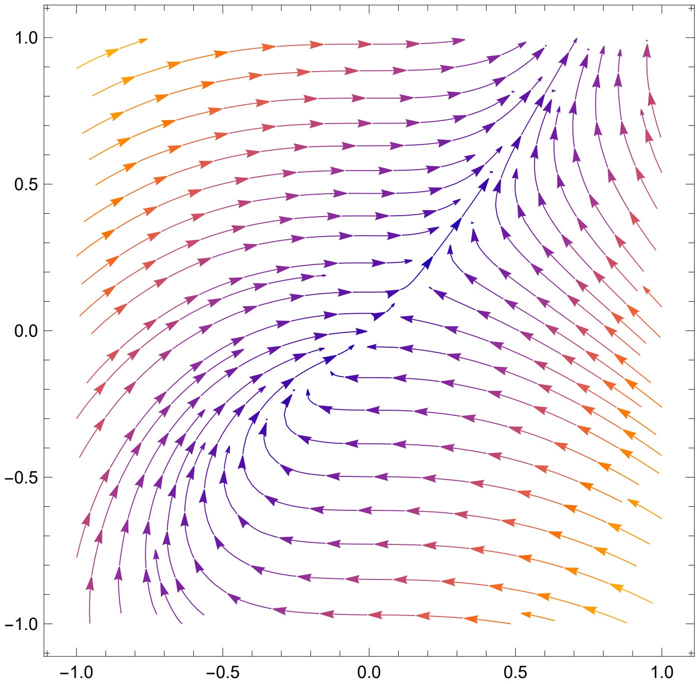
2.1 Index of fixed point

a)

In[ ]:=
```
xdot =.; ydot =.; y =.; x =.; P =.; lim =.;
xdot = y - x;
ydot = x^2;
P = {xdot, ydot};
lim = 1;
StreamPlot[P, { x, -lim, lim}, {y, -lim, lim}]
```

Out[ ]=

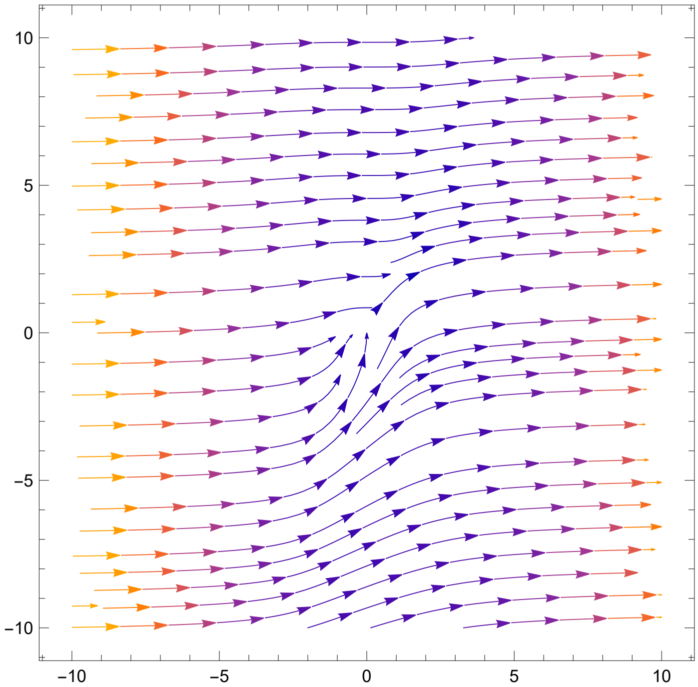


b)

```
xdot =.; ydot =.; y =.; x =.; P =.; lim =.; theta =.;
lim = 10;
xdot = y - x;
ydot = x^2;
P = {xdot, ydot};
P = ToPolarCoordinates[{P}];
StreamPlot[P, {x, -lim, lim}, {y, -lim, lim}]
```

Out[ ]= r Cos[theta] (Sin[theta] + Cos[theta] (-1 + r Sin[theta]))

