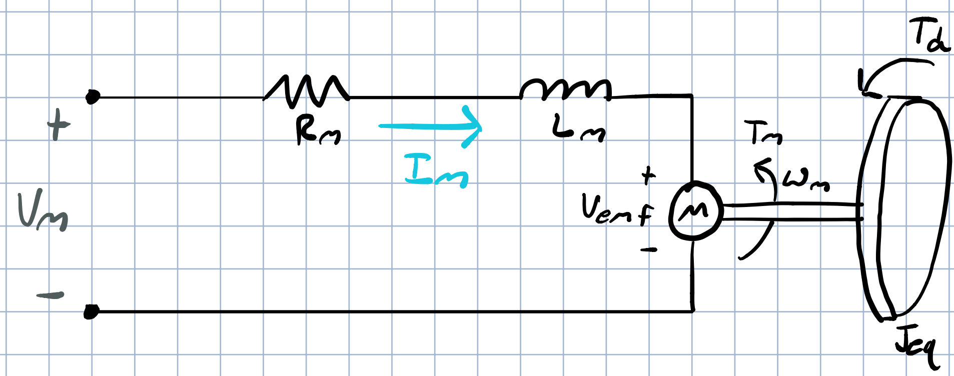
$\dot{I}_m,\ \dot{\omega}_m \longrightarrow I_m, V_m, \omega_m, T_d$



$$\left.\begin{aligned} V_{emf} &= K_m \omega_m (1+\tau_m \omega_m) \\ T_m &= K_t (I_m - I_f) \\ I_f &= I_0\, \mathrm{sgn}(\omega_m) + I_1 \omega_m + I_2 \omega_m^2 \end{aligned}\right\} \quad \tau_m \rightarrow \text{magnetic lag}$$

$$V_m = I_m R_m + L \dot{I}_m + V_{emf}$$

$$J_{eq} \dot{\omega}_m = T_m - T_d = K_t (I_m - I_f) - T_d$$

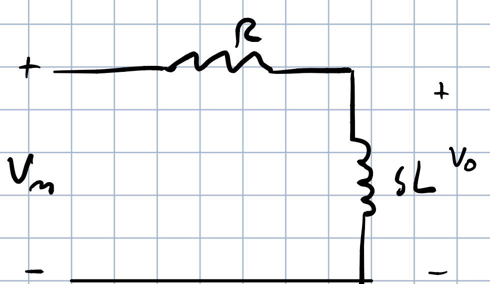
$$= k_t \left(I_m - I_0\, \mathrm{sgn}(\omega_m) - I_1 \omega_m - I_2 \omega_m^2\right) - T_d$$

$$\boxed{\Rightarrow \dot{\omega}_m = \frac{1}{J_{eq}} \left[k_t \left(I_m - I_0\, \mathrm{sgn}(\omega_m) - I_1 \omega_m - I_2 \omega_m^2\right) - T_d \right]}$$

$$L \dot{I}_m = V_m - I_m R_m - V_{emf} = V_m - I_m R_m - K_m \omega_m (1+\tau_m \omega_m)$$

$$\Rightarrow \boxed{\dot{I}_m = \frac{1}{L} \left[V_m - I_m R_m - K_m \omega_m (1+\tau_m \omega_m) \right]}$$

determine τ_e:



$$V_m = V_R + L \frac{dI}{dt} \Rightarrow V_o = V_m \frac{sL}{R+sL}$$

$$\Rightarrow \frac{V_o}{V_m} = \frac{sL}{R+sL} \Leftrightarrow \frac{V_o}{V_m} = \frac{d}{dt} e^{-\frac{R}{L} t}$$

$$\Rightarrow \frac{V_o}{V_m} = -\frac{R}{L} e^{-R/L\, t} \Rightarrow \boxed{\tau_e = \frac{L}{R}}$$

$$\Rightarrow \boxed{\tau_e = \frac{0.82\,\mathrm{mH}}{10.6\,\Omega} = 77.4\,\mu s}$$

