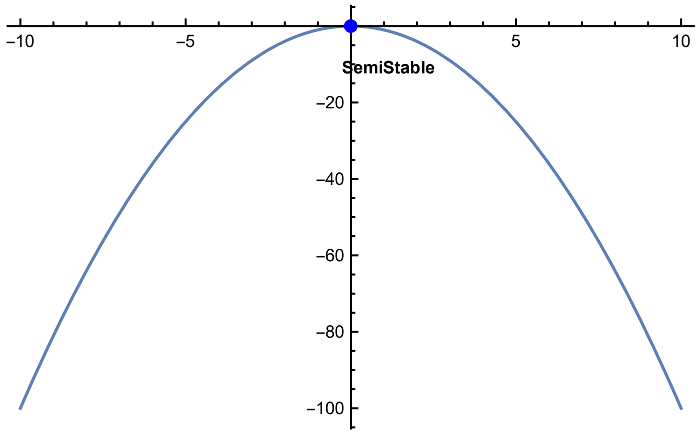
Task 1.1

Task 1.1a

h = 0

```
In[ ]:= h = 0;
r = 0;
roots = Solve[h + x (r - x) == 0, x];
roots = x /. roots;
txt = Text["Unstable", {.7, .7}];
Plot[h + x (r - x), {x, -10, 10},
  Epilog → {Black, PointSize@Large, Point[{{Part[roots, 1], 0}, {Part[roots, 2], 0}}]}];
```



h > 0

```
In[ ]:= h = 5;
r = 0;
roots = Solve[h + x (r - x) == 0, x];
roots = x /. roots;

Show[ListPlot[{{{roots[[1]], 0}}, {{roots[[2]], 0}}},
   PlotMarkers → {Style[○, Black, 20], Style[●, Black, 20]},
   PlotRange → {{-10, 10}, {-10, 10}}], Plot[h + x (r - x), {x, -10, 10},
   PlotStyle → {{ColorData[97][1], Thick}}], ImageSize → Large];
```

h < 0

```
In[ ]:= h = -5;
r = 0;
roots = Solve[h + x (r - x) == 0, x];
roots = rootsForX = x /. roots;
Plot[h + x (r - x), {x, -10, 10}];
```

Task 1.1b

In[ ]:=
```
ContourPlot3D[h + x (r - x) == 0, {x, -10, 10}, {h, -10, 10}, {r, -10, 10}];
```

Task 1.1c

In[ ]:=
```
x =.
r =.
h =.
xstar =.
prim =.

f[x_] := h + x (r - x)
prim = f'[x];
xstar = x /. Solve[prim == 0, x];
hc = h /. Solve[f[xstar] == 0, h]
```

Out[ ]= $\left\{-\frac{r^2}{4}\right\}$

Task 1.1 d

```
M = {{1, -1}, {0, 1}};
Eigenvectors[M];
```

Out[ ]= {{1, 0}, {0, 0}}

1.2 Subcritical pitchfork

1.2a

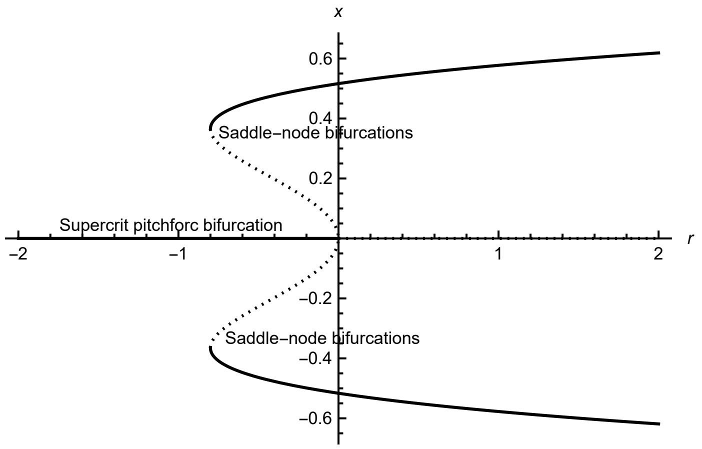
In[ ]:=
```
r =.
x =.
f[x_] := r * x + 4 x^3 - 9 x^5;
roots = x /. Solve[f'[x] == 0, x];
g[r_] := roots[[4]];
rc = Solve[f[g[r]] == 0, r]

p1 = Plot[{roots}, {r, -2, 2}, PlotStyle →
    {{Black, Dashing[Tiny]}, {Black, Dashing[Tiny]}, {Black, Line}, {Black, Line}}];
p2 = Plot[{0}, {r, -2, 0}, PlotStyle → {Black}];
p3 = Plot[{0}, {r, 0, 2}, PlotStyle → {Black, Dashing[Tiny]}];
Show[p1, p2, p3];
```