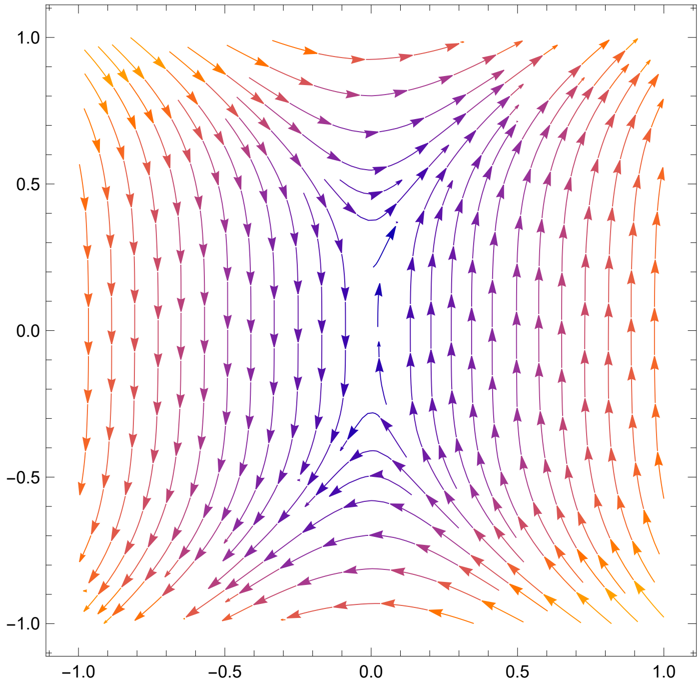
Out[ ]=



c)

In[∘]:=
```
xdot =.; ydot =.; y =.; x =.; P =.; lim =.;
xdot = y^3;
ydot = x;
P = {xdot, ydot};
lim = 1 / 1;
StreamPlot[P, { x, -lim, lim}, {y, -lim, lim}]
```

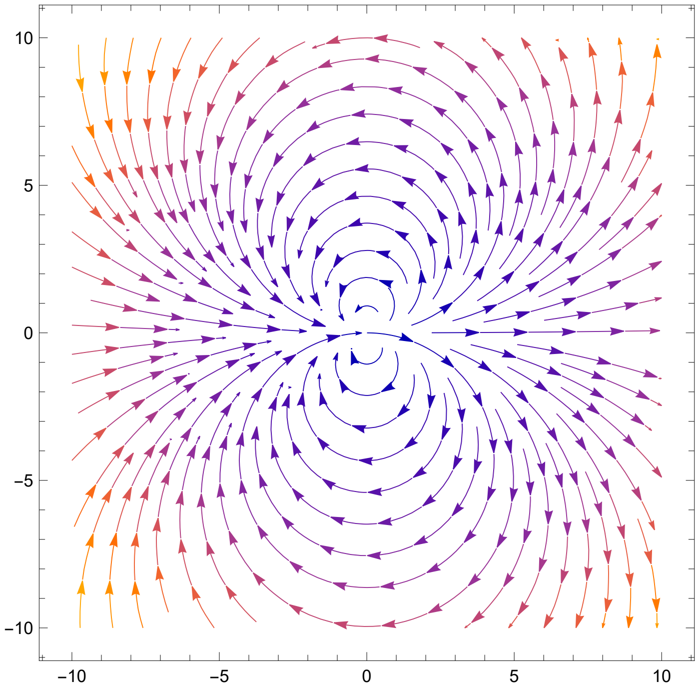
Out[∘]=



d)

In[ ]:=
```
xdot =.; ydot =.; y =.; x =.; P =.; lim =.;
n =.;
xdot = (x^2 + y^2) ^ (Abs[n] / 2) * Cos[n * ArcTan[y / x]] ;
ydot = (x^2 + y^2) ^ (Abs[n] / 2) * Sin[n * ArcTan[y / x]];
lim = 10;
n = 2;

P = {xdot, ydot};
StreamPlot[{(x^2 + y^2) ^ (Abs[n] / 2) * Cos[n * ArcTan[y / x]],
  (x^2 + y^2) ^ (Abs[n] / 2) * Sin[n * ArcTan[y / x]]}, {x, -lim, lim}, {y, -lim, lim}]
```