Determine τ_ω:

$$\dot{\omega}_m = \frac{1}{J_e}\left[K_t\left(I_m - I_1\omega_m\right)\right], \quad I_m = \left(V_m - V_{emf}\right)/R_m$$

$$= \frac{1}{J_e}\left[K_t\left(\frac{V_m - V_{emf}}{R_m} - I_1\omega_m\right)\right], \quad * \text{ Assume } I_1 = 0, \ \tau_m = 0$$

$$= \frac{1}{J_e}\left[K_t\left(\frac{V_m - V_{emf}}{R_m}\right)\right]$$

$$= \frac{1}{J_e}\left[K_t\left(\frac{V_m - K_m\omega_m}{R_m}\right)\right] = \frac{K_t}{J_e}\left(\frac{V_m}{R_m} - \frac{K_m\omega_m}{R_m}\right)$$

$$\longleftrightarrow s\Omega_m = \frac{K_t}{J_m}\left(\frac{V_m}{R_m} - \frac{K_m\Omega_m}{R_m}\right)$$

$$\Rightarrow \Omega_m\left(s + \frac{K_t K_m}{J_m R_m}\right) = \frac{K_t V_m}{J_m R_m}$$

$$\Rightarrow \Omega_m = \frac{K_t V_m}{J_m R_m}\,\frac{1}{s + \frac{K_t K_m}{J_m R_m}} \Rightarrow \omega_m = \frac{K_t V_m}{J_m R_m} e^{-\frac{K_t K_m}{J_m R_m} t}$$

$$\Rightarrow \tau_\omega = \frac{J_{eq} R_m}{K_t K_m}, \quad J_m = 1.16\cdot 10^{-6}, \ R_m = 10.6\,\Omega, \ K_t = K_m = 0.0502$$

$$J_{eq} = J_m + J_{disk}, \quad J_{disk} = 20.91\cdot 10^{-6}$$

$$= \boxed{4.66\cdot 10^{-3}}$$

Because $\tau_\omega \gg \tau_e$ we can ignore τ_e with little loss of system accuracy

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$$\dot{\omega}_m = \frac{1}{J_e}\left[K_t\left(I_m - I_1\omega_m\right)\right], \quad I_m = \left(V_m - V_{emf}\right)/R_m$$

$$\dot{\omega}_m = \frac{1}{J_e}\left(K_t\left[\frac{V_m - K_m\omega_m\left(1 + \tau_m\omega_m\right)}{R_m} - I_1\omega_m\right] - T_D\right)$$

· Assuming I_1 acts as a constant

Re-introduce non-linear terms of I_f

$$= \frac{1}{J_e}\left(K_t\left[\frac{V_m - K_m\omega_m\left(1 + \tau_m\omega_m\right)}{R_m} - \left(I_0\,\mathrm{sgn}(\omega_m) + I_1\omega_m + I_2\omega_m^2\right)\right] - T_D\right)$$

$$\dot{\omega}_m = \frac{1}{J_e}\frac{K_T}{R_m}V_m - \frac{1}{J_e}\frac{K_T K_m}{R_m}\omega_m \cancel{- \frac{1}{J_e}\frac{K_T}{R_m}K_m\tau_m\omega_m^2} \cancel{- \frac{1}{J_e}K_T I_0 \sin(\omega_m)} - \frac{1}{J_e}K_T I_1\omega_m \cancel{- \frac{1}{J_e}K_T I_2\omega_m^2} - \frac{1}{J_e}T_D$$

$$\dot{\omega}_m = \frac{K_T}{J_e R_m}V_m - \frac{K_T K_m + R_m K_T I_0}{J_e R_m}\omega_m - \frac{1}{J_e}T_D$$

$$\Rightarrow \boxed{A = -\frac{K_t K_m + R_m K_t I_o}{J_e R_m}, \quad B = \begin{bmatrix} \frac{K_t}{J_e R_m} \\ -\frac{1}{J_e} \end{bmatrix} \qquad C = 1, \quad D = 0} \qquad \underline{\underline{\begin{bmatrix} V_m \\ T_D \end{bmatrix}}}$$

$$\dot{\theta} = \omega$$

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 1 \\ 0 & -\underbrace{\frac{K_t K_m + R_m K_t I_o}{J_e R_m}}_{\beta} \end{bmatrix}}^{A} \begin{bmatrix} \theta \\ \omega_m \end{bmatrix} + \overbrace{\begin{bmatrix} 0 & 0 \\ \frac{K_t}{J_e R_m} & -\frac{1}{J_e} \end{bmatrix}}^{B} \begin{bmatrix} V_m \\ T_D \end{bmatrix}$$

