$\sin(150) = \sin(90+60) = \; - \quad -$

$\sin(240) = \sin(270-30) =$

Trigonometriska ekvationer (T. E.)

Def Om en ekvation innehåller

Sin, Cos, tan eller Cot

kallas för T. E

Ex $\sin x = 0{,}5$

$\cos x + \sin x = 1$

$\tan x = 2\sin 3x$ O. S. V.

Jas delar T. E i två delar

1. Vanliga ekvationer
2. Ovanliga "

Vanliga

$\sin x = 0.5$ Vad är x?

För vilka x gäller

$\cos x = -1/3$

$$\begin{cases} \sin x = \text{ett tal} \\ \cos x = \\ \tan x = \end{cases} \quad \begin{cases} \sin x = -0.5 \\ \cos x = 0.3 \\ \tan x = 5 \end{cases}$$

Hur löser vi vanliga

$\sin x = 0.5$

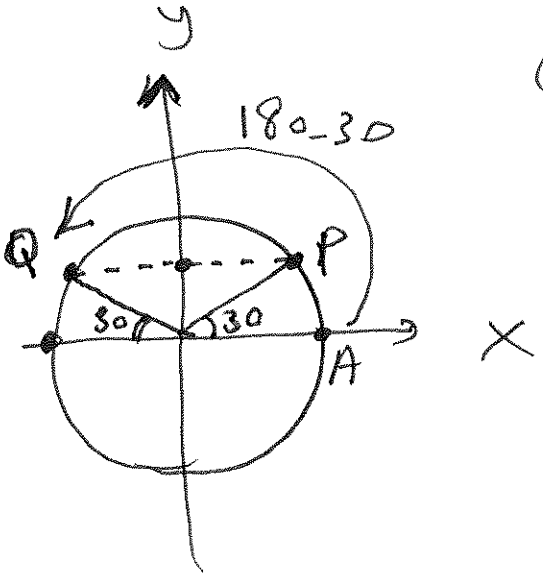
Miniräknare ger $x = 30^\circ = \pi/6$

Men det finns många som har $\sin = 0.5$

t.ex $30 + 360 = 390$

$30 + 720 = 750$

(3)



P och Q har
samma y

detta innebär att
de har samma Sin

Alla vinklar som slutas på P eller Q
har Sin = 0.5

a) Vilka slutas på punkt P?

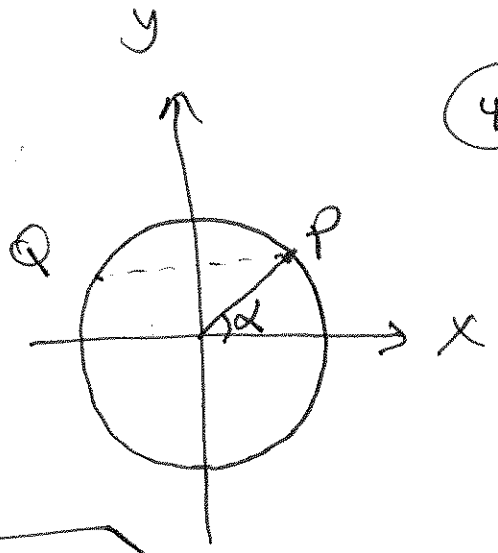
Svar: $360n + 30°$

(b) vilka slutas på Q

$360n + 180 - 30$

Svar:
$$\begin{cases} x = 360n + 30 \\ x = 360n + 180 - 30 \end{cases} \quad n \in \mathbb{Z}$$

I allmänt



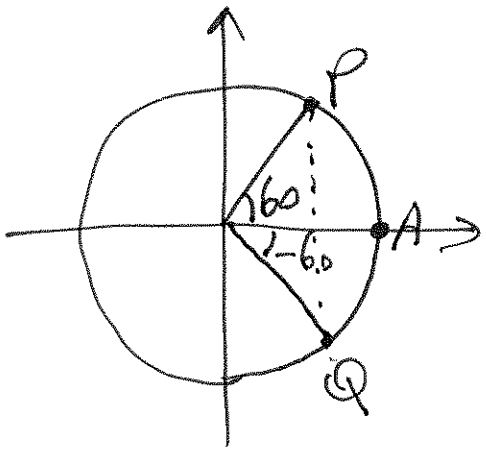
$\sin x = \sin\alpha$

$\Downarrow$

$$\begin{cases} x = 360n + \alpha \\ x = 360n + 180 - \alpha \end{cases} \quad n \in \mathbb{Z}$$

$\cos x = a \rightarrow$ ett tal

$\cos x = 0{,}5 = \cos 60$



P och Q har samma x

alltså samma cos

Vilka slutar på P?

