Paper Clip Fatigue

STAT 466 Project - Technical Report

Katelyn Nelson and Matthew Wilson

1 Introduction

Paper clips, despite their simple design, are common tools used in everyday life. Understanding the factors that influence their performance can provide valuable insights into their longevity and behavior under stress. By exploring how different conditions and material properties affect the fatigue of paper clips, this study seeks to uncover broader understanding of their failure and resilience.

The goal of this project is to investigate the number of times a paper clip can be bent before it breaks. We will examine how temperature and wire gauge (the thickness of the paper clip wire) influence the fatigue, where the number of bends before failure is assumed to follow a Poisson distribution. By manipulating these two covariates, we aim to understand the conditions under which paper clips are most likely to fail. More specifically, we aim to answer the questions:

- How does the temperature (colder, cold, room temperature) affect the number of times a paper clip can be bent before it breaks?
- How does the wire gauge (1mm vs. 0.8mm) influence paper clip fatigue?
- Does the person bending the paper clip have a different affect on paper clip fatigue?

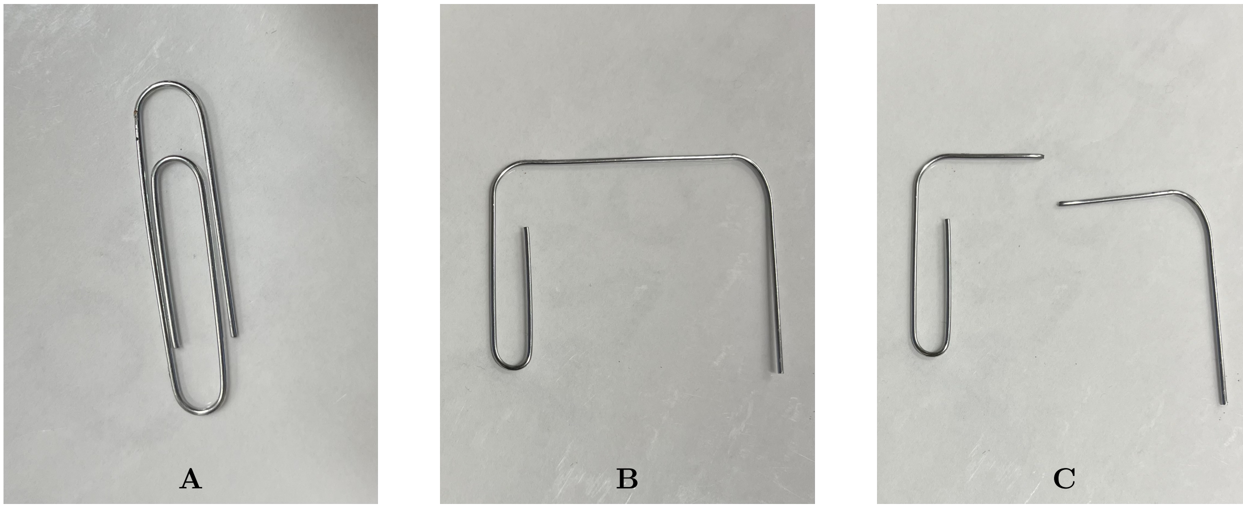
2 Methods

2.1 Data Collection

Paper clips were maintained at the specified temperature (42°F, 53°F, or 74°F) until they were taken out to be tested. Each paper clip was tested as follows:

1. Open paper clip so there is access to the long side of paper clip as shown in Figure 1a and 1b. Hold the paper clip in middle of the long side
2. Bend paper clip to 90^o
3. Bend back to initial position
4. Repeat steps 2 and 3 until paper clip breaks (Figure 1c)
5. Record number of bends until breakage (each bend is counted, ie straightening the paper clip is considered a bend)
6. Record the tester who was breaking the paper clip

We collected data on 20 paperclips for each temperature level, half of which were 1mm gauge and the other half were 0.8mm, resulting in 60 total observations.



(a) Original form of paper clip. (b) Opened paper clip. (c) Broken paper clip.

Figure 1: These three images depict different stages in the data collection process.