$$\begin{bmatrix} \theta \\ \omega \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}^{C} \begin{bmatrix} \theta \\ \omega_m \end{bmatrix} \qquad D = 0$$

$$C(sI - A)^{-1} B = C\left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -\beta \end{bmatrix}\right]^{-1} B = C \begin{bmatrix} s & -1 \\ 0 & s+\beta \end{bmatrix}^{-1} B$$

$$\frac{1}{AD - BC} = \frac{1}{s^2 + \beta s + 1}$$

$$= C\left(\frac{1}{s^2+\beta s+1}\begin{bmatrix} s+\beta & +1 \\ 0 & s \end{bmatrix}\right) B = C \begin{bmatrix} \frac{s+\beta}{s^2+\beta s+1} & \frac{1}{s^2+\beta s+1} \\ 0 & \frac{s}{s^2+\beta s+1} \end{bmatrix} B$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{s+\beta}{s^2+\beta s+1} & \frac{1}{s^2+\beta s+1} \\ 0 & \frac{s}{s^2+\beta s+1} \end{bmatrix} B = \begin{bmatrix} \frac{s+\beta}{s^2+\beta s+1} & \frac{1}{s^2+\beta s+1} \\ 0 & \frac{s}{s^2+\beta s+1} \end{bmatrix} B$$

$$= \begin{bmatrix} \frac{s+\beta}{s^2+\beta s+1} & \frac{1}{s^2+\beta s+1} \\ 0 & \frac{s}{s^2+\beta s+1} \end{bmatrix} \begin{bmatrix} \frac{K_t}{J_e R_m} & 0 \\ 0 & -\frac{1}{J_e} \end{bmatrix}$$

$$= \Big[$$

$$\dot{\omega}_m = \frac{K_T}{J_e R_m} V_m - \frac{K_T K_m + R_m K_T I_o}{J_e R_m} \omega_m - \frac{1}{J_e} T_D$$

$$\rightarrow s\Omega = \frac{K_T}{J_e R_m} V_m - \frac{K_T K_m + R_m K_T I_o}{J_e R_m} \Omega - \frac{1}{J_e} T_D$$

$$\Rightarrow \Omega \left(s + \frac{K_T K_m + R_m K_T I_o}{J_e R_m} \right) = \frac{K_T}{J_e R_m} V_m - \frac{1}{J_e} T_D$$

$$\Rightarrow \Omega = \frac{K_T}{J_e R_m} \left(\frac{1}{s + \frac{K_T K_m + R_m K_T I_o}{J_e R_m}} \right) V_m - \frac{1}{J_e} \left(\frac{1}{s + \frac{K_T K_m + R_m K_T I_o}{J_e R_m}} \right) T_D$$

$$= \begin{bmatrix} \frac{K_T}{J_e R_m} \left(\frac{1}{s + \frac{K_T K_m + R_m K_T I_o}{J_e R_m}} \right) & -\frac{1}{J_e} \left(\frac{1}{s + \frac{K_T K_m + R_m K_T I_o}{J_e R_m}} \right) \end{bmatrix} \begin{bmatrix} V_m \\ T_D \end{bmatrix}$$

$$= \begin{bmatrix} \frac{K}{\tau s + 1} & \frac{K_{T_d}}{\tau T_d s + 1} \end{bmatrix} \begin{bmatrix} V_m \\ T_D \end{bmatrix}$$

$$K = \frac{1}{k_m + R_m I_o}, \quad \tau = \frac{J_e R_m}{k_t k_m + R_m k_T I_o}$$

$$K_{T_d} = \frac{-R_m}{k_t k_m + R_m k_T I_o}, \quad \tau_{T_d} = \frac{J_e R_m}{k_t k_m + R_m k_T I_o}$$

