G.V.

①

$$\lim_{x \to 0} \frac{\cos 2x - 1}{x^2} \quad \left(\frac{0}{0}\right)$$

$$\lim_{x \to 0} \frac{-2\sin 2x}{2x} \quad \left(\frac{0}{0}\right) = -2 \underbrace{\lim_{x \to 0} \frac{\sin 2x}{2x}}_{1} = -2$$

$$= \lim_{x \to 0} \frac{-4\cos 2x}{2} = \frac{-4}{2} = -2$$

Maclaurin

$$\cos t = 1 - \frac{t^2}{2} + \frac{t^4}{24} \cdots$$

$$\cos 2x = \underbrace{1 - \frac{4x^2}{2} + \frac{(2x)^4}{24} + \cdots}$$

$$\lim_{x \to 0} \frac{\cos 2x - 1}{x^2} = \lim \frac{\cancel{1} - \frac{4x^2}{2} + x^4 \cdot h(x) - \cancel{1}}{x^2}$$

$$\lim_{x \to 0} \frac{\cancel{x^2}\left[-2 + x^2 h(x)\right]}{\cancel{x^2}} = -2$$

$$\lim_{x \to 3} \frac{\sqrt{x-2} - 1}{x^2 - 9} = \left(\frac{0}{0}\right)$$

$$\left\{ \begin{array}{l} \sqrt{\cancel{x}} \to \frac{1}{2\sqrt{x}} \\ \sqrt{u} = \frac{u'}{2\sqrt{u}} \\ \sqrt{x-2} \to \frac{1}{2\sqrt{x-2}} \end{array} \right.$$

$$\lim_{x \to 3} \frac{\frac{1}{2\sqrt{x-2}}}{2x} \quad \frac{1}{2.2.3} = 1/12$$

$$\frac{(\sqrt{x-2} - 1)(\sqrt{x-2} + 1)}{(x^2-9)(\sqrt{x-2} + 1)} \qquad a^2 - b^2$$

$$\frac{x-2 - 1}{(x^2-9)(\sqrt{x-2} + 1)} = \frac{\cancel{(x-3)}\, 1}{\cancel{(x-3)}(x+3)(\sqrt{x-2} + 1)}$$

$\underline{\underline{x \to 3}}$

$$\frac{1}{6 \cdot 2} = 1/12$$

$$\lim_{x \to 2} \frac{x^2 - 2x}{2x - 4} \left(\frac{0}{0}\right) \quad \frac{x\cancel{(x-2)}}{2\cancel{(x-2)}} = 1$$

$x \to 2$

$(x-2) \to 0$ $\qquad x - 2 = t$

$\boxed{t \to 0}$ $\qquad \boxed{x = t + 2}$

$$\lim_{t \to 0} \frac{(t+2)^2 - 2(t+2)}{2(t+2) - 4} =$$

$$\frac{t^2 + 4t + \cancel{4} - 2t - \cancel{4}}{2t + \cancel{4} - \cancel{4}} = \frac{t^2 + 2t}{2t}$$

$$\lim_{t \to 0} \frac{t+2}{2} = 1$$

$$\sin x = \cos 2x$$

$$\cos\underbrace{(\pi/2 - x)}_{\alpha} = \cos \underbrace{2x}_{x}$$

$$\begin{cases} 2x = 2n\pi + \pi/2 - x \\ 2x = 2n\pi - \pi/2 + x \end{cases}$$

$$\cos x = \cos \alpha$$

$$x = 2n\pi \pm \alpha$$

$$\sin x = \underbrace{\cos^2 x}_{1-\sin^2 x} - \underbrace{\sin^2 x}_{1-\cancel{\cos^2 x}}$$