2.2 Model Specifications

We determined that Poisson regression was the most appropriate to model the number of bends before a paper clip breaks and to investigate our three predictors: air temperature, wire gauge, and the tester who bends the paperclip. In the model specification shown in Equations 1 and 2, the number of bends y_i follows a Poisson distribution with rate λ_i. Furthermore, λ_i is a linear function of the predictors and the log-link function ensures that the rate is positive. Each regression coefficient β_i is given a diffuse normal prior with mean 0 and variance 100 in Equation 3.

$$y_i \sim \text{Poisson}(\lambda_i) \tag{1}$$

$$log(\lambda_i) = \beta_0 + \beta_1 \text{temp}_i + \beta_2 \text{gauge}_i + \beta_3 \text{tester}_i \qquad \text{where } i = 1, 2, \ldots, 60 \tag{2}$$

$$\beta_j \sim \text{Normal}(0, 100) \qquad \text{where } j = 0, 1, 2, 3 \tag{3}$$

The model was implemented in JAGS using the `R2jags` package in R. In order to get 100,000 posterior samples, 5 Markov Chain Monte Carlo (MCMC) chains were run for 22,000 samples, with a burn-in of 2,000 samples each.

3 Results

3.1 Model Assessment

In order to check the quality of our model and posterior samples we looked at several diagnostics. The trace plots in Figure 2 show good mixing and the Gelman-Rubin diagnostic upper confidence limits are all 1. This indicates that the posterior samples of the regression coefficients, $\beta_0, \beta_1, \beta_2, \beta_3$, have converged.

Table 1: Effective sample size of Poisson regression coefficient parameters

Parameter	β_0	β_1	β_2	β_3
Effective Sample Size	100,000	100,000	100,000	98,219

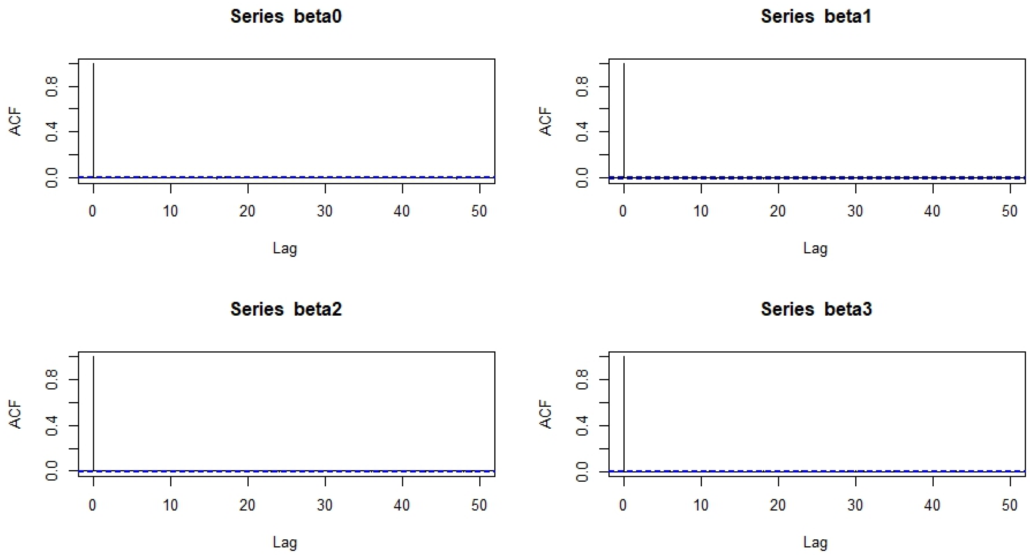


Figure 3: Auto-correlation plots for each β_i.

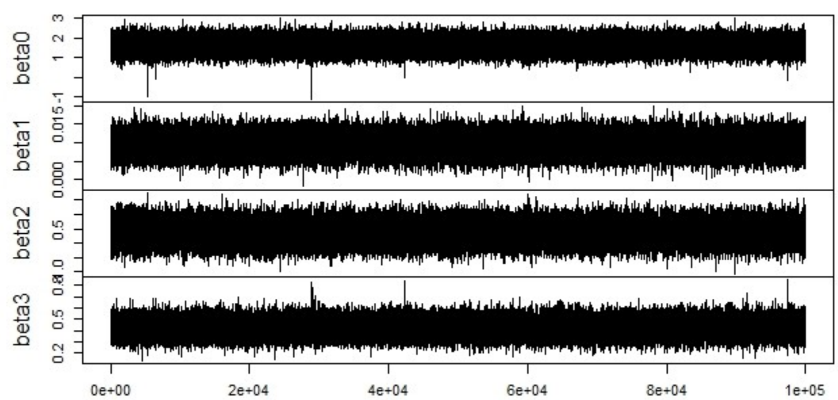


Figure 2: Trace plots for each β_i in the Poisson regression model. All show good mixing.

