Några Partial (Speciell) ①

$\int \ln x \, dx \qquad \int \arctan x \, dx \qquad \int \arcsin x \, dx$

$$\int \underbrace{1}_{f'} \cdot \underbrace{\ln x}_{g} \, dx \qquad \begin{bmatrix} f' = 1 \to f = x \\ g = \ln x \to g' = 1/x \end{bmatrix}$$

$$= x \ln x - \int 1 \, dx = x \ln x - x + C$$

$$\int \underbrace{1}_{f'} \cdot \underbrace{\arctan x}_{g} \, dx \qquad f = x \qquad g' = \frac{1}{1+x^2}$$

$$\boxed{\int f'g = fg - \int fg'}$$

$$= x \arctan x - \int \frac{x}{1+x^2} \, dx$$

$$= x \arctan x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx$$

$$= x \arctan x - \frac{1}{2} \ln(1+x^2) + C$$

$$\int \underbrace{1}_{f'} \cdot \underbrace{\arcsin x}_{g} \, dx \qquad \left[\begin{array}{l} f = x \\ g' = \dfrac{1}{\sqrt{1-x^2}} \end{array} \right]$$

(2)

$$= fg - \int fg'$$

$$= x \cdot \arcsin x - \underbrace{\int \frac{x}{\sqrt{1-x^2}} \, dx}_{\text{Variabel substitution}}$$

Beräkna $\int e^x \sin x \, dx$

Sätt $I = \int \underbrace{e^x}_{f'} \underbrace{\sin x}_{g} \, dx$

$$\left[\begin{array}{l} f' = e^x \longrightarrow f = e^x \\ g = \sin x \longrightarrow g' = \cos x \end{array} \right]$$

$$\boxed{I = e^x \sin x - \int e^x \cos x \, dx} \quad *$$

$$\int \underbrace{e^x}_{f'} \underbrace{\cos x}_{g} \, dx \qquad f' = e^x \longrightarrow f = e^x$$

$$g = \cos x \longrightarrow g' = -\sin x$$

$$= e^x \cos x + \underbrace{\int e^x \sin x \, dx}_{I}$$

$$\boxed{\int e^x \cos x \, dx = e^x \cos x + I}$$

$$* \Rightarrow \quad I = e^x \sin x - e^x \cos x - I$$

$$2I = e^x \sin x - e^x \cos x$$

$$\boxed{I = \frac{e^x \sin x - e^x \cos x}{2}}$$

Metod 2

Variabel substitution
(V. S)

Två frågor

1. När används V. S ?
2. Hur 〃 〃 ?

Svar på 1 När vi ser

Derivatan framför

använder vi V. S.

Ex

① $\int \frac{1}{x} \cdot \ln x \, dx$

② $\int \sin x \left(\frac{1 + \cos x}{3} \right) dx$

③ $\int x \sqrt{1 - x^2} \, dx$

(5)

(4) $\int x e^{x^2} \, dx$

(5) $\int \frac{x^3}{1+5x^4} \, dx = \int x^3 \cdot \frac{1}{1+5x^4} \, dx$

(6) $\int x^2 (2+5x^3)^{100} \, dx$

(7) $\int \frac{\arctan x}{1+x^2} \, dx = \int \frac{1}{1+x^2} \cdot \arctan x \, dx$

$\int \frac{1}{x} \ln x \, dx$

3 st. villkor

(a) $\frac{1}{x}$ står framför

(b) $\frac{1}{x}$ är derivatan av $\ln x$

(c) Vi har $\ln x$

$\boxed{\int \frac{1}{x} \sin x \, dx \quad \text{ej v.s.}}$

Anm

V. S. funkar då derivatan står framför.

Hur?

$\frac{1}{x}$ är derivatan av $\ln x$

Sätt $\ln x = t$

$$\left(\begin{array}{l} \ln x = t \\ \frac{1}{x}\, dx = dt \end{array} \right)$$

$$t = \ln x$$

$$\frac{dt}{dx} = 1/x$$

$$dt = \frac{1}{x}\, dx$$

$$\int \cancel{\frac{1}{x}} \cdot \underbrace{t} \cdot \underbrace{\cancel{x}\, dt}$$

$$= \int t \, dt = \frac{t^2}{2} + C$$

$$= \frac{1}{2} \ln^2 x + C$$

⑦

$$\int \sin x \left(\frac{1 + \cos x}{3} \right) dx$$

$$\left(\begin{aligned} \cos x &= t \\ -\sin x \, dx &= dt \end{aligned} \right) \rightarrow dx = -\frac{dt}{\sin x}$$