$$\sin 2x = \cos 5x$$

$$\cos\underbrace{(\pi/2 - 2x)}_{} = \cos \underbrace{5x}_{}$$

$$\underbrace{lg(x^2+1) - lg(x)} = 1$$

$$lg\left(\frac{x^2+1}{x}\right) = 1$$

$$\frac{x^2+1}{x} = 10 \qquad x^2+1 = 10x$$

$$\underbrace{lg(x^2+1) - lg\,x} = lg(2x-4)$$

$$\cancel{lg}\,\frac{x^2+1}{x} = \cancel{lg}(2x-4)$$

$$\boxed{\frac{x^2+1}{x} = 2x-4}$$

$$\log_{100}(x-6) + 1 = \boxed{\log_{10}(x-30)} \qquad X$$

$$lg(x-30) = t$$

$$x-30 = 10^t \qquad 10^{2t} \qquad 100^t$$

$$(x-30)^2 = 100^t$$

$$\log_{100}(x-30)^2 = t$$

$$\log_{100}(x-6)+1=\log_{100}(x-30)^2$$

$$1=\log_{100}(x-30)^2-\log_{100}(x-6)$$

$$\log_{100}\frac{(x-30)^2}{x-6}=1 \Rightarrow$$

$$\frac{(x-30)^2}{x-6}=\frac{100}{1}$$

$$\boxed{\log_a x=\log_{a^2} x^2}$$

$$\int x^2 \ln(1+x^3)\,dx \qquad \left(\begin{array}{l} 1+x^3 = t \\ 3x^2\,dx = dt \end{array} \right)$$

$$\int \cancel{x^2} \cdot \ln t \cdot \frac{dt}{3\cancel{x^2}}$$

$$\frac{1}{3} \underbrace{\int \ln t \,dt}$$

$$\int \underbrace{1}_{f'} \underbrace{\ln t}_{g} \,dt \qquad \begin{array}{l} f = t \\ g' = 1/t \end{array}$$

$$= t \ln t - \int 1 \,dt = \underline{t \ln t - t}$$

$$\frac{1}{3}(t \ln t - t) + C$$

$$\int \frac{x}{x^2+2x-3}\,dx$$

$$x^2+2x-3=0$$

$$x=-1\pm\sqrt{1+3}$$

$$x=-1\pm 2 \begin{cases} 1 \\ -3 \end{cases}$$

$$(x-1)(x+3)$$

$$\frac{x}{(x-1)(x+3)}$$

$$=\frac{A}{x-1}+\frac{B}{x+3}$$

$$\int \frac{x}{x^2+2x+1}\,dx =$$

$$x^2+2x+1=0$$

$$x=-1$$

$$(x+1)(x+1)$$

$$\int \frac{x}{(x+1)^2}\,dx$$

$$\boxed{\begin{matrix} x+1=t & x=t-1 \\ dx=dt & \end{matrix}}$$

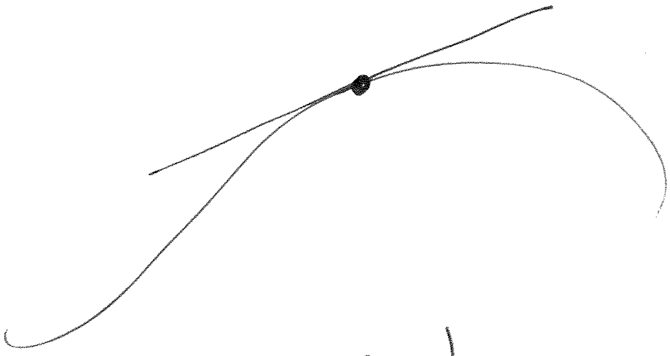
$$\int \frac{t-1}{t^2}\,dt = \int \frac{1}{t}-\frac{1}{t^2}$$

$$\int \frac{1}{t} - \int t^{-2}\,dt$$

Tangentens ekv.

