the size of the integer work array *in bytes* that the user provides in <code>intWork</code>. If <code>intWorkSize</code> is 0, the code will allocate the required space, which can be freed later by calling <code>primme_svds_free()</code>.

Input/output:

```
primme_svds_initialize() sets this field to 0;
this field is read and written by dprimme svds() and zprimme svds().
```

size trealWorkSize

If dprimme_svds() or zprimme_svds() is called with all arguments as NULL except for primme_svds_params then it returns immediately with realWorkSize containing the size in bytes of the real workspace that will be required by the parameters set.

Otherwise if realWorkSize is not 0, it should be the size of the real work array in bytes that the user provides in realWork. If realWorkSize is 0, the code will allocate the required space, which can be freed later by calling primme_svds_free().

Input/output:

```
primme_svds_initialize() sets this field to 0;
this field is read and written by dprimme_svds() and zprimme_svds().
```

int *intWork

Integer work array.

If NULL, the code will allocate its own workspace. If the provided space is not enough, the code will return the error code -21.

Input/output:

```
primme_svds_initialize() sets this field to NULL;
this field is read and written by dprimme_svds() and zprimme_svds().
```

void *realWork

Real work array.

If NULL, the code will allocate its own workspace. If the provided space is not enough, the code will return the error code -2.0.

Input/output:

```
primme_svds_initialize() sets this field to NULL;
this field is read and written by dprimme_svds() and zprimme_svds().
```

PRIMME INT iseed

The PRIMME_INT iseed[4] is an array with the seeds needed by the LAPACK dlarnv and zlarnv.

The default value is an array with values -1, -1, -1 and -1. In that case, iseed is set based on the value of procID to avoid every parallel process generating the same sequence of pseudorandom numbers.

Input/output:

```
primme\_svds\_initialize() sets this field to [-1,-1,-1,-1]; this field is read and written by dprimme\_svds() and zprimme\_svds().
```

void *matrix

This field may be used to pass any required information in the matrix-vector product matrixMatvec.

```
Input/output:
```

```
primme svds initialize() sets this field to NULL;
```

void *preconditioner

This field may be used to pass any required information in the preconditioner function applyPreconditioner.

Input/output:

```
primme_svds_initialize() sets this field to NULL;
```

int precondition

Set to 1 to use preconditioning. Make sure applyPreconditioner is not NULL then!

Input/output:

```
primme_svds_initialize() sets this field to 0;
this field is read and written by primme_svds_set_method() (see Preset Methods);
this field is read by dprimme_svds() and zprimme_svds().
```

primme_svds_op_operator method

Select the equivalent eigenvalue problem that will be solved:

```
 \begin{split} & \texttt{•primme\_svds\_op\_AtA:} \ A^*Ax = \sigma^2 x, \\ & \texttt{•primme\_svds\_op\_AAt:} \ AA^*x = \sigma^2 x, \\ & \texttt{•primme\_svds\_op\_augmented:} \left( \begin{array}{cc} 0 & A^* \\ A & 0 \end{array} \right) x = \sigma x. \end{split}
```

The options for this solver are stored in primme

Input/output:

```
primme_svds_initialize() sets this field to 0;
this field is read and written by primme_svds_set_method() (see Preset
Methods);
this field is read by dprimme_svds() and zprimme_svds().
```

primme svds op operator methodStage2

Select the equivalent eigenvalue problem that will be solved to refine the solution. The allowed options are primme_svds_op_none to not refine the solution and primme_svds_op_augmented to refine the solution by solving the augmented problem with the current solution as the initial vectors. See method.

The options for this solver are stored in primmeStage2.

Input/output:

```
primme_svds_initialize() sets this field to 0;
this field is read and written by primme_svds_set_method() (see Preset
Methods);
this field is read by dprimme_svds() and zprimme_svds().
```

primme_params primme

Parameter structure storing the options for underneath eigensolver that will be called at the first stage. See *method*.