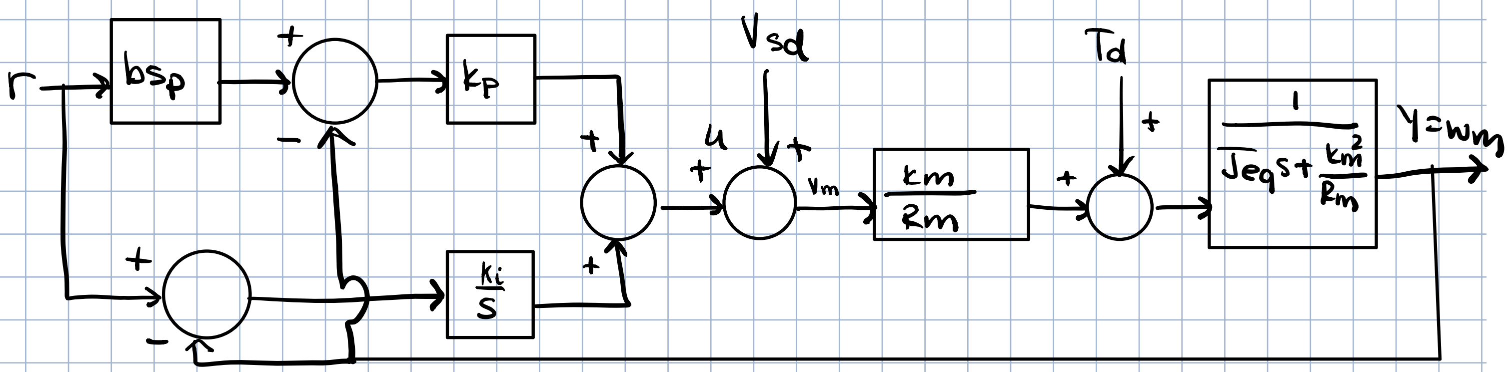
$J_{eq} = 22.1 \cdot 10^{-6}$

$k_m = k_t = 0.0502$

$R_m = 10.6$

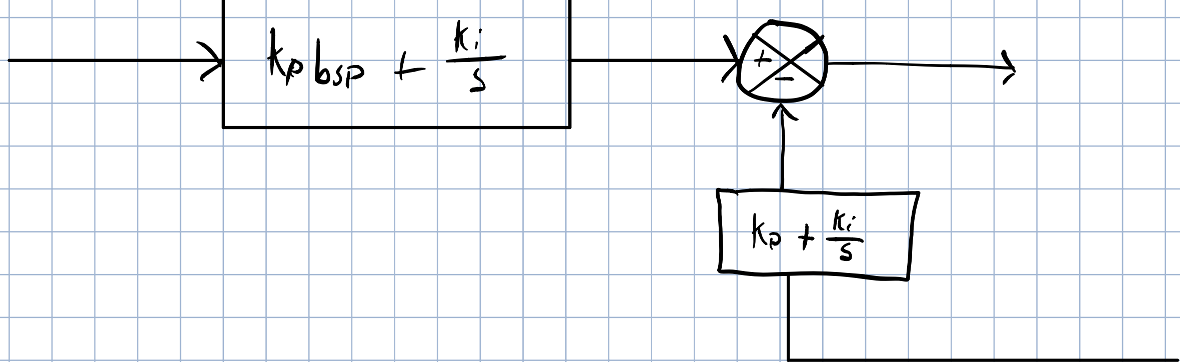
$I_o = 0$

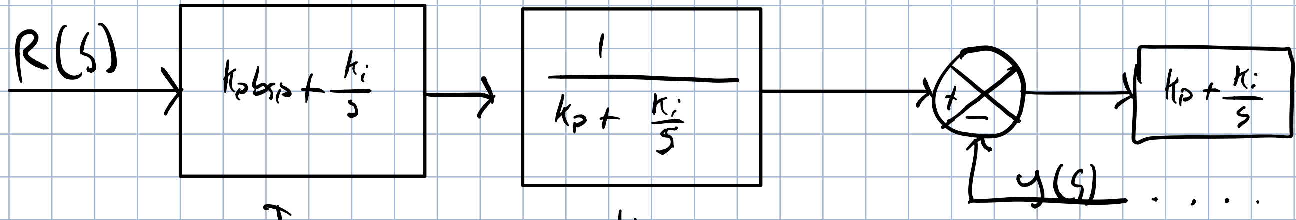
$$K = 19.92, \quad \tau = 0.0928, \quad k_{TD} = -4.2006 \cdot 10^3, \quad \tau_{tD} = 0.0928$$



$$U(s) = k_p(b_{sp} R(s) - Y(s)) + k_i \frac{R(s) - Y(s)}{s}$$

$$= R(s)\left[k_p b_{sp} + \frac{k_i}{s}\right] - Y(s)\left[k_p + \frac{k_i}{s}\right]$$