$$y = \arctan \sqrt{x+1}$$

$$(0, \pi/4)$$



$$y' = \frac{\frac{1}{2\sqrt{x+1}}}{1+1+x}$$

$$\arctan x \to \frac{1}{1+x^2}$$

$$\arctan u \to \frac{u'}{1+u^2}$$

$$u = \sqrt{x+1} \quad u^2 = 1+x$$

$$u' = \frac{1}{2\sqrt{x+1}}$$

$$K = \frac{\frac{1}{2}}{2} = 1/4$$

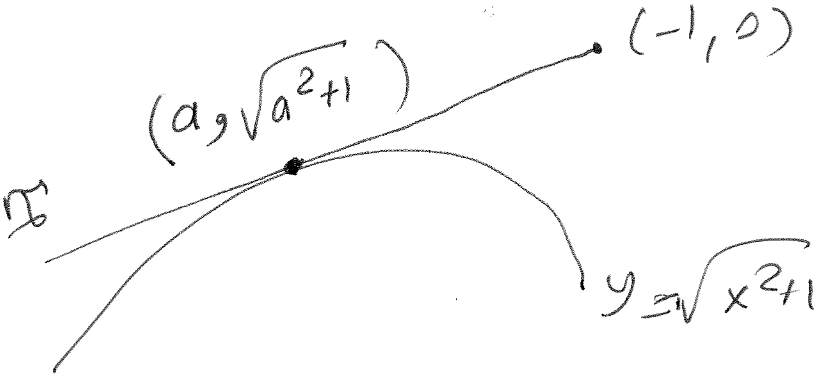
$$(0, \pi/4)$$

$$y - \pi/4 = 1/4 (x - 0)$$

$$\boxed{y = 1/4\, x + \pi/4}$$

$$y=\sqrt{x^2+1} \qquad (-1,0)$$

$$x=-1 \Rightarrow y=\sqrt{(-1)^2+1}=\sqrt{2} \neq 0$$



$$y'=\frac{x}{\sqrt{x^2+1}} \qquad y=\sqrt{u} \to \frac{u'}{2\sqrt{u}}$$

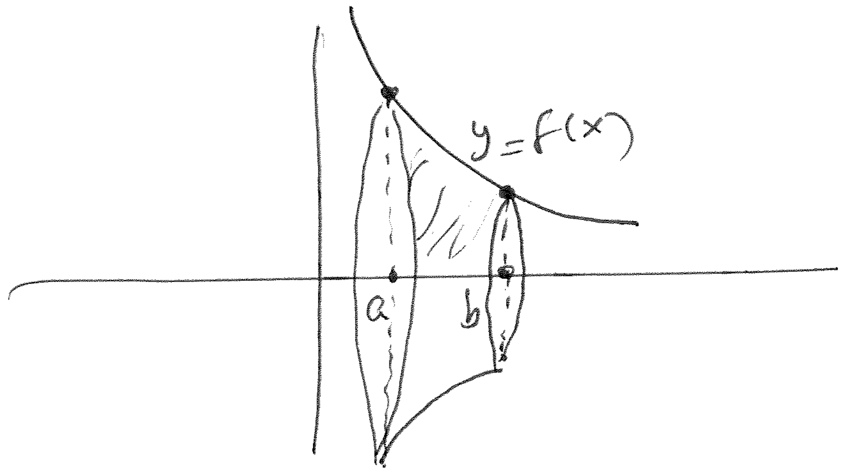
$$K=\frac{a}{\sqrt{a^2+1}} \qquad K=\frac{y_2-y_1}{x_2-x_1}=\frac{\sqrt{a^2+1}}{a+1}$$

$$\frac{\sqrt{a^2+1}}{a+1}=\frac{a}{\sqrt{a^2+1}} \qquad \cancel{a^2}+1=\cancel{a^2}+a$$