Out[ ]=


2.2e

In[1]:=
```
y =.; x =.; sigma =.; t =.; sigma = 2; ClearAll[y]; ClearAll[x];
s = NDSolve[{x'[t] == y[t], y'[t] == -Sin[x[t]] - sigma * y[t], x[0] == y[0] == 0},
  {x[t], y[t]}, {t, 0, 10}]
lim = Pi;
Manipulate[StreamPlot[{y, -Sin[x] - a * y},
   {x, -lim + Pi / 2, lim + Pi / 2}, {y, -lim, lim}], {a, 0, 3}];

sigma =.;
M = {{0 , 1}, {-Cos[x], -sigma}};
x = 0;
lambda1 = Eigenvalues[M]
x = Pi;
lambda2 = Eigenvalues[M]
```

Out[2]= {{x[t] → InterpolatingFunction[Domain: {{0., 10.}} Output: scalar][t],

y[t] → InterpolatingFunction[Domain: {{0., 10.}} Output: scalar][t]}}

Out[8]= $\left\{\frac{1}{2}\left(-\text{sigma}-\sqrt{-4+\text{sigma}^2}\right), \frac{1}{2}\left(-\text{sigma}+\sqrt{-4+\text{sigma}^2}\right)\right\}$

Out[10]= $\left\{\frac{1}{2}\left(-\text{sigma}-\sqrt{4+\text{sigma}^2}\right), \frac{1}{2}\left(-\text{sigma}+\sqrt{4+\text{sigma}^2}\right)\right\}$

2.3

```
xdot =.; ydot =.; y =.; x =.; sigma =.; mu =.; xdot2 =.; ydot2 =.;
t =.; v =.; u =.;

mu = 0;
x0 = {v};
y0 = {u};
(*NDSolve[{x'[t]==-{5,-1}*y[t]+{-x[t]^3,3*y[t]^3},
  y'[t]=={5,-1}*x[t] + {-x[t]^2,2*x[t]^2}, x[0]==x0,y[0]==y0},{x,y},{t}]*)
```

NDSolve: Range specification t is not of the form {x, xend} or {x, xmin, xmax}.

NDSolve: Initial condition {v} is not a number or a rectangular array of numbers.

Out[•]= NDSolve[{x′[t] == {−x[t]³ − 5 y[t], y[t] + 3 y[t]³},
y′[t] == {5 x[t] − x[t]², −x[t] + 2 x[t]²}, x[0] == {v}, y[0] == {u}}, {x, y}, {t}]

