Kapitel fyra:

(4.1) a) $y = x^2 + 3x - 4$

Vi löser $0 = x^2 + 3x - 4$

$$x = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{16}{4}}$$

$$= -\frac{3}{2} \pm \sqrt{\frac{25}{4}}$$

$$= -\frac{3}{2} \pm \frac{5}{2}$$

$$x_1 = \frac{-3+5}{2} = 1$$

$$x_2 = \frac{-3-5}{2} = -4$$

ELLER $0 = x^2 + 3x - 4$

Man ser att en lösning är $x_1 = 1$

eftersom $1 + 3 - 4 = 0$

den andra lösningen är

konstanten i ekvationen

$x_2 = -4$

$\blacksquare$

b) $y = 3x - 1 - 2x^2$

$3 - 1 - 2 = 0$

eller $0 = 3x - 1 - 2x^2$

$0 = -\frac{3}{2}x + \frac{1}{2} + x^2$ (dele med -2)

$x_1 = 1$

$x_2 = 1/2$

c) $y = 3x - 2x^2 - 2$

$0 = 3x - 2x^2 - 2$

$0 = -\frac{3}{2}x + x^2 + 1$

$x = \frac{3}{4} \pm \sqrt{\frac{9}{16} - \frac{16}{16}}$

Saknas

d) $y = \frac{3}{2}x^2 - \frac{1}{2}x - 1$

$\frac{3}{2} - \frac{1}{2} - 1 =$

$\frac{2}{2} - 1 = 0$

$0 = \frac{3}{2}x^2 - \frac{1}{2}x - 1$

$0 = 3x^2 - x - 2$

$0 = x^2 - \frac{1}{3}x - \frac{2}{3}$

$x_1 = 1$

$x_2 = -2/3$

4.2

a) $y = x^2 + 3x - 4$ $\quad x_1 = 1 \quad x_2 = -4$

$y = (x-1)(x+4)$

b) $y = -2x^2 + 3x - 1$ $\quad x_1 = 1$

$x_2 = 1/2$

$y = -2(x-1)(x-\frac{1}{2})$

$y = (x-1)(-2x+1)$

c) går ej

d) $y = \frac{3}{2}x^2 - \frac{1}{2}x - 1$ $\quad x_1 = 1$

$x_2 = -2/3$

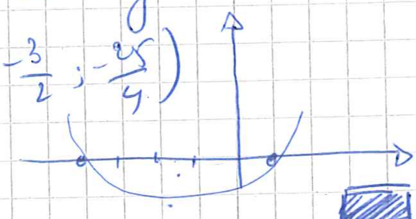
$y = \frac{3}{2}(x-1)(x+\frac{2}{3})$

$y = (x-1)(\frac{3x}{2}+1)$

4.3

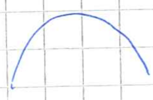
a) $\frac{1-4}{2} = \frac{-3}{2}$ $\quad x = -\frac{3}{2} \quad y = -25/4$

glad funktion $\Rightarrow$ min $(-\frac{3}{2} ; -\frac{25}{4})$



b) $\frac{1+\frac{1}{2}}{2} = \frac{3}{4}$

$y = 1/8$



Max $(\frac{3}{4} ; \frac{1}{8})$

c) $y = 3x - 2x^2 - 2$

$y = -2\left(x^2 - \frac{3x}{2}\right) - 2$

$= -2\left[\left(x - \frac{3}{4}\right)^2 - \frac{9}{16}\right] - 2$

$x^2 + \cancel{\frac{9}{16}} - \frac{3 \cdot x \cdot \cancel{2}}{4} - \cancel{\frac{9}{16}}$

$= -2\left[x - \frac{3}{4}\right]^2 + \frac{9}{8} - 2$

$= -2\left(x - \frac{3}{4}\right)^2 + \frac{9-16}{8}$

$= -2\left(x - \frac{3}{4}\right)^2 - 7/8$

$x = 3/4 \qquad y = -7/8 \qquad$ Max

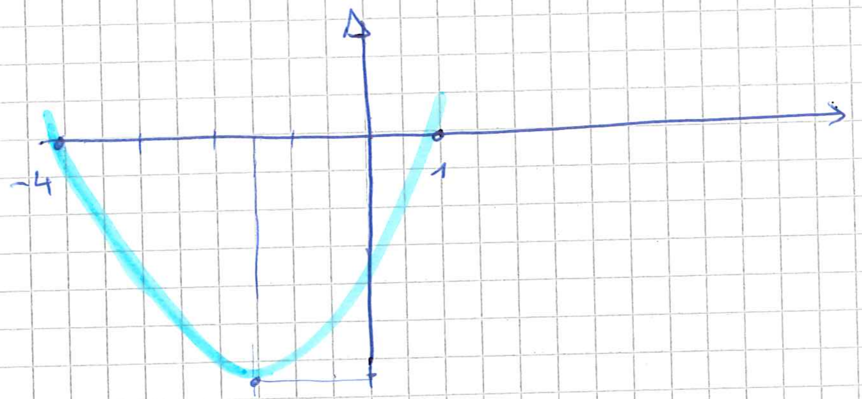
d) $\frac{-\frac{2}{3} + 1}{2} = \frac{1}{6} = x$

$y = -25/24 \qquad \left(\frac{1}{6}, -\frac{25}{24}\right)$

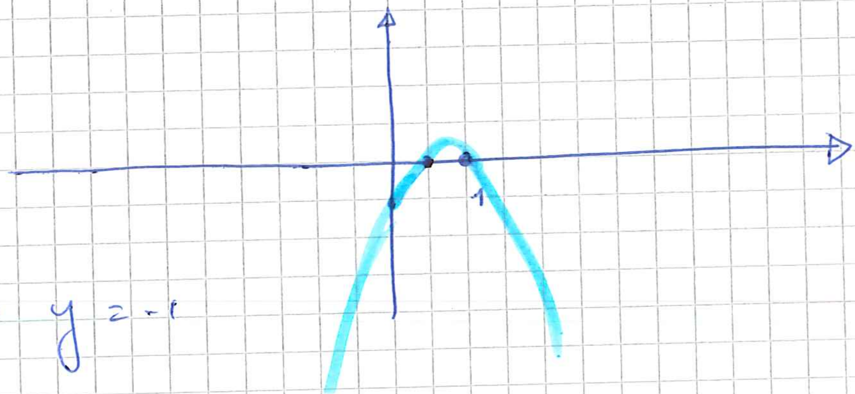
$\Rightarrow$ Minimum.

4.4

a) $x_1 = 1$ $\quad x_1 = -4$ $\quad$ Min $\left(-3/2 \,;\, -\frac{25}{4}\right)$



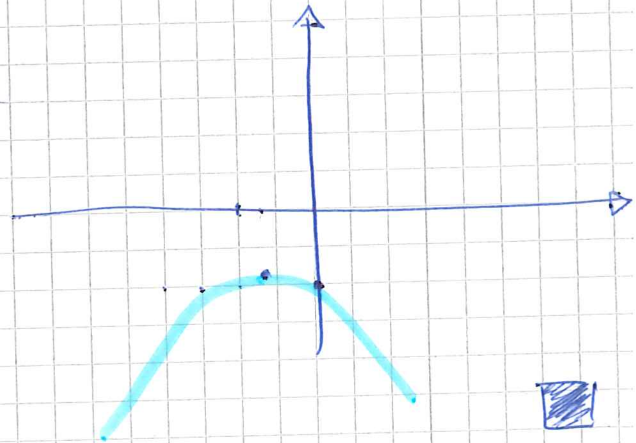
b) $x_1 = 1$ $\quad x_2 = 1/2$ $\quad$ Max $\left(3/4 \,;\, 1/8\right)$



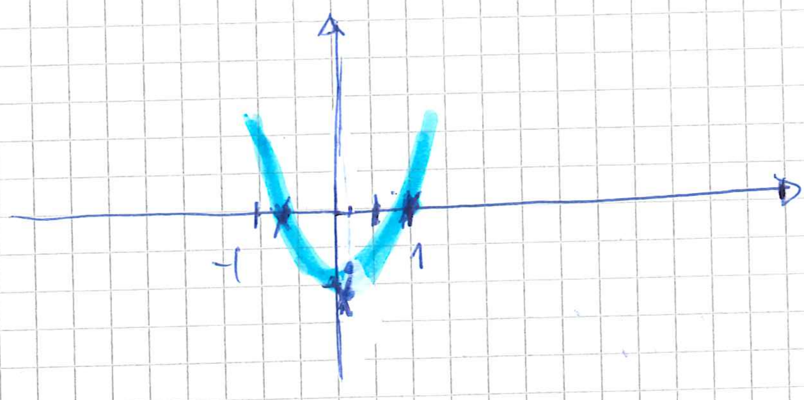
$x = 0$ $\quad y = -1$

c) $x = \frac{3}{4}$ $\quad y = -7/8$ $\quad$ Max

$x = 0$ $\quad y = -2$



d) $x_1 = 1$ $\quad x_2 = -2/3$ $\quad$ Max $\left(\frac{1}{6} \,;\, \frac{-25}{24}\right)$



$x = 0$

$y = -1$

(47) a) $\dfrac{z^3 - 2z + 1}{z - 1}$

$$
\begin{array}{l|l}
z^3 - 2z + 1 & z - 1 \\
\hline
\end{array}
\quad z^2 + z - 1
$$

$$
\begin{array}{r}
z^3 - 2z + 1 \\
z^3 - z^2 \\
\hline
z^2 - 2z + 1 \\
z^2 - z \\
\hline
-z + 1 \\
-z + 1 \\
\hline
0
\end{array}
$$

$$\frac{z^3 - 2z + 1}{z - 1} = z^2 + z - 1$$

Also

$$z^3 - 2z + 1 = (z^2 + z - 1)(z - 1)$$

■

b)

$$\begin{array}{l} \quad x^3 + 3x + 2 \\ \hline x^5 + 4x^3 + 2x^2 + 1 \quad \boxed{x^2+1} \\ x^5 + x^3 \\ \hline \quad 3x^3 + 2x^2 + 1 \\ \quad 3x^3 + 3x \\ \hline \quad 2x^2 - 3x + 1 \\ \quad 2x^2 + 2 \\ \hline \quad -3x - 1 \end{array}$$

