

Ex - Problem Set 2 - Task 2

Friday, February 17, 2023 3:12 PM

$$\frac{\partial u}{\partial t} = f(u, v) + D_u \nabla^2 u$$

$$\frac{\partial v}{\partial t} = g(u, v) + D_v \nabla^2 v$$

b) Lecture notes 7.3

① Find stable steady state(s) where no diffusion

② Consider spatio-temporal pert.

$$\vec{n} = \vec{n}^* + \delta \vec{n}(\vec{r}, t)$$

Linearize equation, 2nd order negligible

③ Insert expression in original expression,

keep 1st order δn terms

④ Obtain diff eq. with solution

$$\delta \vec{n}(\vec{r}, t) = T(t) R(\vec{r}) \delta n_0, \quad \delta n_0 = \text{const}$$

$$⑤ \delta \vec{n}(\vec{r}, t) \sim e^{\lambda t + i \vec{k} \cdot \vec{r}} \quad \lambda > 0, \text{ perturbation}$$

$$\lambda \delta n_0 = [\mathcal{J}(\vec{n}^*) - k^2 D] \delta \vec{n}_0 \quad \text{grows over time}$$

find at least one positive
eigenvalue

condition:

$$\text{let } d = \frac{D_v}{D_u}$$

$$d J_{11} + J_{12} > 0$$

$$(d J_{11} + J_{12})^2 - 4 d \det \mathcal{J} > 0$$

c) Simulation

Laplacian:

$$\nabla^2 u = \frac{u(x+h, y, t) + u(x-h, y, t) + u(x, y+h, t) + u(x, y-h, t) - 4u(x, y, t)}{h^2}$$

Boundary;

- use modulus