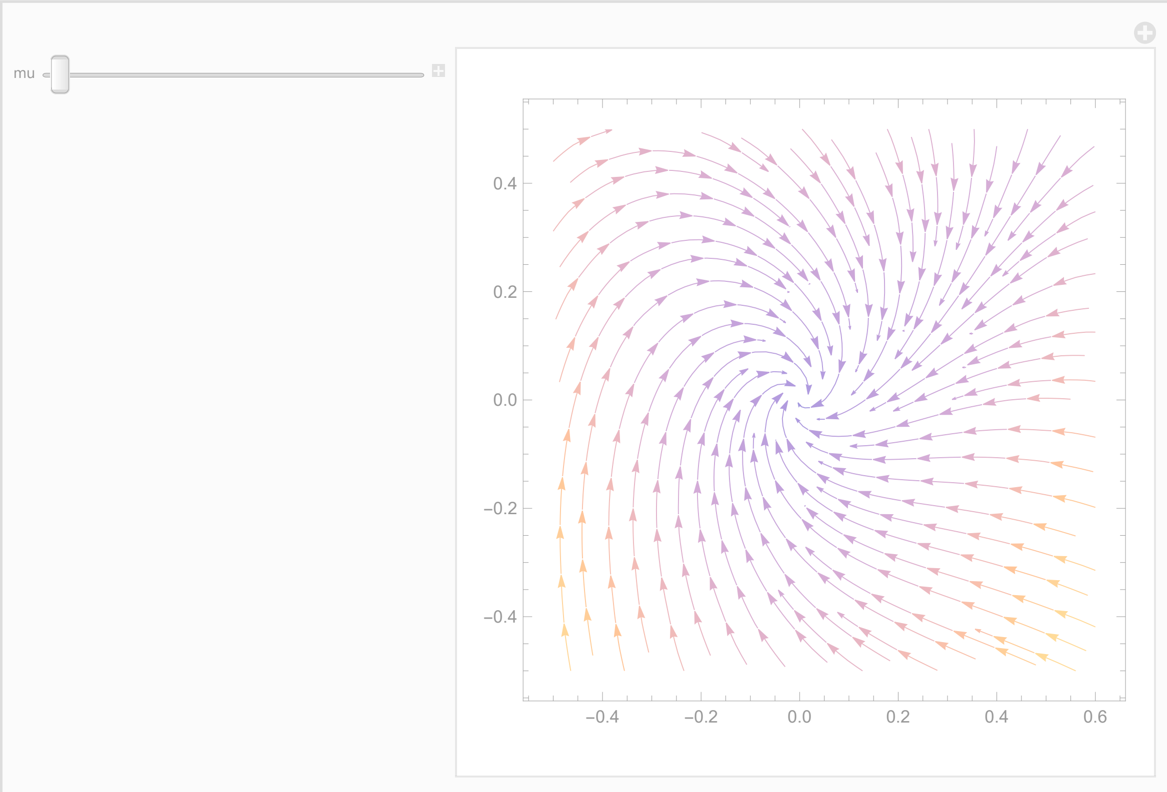
2.4

a)

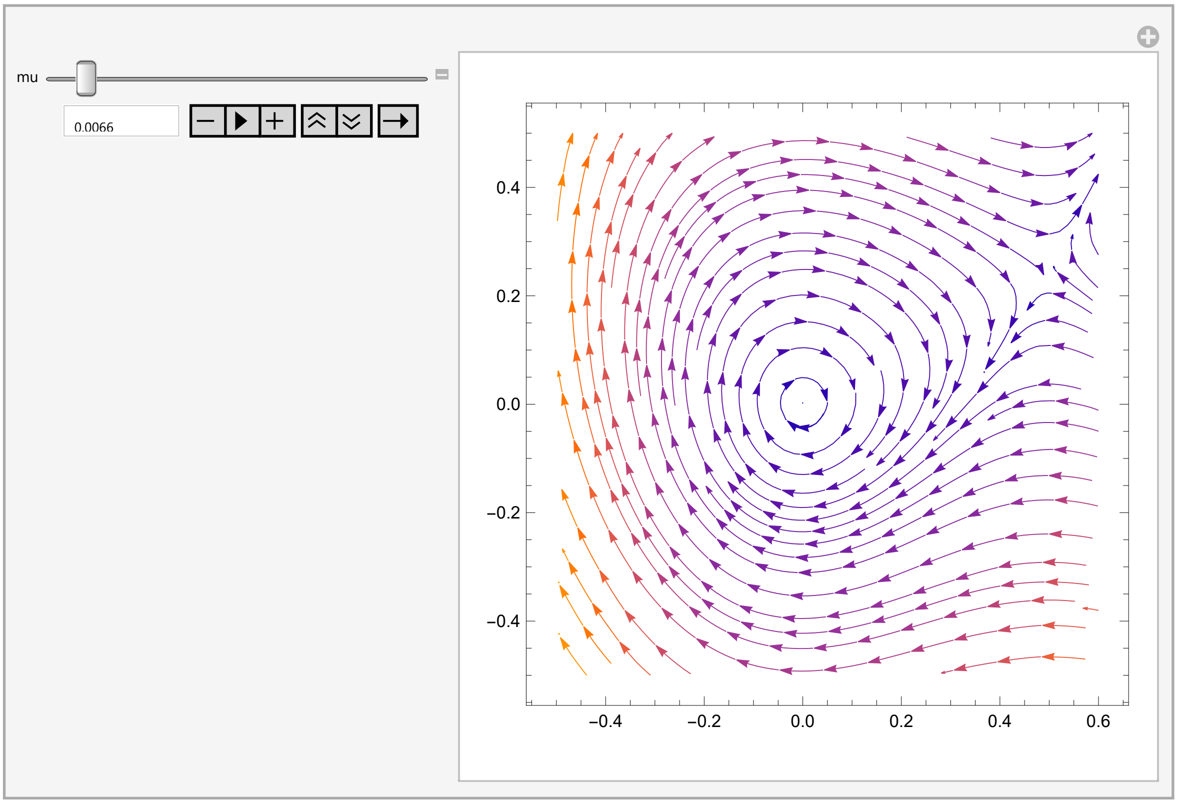
```
xdot = .; ydot = .; x = .; y = .;
lim = 1 / 2;
Manipulate[StreamPlot[{mu x + y - x^2, -x + y mu + 2 x^2},
  {x, -lim, lim + 1 / 10}, {y, -lim, lim}], {mu, -1, .1}]
```