$$(x^5 + 4x^3 + 2x^2 + 1) / (x^2+1) =$$

$$x^3 + 3x + 2 - \frac{3x+1}{x^2+1}$$

eller $x^5 + 4x^3 + 2x^2 + 1 =$

$$(x^3 + 3x + 2)(x^2+1) - (3x+1)$$

4.8

a) Låt $x^2 = t$

$$2t^2 - t - 6 = 0$$

$$t^2 - \frac{1}{2}t - 3 = 0$$

$$t = \frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{48}{16}}$$

$$t = \frac{1}{4} \pm \sqrt{\frac{49}{16}}$$

$$t = \frac{1}{4} \pm \frac{7}{4}$$

$$t_1 = 2$$

$$t_2 = -\frac{6}{4} = -3/2$$

dvs $x^2 = 2$

$x^2 = -3/2$ Nej

så $x = \pm\sqrt{2}$

■

b) $x^3 = t$

$t^2 - t - 2 = 0$

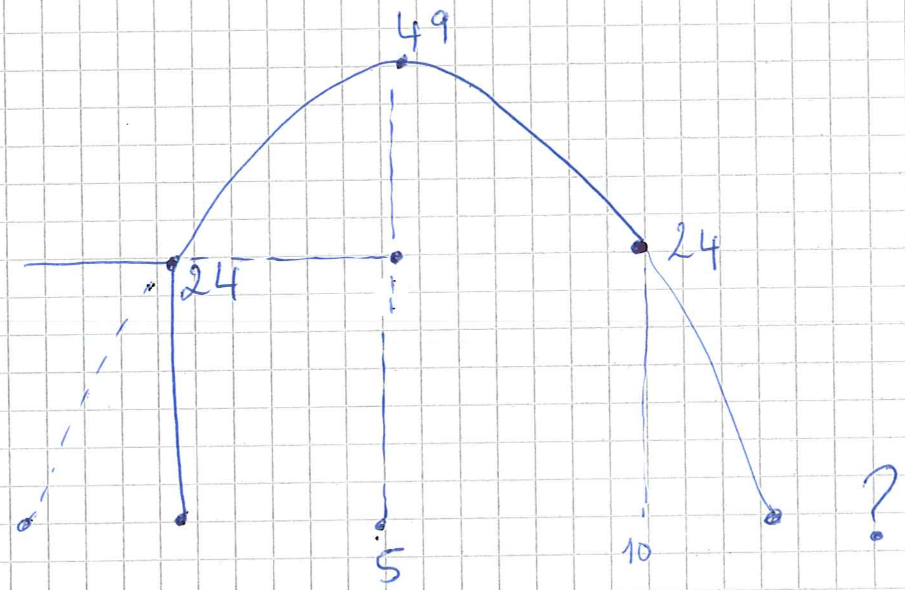
$t_1 = -1$ eftersom $1 - (-1) - 2 = 0$

och $t_2 = 2$ $\left(\frac{\text{konstanten}}{-1} \right)$

dvs $x^3 = -1$ $\quad x = -1$

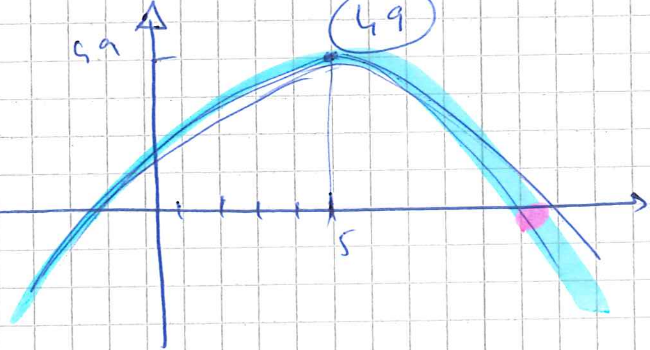
$x^3 = 2$ $\quad x = \sqrt[3]{2}$

4.9



Betrakta

$\boxed{y = -(x-5)^2 + 49}$



När är $y = 0$

$49 = (x-5)^2$

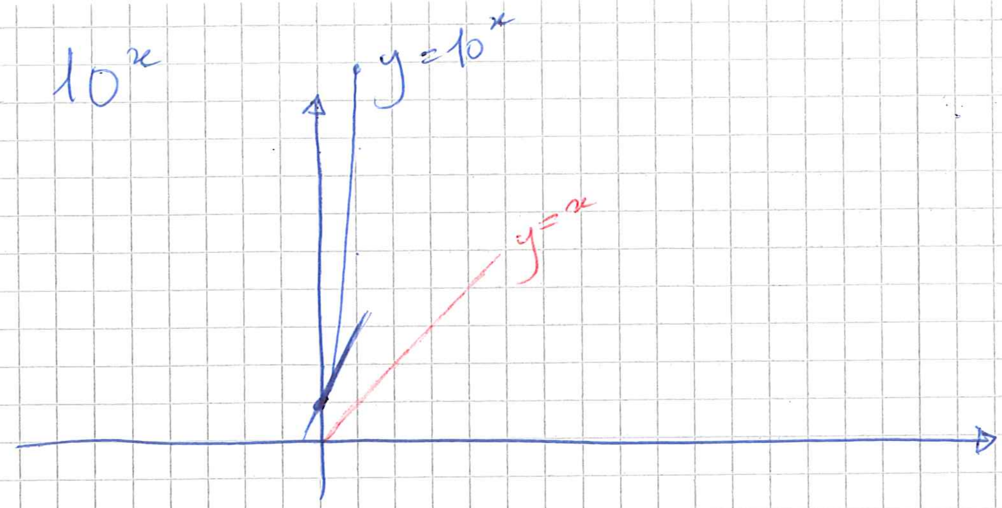
$\pm 7 = x - 5$

$\boxed{x_1 = 7 + 5}$ $\quad x_2 = -7 + 5$

12 meter

4.10

$y = 10^x$



$x = 0 \quad y = 1$

$x = 1 \quad y = 10$

$x = 2 \quad y = 100$

i punkten (0;1) är kurvan växande dvs riktningskoeff. är positiv

jag ritar $y = x$ $(k = 1)$

$y = 10^x$ brantare än $y = x$ i $(0;1)$

dvs riktningskoefficient > 1.

4.11

a) $2^x = e^{\ln 2^x} = e^{x \cdot \ln 2}$

b) $0{,}5^x = e^{\ln \frac{1}{2}^x} = e^{x \ln \frac{1}{2}} = e^{-x \ln 2}$

c) $e \cdot 3^{-x} = e \cdot e^{\ln 3^{-x}}$

$= e^1 \cdot e^{-x \ln 3} = e^{-x \ln 3 + 1}$

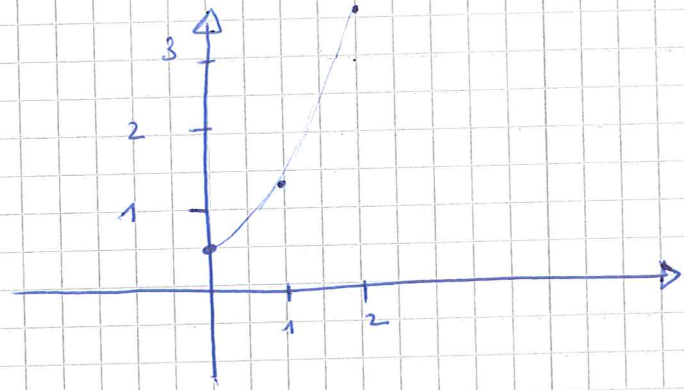
d) $4 \cdot 10^x = e^{\ln 4} \cdot e^{\ln 10^x}$