The effective sample sizes shown in Table 1 measure the number of independent samples from the MCMC chains. They are all close or equal to the number of samples, so we can reasonably trust the independence of the samples. Auto-correlation plots in Figure 3 show that the samples show no auto-correlation confirming what we learned from the effective sample size.

We used a chi-square goodness-of-fit test to assess how well the model fits the observed data. For each posterior sample, fitted quantiles were generated using the Poisson cumulative distribution function (`ppois` in R). These quantiles were then compared with the observed values by computing observed frequencies within predicted intervals and comparing observed frequencies to expected frequencies under the model. The goodness-of-fit p-values for each sample were summarized, and

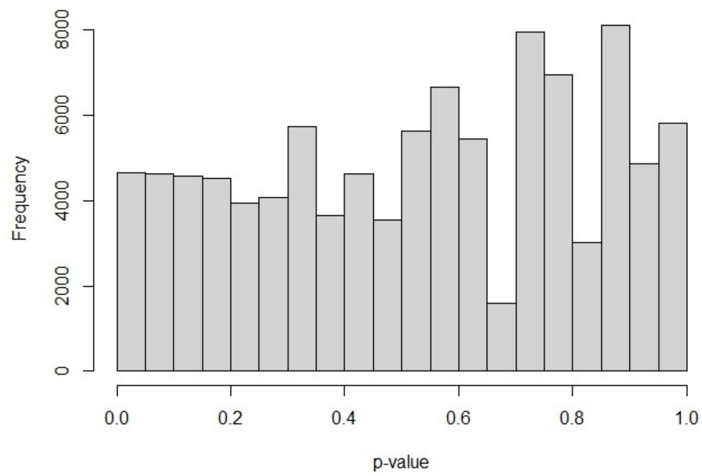


Figure 4: Histogram Summary of the goodness-of-fit p-values. Appears fairly uniform.

a histogram of these values is plotted in Figure 4. A somewhat uniform distribution of p-values is expected under a well-fitting model. Additionally, the mean proportion of samples with p-values less than 0.05 was computed as 0.047. So we conclude that our model fit the data well. We considered other models that included the interaction of gauge size and temperature, as well as models that did not include certain covariates and we found that this model had the best DIC, and we decided to use this one.

3.2 Posterior Inference

Upon analyzing our draws for each β with 95% Confidence we found the following intervals for each β. $\beta_1 : (0.005, 0.014), \beta_2 : (-0.266, 1.009), \beta_3 : (0.296, 0.554)$. Thus we can conclude that β_1, and β_3 have an impact on the number of bends until paperclip breakage. β_1 corresponds to temperature and thus we conclude that as temperature increases, so does the number of bends until failure. In other words, colder metals are more brittle. β_3 correlates to the tester. This causes us to conclude that there was a difference in the expected number of bends until failure between the two testers. Due to modeling $log(\lambda)$, it becomes more difficult to define a linear relationship between a β coefficient and λ, so we will investigate more specific situations using posterior predictive distributions.

With our draws we can then compare the posterior predictive distribution (PPD) of different covariates to see the expected difference in bends until failure. When comparing the two testers at room temperature with a 0.8mm gauge paper clip, we find that one has a mean time to failure (MTTF) of 22 bends and 90% of their PPD draws are in the interval (14,30) and the other has a MTTF of 14 bends with 90% of their PPD draws from (8,21). A PPD of the difference between the bends until breakage between the two testers (shown in Figure 5) has 90% of its draws on the interval (-2,18). This would cause us to be unable to conclude that there is a significant difference between the two testers, although we would suggest caution and be interested in collecting more data to investigate this further.

When taking PPD draws of the difference between bends until breakage of metal at temperatures 74°F and 42°F (shown in Figure 6a) we find that 90% of the draws were between (-5,11). The analysis of our β coefficient for temperature causes us to believe that temperature has an effect on the number of bends until failure, but we would have to extrapolate the data to determine at what point it would have a greater impact. If we consider the difference between bends until breakage of metal at temperatures 100°F and 30°F, in Figure 6b we can see that this difference in temperature

