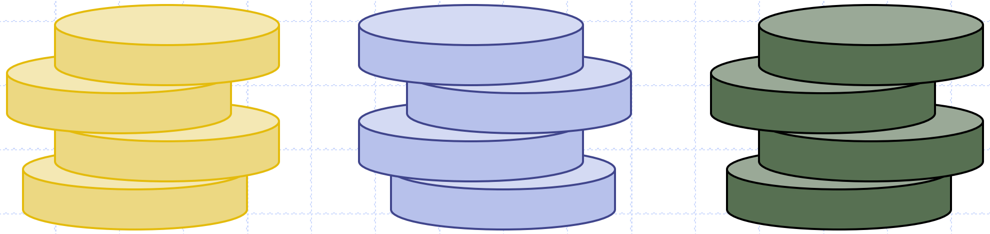
Stacks

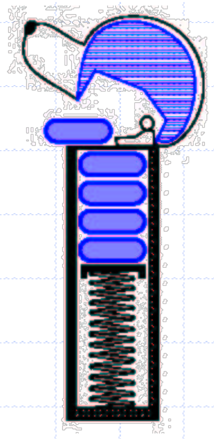


Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations

- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

The Stack ADT



- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element
- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer len(): returns the number of elements stored
  - boolean is_empty(): indicates whether no elements are stored

Stacks

- Last In First Out (LIFO)

  Remove the **most** recently added item

  Analogy: cafeteria trays

Example

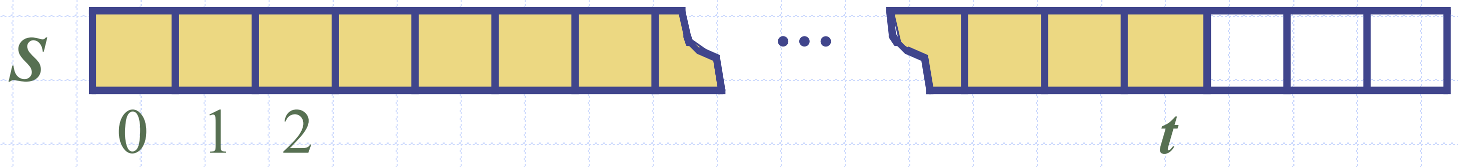
Operation	Return Value	Stack Contents
S.push(5)	–	[5]
S.push(3)	–	[5, 3]
len(S)	2	[5, 3]
S.pop()	3	[5]
S.is_empty()	False	[5]
S.pop()	5	[ ]
S.is_empty()	True	[ ]
S.pop()	"error"	[ ]
S.push(7)	–	[7]
S.push(9)	–	[7, 9]
S.top()	9	[7, 9]
S.push(4)	–	[7, 9, 4]
len(S)	3	[7, 9, 4]
S.pop()	4	[7, 9]
S.push(6)	–	[7, 9, 6]
S.push(8)	–	[7, 9, 6, 8]
S.pop()	8	[7, 9, 6]

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in a language that supports recursion
  - Parsing in a compiler
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

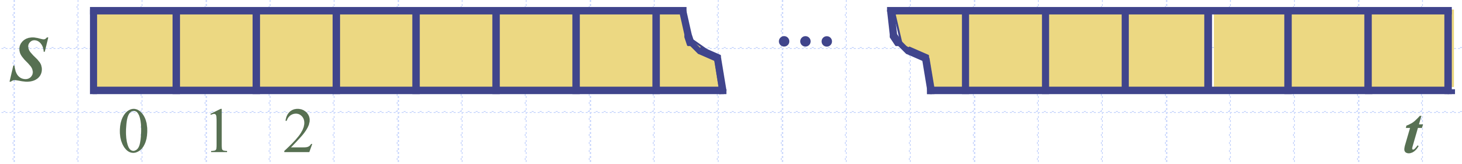
Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element



Array-based Stack (cont.)

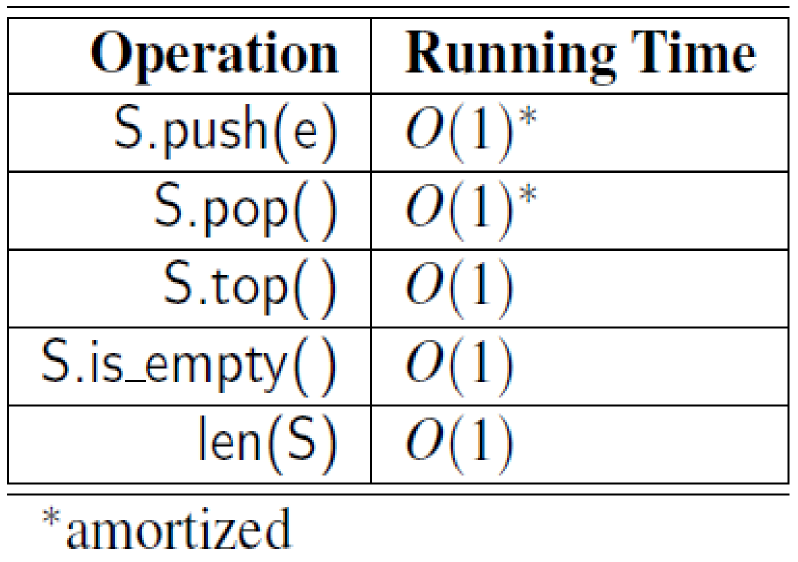
- The array storing the stack elements may become full
- A push operation will then need to grow the array and copy all the elements over.



Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$ (amortized in the case of push & pop due to resizing)

Analyzing the Array-Based Stack



Operation	Running Time
S.push(e)	$O(1)^*$
S.pop()	$O(1)^*$
S.top()	$O(1)$
S.is_empty()	$O(1)$
len(S)	$O(1)$

*amortized

Parentheses Matching (Application of Stack)

- Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “[”
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )){([( )])}))
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

Parentheses Matching Algorithm

```
Algorithm ParenMatch(X,n):
Input: An array X of n tokens, each of which is either a grouping symbol, a
variable, an arithmetic operator, or a number
Output: true if and only if all the grouping symbols in X match
Let S be an empty stack
for i=0 to n-1 do
    if X[i] is an opening grouping symbol then
        S.push(X[i])
    else if X[i] is a closing grouping symbol then
        if S.is_empty() then
            return false {nothing to match with}
        if S.pop() does not match the type of X[i] then
            return false {wrong type}
if S.isEmpty() then
    return true {every symbol matched}
else return false {some symbols were never matched}
```

Parentheses Matching in Python

```
def is_matched(expr):
  """Return True if all delimiters are properly match; False otherwise."""
  lefty = '({['                                   # opening delimiters
  righty = ')}]'                                  # respective closing delims
  S = ArrayStack()
  for c in expr:
    if c in lefty:
      S.push(c)                                   # push left delimiter on stack
    elif c in righty:
      if S.is_empty():
        return False                              # nothing to match with
      if righty.index(c) != lefty.index(S.pop()):
        return False                              # mismatched
  return S.is_empty()                             # were all symbols matched?
```

HTML Tag Matching (Application of Stack)

- For fully-correct HTML, each <name> should pair with a matching </name>

```
<body>
<center>
<h1> The Little Boat </h1>
</center>
<p> The storm tossed the little
boat like a cheap sneaker in an
old washing machine. The three
drunken fishermen were used to
such treatment, of course, but
not the tree salesman, who even as
a stowaway now felt that he
had overpaid for the voyage. </p>
<ol>
<li> Will the salesman die? </li>
<li> What color is the boat? </li>
<li> And what about Naomi? </li>
</ol>
</body>
```

The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

Tag Matching Algorithm in Python

```
def is_matched_html(raw):
  """Return True if all HTML tags are properly match; False otherwise."""
  S = ArrayStack()
  j = raw.find('<')                          # find first '<' character (if any)
  while j != −1:
    k = raw.find('>', j+1)                   # find next '>' character
    if k == −1:
      return False                           # invalid tag
    tag = raw[j+1:k]                         # strip away < >
    if not tag.startswith('/'):              # this is opening tag
      S.push(tag)
    else:                                    # this is closing tag
      if S.is_empty():
        return False                         # nothing to match with
      if tag[1:] != S.pop():
        return False                         # mismatched delimiter
    j = raw.find('<', k+1)                   # find next '<' character (if any)
  return S.is_empty()                        # were all opening tags matched?
```

Evaluating Arithmetic Expressions

Slide by Matt Stallmann included with permission.

14 – 3 * 2 + 7 = (14 – (3 * 2)) + 7

Operator precedence

* has precedence over +/–

Associativity

operators of the same precedence group evaluated from left to right

Example: (x + y) + z rather than x + (y + z)

Idea: **1)** push each operator on the stack, but first pop and perform higher and *equal* precedence operations.

2) Use 2 stacks: one to hold operands; another one to hold operators

Algorithm for Evaluating Expressions

Slide by Matt Stallmann included with permission.