$= e^{\ln 4} \cdot e^{x \ln 10}$

$= e^{\ln 4 + x \ln 10}$

4.12

$$y = \frac{e^{x}}{2}$$

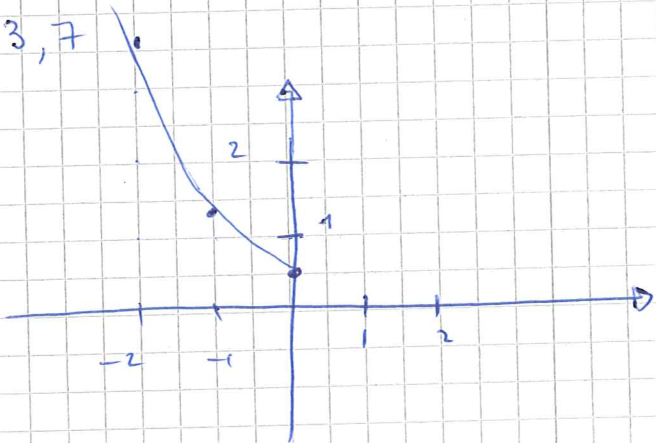


$x = 0 \qquad y = 1/2$

$x = 1 \qquad y = \frac{e}{2} \approx 1{,}4$

$x = 2 \qquad y = \frac{e^{2}}{2} = 3{,}7$

$$y = \frac{e^{-x}}{2}$$

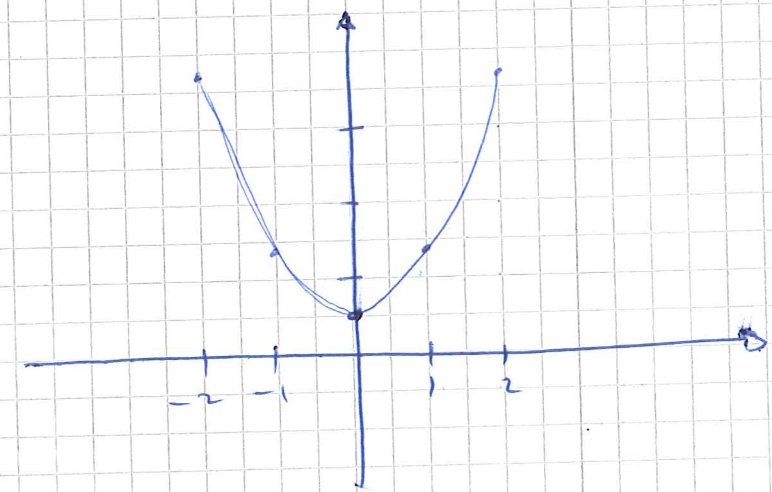


$x = 0 \qquad y = 1/2$

$x = -1 \qquad y = 1{,}4$

$x = -2 \qquad y = 3{,}7$

$$\begin{cases} y = \dfrac{e^{x} + e^{-x}}{2} \\ \\ y = \cosh x \end{cases}$$



Cosine hyperbolic

4.14 $y = \log_8 x^2$

$2 = 8^{1/3}$

a) $x = 2$

$y = \log_8 2^2 = \log_8 (8^{1/3})^2$

$= \log_8 8^{2/3} = \frac{2}{3} \log_8 8$

$= \frac{2}{3} \cdot 1 = 2/3$ ■

b) $x = 1/2$

$y = \log_8 (1/2)^2$ $\qquad 2 = 8^{1/3}$

$\frac{1}{2} = 2^{-1} = 8^{-1/3}$

$y = \log_8 (8^{-1/3})^2$

$y = \log_8 8^{-2/3} = -\frac{2}{3} \log_8 8 = -2/3$ ■

c) $x = -1$

$y = \log_8 (-1)^2 = \log_8 1 = 0$ ■

d) $x = 32$

$32 = 2^5$

$= (8^{1/3})^5$

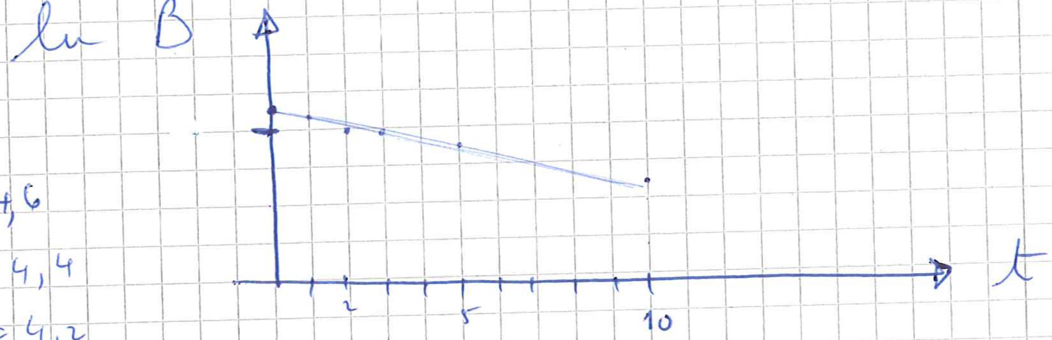
$= 8^{5/3}$

$y = \log_8 (32)^2$

$y = \log_8 (2^{5/3})^2$

$y = \log_8 8^{\frac{10}{3}} = \frac{10}{3} \log_8 8 = 10/3$ ■

4.16



$\ln 100 = 4{,}6$

$\ln 79 = 4{,}4$

$\ln 68 = 4{,}2$

$B = a e^{bt}$ $\qquad (0;100) \Rightarrow 100 = a \cdot e^{b \cdot 0}$

$a = 100$

$B = 100 e^{bt}$ $\qquad (1,79) = 79 = 100 \cdot e^{b \cdot 1}$

$\underline{\text{dvs}} \quad \frac{79}{100} = e^{b}$

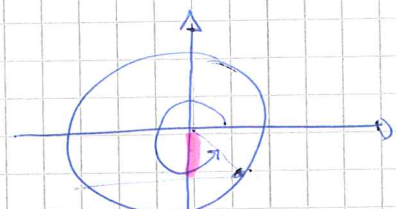
$\ln \frac{79}{100} = \ln e^{b}$

$\ln\left(\frac{79}{100}\right) = b \qquad b = -0{,}236$

och $\boxed{B = 100 \cdot e^{-0{,}236t}}$

4.17

a) $\sin v = -\frac{1}{2}$



$v = \arcsin(-1/2)$

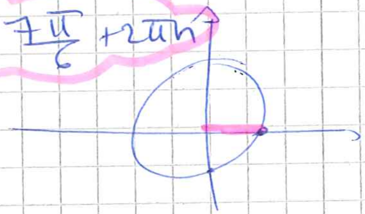
$v_1 = \frac{11\pi}{6} + 2\pi n$ $\qquad 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$v_2 = \pi - v_1 = \pi - \frac{11\pi}{6} = \frac{6\pi - 11\pi}{6} = \frac{-5\pi}{6}$

$v_2 = -\frac{5\pi}{6} + 2\pi + 2\pi n$ (addera ett varv)

$v_1 = 11\pi/6 + 2\pi n \qquad v_2 = \frac{7\pi}{6} + 2\pi n$

b) $\cos v = 1$



$v = \pm 0 + 2\pi n$

dvs $v = 2\pi n$

c) $\sin v = \frac{\sqrt{3}}{2}$

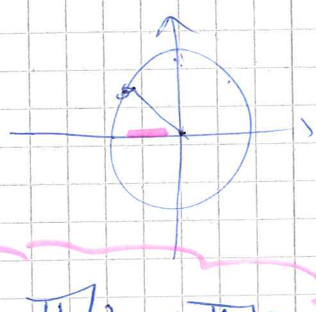
$v_1 = \frac{\pi}{3} + 2\pi n$

$v_2 = \pi - \frac{\pi}{3} + 2\pi n$

$v_1 = \pi/3 + 2\pi n$

$v_2 = \frac{2\pi}{3} + 2\pi n$

d) $\cos v = -\frac{1}{\sqrt{2}}$



$v = \pm \frac{3\pi}{4} + 2\pi n$

e) $\tan v = \sqrt{3}$

$v = \pi/3 + \pi n$

f) $\tan v = \frac{-1}{\sqrt{3}}$

$v = -\pi/6 + \pi n$

tabell
s. 124

(4.18) a) $\tan(2v - \pi) = \frac{1}{\sqrt{3}}$

$$2v = \frac{\pi}{6} + n\pi$$

$$2v = \frac{\pi}{6} + n\pi$$

$$v = \frac{\pi}{12} + \frac{n\pi}{2}$$

(*) $\tan(2v - \pi) = \tan 2v$

eftersom

$$\tan(t - \pi) = \frac{\sin(t-\pi)}{\cos(t-\pi)} = \frac{-\sin t}{-\cos t}$$

$$= \tan t \qquad \blacksquare$$

b) $\sin 2v = \sin 3v$

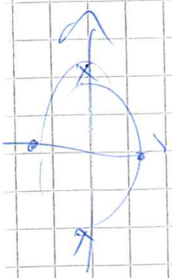
$$\sin 2v - \sin 3v = 0$$

$$2 \cos\frac{5v}{2} \sin\frac{-v}{2} = 0$$

(A21 i boken)

dvs $\cos\frac{5v}{2} = 0$ eller $\sin(-\frac{v}{2}) = 0$

$-\sin(\frac{v}{2}) = 0$



$\frac{5v}{2} = \pm\frac{\pi}{2} + 2\pi n$ eller $+\frac{v}{2} = n\pi$

eller $v = 2\pi n$

dvs

$= \frac{5v}{2} = \frac{\pi}{2} + \pi n$

$$5v = \pi + 2\pi n$$

$v = \frac{\pi}{5} + \frac{2\pi n}{5}$ eller $v = 2\pi n$ $\blacksquare$

c) $\cos(\pi - v) = \sin(2v - \pi)$

$\cancel{-}\cos v = \cancel{-}\sin 2v$

$\cos v - \sin 2v = 0$

$\cos v - 2\sin v \cos v = 0$

$\cos v(1 - 2\sin v) = 0$

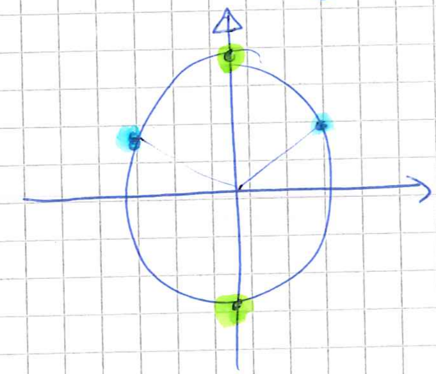
$\cos v = 0$ eller $\sin v = 1/2$

$v = \frac{\pi}{2} + \pi n$

$v_1 = \frac{\pi}{6} + 2\pi n$

$v_2 = \pi - \pi/6 + 2\pi n$

$= \frac{5\pi}{6} + 2\pi n$

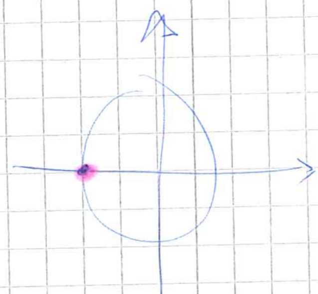


$v = \frac{\pi}{2} + \pi n$

$v = \frac{\pi}{6} + 2\pi n$

$v = \frac{5\pi}{6} + 2\pi n$

■



d) $\cos(3v - \frac{\pi}{5}) = -1$

$3v - \frac{\pi}{5} = \pi + 2\pi n$

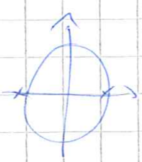
$3v = \frac{6\pi}{5} + 2\pi n$

$v = \frac{2\pi}{5} + \frac{2\pi n}{3}$

■

e) $\frac{\cos v}{\sin v} = \cos v$ $\qquad \sin v \neq 0$

$\qquad\qquad\qquad\qquad v \neq n\pi$



$\cos v = \cos v \sin v$

$\cos v (1 - \sin v) = 0$

$\cos v = 0$ eller $\sin v = 1$

$v_1 = \frac{\pi}{2} + n\pi$ eller $v_2 = \frac{\pi}{2} + 2\pi n$

$\Longrightarrow \quad v = \frac{\pi}{2} + n\pi$

(v_2 ingår i v_1)

f) $\sin\left(v + \frac{2\pi}{5}\right) = \frac{1}{\sqrt{2}}$

$v + \frac{2\pi}{5} = \frac{\pi}{4} + 2\pi n$ eller $v + \frac{2\pi}{5} = \pi - \frac{\pi}{4} + 2\pi n$

$v = \frac{\pi}{4} - \frac{2\pi}{5} + 2\pi n$ eller $v = \frac{3\pi}{4} - \frac{2\pi}{5} + 2\pi n$

$v = \frac{-3\pi}{20} + 2\pi n$ eller $v = \frac{7\pi}{20} + 2\pi n$

Kom ihåg:

$\sin v = a$

$v_1 = \arcsin(a) + 2\pi n$

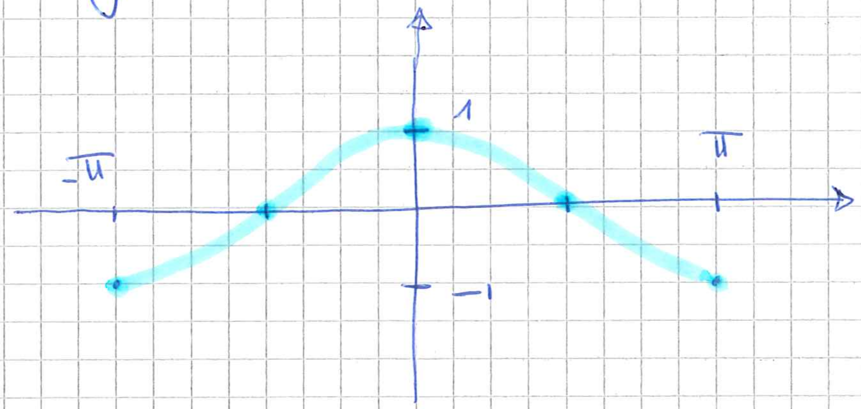
$v_2 = \pi - \arcsin(a) + 2\pi n$

$\cos v = a$

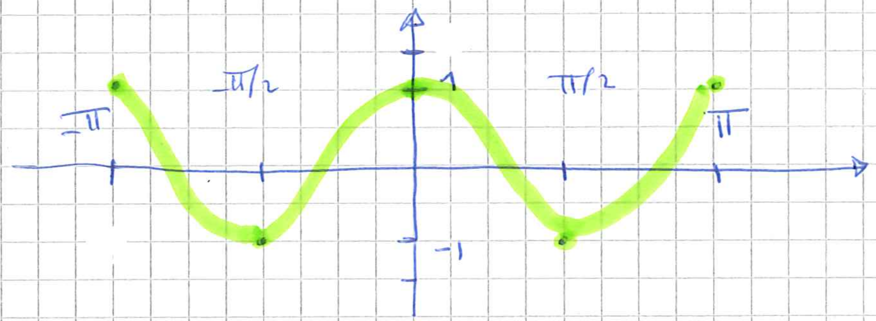
$v = \pm \arccos a + 2\pi n$

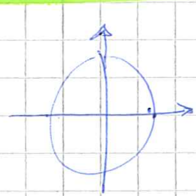
4.19 a) $y = 2\cos 2x$

$y = \cos x$

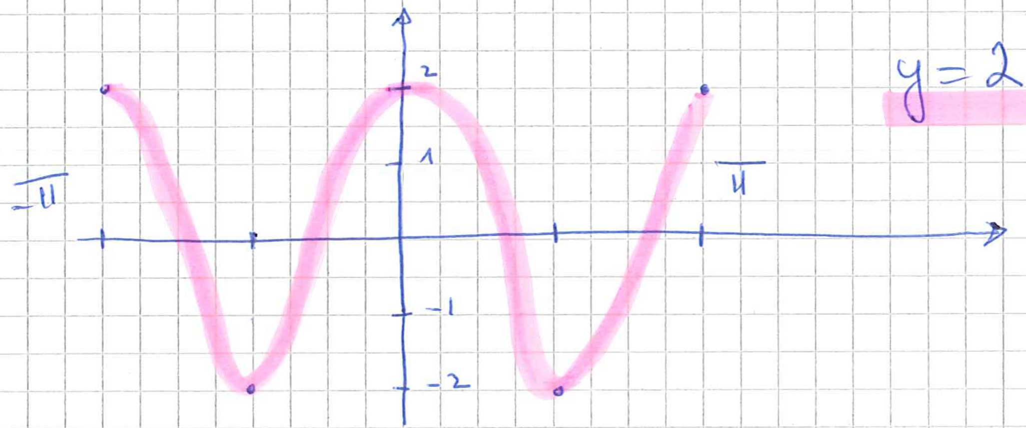


$y = \cos 2x$





$y = 2\cos 2x$



steg 1: $y = \cos x$

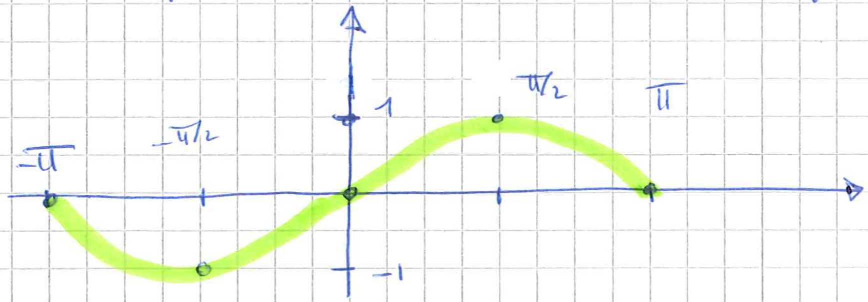
steg 2: dubblere perioden

steg 3: dubblere Amplituden

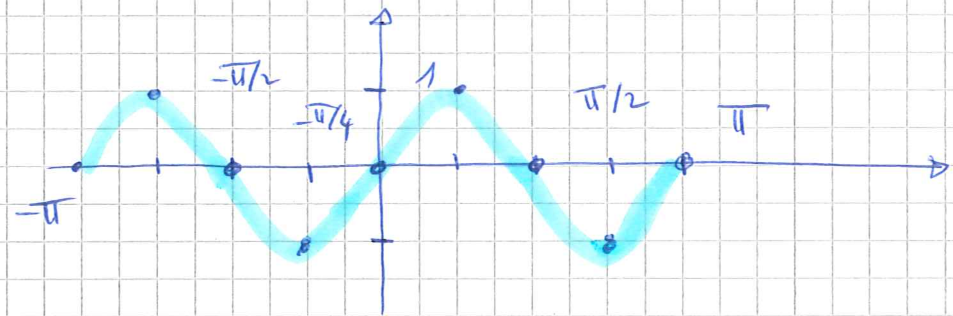
4.19

b) $y = -\sin\left(\frac{\pi}{4} - 2v\right) = \sin\left(2v - \frac{\pi}{4}\right)$

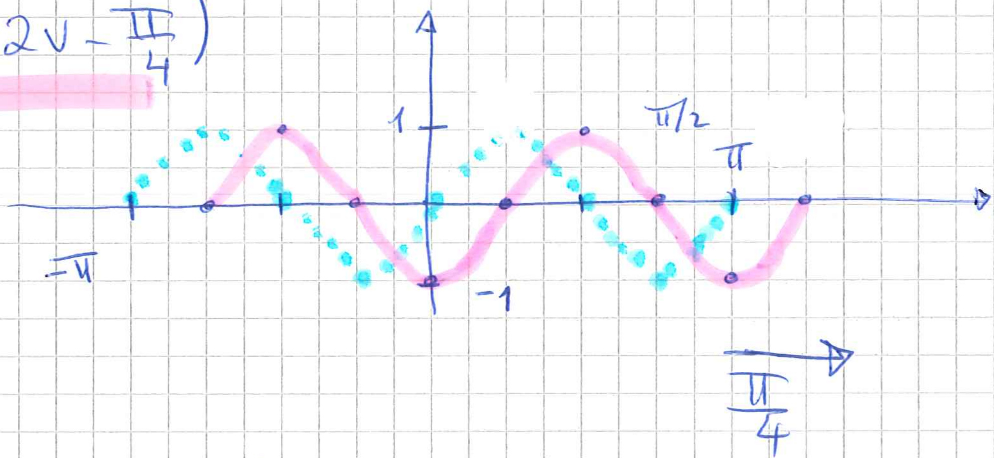
$y = \sin v$



$y = \sin 2v$



$y = \sin\left(2v - \frac{\pi}{4}\right)$

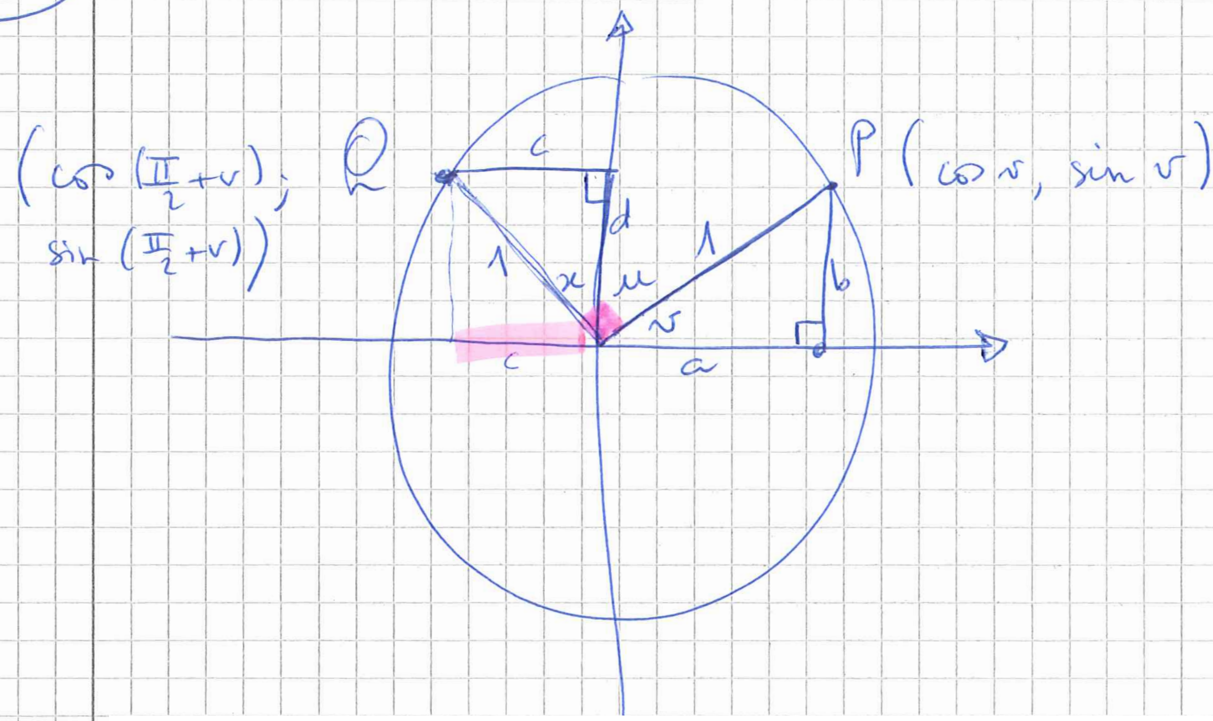


<u>steg 1</u> $y = \sin x$

<u>steg 2:</u> dubblera perioden

<u>steg 3:</u> flytta hela grafen med $\pi/4$ enheter åt höger

4.20 a_1) $\sin\left(\frac{\pi}{2}+v\right) = \cos v$



x = v och trianglarna har båda en vinkel som är 90 grader

dvs trianglarna är likformiga pga 3 vinklar som är lika samt en sida som är radien = 1 ⇒ trianglarna är kongruenta

Detta innebär att alla motsvarande sidor är identiska

$a = d$ och $c = b$ (c är negativ)

$$-\cos\left(v+\frac{\pi}{2}\right) = \sin v$$

dessutom har vi $a = d$ (båda positiva)

$$\cos v = \sin\left(\frac{\pi}{2}+v\right)$$

uppgiften frågan om enhetscirkeln,

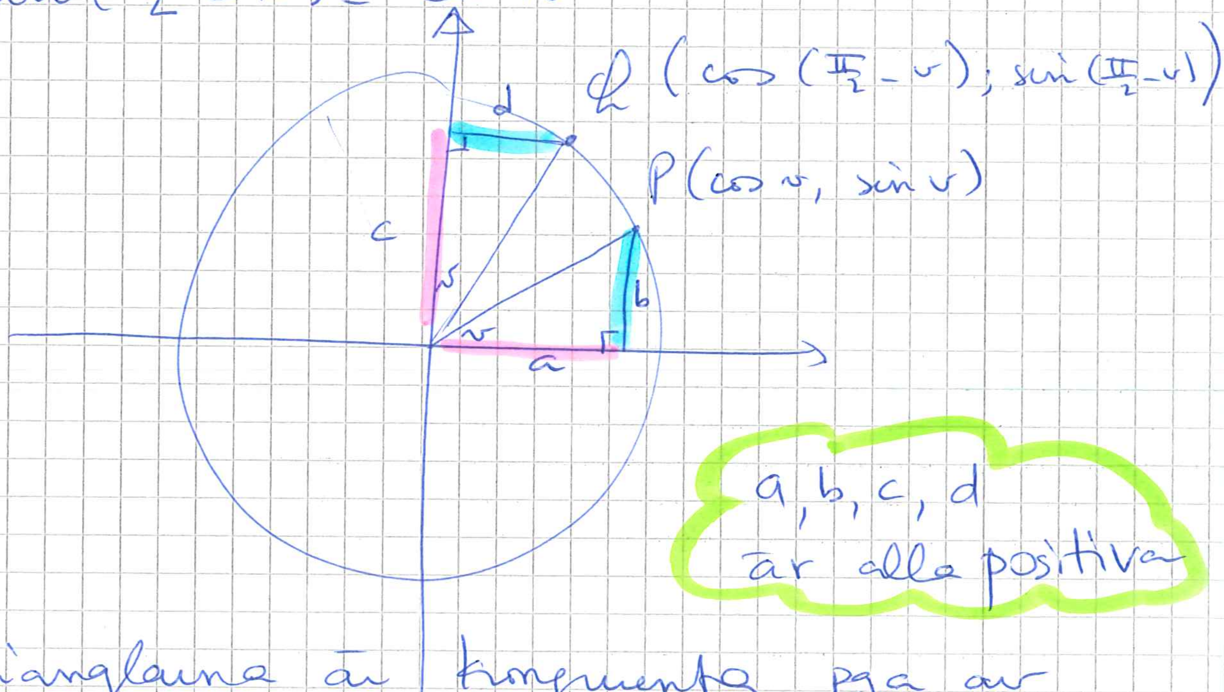
men samma resultat kan man få genom:

$$\sin\left(\frac{\pi}{2}+v\right) = \sin\frac{\pi}{2}\cos v + \sin v \cos\frac{\pi}{2}$$

$$= 1 \cdot \cos v + \sin v \cdot 0$$

$$\sin\left(\frac{\pi}{2}+v\right) = \cos v$$

$a_2)$ $\sin\left(\frac{\pi}{2}-v\right) = \cos v$



a, b, c, d är alla positiva

trianglarna är kongruenta pga av likformighet VVV och radien som ger samma skala.