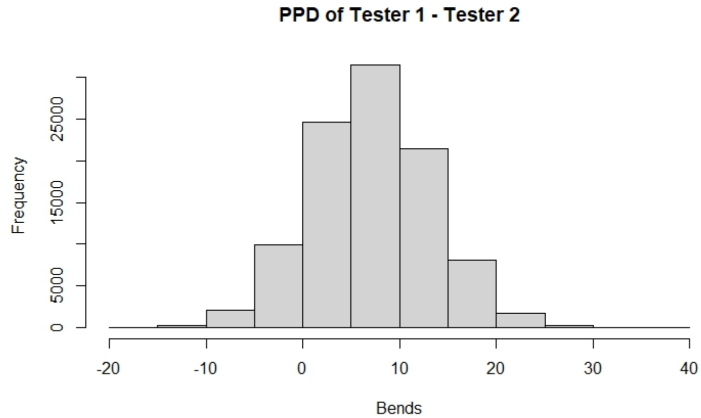
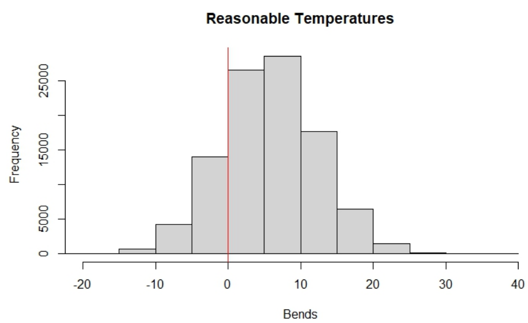
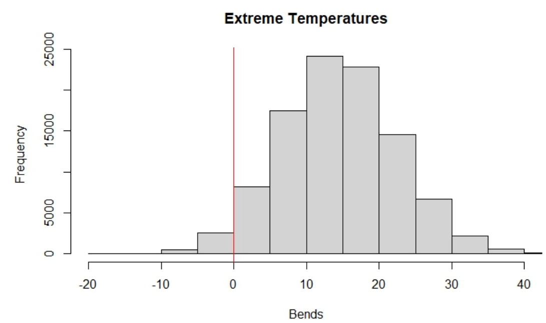


Figure 5: Posterior predictive distribution of the difference in the number of bends until breaks for Testers 1 and 2.



(a) PPD depicting how many more bends we expect a paper clip to withstand at 74°F compared to 42°F.



(b) PPD depicting how many more bends we expect a paper clip to withstand at 100°F compared to 30°F.

Figure 6: Comparing the impact of cold versus hot temperatures in reasonable and extreme situations.

has a greater difference in the bends required to break. Thus we conclude that temperature does have an affect on the number of bends until breakage but the difference is not significant at more realistic temperature levels.

We collected draws from the PPD of bends until breakage at our two difference wire gauges (shown in Figure 7) and we found that the mean difference was 6, but 90% of our draws were contained in the interval (-8,19). Combining this with the analysis of our β coefficient for wire gauge we conclude that this does not significantly affect the number of bends until breakage. While it is possible that more extreme values for wire gauge (very large and small gauges) would see a greater difference, we do not think it is very reasonable to see paper clips of that size.

4 Conclusion

Our results show that temperature has a significant impact: colder temperatures cause paper clips to break after fewer bends, suggesting they become more brittle. However, the difference between

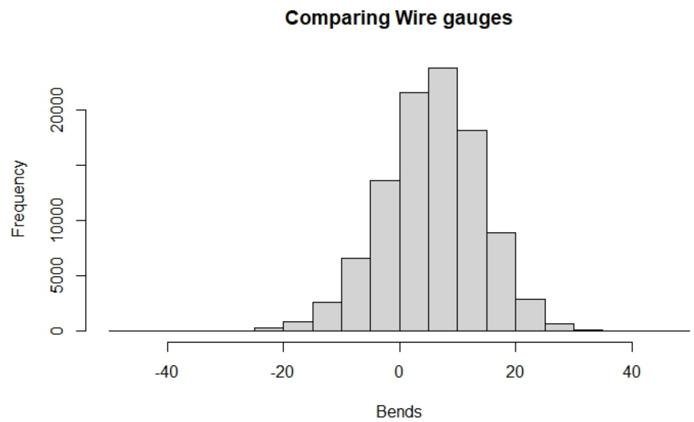


Figure 7: PPD depicting how many more bends it takes to break a 1mm paper clip versus 0.8mm.

reasonable temperatures tested (42°F and 74°F) was not significant. Wire gauge (1mm vs. 0.8mm) did not significantly affect the number of bends until failure. Although there was a small difference, it was not enough to conclude that realistic wire thicknesses play a major role. Finally, the tester had some effect on the results, but the difference was not statistically significant.

In summary, temperature and tester appear to influence paper clip fatigue, while wire gauge does not seem to have a meaningful impact. More data with a broader scope of covariates and a greater variety of temperatures and gauges could help clarify these findings.