Out[1088]= $\frac{1}{2}$

Out[1089]=



Out[43]=



In[2966]:=
```
minx = -2;
miny = -2;
maxx = 2;
maxy = 2;
mu = 1.1;
sol[x0_, y0_] := NDSolve[{x'[t] == mu x[t] + y[t] - x[t]^2,
   y'[t] == -x[t] + y[t] * mu + 2 x[t]^2, x[0] == x0, y[0] == y0}, {x, y}, {t, 0, 10}]
initialC =
  Join[Table[{minx, y}, {y, miny, maxy, 0.1}], Table[{maxx, y}, {y, miny, maxy, 0.1}],
   Table[{x, miny}, {x, minx, maxx, 0.1}], Table[{x, maxy}, {x, minx, maxx, 0.1}]];
ParametricPlot[Evaluate[{x[t], y[t]} /. sol[initialC[[50, 1]], initialC[[50, 2]]]],
 {t, 0, 6}, PlotRange → {{minx, maxx}, {miny, maxy}}]
```

2.4

a)

In[2991]:=
```
x =.; y =.; μ =.; Remove[flow]; Remove[sol];
flow = {μ * x + y - x^2, -x + μ * y + 2 x^2};
fpsols = Solve[flow == 0, {x, y}];
stabmat = D[flow, {{x, y}}] /. %[[2]];
stabmat // MatrixForm;
eigsys = Eigensystem[stabmat];
eigenVal = Eigenvalues[stabmat]
unstabexponent = eigsys[[1, 2]];
unstabdirection = eigsys[[2, 2]];
```

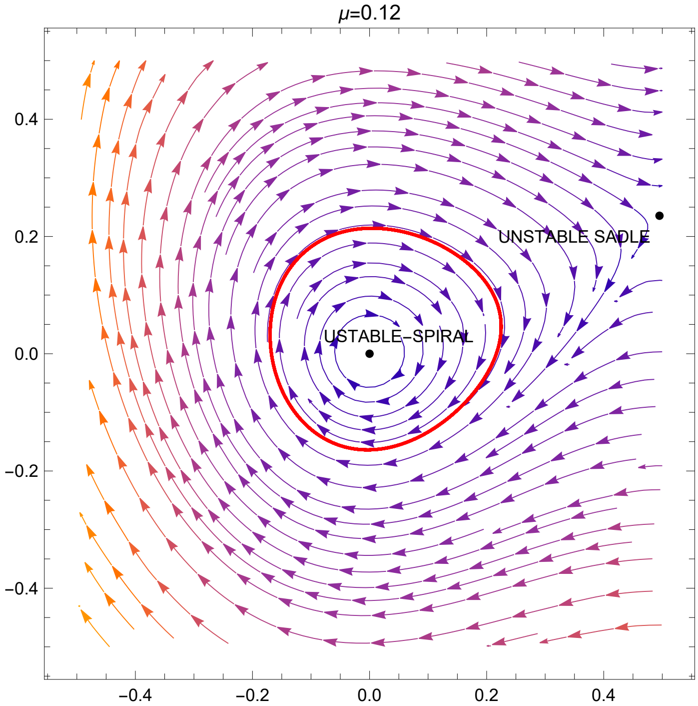
Out[2997]=

$$\left\{\frac{-1+2\mu-\sqrt{5+9\mu^2+4\mu^3+\mu^4}}{2+\mu}, \frac{-1+2\mu+\sqrt{5+9\mu^2+4\mu^3+\mu^4}}{2+\mu}\right\}$$