$a = c$ och $b = d$

$\cos v = \sin\left(\frac{\pi}{2}-v\right)$ och $\sin v = \cos\left(\frac{\pi}{2}-v\right)$

På samma sätt:

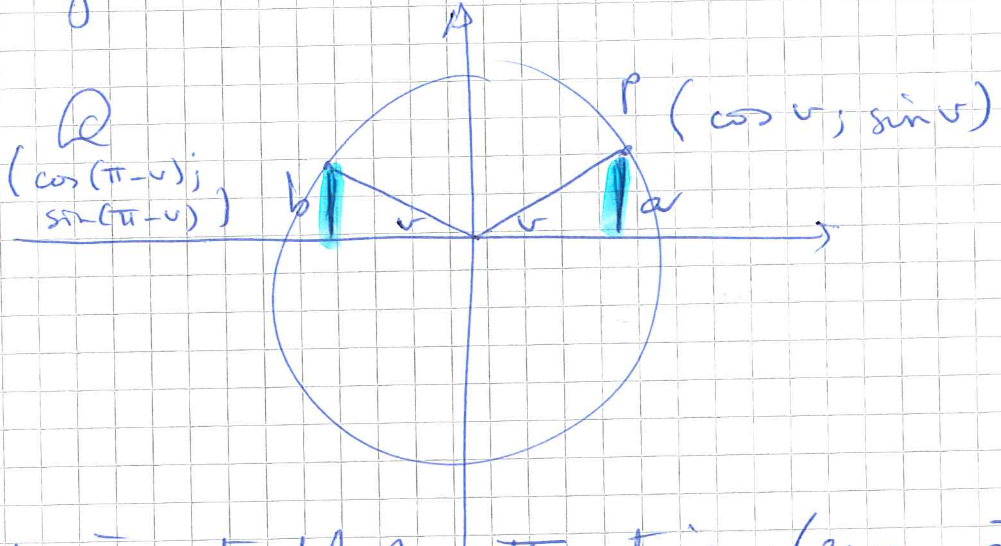
$$\sin\left(\frac{\pi}{2}-v\right) = \sin\frac{\pi}{2}\cos v - \sin v \cos \pi/2$$

$$= \cos v$$

b) $\sin v = \cos\left(\frac{\pi}{2} - v\right)$

har bevisats i förra uppgift a2)

vi gör nu $\sin v = \sin(\pi - v)$



Det är tydligt att trianglerna är kongruenta dvs $a = b$

$$\sin v = \sin(\pi - v)$$

och båda är positiva

stämmer det?

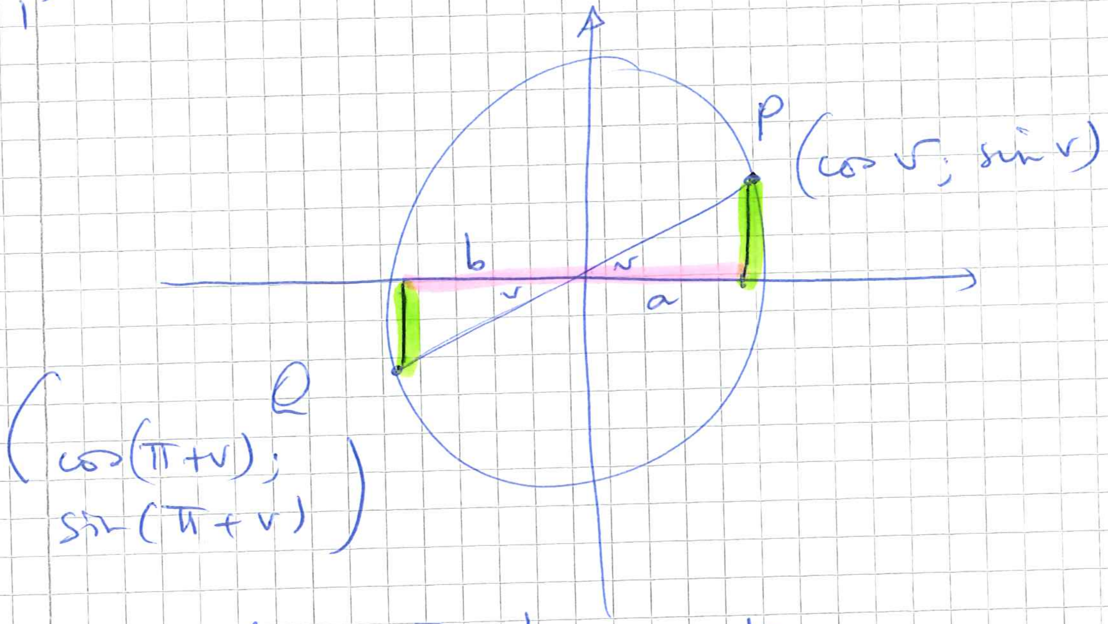
$$\sin(\pi - v) = \sin\pi \cos v - \sin v \cos\pi$$

$$= 0 \cdot \cos v - \sin v(-1)$$

$$= +\sin v$$

$\blacksquare$

c) $\cos(\pi + v) = -\cos v$



trianglarna är kongruenta.

$a = b$ men åt olika håll

$\cos v = -\cos(\pi + v)$

eller $\cos(\pi + v) = -\cos v$

vi har också att $-\sin v = \sin(\pi + v)$

stämmer det?

$\cos(\pi + v) = \cos\pi \cos v - \sin\pi \sin v$

$= -\cos v$

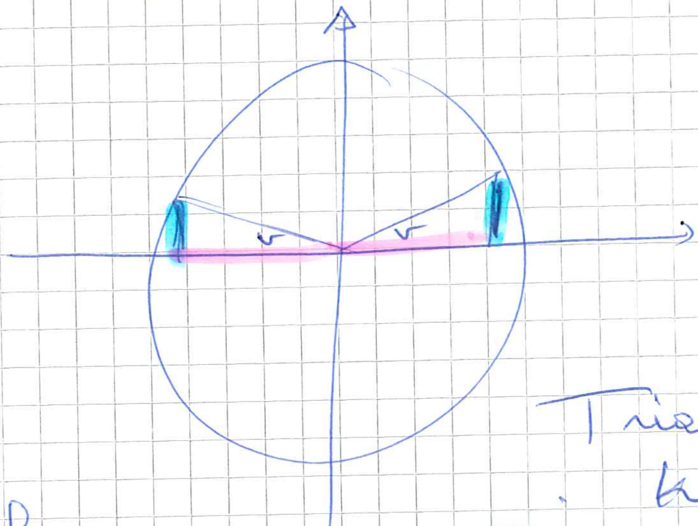
och

$\sin(\pi + v) = \sin\pi \cos v + \cos\pi \sin v$

$= -\sin v$

■

(c) $\cos(\pi - v) = -\cos v$



Trianglarna är kongruenta.

De har samma sinus
men motsatta cosinus

<u>dvs</u> $\cos(\pi - v) = -\cos v$

och $\sin(\pi - v) = \sin v$

stämmer det?

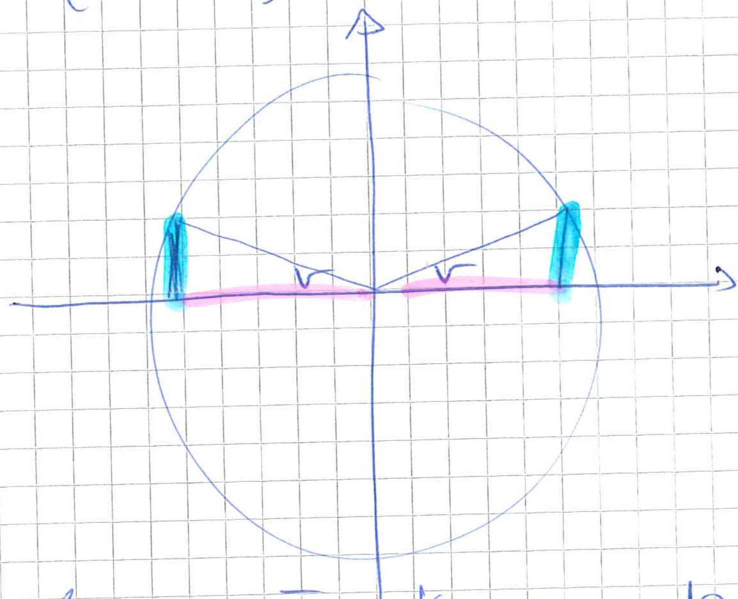
$$\cos(\pi - v) = \cos\pi \cos v + \sin\pi \sin v$$
$$= -\cos v$$

och

$$\sin(\pi - v) = \sin\pi \cos v - \sin v \cos\pi$$
$$= 0 - \sin v(-1)$$
$$= \sin v$$

■

d) $\tan(\pi - v) = -\tan v$



triangulerna är kongruenta

de har:

samma sinus: $\sin v = \sin(\pi - v)$

olika cosinus: $\cos(\pi - v) = -\cos v$

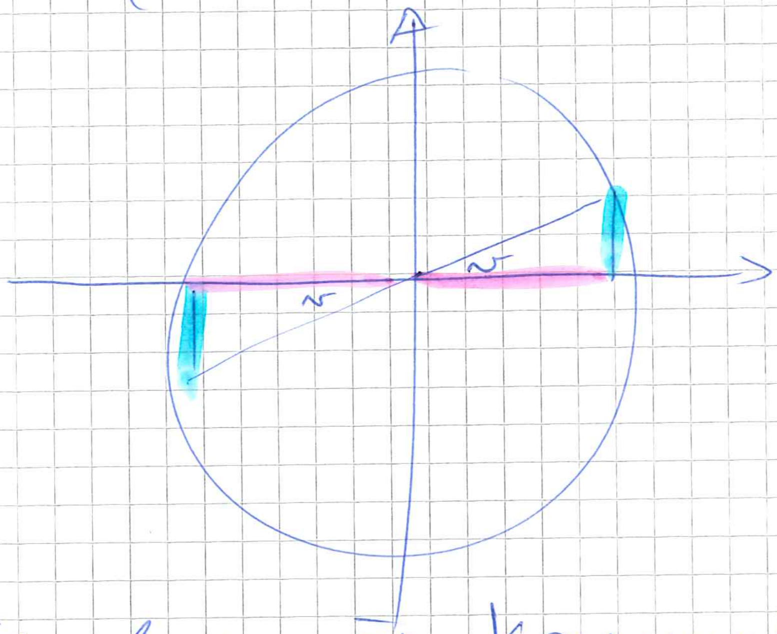
(samma men olika håll, olika tecken!)

$$\tan(\pi - v) = \frac{\sin(\pi - v)}{\cos(\pi - v)} = \frac{\sin v}{-\cos v}$$

$$= -\frac{\sin v}{\cos v} = -\tan v$$

$\blacksquare$

e) $\tan(\pi + v) = \tan v$



trianglarna är kongruenta
de har:
samma sinus fast olika håll dvs
olika tecken $\sin(\pi + v) = -\sin v$

