Kapitel tre:

3.1

a) $y = x + 1$ $\quad D_f = \mathbb{R}$

b) $y = \frac{1}{x+1}$ $\quad D_f :]-\infty; -1[\cup]-1; +\infty[$

c) Nej

d) $y = \sqrt[3]{x^2+1}$ $\quad D_f = \mathbb{R}$

e) $x^2 + 1 = y$ $\quad D_f = [0; \infty[$

f) Nej

g) $y = 1/3$

h) Nej

i) $y = \frac{x}{x + 2x^2}$ $\quad D_f :]-\infty; -\frac{1}{2}[\cup]-\frac{1}{2}; 0[\cup]0; +\infty[$

$y = \frac{1}{1 + 2x}$

3.2

a) $D_f = \mathbb{R}$ $\quad V = \mathbb{R}$

b) $D_f = \mathbb{R} - \{-1\}$ $\quad V = \mathbb{R} - \{0\}$

d) $D_f = \mathbb{R}$ $\quad V = [1; \infty[$

e) $D_f = \mathbb{R}^+$ $\quad V = [1; \infty[$

g) $D_f = \mathbb{R}$ $\quad V = \{1/3\}$

i) $D_f =]-\infty; -1/2[\cup]-1/2; 0[\cup]0; +\infty[$

$V =]-\infty; 0[\cup]0; +\infty[$

3.3

$V = K \cdot T$ och $V = 20T$

a) $V(2) = 20 \cdot 2 = 40$

$V(25) = 20 \cdot 25 = 500$

$V: \quad 40 \leq V \leq 500$

b) $D_f: \quad]0; +\infty[$

3.4 $f(x) = x - 2x^2$

a) $f(0) = 0 - 0 = 0$

b) $f(-3) = -3 - 2 \cdot 9 = -3 - 18 = -21$

c) $f(2-a) = 2 - a - 2(4 + a^2 - 4a)$

$= 2 - a - 8 - 2a^2 + 8a$

$= -2a^2 + 7a - 6$

d) $f(x^2) = x^2 - 2(x^2)^2 = x^2 - 2x^4$

e) $f\left(\frac{1}{2x+1}\right) = \frac{1}{2x+1} - \frac{2}{(2x+1)^2}$

$= \frac{2x+1-2}{(2x+1)^2} = \frac{2x-1}{(2x+1)^2}$

f) $f(x+1) = (x+1) - 2(x^2 + 1 + 2x)$

$f(x-1) = (x-1) - 2(x^2 + 1 - 2x)$

$f(x+1) - f(x-1) =$

$x + 1 - 2x^2 - 2 - 4x$

$- x + 1 + 2x^2 + 2 - 4x =$

$-8x + 2$

(3.5) $f(g(x)) = f(x^2)$

$= (x^2)^2 + 1 = x^4 + 1$

(3.6) $f(y) = \sqrt{y}$

om |x|=x dvs om x>=0

$f(g(x)) = x\sqrt{1-x} = \sqrt{x^2(1-x)}$

dvs $g(x) = x^2(1-x) = x^2 - x^3$

$g(x)$ bör vara positiv dvs $x \geq 0$ (*)

och $1 - x \geq 0$

dvs $x^2(1-x) \geq 0$

$1 - x \geq 0$

$x \leq 1$ (*) (*)

D_f : $0 \leq x \leq 1$ (från (*) och (*)(*))

(3.7) $f(x) = 2x^3$ $g(x) = \frac{1}{x^3}$

a) $f \circ g = f(g(x)) = f\left(\frac{1}{x^3}\right) = 2\left(\frac{1}{x^3}\right)^3 = \frac{2}{x^9}$

b) $g \circ f = g(f(x)) = g(2x^3) = \frac{1}{(2x^3)^3} = \frac{1}{8x^9}$

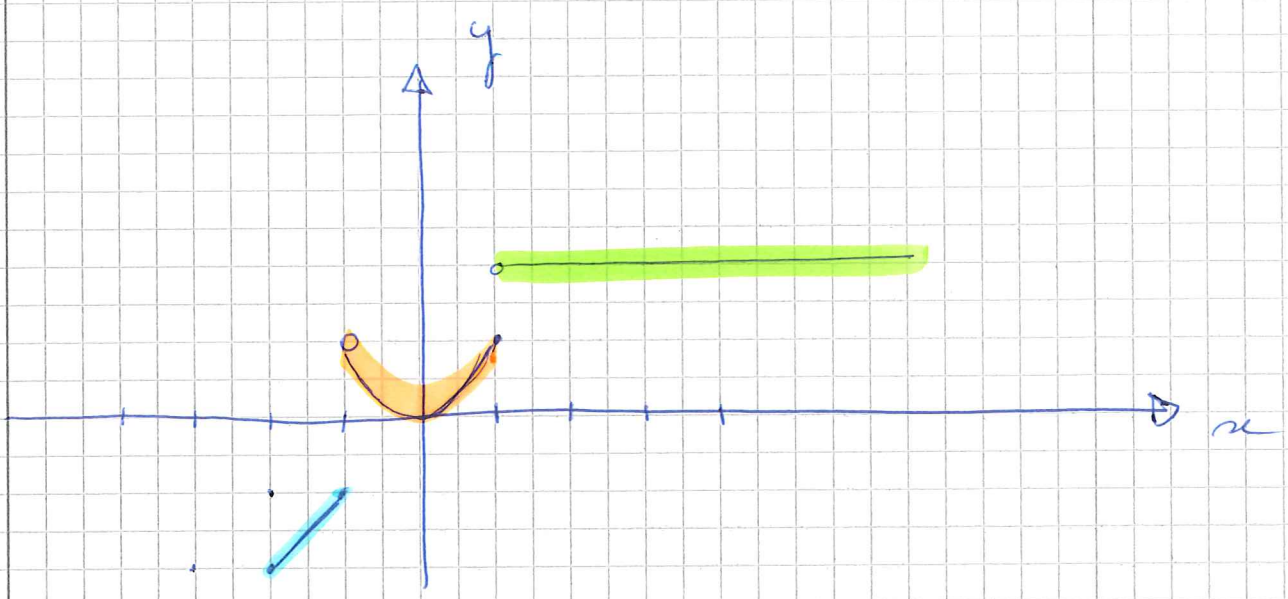
c) $f \circ f = f(2x^3) = 2(2x^3)^3 = 2 \cdot 8x^9 = 16x^9$

