HW3

(1) Derive the state space model of the space ship motion (40 points).
(2) Propagate the galaxy motion and ship motion as the animation presented in class (40 points) (hint plot the trajectory in phase plane (ex. x1 vs x2) but not in time plane).
(3) Linearize the ship motion dynamics at equilibrium points and analysis the stability of different equilibrium points (20 + 10 bonus)

The problem statement is attached below:

A central-force law governs the motion of both the stars and the ships. The ephemerides of the stars are available analytically because they are assumed to follow circular orbits. The central-force law assumed for the competition approximately models the circular motion observed for actual stars in our Milky Way. Specifically, the circular orbit speed, v_c, for a body at radius r from the galactic center, and the acceleration, f, directed towards the galactic center are

$$v_c(r) = \frac{1}{k_8 r^8 + k_7 r^7 + k_6 r^6 + k_5 r^5 + k_4 r^4 + k_3 r^3 + k_2 r^2 + k_1 r + k_0} \tag{1}$$

$$f(r) = \frac{v_c^2}{r} \tag{2}$$

where the k_i $(i = 0, \ldots, 8)$ are constants defined in the "Constants and Conversions" section. The Cartesian position coordinates of any body (x, y, z) obey the differential equations with respect to time, t:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\frac{x}{r} f(r) \tag{3}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = -\frac{y}{r} f(r) \tag{4}$$

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = -\frac{z}{r} f(r) \tag{5}$$

where $r = \sqrt{x^2 + y^2 + z^2}$.

The motion of the stars also obeys these differential equations, but, due to their orbits being circular, the motion can be expressed analytically and so the following equations should be used to determine position and velocity for the stars as a function of time past Year Zero, t:

$$n \equiv v_c(R)/R \tag{6}$$

$$x = R[\cos(nt + \phi)\cos\Omega - \sin(nt + \phi)\cos i \sin\Omega] \tag{7}$$

$$y = R[\cos(nt + \phi)\sin\Omega + \sin(nt + \phi)\cos i \cos\Omega] \tag{8}$$

$$z = R[\sin(nt + \phi)\sin i] \tag{9}$$

$$v_x = v_c(R)[-\sin(nt + \phi)\cos\Omega - \cos(nt + \phi)\cos i \sin\Omega] \tag{10}$$

$$v_y = v_c(R)[-\sin(nt + \phi)\sin\Omega + \cos(nt + \phi)\cos i \cos\Omega] \tag{11}$$

$$v_z = v_c(R)[\cos(nt + \phi)\sin i] \tag{12}$$

where the values of R, i, Ω, ϕ for each star are specified in the file `stars.txt` which has one header line followed by one line for each star, including Sol. Stars are assigned integer identifiers (IDs) in the file, starting with 0 for Sol and progressing to 100,000. The final column in the file, given for convenience, is the final polar angle of the star which is a derived quantity: $\theta_f = \mathrm{atan2}(y(t_f), x(t_f))$, $t_f = 90$ Myr. It should be noted that we are following common galactic conventions: We take the $+z$ direction as the Galactic North direction and so the stars are orbiting in a retrograde direction around this axis (clockwise when looking down on the galaxy from Galactic North); that is why, for the above equations, the inclinations, i, listed in the file approach 180 degrees.

The values for the necessary constants and unit conversions are shown in Table 2. Two of the less customary units used, at least in the field of spacecraft trajectory design, are Myr (million years) and kpc (kiloparsecs).

Table 2: **Constants and unit conversions**

Constant	Value	Units
k_8	-1.94316e-12	$(\text{km/s})^{-1}/\text{kpc}^8$
k_7	3.7516e-10	$(\text{km/s})^{-1}/\text{kpc}^7$
k_6	-2.70559e-08	$(\text{km/s})^{-1}/\text{kpc}^6$
k_5	9.70521e-07	$(\text{km/s})^{-1}/\text{kpc}^5$
k_4	-1.88428e-05	$(\text{km/s})^{-1}/\text{kpc}^4$
k_3	0.000198502	$(\text{km/s})^{-1}/\text{kpc}^3$
k_2	-0.0010625	$(\text{km/s})^{-1}/\text{kpc}^2$
k_1	0.0023821	$(\text{km/s})^{-1}/\text{kpc}$
k_0	0.00287729	$(\text{km/s})^{-1}$
1 kpc	30856775814671900	km
1 Myr	10^6	yr
1 yr	31557600	s