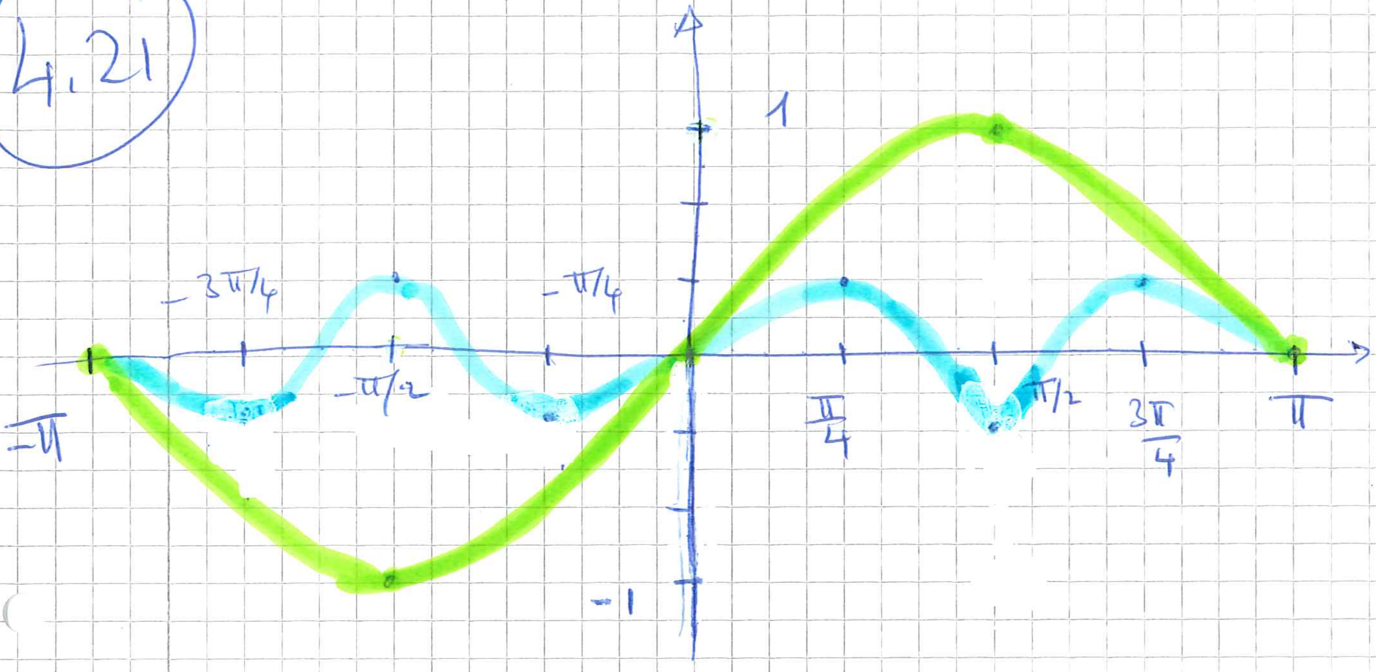
samma cosinus fast åt olika håll dvs
olika tecken $\cos(\pi + v) = -\cos v$

$$\tan(\pi + v) = \frac{\sin(\pi + v)}{\cos(\pi + v)} = \frac{-\sin v}{-\cos v}$$

$$= \frac{\sin v}{\cos v} = \tan v$$

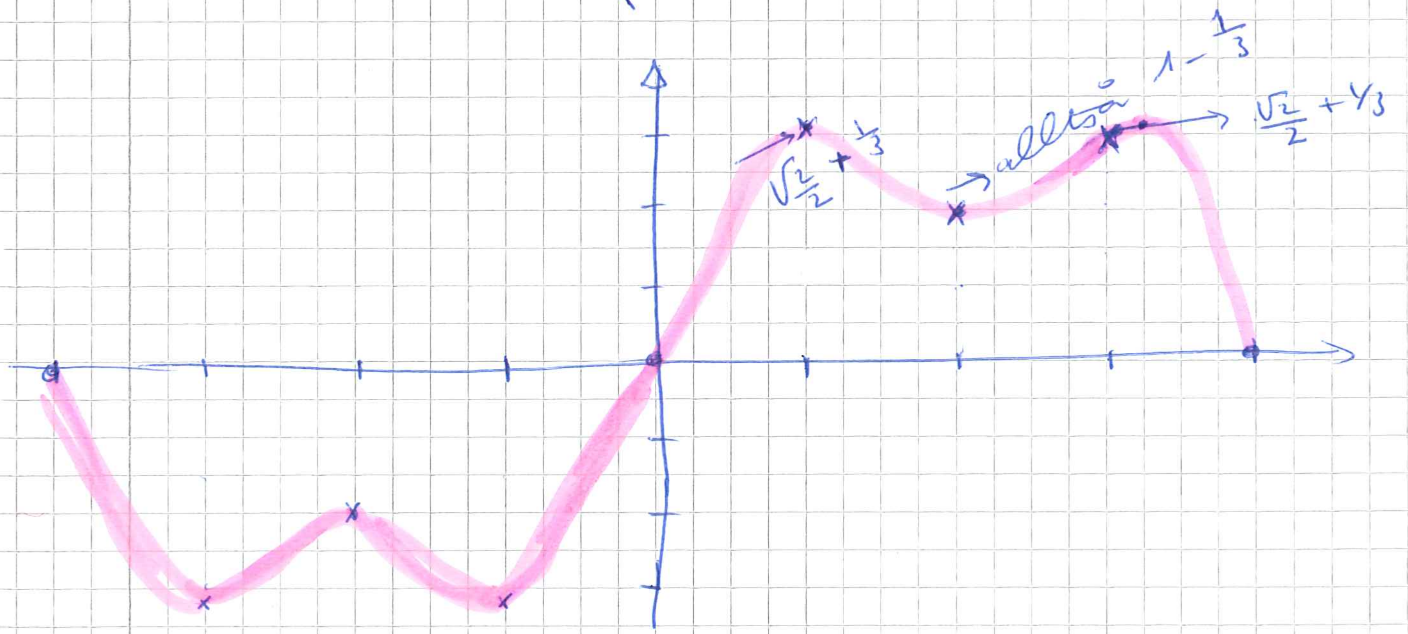
■

4.21

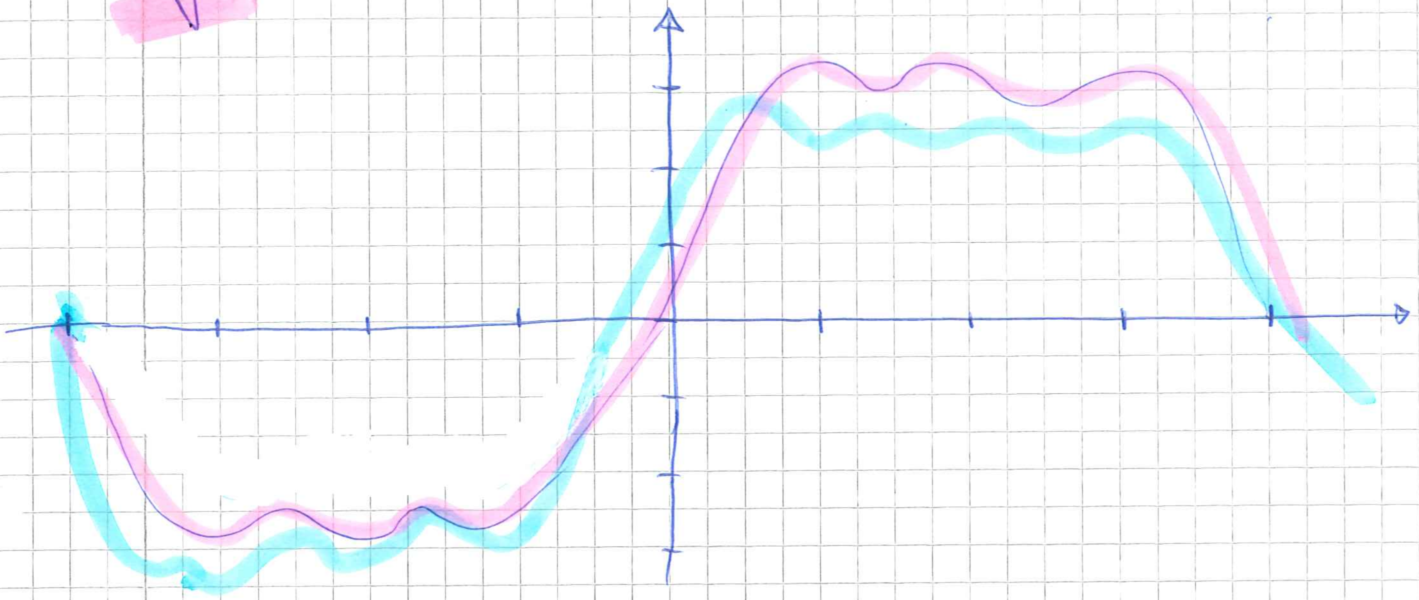


$y = \sin x$ $y = \frac{1}{3} \sin 3x$

Dessutom $y = \sin x + \frac{1}{3} \sin 3x$



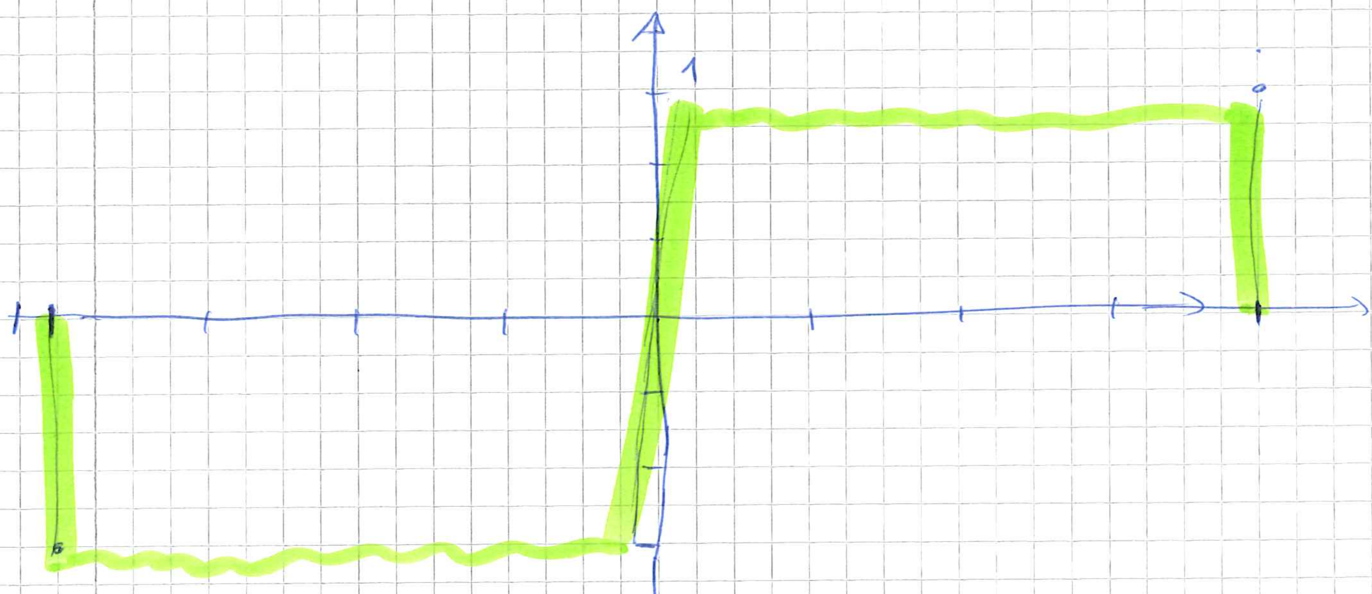
$$y = \sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x$$



$$y = \sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x$$

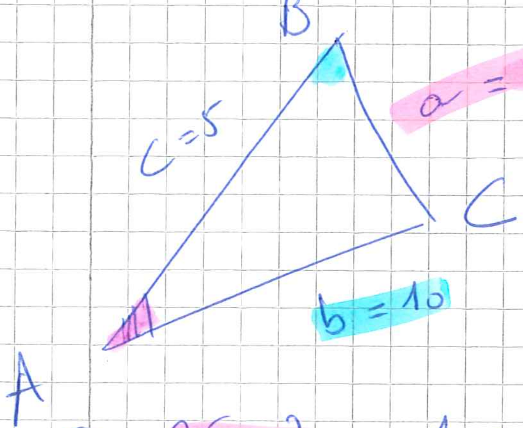
osv -- osv --

i slutändan kommer det att se ut:



4.22 Cosinussatsen

$$a^2 = b^2 + c^2 - 2bc\cos A$$



(*) $25 \cdot 3 = 100 + 25 - 2 \cdot 10 \cdot 5 \cos A$

$$\frac{75 - 125}{-100} = \cos A$$

$\cos A = \frac{1}{2}$ $\qquad A = \frac{\pi}{3}$ $\qquad A = 60^\circ$

eftersom $0 \leq A \leq \pi$

$0 \leq A \leq 180^\circ$

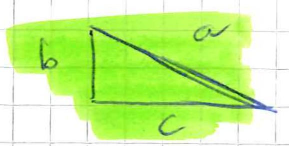
(*) $100 = 25 + 25 \cdot 3 - 2 \cdot 5 \cdot 5\sqrt{3} \cos B$

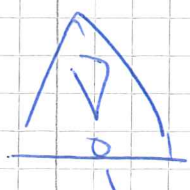
$100 - 25 - 75 = -50\sqrt{3}\cos B$

$0 = \cos B$ $\qquad B = \frac{\pi}{2} = 90^\circ$

dvs $C = 30^\circ$

$C = \frac{\pi}{6}$.