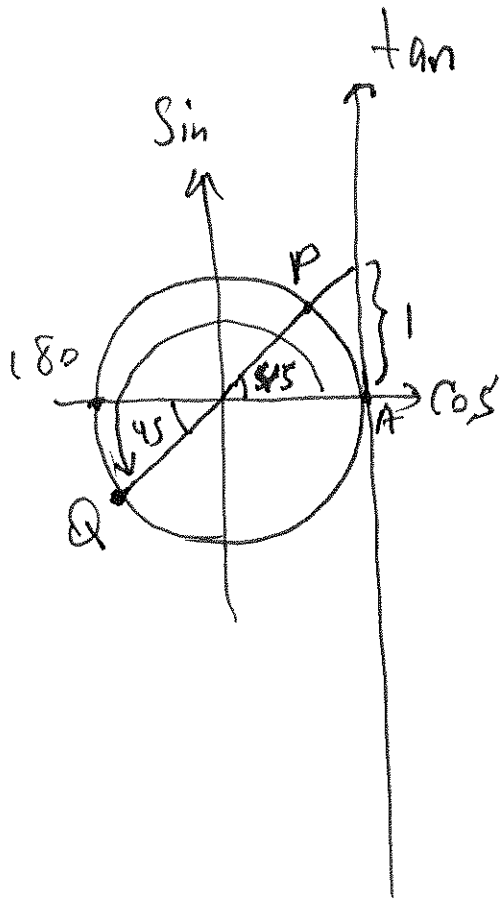
Svar: $360n + 60$

Vilka slutar på Q

Svar: $360n - 60$

$x = 360n \pm 60$

$\tan x = 1 = \tan 45$



Vilka slutar på P?

Svar: $360n + 45$

Vilka slutar på Q

Svar: $360n + 180 + 45$

$$\begin{cases} x = 360n + 45 \\ x = 360n + 180 + 45 \end{cases} \quad n \in \mathbb{Z}$$

~~$x = 360$~~ ~~$x = 2(180n) + 45$~~

$x =$

$$\begin{cases} x = 2n(180) + 45 \\ x = (2n+1)(180) + 45 \end{cases}$$

$$x = 180n + 45$$

Samman fattning

$\sin x = \sin \alpha$,

$\tan x = \tan \alpha \Rightarrow \underline{x = 180n + \alpha}$

Några speciella

$$\begin{cases} \sin x = 0 \rightarrow x = 180n \\ \cos x = 0 \rightarrow \\ \sin x = 1 \end{cases}$$

$\sin x = 0 = \sin 0$

$$\begin{cases} x = 360n + 0 \\ x = 360n + 180 - 0 \end{cases}$$

$x = 2n \cdot 180$

$x = (2n+1) \cdot 180$ $\Rightarrow$ $\boxed{x = 180n}$

$\cos x = 1 = \cos 0$

$x = 360n$

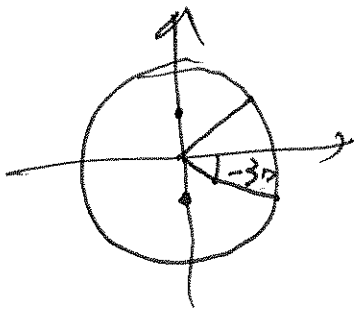
$\sin x = 1 = \sin 90$

$x = 360n + 90$

~~$x = 360n + 180 - 90$~~

$\sin x = -0.5 = \sin(-30)$

$\begin{cases} x = 360n - 30 \\ x = 360n + 180 + 30 \\ \qquad\quad 210 \end{cases}$