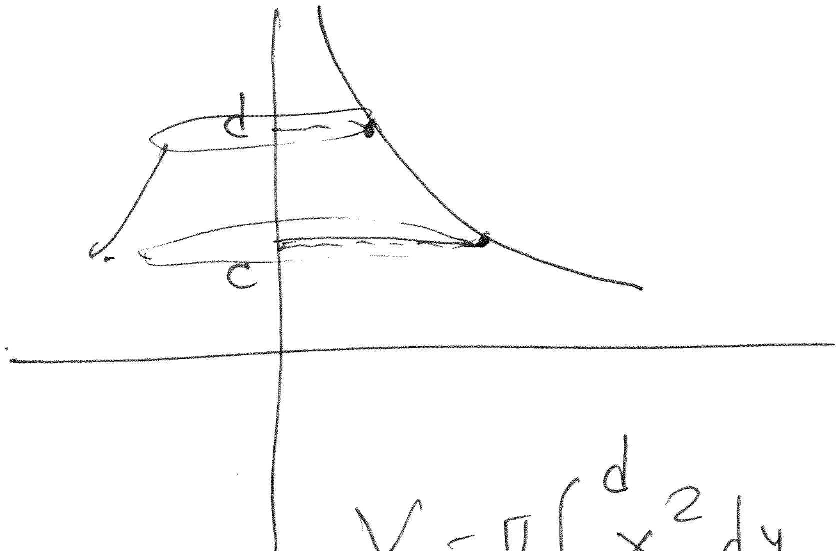
$$a=1$$

$$\underline{K=\frac{1}{\sqrt{2}}} \qquad \underline{(-1,0)}$$

$$\boxed{y-0=\frac{1}{\sqrt{2}}(x+1)}$$

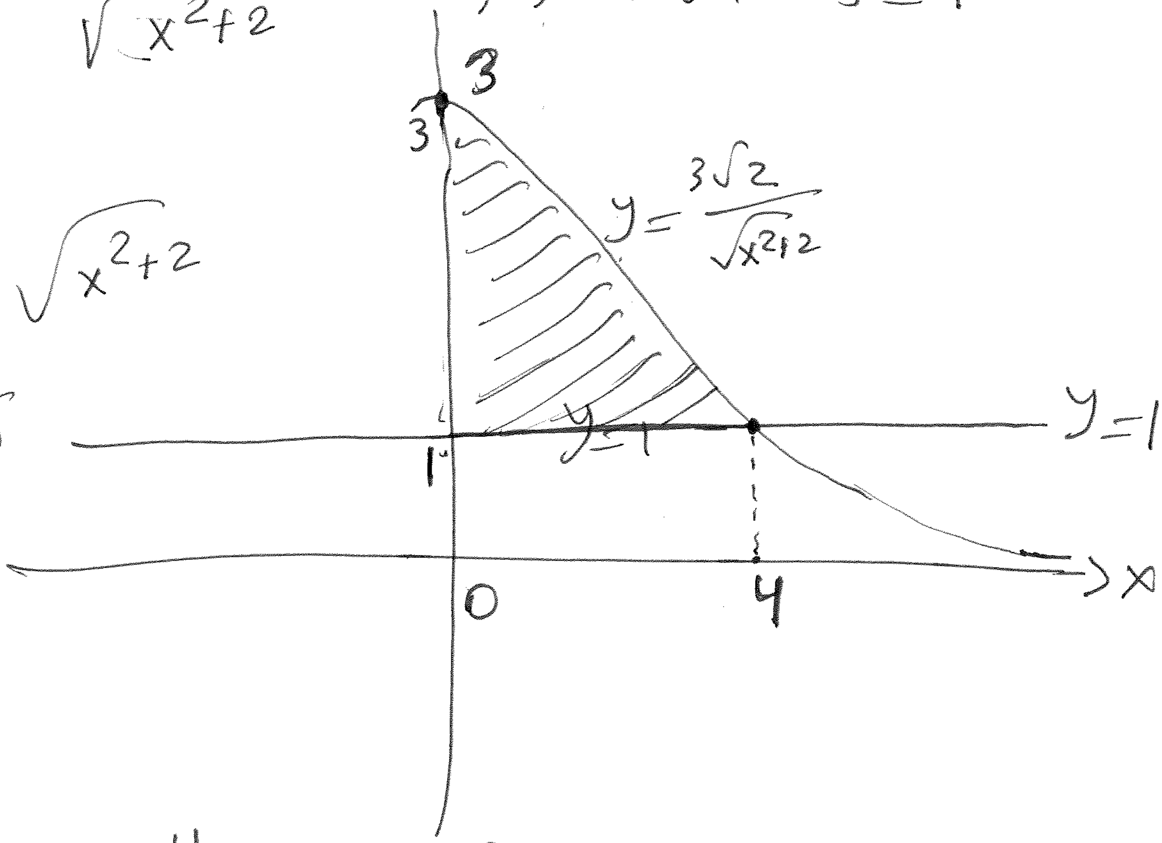


$$V = \pi \int_a^b y^2 \, dx$$



$$V = \pi \int_c^d x^2 \, dy$$

$$y = \frac{3\sqrt{2}}{\sqrt{x^2+2}}, \quad y\text{-axeln} \quad y = 1$$



$$3\sqrt{2} = \sqrt{x^2+2}$$

$$x^2+2 = 18$$

$$x^2 = 16$$

$$x = \pm 4$$

$$V_x = \pi \int_0^4 \left[\left(\frac{3\sqrt{2}}{\sqrt{x^2+2}} \right)^2 - 1^2 \right] dx$$

$$V_x = \pi \int_0^4 \left(\frac{18}{x^2+2} - 1 \right) dx$$

$$\cancel{x^2+2=0}$$

$$\frac{18}{2+x^2} = \frac{\cancel{18}}{2\left[1+\frac{x^2}{2}\right]}$$

$$9 \int \frac{1}{1+\frac{x^2}{2}} \, dx \qquad \frac{x^2}{2} = t^2$$

$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx$$

$$\int \frac{1}{(x+1)^2}\,dx = \int (x+1)^{-2}\,dx$$

$$\int \frac{1}{(2x-1)^2}\,dx = \int (2x-1)^{-2}$$

$$\frac{1}{2} \cdot \frac{1}{-2+1}\,(2x-1)^{-2+1}$$