Pythagoras sats är ett speciellt fall av Cosinussatsen när $\cos v = 0$

$\Longrightarrow a^2 = b^2 + c^2$

4.23

$$\frac{\tan^2 v}{1+\tan^2 v} = \frac{\frac{\sin^2 v}{\cos^2 v}}{\frac{\cos^2 v}{\cos^2 v} + \frac{\sin^2 v}{\cos^2 v}}$$

$$= \frac{\sin^2 v}{\cancel{\cos^2 v}} \cdot \frac{\cancel{\cos^2 v}}{(\cos^2 v + \sin^2 v)}$$

$$= \frac{\sin^2 v}{1} = \sin^2 v \qquad \blacksquare$$

4.24 $\cos 3x = \cos(2x + x)$ A18

$$= \cos 2x \cos x - \sin 2x \cdot \sin x$$

$$= (1 - 2\sin^2 x)\cos x - 2\sin x \cos x \cdot \sin x$$

A13 A12

$$= \cos x - 2\sin^2 x \cos x - 2\cos x \sin^2 x$$

$$= -4\sin^2 x \cdot \cos x + \cos x$$

$$= -4(1 - \cos^2 x) \cdot \cos x + \cos x$$

$$= -4\cos x + 4\cos^3 x + \cos x$$

$$= -3\cos x + 4\cos^3 x$$

$\blacksquare$

trigonometriske ettan!

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

4.25

$$\frac{1-\cos^2 x}{\sin 2x} = \frac{\sin^2 x}{2\sin x \cos x}$$

$$= \frac{\sin x \cdot \cancel{\sin x}}{2 \cdot \cancel{\sin x} \cdot \cos x}$$

$$= \frac{1}{2} \frac{\sin x}{\cos x} = \frac{1}{2} \tan x$$

4.26 $\sqrt{2} \sin\left(x + \frac{\pi}{4}\right) =$

a) $\sqrt{2}\left[\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}\right] =$

$\sqrt{2}\left[\sin x \cdot \frac{1}{\sqrt{2}} + \cos x \frac{1}{\sqrt{2}}\right] =$

$\sin x + \cos x$

b) $\sin x + \cos x = 1$ Kan ersättas med

$$\sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = 1$$

$$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$x + \frac{\pi}{4} = \frac{\pi}{4} + 2\pi n$ eller $x + \frac{\pi}{4} = \pi - \frac{\pi}{4} + 2\pi n$

$x_1 = 2\pi n$ eller $x_2 = \frac{2\pi}{4} + 2\pi n$

$x_2 = \frac{\pi}{2} + 2\pi n$

4.28

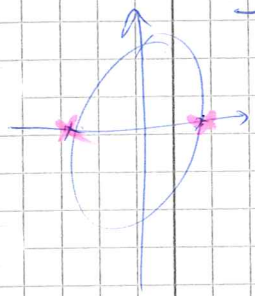
$\sin 2x \cos x = 2 \sin^3 x$

$2 \cdot \cos x \cdot \sin x \cdot \cos x - 2 \sin^3 x = 0$

$2 \sin x \left(\cos^2 x - \sin^2 x \right) = 0$

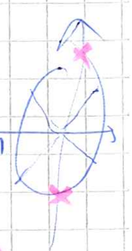
$2 \sin x \cdot \cos 2x = 0$

<u>dvs</u> $\sin x = 0$ eller $\cos 2x = 0$



$x = n\pi$

$2x = \pm \frac{\pi}{2} + 2\pi n$

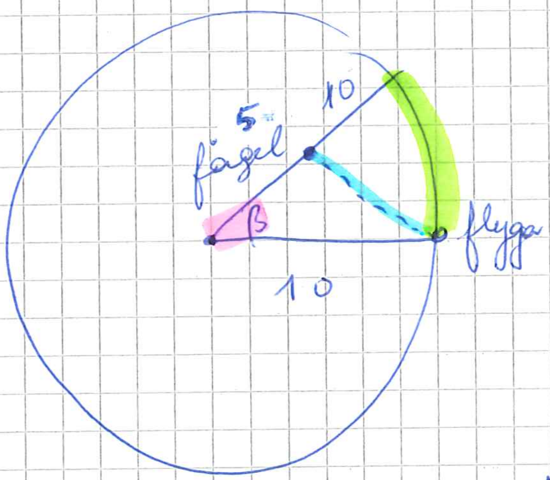


$x = \pm \frac{\pi}{4} + \pi n$

eller $x = \frac{\pi}{4} + \frac{\pi n}{2}$

(det är samma vinklar)

4.31



fågeln rör sig 5 m

$\Rightarrow$ flygan 2,5 m

eftersom fågeln rör sig dubbelt så fort

$b = 2,5$ m

$b = r\beta \quad \beta = \frac{b}{r} = \frac{2,5}{10} = \frac{1}{4} \quad \beta = \frac{1}{4}$

x = avståndet

Cosinussatsen

$x^2 = 5^2 + 10^2 - 2 \cdot 5 \cdot 10 \cdot \cos \frac{1}{4}$

$= 125 - 100 \cdot \cos \frac{1}{4}$

$x^2 = 28,2$

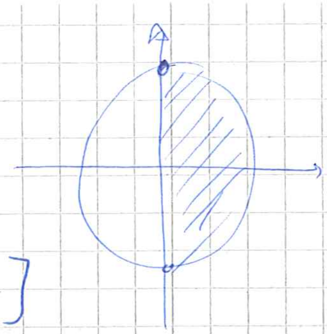
$x = 5,3$ m

Använd radian här

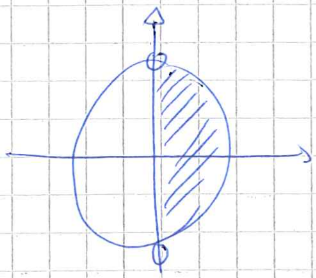
4.33

a) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

eftersom $-\frac{\pi}{6} \in \left[-\pi/2 ; \pi/2\right]$



b) $\arctan\left(-\frac{1}{\sqrt{3}}\right) = -\pi/6$



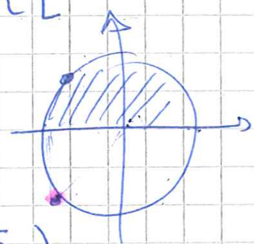
c) $\arctan\left(\tan \frac{5\pi}{3}\right) =$

$\arctan\left(\tan\left(\pi + \frac{2\pi}{3}\right)\right) =$

$\arctan\left(\tan\left(2\pi - \frac{\pi}{3}\right)\right) = -\pi/3$

eftersom $-\pi/3 \in \left]-\pi/2 ; \pi/2\right[$

d) $\arccos 0 = \pi/2$ (ikke $-\pi/2$)



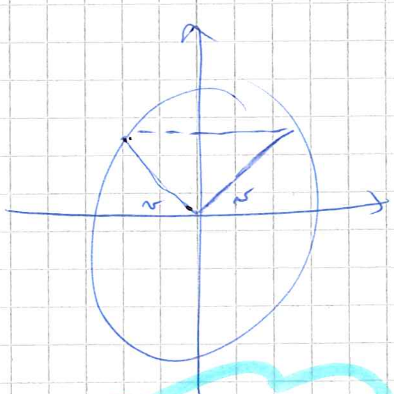
e) $\arccos\left(\sin -\frac{3\pi}{4}\right) = \arccos\left(-\frac{\sqrt{2}}{2}\right)$