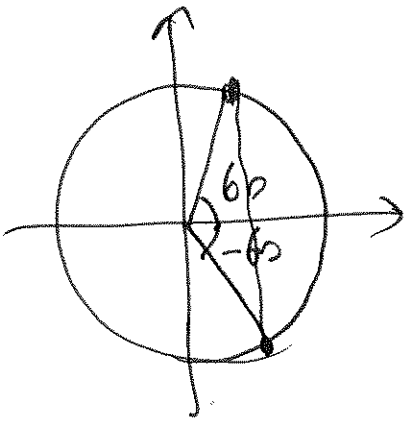
$\boxed{\alpha = -30}$

$\cos x = -0.5$

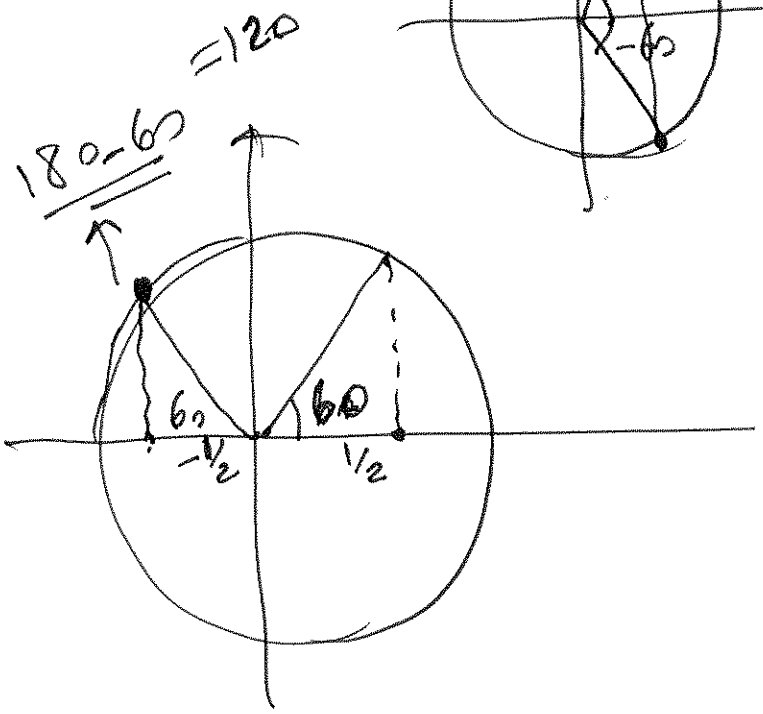
Obs: $\cos 60 = 0.5$

Fel — det är fel att ta $\underline{\underline{-60}}$

$\cos(-60) = +0.5$



Rätt



$\cos(120) = -0.5$

$\cos(180-\alpha) = -\cos\alpha$

Svar $\boxed{x = 360n \pm 120}$

$$\sin x = \sin\alpha \Rightarrow \begin{cases} x = 360n + \alpha \\ x = 360n + 180 - \alpha \end{cases}$$

$$\cos x = \cos\alpha \Rightarrow \begin{cases} x = 360 \pm \alpha \end{cases}$$

$$\tan x = \tan\alpha \Rightarrow x = 180n + \alpha$$

$360n = 2\pi n$

$\sin x = 0.5 = \sin \pi/6$

$$\begin{cases} x = 2n\pi + \pi/6 \\ x = 2n\pi + \pi - \pi/6 \end{cases} \qquad n \in \mathbb{Z}$$

Övanliga

Typ 1

Ekvationer som innehåller båda tan och cot av samma vinkel.

$$\tan x + \cot x = 2$$

Metod Multiplicera med $\tan x$

$$\tan^2 x + \tan x \cdot \cot x = 2 \tan x$$

$$\tan^2 x + 1 = 2 \tan x$$

$$\boxed{\tan x = t}$$

$$t^2 + 1 = 2t$$

$$t^2 - 2t + 1 = 0 \Rightarrow t = 1$$

$\Rightarrow \tan x = 1$ (vanlig)

$= \tan 45$

$\Rightarrow \boxed{x = 180n + 45 \quad n \in \mathbb{Z}}$

Anm $\tan(3x) + \cot(3x) = 2$

Mult. $\tan(3x)$

$\tan^2(3x) + 1 = 2\tan 3x$

$\tan(3x) = 1 = \tan 45$

$3x = 180n + 45$

$\boxed{x = 60n + 45/3}$

Typ 2

Ekvationen av formen

$$a\sin x + b\cos x = c$$

t.ex $\sin x + \cos x = \sqrt{2}$ $\begin{cases} a=1 \\ b=1 \\ c=\sqrt{2} \end{cases}$

eller

$\sqrt{3}\sin x + \cos x = 1$ $\begin{cases} a=\sqrt{3} \\ b=1 \\ c=1 \end{cases}$

Generell metod

dela ekvationen med $\sqrt{a^2+b^2}$

$$\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x = \frac{c}{\sqrt{a^2+b^2}}$$

$\frac{a}{\sqrt{a^2+b^2}} = \cos\alpha$

$\frac{b}{\sqrt{a^2+b^2}} = \sin\alpha$

Det måste finnas en vinkel α

så att

$$\begin{cases} \cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}} \\ \sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}} \end{cases}$$

eftersom $\left(\dfrac{a}{\sqrt{a^2+b^2}}\right)^2 + \left(\dfrac{b}{\sqrt{a^2+b^2}}\right)^2 = 1$

ekvationen blir

$$\underbrace{\cos\alpha \sin x + \sin\alpha \cos x} = \frac{c}{\sqrt{a^2+b^2}}$$