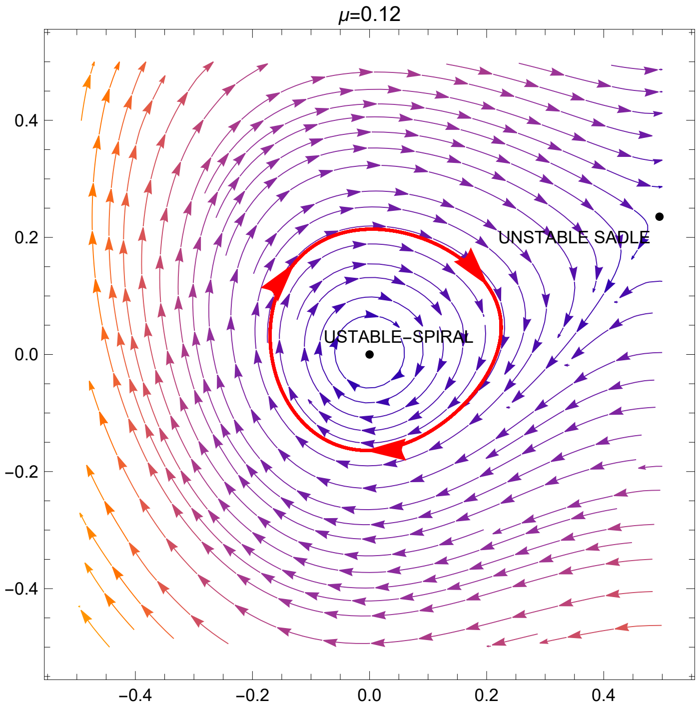
In[3000]:=
```
dynsys = {x'[t] == μ * x[t] + y[t] - x[t]^2, y'[t] == -x[t] + μ * y[t] + 2 x[t]^2};
sol[μdummy_, T_] :=
  Quiet@NDSolve[Join[dynsys, Thread[{x[0], y[0]} == {(1 + μ^2)/(2 + μ), (1 - 2 μ + μ^2 - 2 μ^3)/(2 + μ)^2} -
        0.001 * unstabdirection]] /. μ → μdummy, {x[t], y[t]}, {t, 0, T}];
```

In[3002]:=
```
μval = 0.02;
Tmin = 800;
Tmax = 1000;
text1 = Graphics[Text["", {-0.4, -0.55}]];
text2 = Graphics[Text["UNSTABLE SADLE", {0.35, 0.2}]];
text3 = Graphics[Text["USTABLE-SPIRAL", {0.05, 0.03}]];
text4 = Graphics[Text["", {0.05, 0.4}]];
p = Show[StreamPlot[flow /. μ → μval, {x, -0.5, 0.5}, {y, -0.5, 0.5}],
  ParametricPlot[Evaluate[{x[t], y[t]} /. sol[μval, Tmax]],
   {t, Tmin, Tmax}, PlotStyle → Red, PlotRange → {{-0.5, 2.5}, {-0.5, 6.5}}],
  ListPlot[{x, y} /. fpsols /. μ → μval, PlotRange → {{-1.5, 1.5}, {-1.5, 1.5}},
   PlotStyle → Black], text1, text2, text3, text4, PlotLabel → "μ=0.12"]
(p // Normal) /. Line[x_] :> {Arrowheads[{0.05, 0.05, 0.05}], Arrow[x]}
```