Out[ ]= $\left\{\left\{r \to -\frac{4}{9}\right\}\right\}$

```
Show[%2586, AxesLabel → {HoldForm[r], HoldForm[x]},
  PlotLabel → None, LabelStyle → {GrayLevel[0]}];
```

Out[ ]=



In[ ]:=

```
Manipulate[Plot[r * x + 4 x^3 - 9 x^5, {x, -1, 1}], {r, -1, 1}];
```

TASK 1.3

In[ ]:=
```
sigma =.
ClearAll[x]
ClearAll[y]


M = {{sigma + 3, 4}, {-9 / 4, sigma - 3}}
Eigenvalues[M];
-Eigenvectors[M][[1]];
Inverse[M];

M = {{sigma + 3, 4}, {-9 / 4, sigma - 3}}
plot = {M[[1, 1]] x + M[[1, 2]] y, M[[2, 1]] x + M[[2, 2]] y}

st1 = StreamPlot[plot /. sigma → -1,
   {x, -3, 3}, {y, -3, 3}, PlotLabel → "Stable for sigma=-1"];
st2 = StreamPlot[plot /. sigma → 0, {x, -3, 3},
   {y, -3, 3}, PlotLabel → "Uniform motion for sigma=0"];
st3 = StreamPlot[plot /. sigma → 1,
   {x, -3, 3}, {y, -3, 3}, PlotLabel → "Unstable sigma=1"];
GraphicsRow[{st1, st2, st3}];

sigma = .;
c = 3 / 2;
d = -2;
K = {{sigma - c d, 4}, {-9 / 4, sigma - 3}}
Eigenvalues[K]

sigma = 0;
c =.;
d =.;
K = {{sigma - c d, d^2}, {-c^2, sigma + c d}};
Eigenvectors[K]
```

Out[ ]= $\left\{\{3+\text{sigma},4\},\left\{-\frac{9}{4},-3+\text{sigma}\right\}\right\}$

Out[ ]= $\left\{\{3+\text{sigma},4\},\left\{-\frac{9}{4},-3+\text{sigma}\right\}\right\}$

Out[ ]= $\left\{(3+\text{sigma})\,x+4\,y,-\frac{9\,x}{4}+(-3+\text{sigma})\,y\right\}$

Out[ ]= $\left\{\{3+\text{sigma},4\},\left\{-\frac{9}{4},-3+\text{sigma}\right\}\right\}$

Out[ ]= $\{\text{sigma},\text{sigma}\}$

Out[ ]= $\left\{\left\{\frac{d}{c},1\right\},\{0,0\}\right\}$

Out[ ]= $\left\{\{3+\text{sigma},4\},\left\{-\frac{9}{4},-3+\text{sigma}\right\}\right\}$

Out[ ]= {sigma, sigma}

Out[ ]= $\left\{\left\{\frac{d}{c},1\right\},\{0,0\}\right\}$

Task 1.4

1.4b) Generate the function for x and y for x0 and y0

In[ ]:=
```
sigma =.; t =.; x =.; y =.; sol =.; x0 =.; y0 =.; u =.; v =.;
ClearAll[y]
ClearAll[x]
x0 = u;
y0 = v;
sol[x0_, y0_] := DSolve[{x'[t] == ((sigma + 1) x[t] + 3 y[t]) ,
    y'[t] == (-2 x[t] + (sigma - 1) y[t]), x[0] == x0, y[0] == y0}, {x, y}, {t}];
sol[x0, y0]
```

Out[ ]= $\left\{\left\{x\to\text{Function}\left[\{t\},\frac{1}{5}e^{\text{sigma}\,t}\left(5\,u\,\text{Cos}\left[\sqrt{5}\,t\right]+\sqrt{5}\,u\,\text{Sin}\left[\sqrt{5}\,t\right]+3\sqrt{5}\,v\,\text{Sin}\left[\sqrt{5}\,t\right]\right)\right],\right.\right.$
$\left.\left.y\to\text{Function}\left[\{t\},-\frac{1}{5}e^{\text{sigma}\,t}\left(-5\,v\,\text{Cos}\left[\sqrt{5}\,t\right]+2\sqrt{5}\,u\,\text{Sin}\left[\sqrt{5}\,t\right]+\sqrt{5}\,v\,\text{Sin}\left[\sqrt{5}\,t\right]\right)\right]\right\}\right\}$