I H

TF:

Plant: $\frac{K}{\tau s+1}$, controller: $k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s}$

$$G(s)_{OL} = \frac{K(k_p s + k_i)}{s(\tau s + 1)}$$

$$G(s)_{cl} = \frac{K(k_p s + k_i)}{K(k_p s + k_i) + s(\tau s + 1)} = \frac{K K_p s + K K_i}{\tau s^2 + (K K_p + 1)s + K K_i}$$

$$H \cdot G = \frac{s}{K_p s + K_i} \cdot \underbrace{\frac{K K_p s + K K_i}{\tau s^2 + (K K_p + 1)s + K K_i}} = \frac{sK}{\tau s^2 + (K K_p + 1)s + K K_i}$$

$$I \cdot H \cdot G = \frac{K_p b_{sp} s + K_i}{s} \cdot \frac{sK}{\tau s^2 + (K K_p + 1)s + K K_i}$$

$$= \boxed{\frac{K K_p b_{sp} s + K K_i}{\tau s^2 + (K K_p + 1)s + K K_i}}$$

$$\frac{1}{\tau}\left[K k_p + 1\right] = 2\zeta\omega_0 \Rightarrow \frac{K k_p}{\tau} = 2\zeta\omega_0 - \frac{1}{\tau}$$

$$\Rightarrow \boxed{k_p = \frac{2\zeta\omega_0\tau - 1}{K}}$$

$$\frac{1}{\tau}\left[K k_i\right] = \omega_0^2$$

$$\Rightarrow \boxed{k_i = \frac{\omega_0^2 \tau}{K}}$$

$$\frac{1}{2} J \omega_{nom}^2 + \frac{1}{2} J \omega_{nom}^2 = J \omega_{nom}^2$$

$V_{emf} \approx K_m \omega_{nom}$

Assume system is $\frac{K}{s\tau+1}$

$V_{in} = IR - V_{emf}$

$I_m = \frac{\tau_m}{k_t}$

$$\frac{2\omega_{norm}}{\omega_{max} + \omega_{norm}} = e^{-t/\tau}$$

$$\Rightarrow -\frac{t}{\tau} = \ln\left|\frac{2\omega_{norm}}{\omega_{max} + \omega_{norm}}\right|$$

$$\Rightarrow t = -\tau \ln\left|\frac{2\omega_{norm}}{\omega_{max} + \omega_{norm}}\right| = \boxed{-(90\,ms) \ln\left|\frac{2\omega_{norm}}{\omega_{max} + \omega_{norm}}\right|}$$

ω_{max}: $V_{emf} = V_{max}$

$$\Rightarrow \omega_{max} \approx \frac{15}{K_m} = \frac{15}{0.0575} = 260.9 \text{ rad/s} = 41.5 \text{ RPS}$$

$$= 2,492 \text{ RPM}$$

$$\frac{K K_p b_{sp} s + K K_i}{\tau s^2 + (K K_p + 1) s + K K_i}$$

$$\frac{K K_p b_{sp} s + K K_i}{\tau s^2 + (K K_p + 1) s + K K_i} \Rightarrow \frac{K K_p + 1}{\tau} = 2\omega_0, \quad \frac{K K_i}{\tau} = \omega_0^2$$

$$\Rightarrow K K_p + 1 = 2 K K_i$$

$$\frac{-(K K_p + 1) \pm \sqrt{(K K_p + 1)^2 - 4\tau K K_i}}{2\tau} = -\omega_0 \pm \frac{\sqrt{(K K_p + 1)^2 - 4\tau K K_i}}{2\tau}$$

$$= -\omega_0 \pm \sqrt{\left(\frac{(K K_p + 1)}{2\tau}\right)^2 - \left(\frac{K K_i}{1}\right)}$$

$$= -\omega_0 \pm \sqrt{\frac{\omega_0^2}{1} - \frac{\omega_0^2}{1}}$$

$$= -\omega_0, \quad \text{multiplicity } 2$$

$$\Rightarrow \frac{\left(s + \frac{K K_i}{K K_p b_{sp}}\right) K K_p b_{sp}}{(s + \omega_0)(s + \omega_0)\tau}$$