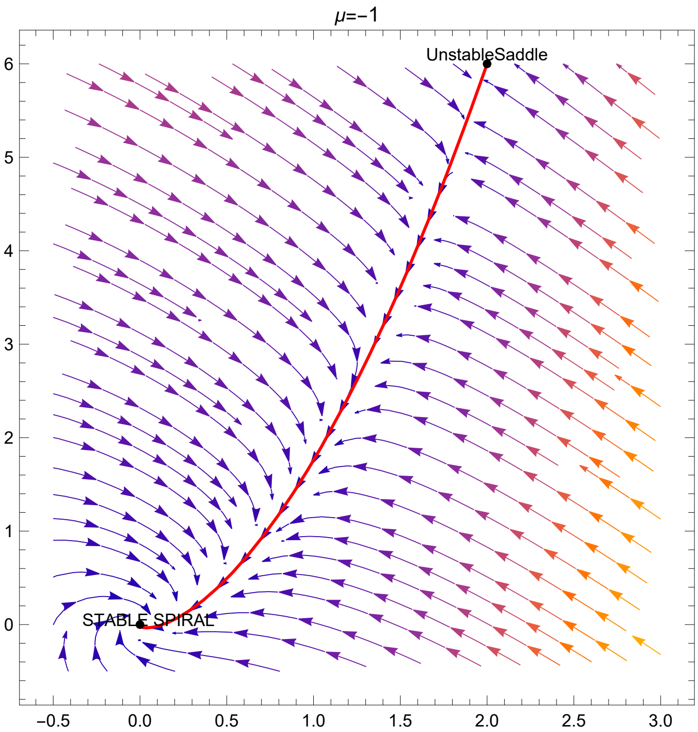
Out[3009]=



Out[3010]=



In[2450]:=

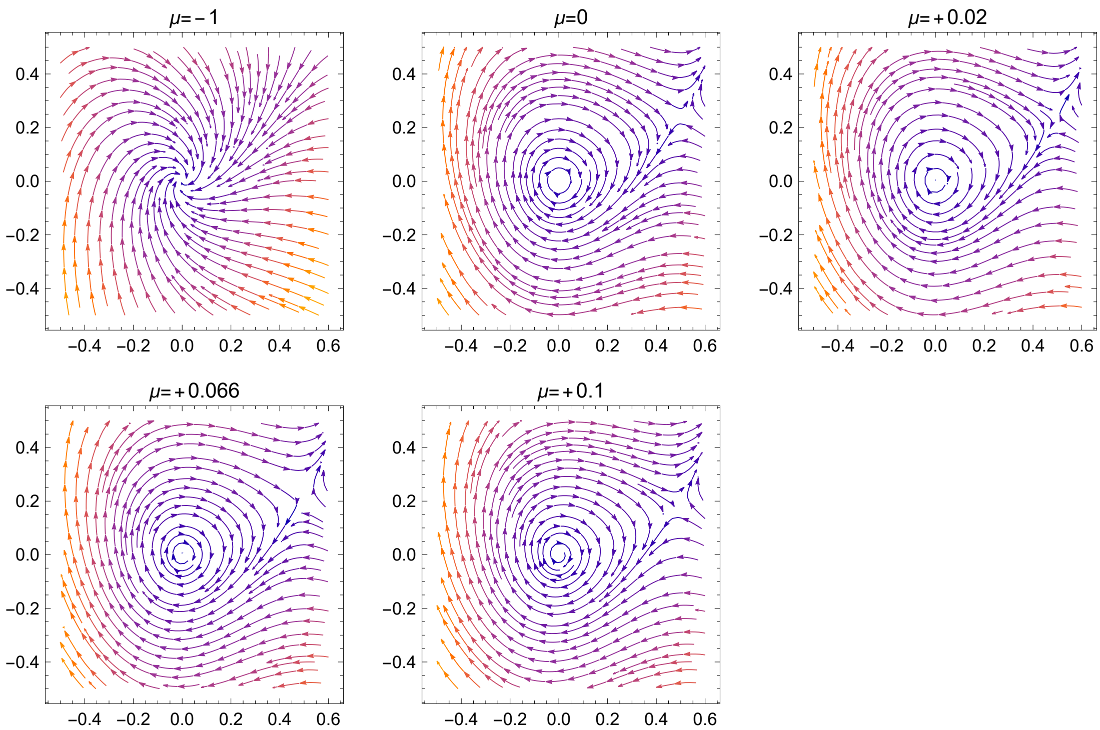

μ=−1
UnstableSaddle
STABLE SPIRAL
6
5
4
3
2
1
0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0