Case: Sigma = 0

In[ ]:=
```
t =.
min = -2;
max = 2;
dist = 10;
sigma = 0;
u = 1;
v = 1;
sol[x0, y0]


M = {{sigma + 1, 3}, {-2, sigma - 1}};
Re[Eigenvalues[M]];

inits = Join[Table[ {min, y}, {y, min, max, dist}], Table[ {max, y}, {y, min, max, dist}],
   Table[ {x, min}, {x, min, max, dist}],
   Table[ {x, max}, {x, min, max, dist}]];


text = Graphics[Text["Undamped for sigma = 0", {.3, .7}]];
plot1 =
 Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[inits[[i, 1]], inits[[i, 2]]]],
    {t, 0, 50}], {i, Length[inits]}], text];
p1 = (plot1 // Normal) /. Line[x_] :> {Arrowheads[{0, 0.1, 0}], Arrow[x]};
```

Out[ ]= $\left\{\left\{x \to \text{Function}\left[\{t\}, \frac{1}{5}\left(5\,\text{Cos}\left[\sqrt{5}\,t\right] + 4\sqrt{5}\,\text{Sin}\left[\sqrt{5}\,t\right]\right)\right],\right.\right.$
$\left.\left. y \to \text{Function}\left[\{t\}, \frac{1}{5}\left(5\,\text{Cos}\left[\sqrt{5}\,t\right] - 3\sqrt{5}\,\text{Sin}\left[\sqrt{5}\,t\right]\right)\right]\right\}\right\}$

Case: Sigma = -1/10

In[ ]:=
```
sigma = -1 / 10;

M = {{sigma + 1, 3}, {-2, sigma - 1}};
a = Re[Eigenvalues[M]]

text = Graphics[Text[Style["Stable for sigma = -1/10", Bold, Large], {0, 1}]];
plot2 =
 Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[inits[[i, 1]], inits[[i, 2]]]],
    {t, 0, 50}], {i, Length[inits]}], text];
p2 = (plot2 // Normal) /. Line[x_] :→ {Arrowheads[{0, -a[[1]], 0}], Arrow[x]};
```

Out[ ]= $\left\{-\frac{1}{10}, -\frac{1}{10}\right\}$

Sigma = 1/10

In[ ]:=
```
sigma = 1 / 10;

M = {{sigma + 1, 3}, {-2, sigma - 1}};
a = Re[Eigenvalues[M]]

text = Graphics[Text[Style["Unstable for sigma = 1/10", Bold, Large], {50, 150}]];
plot3 =
 Show[Table[ParametricPlot[Evaluate[{x[t], y[t]} /. sol[inits[[i, 1]], inits[[i, 2]]]],
    {t, 0, 50}], {i, Length[inits]}], text];
p3 = (plot3 // Normal) /. Line[x_] :→ {Arrowheads[{0, a[[1]], 0}], Arrow[x]};
```

Out[ ]= $\left\{\frac{1}{10}, \frac{1}{10}\right\}$

1.4 d

In[ ]:=
```
period = 2 Pi;
argument = Sqrt[5];
periodTime = period / argument
```

Out[ ]= $\frac{2\pi}{\sqrt{5}}$

1.4 e

In[ ]:=
```
sigma =.; t =.; x =.; y =.; sol =.; x0 =.; y0 =.; u =.; v =.; a =.;
ClearAll[y]
ClearAll[x]
sigma = 0;
u = 1;
v = 1;
x0 = u;
y0 = v;
sol[x0_, y0_] := DSolve[{x'[t] == ((sigma + 1) x[t] + 3 y[t]),
   y'[t] == (-2 x[t] + (sigma - 1) y[t]), x[0] == x0, y[0] == y0}, {x, y}, {t}]
fkn[t_] := sol[x0, y0]
```

$$\text{vec} = \left\{\frac{1}{5}\left(5\,\text{Cos}\left[\sqrt{5}\ t\right] + 4\sqrt{5}\ \text{Sin}\left[\sqrt{5}\ t\right]\right),\ \frac{1}{5}\left(5\,\text{Cos}\left[\sqrt{5}\ t\right] - 3\sqrt{5}\ \text{Sin}\left[\sqrt{5}\ t\right]\right)\right\};$$

```
max = FindMaximum[Norm[vec], t];
min = FindMinimum[Norm[vec], t];
kvot = max / min
angle = Solve[Tan[theta] == kvot[[1]], theta];
angle = theta /. angle;
vec = {Sin[angle], -Cos[angle]}
```

Out[ ]= $\left\{1.61803,\ \left\{\frac{t \to 0.637691}{t \to 1.34017}\right\}\right\}$

Out[ ]= {{0.850651}, {-0.525731}}