Comments with "Tight Collision Probability for UAV Motion Planning in Uncertain Environment"

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***Abstract*— This is the document accompanying the paper "Tight Collision Probability for UAV Motion Planning in Uncertain Environment" to clarify two issues related to [1] and [2].**

I. Issue I

Consider two ellipsoids $\mathcal{E}_1 = \mathcal{E}^n(B, \mathbf{b})$ and $\mathcal{E}_2 = \mathcal{E}^n(C, \mathbf{c})$, the collision status of them is identified by:

$$\mathbf{y}^T A \mathbf{y} \leq 1/\lambda_0^2(M'), \tag{1}$$

where $\mathbf{y} = \mathbf{b} - \mathbf{c}$, $\lambda_0(\cdot)$ is to calculate the minimal eigenvalue, and $A, M' \in \mathbb{R}^{2n \times 2n}$ are functions of $B, C, \mathbf{b}, \mathbf{c}$. The equation (1) can be written explicitly as

$$\mathbf{y}^T A(B, C, \mathbf{b}, \mathbf{c})\mathbf{y} \leq 1/\lambda_0^2\left(M'(B, C, \mathbf{b}, \mathbf{c})\right), \tag{2}$$

Thomas et al. [1] claimed to find the exact solution of the collision probability of $\mathcal{E}_1$ and $\mathcal{E}_2$ by reformulate (1) as

$$\begin{aligned} P\left(\mathbf{y}^T A \mathbf{y} \leq 1/\lambda_0^2(M')\right) &= P\left(v \leq 1/\lambda_0^2(M')\right) \\ &= F_v\left(1/\lambda_0^2(M')\right) \end{aligned} \tag{3}$$

By regarding $\mathbf{b}$ and $\mathbf{c}$ as Gaussian distributed random variables, the collision probability is transformed into the CDF of the quadratic form in the random variable $\mathbf{y}$. However, this transformation is only valid if A and $1/\lambda_0^2(M')$ are deterministic variables. Unfortunately, [1] seems to overlook the fact that A and $1/\lambda_0^2(M')$ are functions of $\mathbf{b}$ and $\mathbf{c}$, and therefore, are random variables. Instead, they use the mean values of $\mathbf{b}$ and $\mathbf{c}$ to calculate A, $1/\lambda_0^2(M')$, and (3).

II. Issue II

Our iterative trajectory optimization algorithm is benchmarked with [2]–[5]. For the Scene I in our paper, [2] gives the results presented in Figure 1, using their open source code (https://rebrand.ly/castillo_RAL2020benchmark). However, they carelessly took the variance as the standard deviation in the implementation of [2], [3], [5]. The rectified results is shown in Figure 2. The issue has been confirmed with the author of [2].

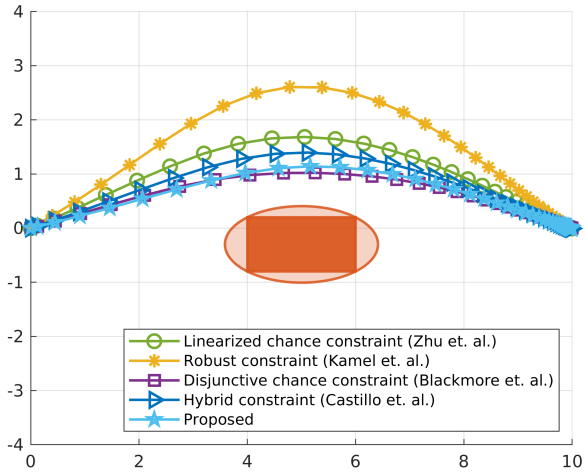

Fig. 1. Benchmark result of [2]

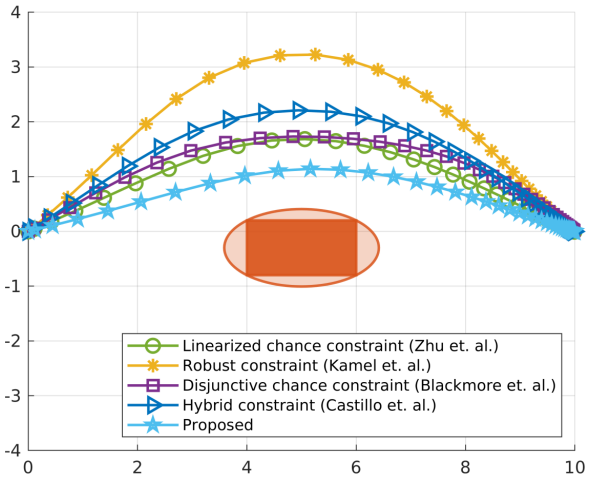

Fig. 2. Benchmark result of our implementation

References

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