d) $f \circ g \circ g = f \circ g(g(x)) = f \circ g\left(\frac{1}{x^3}\right)$

$= f\left(\frac{1}{(1/x^3)^3}\right) = f(x^9) = 2(x^9)^3$

$= 2x^{27}$

3.8



$$f(x) = \begin{cases} x & x \leq -1 \\ x^2 & -1 < x \leq 1 \\ 2 & x > 1 \end{cases}$$

Växande: $-1 \leq x$

$x \geq 0$

avtagande: $-1 < x \leq 0$

$x > 1$

Strängt växande: $x \leq -1$

$0 \leq x \leq 1$

Strängt avtagande: $-1 < x \leq 0$

3.10

a) $y = 2x \qquad x = \frac{y}{2}$

$f^{-1}(x) = \frac{x}{2}$

b) $f(x) = \frac{1}{x^2} \qquad y = \frac{1}{x^2}$

existerar inte eventuellt

eftersom $x = \pm\sqrt{\frac{1}{y}}$

men vi vet att $x > 0$ så

att $x = \sqrt{\frac{1}{y}}$

och $f^{-1}(x) = \sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}} = f^{-1}(x)$

c) $f(x) = 2 - \frac{1}{x} \qquad x > 0$

$y = 2 - \frac{1}{x}$

$\frac{1}{x} = 2 - y$

$x = \frac{1}{2-y}$

$f^{-1}(x) = \frac{1}{2-x}$

med $\frac{1}{2-x} > 0$

$2 - x > 0$

$x < 2$

3.12

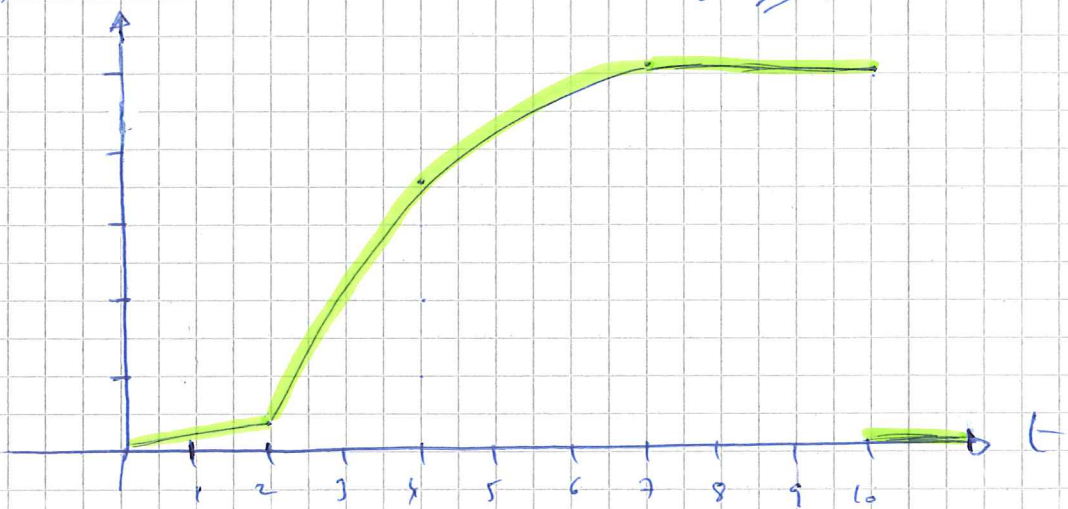
$N(0) = 10$

$N(t) = 10t + 10 \quad 0 \leq t < 2$

$N(t) = 312t - 24t^2 - 498 \quad 2 \leq t < 7$

$N(t) = 510 \quad 7 \leq t < 10$

$N(t) = 0 \quad t \geq 10$



$N(4) = 366$

t
1
2
3
4
5
6
6,5
7

Hier !

3.13

$$f(x) = 2e^{3x}$$

$$f(\ln x) = 2e^{3 \cdot \ln x}$$

$$= 2e^{\ln x^3}$$

$$= 2 \cdot x^3$$

Kom ihåg logaritmlagarna

$$\ln a^p = p \cdot \ln a$$