Two stacks:

- opStk holds operators
- valStk holds values
- Use $ as special "end of input" token with lowest precedence

Algorithm doOp()

```
x ← valStk.pop();
y ← valStk.pop();
op ← opStk.pop();
valStk.push( y op x )
```

Algorithm repeatOps(refOp):

```
 while ( valStk.size() > 1 ∧
         prec(refOp) ≤
         prec(opStk.top())
   doOp()
```

Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)

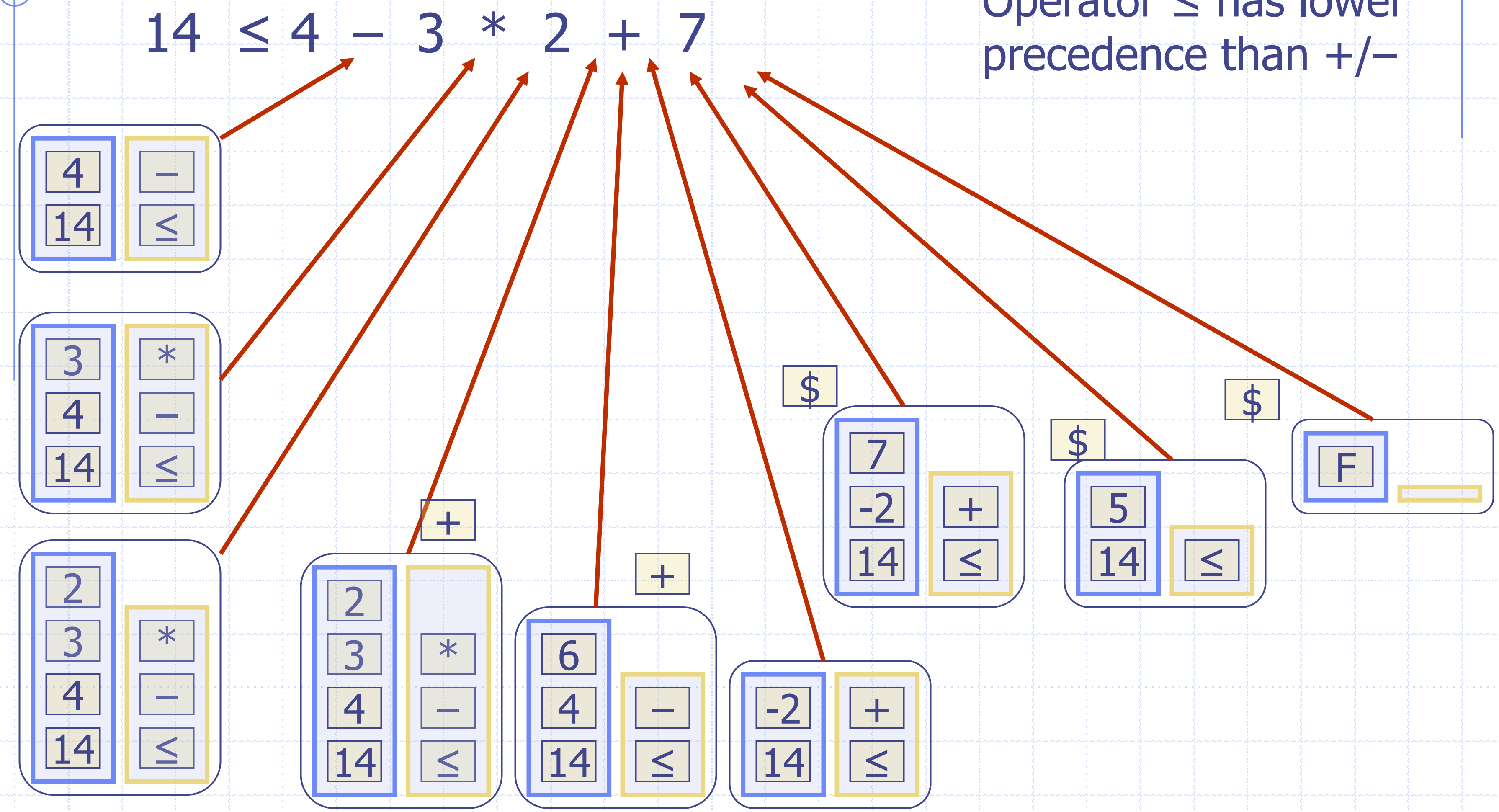
Output: the value of the expression

```
while there's another token z
    if isNumber(z) then
        valStk.push(z)
    else
        repeatOps(z);
        opStk.push(z)
repeatOps($);
return valStk.top()
```

Algorithm on an Example Expression

Slide by Matt Stallmann included with permission.

Operator ≤ has lower precedence than +/–



Let’s Implement a simple Stack using Python’s List Class

- Open array_stack_student.py from brightspace.nyu.edu
- Fill in the #to do part to implement a stack
- We will implement ArrayStack Class.
- Submit your code to Gradescope

Let’s use our simple Stack to reverse a file

- Open reverse_file_student.py from Brightspace
- Fill in the #to do part to reverse the file contents of DSSyllabus.txt
- Use ArrayStack class in array_stack_student.py file to implement your solution
- Submit the 3 files to Gradescope