```
 1 import os
 2 from scipy.integrate import odeint
 3 import numpy as np
 4 import matplotlib.pyplot as plt
 5
 6
 7 def plot_dynamical_system(fun):
 8     fig = plt.figure()
 9     ax = plt.axes(projection='3d')
10     ax.plot3D(fun[:, 0], fun[:, 1], fun[:, 2], 'green
   ')
11     ax.set_title('Lorenz flow')
12     plt.show()
13
14
15 def model(t, state, sigma, b, r):
16     x, y, z = state
17     xdot = sigma*(y-x)
18     ydot = r*x - y - x*z
19     zdot = x*y - b*z
20     return [xdot, ydot, zdot]
21
22
23 def compute_eig(points, sigma, b, r, dt, t_max, N):
24     eigenvalues = np.zeros(3)
25     Q = np.eye(3)
26     lambda_history = np.zeros((N, 3))
27
28     for index in range(N):
29         M = np.eye(3) + np.array([[-sigma, sigma, 0],
30                                   [r-points[index, 2
   ], -1, -points[index, 0]],
31                                   [points[index, 1],
   points[index, 0], -b]])*dt
32         Q, R = np.linalg.qr(np.matmul(M, Q))
33         eigenvalues += np.log(np.abs(np.array([R[0, 0
   ], R[1, 1], R[2, 2]])))
34         lambda_history[index] = eigenvalues/(index+1)
35     eigenvalues /= t_max
36     lambda_history /= dt
37     print(sorted(eigenvalues, reverse=True))
```

```
38
39      return eigenvalues, lambda_history
40
41
42  def plot_eigenvalues(lambda_history):
43      fig, ax = plt.subplots(1, 1, figsize=(10, 10))
44      ax.set_xscale('log')
45      ax.plot(lambda_history[:, 0], 'b', label=r'$\
    lambda_1$')
46      ax.plot(lambda_history[:, 1], 'b', label=r'$\
    lambda_2$')
47      ax.plot(lambda_history[:, 2], 'b', label=r'$\
    lambda_3$')
48      plt.title("Eigenvalues over time")
49      plt.xlabel("Time t")
50      plt.ylabel("Eigenvalues")
51      fig.tight_layout()
52      plt.show()
53
54
55  def main():
56      t_max = 100
57      dt = 10**-4
58      sigma = 10
59      b = 3
60      r = 28
61      N = int(t_max/dt)
62
63      # Create initial transient and take last point
64      start_point = np.array([1, 1, 1])
65      t = np.linspace(0, 30, 1000)
66      points = odeint(model, start_point, t, (sigma, b
    , r), tfirst=True, full_output=0)
67      start_point = points[-1]
68
69      t = np.linspace(0, t_max, N)
70      points = odeint(model, start_point, t, (sigma, b
    , r), tfirst=True)
71      plot_dynamical_system(points)
72      eigenvalues, lambda_history = compute_eig(points
    , sigma, b, r, dt, t_max, N)
```

```
    plot_eigenvalues(lambda_history)


if __name__ == '__main__':
    os.chdir(os.path.dirname(__file__))
    main()
```