$$\int \frac{1}{(2x-1)^2}\,dx \qquad 2x-1=t$$

$$2dx = dt$$

$$\int \frac{1}{t^2} \cdot \frac{dt}{2} \qquad \frac{1}{2}\int t^{-2}\,dt$$

$$\frac{1}{2}\ \frac{1}{-2+1}\ t^{-2+1}$$

$\arcsin\left(\cos\frac{\pi}{13}\right)$, $\cos\left(2\arcsin\left(-\frac{1}{5}\right)\right)$

① Sätt arc del = t

② Komma till Sin, Cos,

③ Vad ska beräknas

$$\arcsin\left(\cos\frac{\pi}{13}\right) = t$$

$$\sin t = \cos\frac{\pi}{13} \qquad \boxed{-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}}$$

$$\sin t = \sin\left(\frac{\pi}{2} - \frac{\pi}{13}\right)$$

$$\sin t = \sin\left(\frac{11\pi}{26}\right)$$

$$\boxed{t = \frac{11\pi}{26}}$$

$$\cos\left(2\left(\arcsin\left(-1/5\right)\right)\right)$$

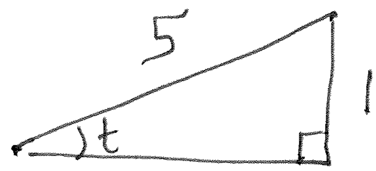
$$\arcsin(-1/5) = t$$

$$\sin t = -1/5$$

$\cos 2t$ ska beräknas ?

$$\Downarrow$$

$$\cos^2 t - \sin^2 t$$



$$\sqrt{24} = 2\sqrt{6}$$

$$\cos 2t = \frac{24}{25} - \frac{1}{25} = \frac{23}{25}$$

$$\sin t = -1/5$$

$$\cos t = \frac{\sqrt{24}}{5}$$