$$= \int \cancel{\sin x} \cdot \left(\frac{1+t}{3} \right) \left(-\frac{dt}{\cancel{\sin x}} \right)$$

$$= -1/3 \int (1+t) \, dt$$

$$= -1/3 \left(t + \frac{t^2}{2} \right) + C$$

$$= -1/3 \cos x - 1/6 \cos^2 x + C$$

$$\int x\sqrt{1-x^2}\,dx$$

Viktig x är derivatan av x^2

Enligt regeln ska vi sätta $x^2 = t$

Men vi sätter $1-x^2 = t$

$$\left(\begin{array}{l} 1-x^2 = t \\ -2x\,dx = dt \end{array}\right) \rightarrow dx = \frac{dt}{-2x}$$

$$\int \cancel{x} \cdot \sqrt{t} \cdot \frac{dt}{-2\cancel{x}}$$

$$= -\frac{1}{2}\int \sqrt{t}\,dt = -\frac{1}{2}\int t^{1/2}\,dt$$

$$-\frac{1}{2}\,\frac{1}{1+\frac{1}{2}}\,t^{\frac{1}{2}+1} + C = -\frac{1}{3}\,t^{3/2} + C$$

$$\int x\sqrt{1-x^2}\,dx = \underbrace{-\tfrac{1}{3}(1-x^2)^{3/2} + C}$$

$\downarrow$ Derivera

$$-\tfrac{1}{\cancel{3}}\cdot\tfrac{\cancel{3}}{2}(1-x^2)^{3/2-1}\cdot(-2x)$$

$$x(1-x^2)^{1/2}$$

$$x\sqrt{1-x^2}$$

$$\int x e^{x^2}\,dx \qquad (\text{ej Partial})$$

$$\begin{pmatrix} x^2 = t \\ 2x\,dx = dt \end{pmatrix} \rightarrow dx = \frac{dt}{2x}$$

$$\int \cancel{x}\cdot e^t\cdot\frac{dt}{2\cancel{x}} = \tfrac{1}{2}e^t + c$$

$$\tfrac{1}{2}e^{x^2} + c$$

10

$$\int x^3 \frac{1}{1+5x^4}\,dx$$

$$\left(\begin{array}{l} 1+5x^4 = t \\ 20x^3\,dx = dt \end{array}\right)$$

$$= \int \cancel{x^3} \cdot \frac{1}{t} \cdot \frac{dt}{20\cancel{x^3}}$$

$$= \frac{1}{20}\int \frac{1}{t}\,dt = \frac{1}{20}\ln|t| + C$$

$$= \frac{1}{20}\ln\left(1+5x^4\right) + C$$

$$\int x^2 \left(2+5x^3\right)^{100} dx$$

$\underline{\underline{x^2}}$ är derivatan av $\underline{\underline{2+5x^3}}$

$$\left(\begin{array}{l} 2+5x^3 = t \\ 15x^2\, dx = dt \end{array}\right)$$

$$\int \cancel{x^2} \cdot t^{100} \cdot \frac{dt}{15\cancel{x^2}}$$

$$\frac{1}{15}\int t^{100}\, dt = \frac{1}{15}\,\frac{1}{101}\, t^{101} + C$$

$$\frac{1}{15}\cdot\frac{1}{101}\left(2+5x^3\right)^{101} + C$$

$$\int \frac{\arctan^2 x}{1+x^2}\, dx \qquad \int \frac{1}{1+x^2} \cdot \arctan^2 x\, dx$$

$$\left(\begin{array}{l} \arctan x = t \\ \frac{1}{1+x^2}\,dx = dt \end{array} \right) \to dx = (1+x^2)\,dt$$

$$= \int \cancel{\frac{1}{1+x^2}} \cdot t^2 \cdot \cancel{(1+x^2)}\,dt$$

$$= \tfrac{1}{3} t^3 + C = \tfrac{1}{3} \arctan^3 x + C$$

Viktig AnM

$$\arctan^2 x = t \quad \text{fel}$$

$$\downarrow$$

$$2 \arctan x \cdot \left(\frac{1}{1+x^2}\right)$$

$$u^2$$

$$2u \cdot u'$$

Viktig

Tänk på O framför innan Partial.

När ska ni fixa en uppgift.

tänk på V. S. först. och

Partial sen.