2.4 b)

```
xdot = .; ydot = .; x = .; y = .;
lim = 1 / 2;
Remove[p1]
Remove[p2]
Remove[p3]
Remove[p4]
Remove[p5]

mu = -1;
p1 = StreamPlot[{mu x + y - x^2, -x + y mu + 2 x^2},
    {x, -lim, lim + 1 / 10}, {y, -lim, lim}, PlotLabel → "μ=" + mu];
mu = 0;
p2 = StreamPlot[{mu x + y - x^2, -x + y mu + 2 x^2},
    {x, -lim, lim + 1 / 10}, {y, -lim, lim}, PlotLabel → "μ=0"];
mu = 0.02;
p3 = StreamPlot[{mu x + y - x^2, -x + y mu + 2 x^2},
    {x, -lim, lim + 1 / 10}, {y, -lim, lim}, PlotLabel → "μ=" + mu];
mu = 0.066;
p4 = StreamPlot[{mu x + y - x^2, -x + y mu + 2 x^2},
    {x, -lim, lim + 1 / 10}, {y, -lim, lim}, PlotLabel → "μ=" + mu];
mu = 0.1;
p5 = StreamPlot[{mu x + y - x^2, -x + y mu + 2 x^2},
    {x, -lim, lim + 1 / 10}, {y, -lim, lim}, PlotLabel → "μ=" + mu];
GraphicsGrid[{{p1, p2, p3}, {p4, p5}}]
```

Out[2927]=



2.4 d)

In[2884]:=
```
x =.; y =.; μ =.; Remove[flow]; Remove[sol];
flow = {μ * x + y - x^2, -x + μ * y + 2 x^2};
fpsols = Solve[flow == 0, {x, y}];
stabmat = D[flow, {{x, y}}] /. %[[2]];
stabmat // MatrixForm;

(* Unstable is + Sqrt *)
eigenVal = Eigenvalues[stabmat][[2]]
```

Out[2889]=
$$\frac{-1 + 2\,\mu + \sqrt{5 + 9\,\mu^2 + 4\,\mu^3 + \mu^4}}{2 + \mu}$$

2.4 e)

In[3681]:=
```
x =.; y =.;
lim = 1 / 2;
mu = 0.05;
StreamPlot[{mu x + y - x^2, -x + y mu + 2 x^2}, {x, -lim, lim + 1 / 10}, {y, -lim, lim}];
dynsys = {x'[t] == μ * x[t] + y[t] - x[t]^2, y'[t] == -x[t] + μ * y[t] + 2 x[t]^2};
sol[μdummy_, T_] :=
  Quiet@NDSolve[Join[dynsys, Thread[{x[0], y[0]} == {(1 + μ^2)/(2 + μ), (1 - 2 μ + μ^2 - 2 μ^3)/(2 + μ)^2} -
        0.001 * unstabdirection]] /. μ → μdummy, {x[t], y[t]}, {t, 0, T}];
μval = 0.05;
Tmin = 0;
Tmax = 100;
text1 = Graphics[Text["", {-0.4, -0.55}]];
text2 = Graphics[Text["", {0.35, 0.2}]];
text3 = Graphics[Text["", {0.05, 0.03}]];
text4 = Graphics[Text["", {0.05, 0.4}]];
p = Show[StreamPlot[flow /. μ → μval, {x, -0.5, 0.5}, {y, -0.5, 0.5}],
   ParametricPlot[Evaluate[{x[t], y[t]} /. sol[μval, Tmax]], {t, Tmin, Tmax},
    PlotStyle → Red, PlotRange → {{-0.5, 2.5}, {-0.5, 6.5}}],
   ListPlot[{x, y} /. fpsols /. μ → μval, PlotRange → {{-1.5, 1.5}, {-1.5, 1.5}},
    PlotStyle → Black], text1, text2, text3, text4, PlotLabel → "μ=0.12"];
(p // Normal) /. Line[x_] :→ {Arrowheads[{0.05, 0.05, 0.05}], Arrow[x]};

a =.;
a = Evaluate[{x[t], y[t]} /. sol[μval, Tmax]];
Plot[a〚1, 1〛, {t, 0, 30}]
```

Out[3698]=