Input/output:

```
primme_svds_initialize() initialize this structure;
this field is read and written by primme_svds_set_method() (see Preset
Methods);
this field is read and written by dprimme_svds() and zprimme_svds().
```

primme_params primmeStage2

Parameter structure storing the options for underneath eigensolver that will be called at the

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```
second stage. See methodStage2.
   Input/output:
       primme_svds_initialize() initialize this structure;
       this field is read and written by primme_svds_set_method() (see Preset
       Methods):
       this field is read and written by dprimme_svds() and zprimme_svds().
PRIMME_INT stats.numOuterIterations
   Hold the number of outer iterations.
   Input/output:
       primme_svds_initialize() sets this field to 0;
       written by dprimme svds() and zprimme svds().
PRIMME_INT stats.numRestarts
   Hold the number of restarts.
   Input/output:
       primme_svds_initialize() sets this field to 0;
       written by dprimme_svds() and zprimme_svds().
PRIMME_INT stats.numMatvecs
   Hold how many vectors the operator in matrixMatvec has been applied on.
   Input/output:
       primme_svds_initialize() sets this field to 0;
       written by dprimme_svds() and zprimme_svds().
PRIMME_INT stats.numPreconds
   Hold how many vectors the operator in applyPreconditioner has been applied on.
   Input/output:
       primme_svds_initialize() sets this field to 0;
       written by dprimme_svds() and zprimme_svds().
double stats.elapsedTime
   Hold the wall clock time spent by the call to dprimme_svds() or zprimme_svds().
   Input/output:
       primme_svds_initialize() sets this field to 0;
       written by dprimme_svds() and zprimme_svds().
```

Error Codes

The functions <code>dprimme_svds()</code> and <code>zprimme_svds()</code> return one of the next values:

- 0: success,
- 1: reported only amount of required memory,
- -1: failed in allocating int or real workspace,
- -2: malloc failed in allocating a permutation integer array,
- -3: main_iter() encountered problem; the calling stack of the functions where the error occurred was printed in 'stderr',
- -4: primme_svds is NULL,
- -5: Wrong value for m or n,

- -6: Wrong value for numProcs,
- -7: matrixMatvec is not set,
- -8: applyPreconditioner is not set but precondition == 1,
- -9: numProcs >1 but globalSumReal is not set,
- -10: Wrong value for numSvals, it's larger than min(m, n),
- -11: Wrong value for numSvals, it's smaller than 1,
- -13: Wrong value for target,
- -14: Wrong value for method,
- -15: Not supported combination of method and methodStage2,
- -16: Wrong value for printLevel,
- -17: svals is not set,
- -18: svecs is not set,
- -19: resNorms is not set
- -20: not enough memory for realWork
- -21: not enough memory for intWork
- -100 up to -199: eigensolver error from first stage; see the value plus 100 in Error Codes.
- -200 up to -299: eigensolver error from second stage; see the value plus 200 in Error Codes.

Preset Methods

primme_svds_preset_method

primme_svds_default

Set as primme_svds_hybrid.

primme_svds_normalequations

Solve the equivalent eigenvalue problem $A^*AV = \Sigma^2 V$ and computes U by normalizing the vectors AV. If m is smaller than n, AA^* is solved instead.

With primme_svds_normalequations primme_svds_set_method() sets method to primme_svds_op_AtA if m is larger or equal than n, and to primme_svds_op_AAt otherwise; and methodStage2 is set to primme_svds_op_none.

primme_svds_augmented

Solve the equivalent eigenvalue problem $\begin{pmatrix} 0 & A^* \\ A & 0 \end{pmatrix} X = \sigma X$ with $X = \begin{pmatrix} V \\ U \end{pmatrix}$.

With primme_svds_augmented primme_svds_set_method() sets method to primme_svds_op_augmented and methodStage2 to primme_svds_op_none.

primme_svds_hybrid

First solve the equivalent normal equations (see primme_svds_normalequations) and then refine the solution solving the augmented problem (see primme_svds_augmented).

With primme_svds_normalequations primme_svds_set_method() sets method to primme_svds_op_AtA if m is larger or equal than n, and to primme_svds_op_AAt otherwise; and methodStage2 is set to primme_svds_op_augmented.

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