Anm Ser ni $\sqrt{x}$ i integralen

gör V. S. $x = t^2$ innan

ni tänker på metod.

Ex: $\int \sin\sqrt{x}\,dx$

$$\left(\begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right)$$

$$\int (\sin t) \cdot 2t \, dt$$

$$= 2 \int t \cdot \sin t \, dt$$

Nu tänker vi: på metod.

Derivatan framför? Nej

Partial? Ja

$$\int \underbrace{t}_{f} \cdot \underbrace{\sin t}_{g'} \, dt \qquad f' = 1 \quad g = -\cos t$$

$$= -t\cos t + \int \cos t \, dt$$

$$-t\cos t + \sin t + C$$

Svar: $-2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$

Några integraler

$$\int \sqrt{1-x^2}\, dx$$

Variabel substitutionen $\underline{x = \sin t}$ fungerar.

$$\left(\sqrt{1-x^2} = \sqrt{1-\sin^2 x} = \sqrt{\cos^2 x} = \cos x\right)$$

$$\left(\begin{array}{l} x = \sin t \\ dx = \cos t\, dt \end{array}\right)$$

$$= \int \cos t \cdot \cos t\, dt = \int \cos^2 t\, dt$$

$$\begin{cases} \cos^2 x = \dfrac{1+\cos 2x}{2} \\ \sin^2 x = \dfrac{1-\cos 2x}{2} \end{cases}$$

integral formler

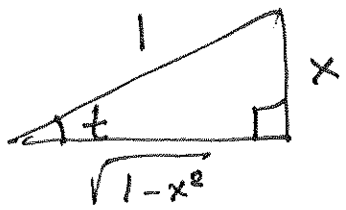
$$\int \cos^2 t \, dt = \int \frac{1+\cos 2t}{2} \, dt$$

$$= \frac{1}{2} \int (1+\cos 2t) \, dt$$

$$= \frac{1}{2} \left[t + \frac{1}{2} \sin 2t \right] + C$$

$\sin t = x \rightarrow t = \arcsin x$

$\cos t = \sqrt{1-x^2}$



$\sin 2t = 2 \sin t \cdot \cos t$

$= 2 \cdot x \cdot \sqrt{1-x^2}$

$$= \frac{1}{2} \left[\arcsin x + x\sqrt{1-x^2} \right] + C$$

Integral av trigonometriska.

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} \, dx$$

$$= \tfrac{1}{2}x - \tfrac{1}{4}\sin 2x + C$$

$$\int \sin^3 x \, dx = \int \sin x \,(1-\cos^2 x) \, dx$$

D framför

$$\left(\begin{array}{l} \cos x = t \\ -\sin x \, dx = dt \end{array} \right)$$

$$= \int \cancel{\sin x} \cdot (1-t^2) \cdot \frac{dt}{-\cancel{\sin x}}$$

$$= \int (t^2-1) \, dt = t^3/3 - t + C$$

$$= \tfrac{1}{3}\cos^3 x - \cos x + C$$

$$\int \cos^{11} x \, dx$$

$$= \int \cos x \, (\cos x)^{10} \, dx$$

$$= \int \cos x \, (1 - \sin^2 x)^5 \, dx$$

$$\left(\begin{aligned} \sin x &= t \\ \cos x \, dx &= dt \end{aligned} \right)$$

$$= \int \cancel{\cos x} \, (1 - t^2)^5 \cdot \frac{dt}{\cancel{\cos x}}$$

$$\int (1 - t^2)^5 \, dt$$

$\longrightarrow$ Binomial.

$$\int \tan x \, dx = -\int \frac{-\sin x}{\cos x} \, dx$$

$$\boxed{\int \frac{U'}{U} = \ln|U|}$$

$$= -\ln|\cos x| + C$$

$$\int \tan^2 x \, dx = \int (1 + \tan^2 x - 1) \, dx$$

$$= \int (1 + \tan^2 x) \, dx - \int 1 \, dx$$

$$= \tan x - x + C$$

$$\int \sqrt{2 - x^2} \, dx = \int \sqrt{2\left(1 - \frac{x^2}{2}\right)} \, dx$$

$$\sqrt{2} \int \sqrt{1 - \left(x/\sqrt{2}\right)^2} \, dx$$

$$\frac{x}{\sqrt{2}} = \sin t$$

$$\boxed{x = \sqrt{2} \sin t}$$