Ex - Problem Set 2 - Task 1

Friday, February 17, 2023 3:10 PM

$$\frac{\partial n(x,t)}{\partial t} = \underbrace{r n\left(1 - n/k\right) - \frac{A n}{1 + n/B}}_{f(n)\ (x,t)} + D \frac{\partial^2 n}{\partial x^2}$$

growth; inhibition; diffusion (smooths out distribution)

$(x, t-H), (x, t+H)$

1. dedimensionalize, get fixed pts

Simulation

$$\frac{d^2 f}{dx^2} = \frac{f(x+h,t) + f(x-h,t) - 2f(x,t)}{h^2}$$

modify laplacian at boundaries

rewrite

$$= \frac{\frac{f(x+h) - f(x)}{h} - \frac{f(x) - f(x-h)}{h}}{h}$$

derivative at boundary, set to 0

step size 1, $\varepsilon_2 - \varepsilon_1$

at $x = 0$: $\frac{d^2 f}{dx^2} = \frac{f(h) - f(0)}{h^2}$

at $x = L$: $\frac{d^2 f}{dx^2} = -\frac{f(L) - f(L-h)}{h^2}$

Discretized equation of motion:

$$n(x, t+\Delta t) = n(x,t) + \Delta t \left[f(n(x,t)) + D \left\{ \frac{n(x+h,t) + n(x-h,t) - 2n(x,t)}{h^2} \right\} \right]$$

Simulation:

Set $n(x,0) = \ldots$

Use E.O.M to get

$u(x, \Delta t)$

$u(x, 2\Delta t)$

⋮

Divide time into M steps with index m

space 100 p

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for m=1:M
  For p=0:99
    discrete E.O.M   <- care boundary conditions
  end
end
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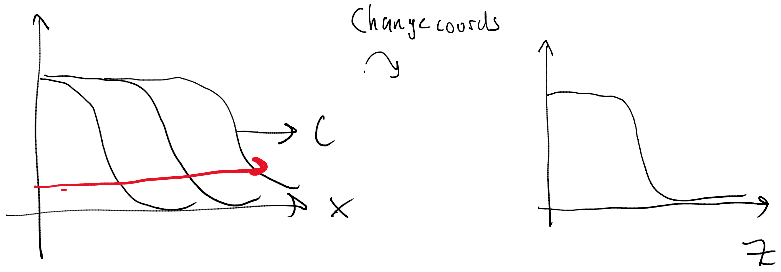
is traveling?

$f(x,t) = f(x-ct) = f(z)$, where $z = x - ct$



look at tips at fps

$\frac{\partial}{\partial t} \rightarrow -c\frac{\partial}{\partial z}$

$\frac{\partial^2}{\partial x^2} \rightarrow \frac{\partial}{\partial z^2}$ ⟶ obtain 2nd order equation in z

Use velocity to compute Jacobian for for eigenvalues