$$\sin(\alpha + x) = \frac{c}{\sqrt{a^2+b^2}}$$

$$\alpha + x = t$$

$$\sin t = \frac{c}{\sqrt{a^2+b^2}} \quad \underline{\text{Vanlig}}$$

Ex

$$\sqrt{3}\sin x + \cos x = 1$$

$$a = \sqrt{3} \quad b = 1 \quad c = 1$$

$$\sqrt{a^2+b^2} = \sqrt{3+1} = \sqrt{4} = 2$$

dela ekvationen med 2

$$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{2}$$

$\frac{\sqrt{3}}{2} = \cos\alpha$ $\quad$ $\frac{1}{2} = \sin\alpha$

$$\alpha = 30$$

$$\cos 30 \cdot \sin x + \sin 30 \cdot \cos x = \tfrac{1}{2}$$

$$\sin(x+30) = \tfrac{1}{2}$$

$\sin(x+30) = \frac{1}{2} = \sin 30$

$x+30 = 360n + 30$

$x+30 = 360n + 150$

$x = 360n$

$x = 360n + 120$

$n \in \mathbb{Z}$

Andra metod

$\sqrt{3}\sin x + \cos x = 1$

$\sin x = t$

$\cos x = \sqrt{1-t^2}$

$\cos^2 x = 1 - \sin^2 x$

$= 1 - t^2$

$\cos x = \sqrt{1-t^2}$

$$\sqrt{3}\,t + \sqrt{1-t^2} = 1$$

rot - ekvation

$$\sqrt{1-t^2} = 1 - \sqrt{3}\,t$$

Kvadrera

$$\left(\sqrt{1-t^2}\right)^2 = \left(1-\sqrt{3}\,t\right)^2$$

$$1 - t^2 = 1 + 3t^2 - 2\sqrt{3}\,t$$

$$4t^2 - 2\sqrt{3}\,t = 0$$

$$2t\left(2t - \sqrt{3}\right) = 0$$

$$\begin{cases} t = 0 \\ 2t - \sqrt{3} = 0 \end{cases} \rightarrow t = \frac{\sqrt{3}}{2}$$

$\sin x = 0$ — vanlig

$\sin x = \frac{\sqrt{3}}{2}$ — vanlig

Typ 3

Ekvationer som innehåller

(a) $\sin^2 x \quad \cos^2 x \quad \cos x$

eller

(b) $\sin^2 x \quad \cos^2 x \quad \sin x$

I (a) kan vi skriva om som

$1 - \cos^2 x \quad \cos^2 x \quad \cos x$

$\cos x = t$

$1 - t^2 \quad t^2 \quad t$

Andra gradsekvation.

I del b ersætter vi:

$\cos^2 x$ med $1-\sin^2 x$

$\sin^2 x$, $1-\sin^2 x$, $\sin x$

Ex

$$\sin^2 x - \cos^2 x - \sin x = 0$$

$$\sin^2 x - (1-\sin^2 x) - \sin x = 0$$

$$\sin^2 x - 1 + \sin^2 x - \sin x = 0$$

$$\boxed{\sin x = t}$$

$$2t^2 - t - 1 = 0$$

$$t = \frac{1 \pm \sqrt{1+8}}{4} \quad \begin{matrix} \nearrow 1 \\ \searrow -1/2 \end{matrix}$$

$\sin x = 1 = \sin 90$

$x = 360n + 90$

$\sin x = -0.5$

$= \sin(-30)$

$x = 360n - 30$

$x = 360n + 180 + 30$

Typ 4

$\sin 5x = \sin 3x$

$\cos 7x = \cos 2x$

$\tan 9x = \tan 6x$

$\underbrace{\sin(5x) = \sin(3x)}_{x \qquad \alpha}$

$5x = 360n + 3x$

$5x = 360n + 180 - 3x$

$\sin x = \sin \alpha$

$x = 360n + \alpha$

$x = 360n + 180 - \alpha$

$5x - 3x = 360n$

$2x = 360n \Rightarrow$ $\boxed{x = 180n}$

$5x = 360n + 180 - 3x$

$8x = 360n + 180$

$\boxed{x = 45n + 45/2}$

$\tan(9x) = \tan(7x)$

$9x = 180n + 7x$

$2x = 180n$

$\boxed{x = 90n, \; n \in \mathbb{Z}}$

$\tan x = \tan \alpha$

$x = 180n + \alpha$

Typ 5

$\sin 5x = \cos 2x$

I typ 4 hade vi $\sin = \sin$

vi ändrar cos i HL till sin

$\cos\alpha = \sin(90-\alpha)$

$\cos(2x) = \sin(90-2x)$

Ekvationen blir

$\sin \underbrace{5x}_{x} = \sin(\underbrace{90-2x}_{\alpha})$ typ 4

$5x = 360n + 90 - 2x$

$5x = 360n + 180 - 90 + 2x$

Undang

$$\sin x + 2\sin x \cos 2x = 0$$

$$\sin x \left(1 + 2\cos 2x\right) = 0$$

$$\sin x = 0 \longrightarrow \boxed{x = 180n}$$

$$1 + 2\cos 2x = 0$$

$$\Downarrow$$

$$\cos(2x) = -0.5 = \cos 120$$

$$2x = 360n \pm 120$$

$$\boxed{x = 180n \pm 60}$$