$$= \frac{K K_p b_{sp}}{(s + \omega_0)\tau}$$

$$\Rightarrow \frac{K_i}{K_p b_{sp}} = \omega_0 \Rightarrow \boxed{b_{sp} = \frac{K_i}{\omega_0 K_p}}$$

$$\frac{\left(s+\frac{K K_i}{K K_p b_{sp}}\right) K K_p b_{sp}}{(s+\omega_0)(s+\omega_0)}$$

using System from above with $b_{sp} = \frac{k_i}{\omega_0 K_p}$

$$\rightarrow \frac{(s+\omega_0)\frac{K K_i}{\omega_0}}{\tau (s+\omega_0)^2} = \frac{\omega_0}{s+\omega_0}$$

Step

$$\rightarrow \frac{1}{s}\frac{\omega_0}{s+\omega_0} = \frac{A}{s} - \frac{B}{s+\omega_0} = \frac{1}{s} + \frac{1}{s+\omega_0} \Leftrightarrow \boxed{1-e^{-\omega_0 t}}$$

$A = -B$

$A\omega_0 = \omega_0 \Rightarrow A = 1,\ B = -1$

t_s is t for $(1-e^{-\omega_0 t}) = 0.98 \Rightarrow 0.02 = e^{-\omega_0 t} \Rightarrow -\omega_0 t = \ln(0.02)$

$$\Rightarrow t_s = -\frac{\ln(0.02)}{\omega_0} \approx \frac{4}{\omega_0}$$

$$\frac{E(s)}{R(s)} = 1 - \frac{W(s)}{R(s)} = 1 - \frac{\omega_0}{s+\omega_0} = \frac{s+\omega_0-\omega_0}{s+\omega_0} = \boxed{\frac{s}{s+\omega_0}}$$

$$\Rightarrow e_{ss_{\text{step}}} = \lim_{s\to 0} \frac{1}{s}\cdot\frac{s^2}{s+\omega_0} = 0$$

$$e_{ss_{\text{ramp}}} = \lim_{s\to 0} \frac{1}{s^2}\frac{s^2}{s+\omega_0} = \boxed{\frac{1}{\omega_0}}$$

$$\begin{bmatrix} \frac{k}{\tau s+1} & \frac{k_{T_d}}{\tau_{T_d} s+1} \end{bmatrix} \begin{bmatrix} V_m \\ T_D \end{bmatrix}$$

$$\boxed{\begin{aligned} K &= \frac{1}{k_m + R_m I_o}, & \tau &= \frac{J_e R_m}{k_t k_m + R_m k_T I_o} \\ k_{T_d} &= \frac{-R_m}{k_t k_m + R_m k_T I_o}, & \tau_{T_d} &= \frac{J_e R_m}{k_t k_m + R_m k_T I_o} \end{aligned}}$$