$= \arccos\left(\cos\left(\pi - \pi/4\right)\right)$

$= \arccos\left(\cos\left(3\pi/4\right)\right)$

$= 3\pi/4$ som $\in [0 ; \pi]$

f) $\arcsin\left(\sin \frac{8\pi}{3}\right) =$



$\arcsin\left(\sin\left(2\pi + \frac{2\pi}{3}\right)\right) =$

$\arcsin\left(\sin \frac{2\pi}{3}\right) =$

$\arcsin\left(\sin\left(\pi - \frac{\pi}{3}\right)\right) =$

$\arcsin\left(\sin\left(\pi/3\right)\right) =$

$\arcsin\left(\sin \pi/3\right) = \pi/3$

$\in \left[-\pi/2 ; \frac{\pi}{2}\right]$

$\sin(\pi - v) = \sin v$

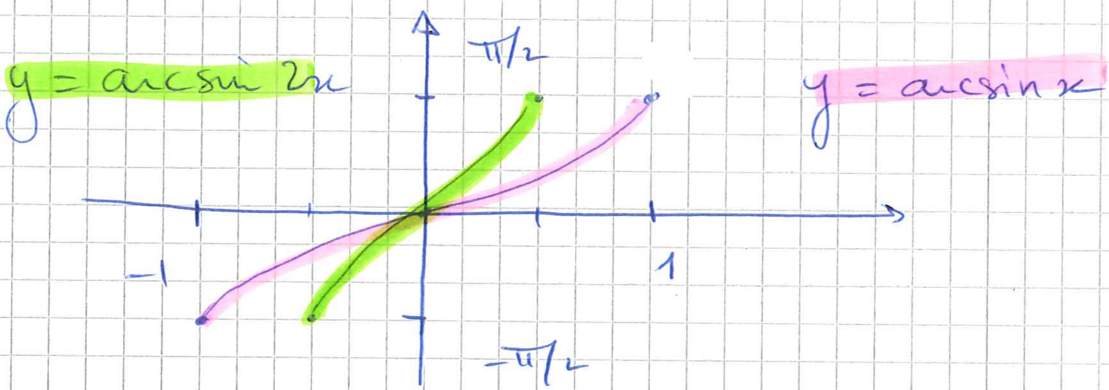
4.34

$y = \arcsin 2x$

Jämför med $y = \arcsin x$

$-1 \leq x \leq 1$

$-\pi/2 \leq y \leq \pi/2$



för $y = \arcsin 2x$

$-1 \leq 2x \leq 1$

$-\frac{1}{2} \leq x \leq \frac{1}{2}$

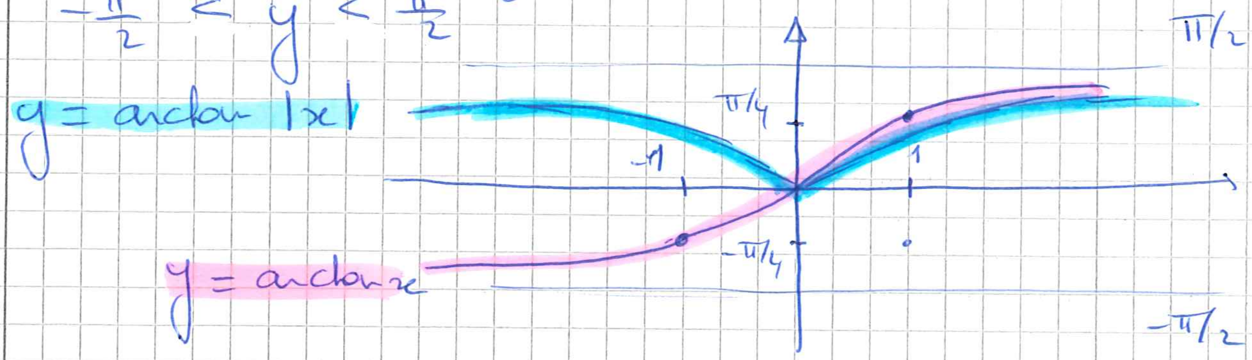
och $-\pi/2 \leq y \leq \pi/2$

4.35

$y = \arctan |x|$

börja med $y = \arctan (x)$

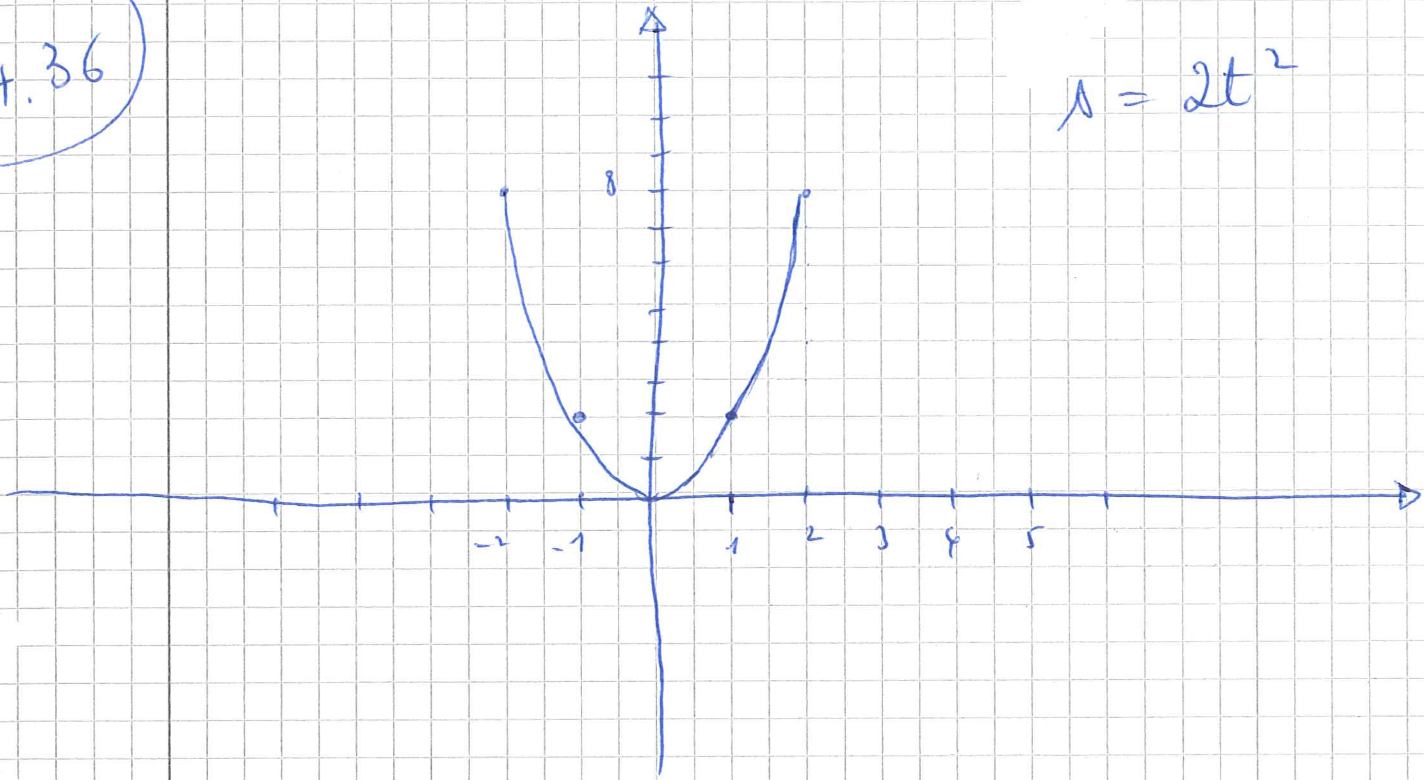
$-\frac{\pi}{2} < y < \frac{\pi}{2}$



$y = \arctan |x|$ låt alla y bli positiva pga alla $|x|$ är positiva

4.36

$s = 2t^2$



$$\begin{cases} a = 4 \\ v = 4t \\ s = 2t^2 \end{cases}$$

(1) $t = 1$ $\quad$ $t = 5$

$s = 1$ $\quad$ $s = 2 \cdot 25 = 50$

$(1,2)$ $\quad$ $(5;50)$

linjen mellan dem

$$k = \frac{50-2}{5-1} = \frac{48}{4} = 12$$

$$s - 2 = 12(t-1) = 12t - 12$$

$$s = 12t - 12 + 2 \qquad \boxed{s = 12t - 10}$$

(2) $t = 1$ $\quad$ $t_2 = 2$

$(1,2)$ $\quad$ $(2;8)$

$$k = \frac{8-2}{2-1} = \frac{6}{1} = 6$$

$$s - 2 = 6(t-1)$$

$$s = 6t - 6 + 2$$

$$\boxed{s = 6t - 4}$$

3 $t = 1$ $t = 1,2$

$s = 2 \cdot 1 = 2$ $s = 2 \cdot 1,2^2 = 2 \cdot 1,44$
$= 2,88$

$(1, 2)$ $(1,2 \, ; 2,88)$

linjen mellan dem

$$k = \frac{2,88 - 2}{1,2 - 1} = \frac{0,88}{0,2} = 4,4$$

$$s - 2 = 4,4(t - 1)$$
$$= 4,4t - 4,4$$

$$\boxed{s = 4,4t - 2,4}$$

linjerna är

$s = 12t - 10$

$s = 6t - 4$

$s = 4,4t - 2,4$

De representerar medelhastigheten mellan två punkter.

4.37

$$A(t) = 1{,}5 \sin\left(\frac{4}{3}\pi t + \frac{\pi}{3}\right)$$

a) $t = 0 \quad A(0) = 1{,}5 \sin\left(\frac{\pi}{3}\right)$

$$= 1{,}5 \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{3}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{4}$$

b) $A(t) = 0{,}75 \implies$

$$0{,}75 = 1{,}5 \sin\left(\frac{4}{3}\pi t + \pi/3\right)$$

$$\frac{0{,}75}{1{,}5} = \sin\left(\frac{4}{3}\pi t + \pi/3\right)$$

$$0{,}5 = \sin\left(\frac{4}{3}\pi t + \frac{\pi}{3}\right)$$

$\frac{4\pi}{3}t + \frac{\pi}{3} = \frac{\pi}{6} + 2\pi n$ eller $\frac{4\pi}{3}t + \frac{\pi}{3} = \pi - \frac{\pi}{6} + 2\pi n$

$$\frac{4t}{3} = \frac{1}{6} - \frac{1}{3} + 2n$$

$$4t = -\frac{3}{6} + 6n$$

$$4t = -\frac{1}{2} + 6n$$

$$t = -\frac{1}{8} + \frac{3}{2}n$$

$$\frac{4}{3}t = 1 - \frac{1}{6} - \frac{1}{3} + 2n$$

$$\frac{4}{3}t = \frac{1}{2} + 2n$$

$$t = \frac{3}{8} + \frac{6n}{4}$$

$$t = \frac{3}{8} + \frac{3n}{2}$$

$t_1 = -\frac{1}{8}$ $(n=0)$

$t_2 = 1{,}375$ $(n=1)$

$t_3 = 2{,}875$ $(n=2)$

$t_4 = 4{,}375$ $(n=3)$

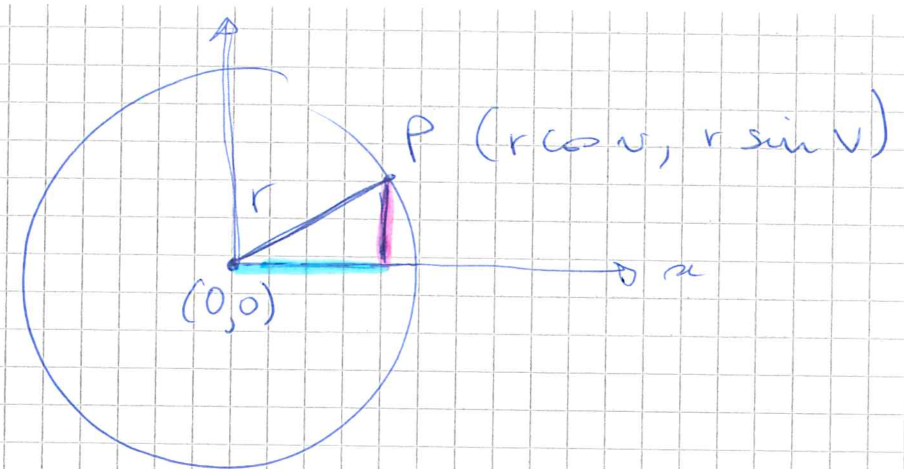
$t_5 = 0{,}375$ $(n=0)$

$t_6 = 1{,}875$ $(n=1)$

$t_7 = 3{,}375$ $(n=2)$

$t_8 = 4{,}875$ $(n=3)$

4.38



$\sin v = \dfrac{y}{r}$ $\qquad y = r \cdot \sin v$

$\cos v = \dfrac{x}{r}$ $\qquad x = r \cdot \cos v$

Stämmer det?

$$x^2 + y^2 = r^2\cos^2 v + r^2 \sin^2 v$$
$$= r^2(\cos^2 v + \sin^2 v)$$
$$= r^2 \cdot (1)$$
$$= r^2$$

(cirkelns ekvation och dessutom: Pythagoras sats i triangeln)

4.39

$$\frac{2x^2 - 4x + 3}{x^2 - 1}$$

(Välj $x = 3 \cdot 10^{10}$) undersök!

eller

$$\frac{\cancel{x^2}\left(2 \cancel{- \frac{4}{x}} \cancel{+ \frac{3}{x^2}}\right)}{\cancel{x^2}\left(1 \cancel{- \frac{1}{x^2}}\right)}$$

$$= \frac{2}{1} = 2$$

går mot "2".

x är jättestor dvs $\frac{1}{x}$ är nästan noll