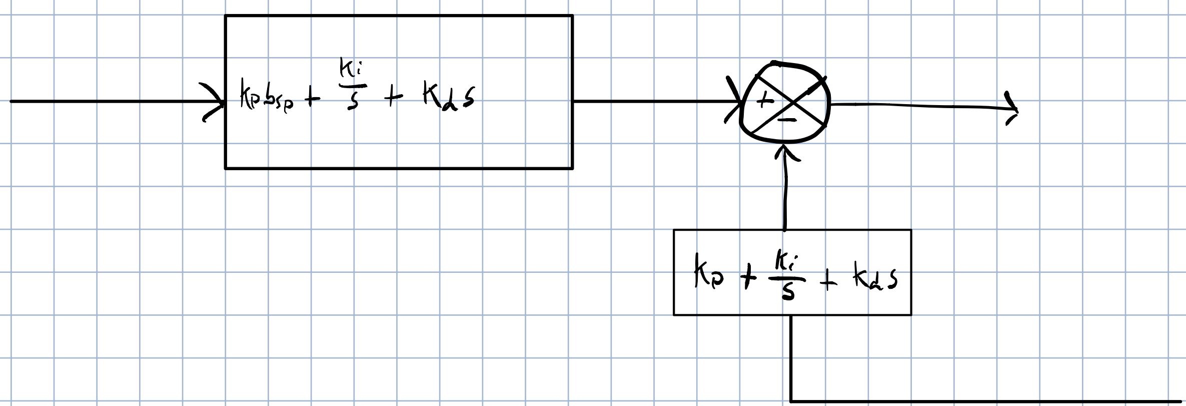
$J_{eq} = 22.1 \cdot 10^{-6}$

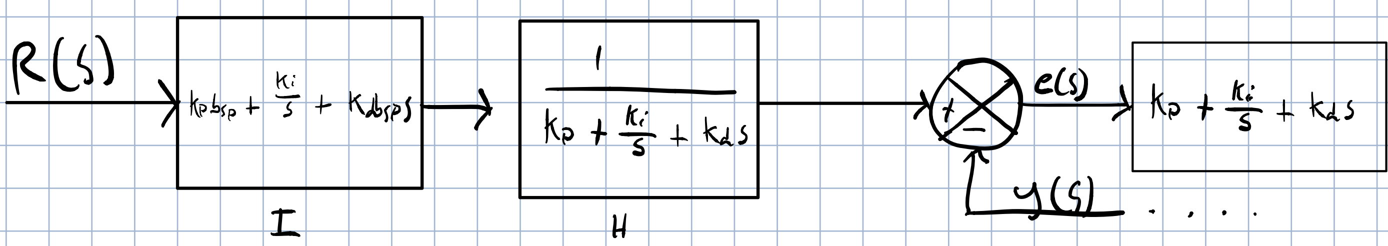
$k_m = k_t = 0.0502$

$R_m = 10.6$

$I_o = 0$

TF from $\frac{\Theta(s)}{R(s)} = \frac{k}{s(\tau s+1)}$, $\frac{\Theta(s)}{T_D(s)} = \frac{k_{td}}{s(\tau_{T_D} s+1)}$





$$\frac{\Theta(s)}{R(s)} = IH \frac{k\left(k_p + \frac{k_i}{s} + k_d s\right)}{s(\tau s+1) + k\left(k_p + \frac{k_i}{s} + k_d s\right)} = \frac{k(k_p s + k_i + k_d s^2)}{\tau s^3 + (1+kk_d)s^2 + s k k_p + k k_i} IH$$

$$\tau s^2 + s + k k_p + \frac{k k_i}{s} + k k_d s$$

$$= \tau s^3 + (1+kk_d)s^2 + s k k_p + k k_i$$

$$= \frac{k_d b_{sp} s^2 + k_p b_{sp} s + k_i}{\cancel{k_d s^2 + k_p s + k_i}} \cdot \frac{k\left(\cancel{k_p s + k_i + k_d s^2}\right)}{\tau s^3 + (1 + k k_d) s^2 + s k k_p + k k_i}$$

$$= \frac{k\left(k_d b_{sp} s^2 + k_p b_{sp} s + k_i\right)}{\tau s^3 + (1 + k k_d) s^2 + s k k_p + k k_i}$$

Disturbance:

$$\frac{K_{TD}\left(k_d b_{sp} s^2 + k_p b_{sp} s + k_i\right)}{\tau_{TD} s^3 + (1 + K_{TD} k_d) s^2 + s K_{TD} k_p + K_{TD} k_i}$$

$$\frac{E(s)}{R(s)} = 1 - \frac{k\left(k_d b_{sp} s^2 + k_p b_{sp} s + k_i\right)}{\tau s^3 + (1 + k k_d) s^2 + s k k_p + k k_i}$$

$$= \frac{\tau s^3 + (1 + k k_d) s^2 + s k k_p + \cancel{k k_i} - k\left(k_d b_{sp} s^2 - k_p b_{sp} s \cancel{- k_i}\right)}{\tau s^3 + (1 + k k_d) s^2 + s k k_p + k k_i}$$

Step:

$$\rightarrow \lim_{s \to 0} \frac{s}{s} \frac{E(s)}{R(s)} = \boxed{0}$$

Ramp

$$\rightarrow \lim_{s \to 0} \frac{s}{s^2} \frac{E(s)}{R(s)} = \frac{k k_p - k k_p b_{sp}}{k k_i} = \boxed{\frac{k_p(1 - b_{sp})}{k_i}}$$