Kapitel sex

6.1

a) $f(x) = 2x + 1$

$$k = \frac{f(x+h) - f(x)}{h} = \frac{2(x+h) + 1 - 2x - 1}{h}$$

$$= \frac{\cancel{2x} + 2h \cancel{+1} - \cancel{2x} \cancel{-1}}{h} = \frac{2h}{h} = 2$$

om $h \to 0$ $\lim_{h \to 0} k = 2$

b) $f(x) = 9x^2 + 3$

$$k = \frac{f(x+h) - f(x)}{h} = 9(x^2 + h^2 + 2xh) + 3 - 9x^2 - 3 \;/ h$$

$$= (\cancel{9x^2} + 9h^2 + 18xh \cancel{+3} - \cancel{9x^2} \cancel{-3}) / h$$

$$= \frac{\cancel{h}(9h + 18x)}{\cancel{h}} = 18x + 9h$$

om $h \to 0$ $\lim_{h \to 0} k = 18x$

c) $f(x) = \frac{1}{x}$

$$K = \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \frac{\cancel{x} - \cancel{x} - h}{h\,x\,(x+h)} = \frac{-\cancel{h}}{\cancel{h}\,x\,(x+h)} = \frac{-1}{x\,(x+h)}$$

$$\lim_{h \to 0} K = \frac{-1}{x^2}$$

d) $f(x) = \frac{1}{\sqrt{x}}$

$$K = \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \frac{\sqrt{x} - \sqrt{x+h}}{h\,(\sqrt{x+h})\,\sqrt{x}}$$

$$= \frac{(\sqrt{x} - \sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}{h\,\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{\cancel{x} - \cancel{x} - h}{h\,\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{-1}{\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x} + \sqrt{x+h})}$$

om $h \longrightarrow 0$

$$\lim_{h \to 0} K = \frac{-1}{\sqrt{x}\,\sqrt{x}\,(\sqrt{x} + \sqrt{x})} = \frac{-1}{2x\sqrt{x}}$$

6.2

a) $f(x) = (x - x^2)\cos x$

$f'(x) = (1-2x)\cos x$
$+ (x - x^2)(-\sin x)$

$f'(x) = (1-2x)\cos x - (x - x^2)\sin x$

b) $f(x) = x^5 - 2x^3 + x^2$

$f'(x) = 5x^4 - 6x^2 + 2x$

c) $f(x) = 3x^2 - 2\tan x$

$f'(x) = 6x - \dfrac{2}{\cos^2 x}$

$= 6x - 2(1 + \tan^2 x)$

d) $f(x) = \cos^2 x + \sin^2 x = 1$

$f'(x) = 0$

e) $f(x) = e^x \sin x$

$f'(x) = e^x \sin x + e^x \cos x$

$= e^x(\sin x + \cos x)$

f) $f(x) = 5x \cdot \ln x$

$f'(x) = 5\ln x + 5x \cdot \dfrac{1}{x}$

$= 5\ln x + 5$

6.3

a) $f(x) = \dfrac{2x}{x-2}$

$f'(x) = [2(x-2) - 1(2x)] / (x-2)^2$

$= \dfrac{\cancel{2x} - 4 - \cancel{2x}}{(x-2)^2} = \dfrac{-4}{(x-2)^2}$

b) $f(x) = \dfrac{\ln x}{x}$

$f'(x) = \left(\frac{1}{x} \cdot x - 1 \cdot \ln x\right) / x^2$

$= \dfrac{1 - \ln x}{x^2}$

c) $f(x) = \dfrac{1}{\tan x} = \dfrac{1}{\frac{\sin x}{\cos x}} = \dfrac{\cos x}{\sin x}$

$f'(x) = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\dfrac{1}{\sin^2 x}$

d) $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$

$f'(x) = [2x(x^2+1) - 2x(x^2-1)] / (x^2+1)^2$

$= (\cancel{2x^3} + 2x - \cancel{2x^3} + 2x) / (x^2+1)^2$

$= \dfrac{4x}{(x^2+1)^2}$

e) $f(x) = \dfrac{e^x}{e^{x-1}} = \dfrac{e^x \cdot e^1}{e^{x-1} \cdot e^1}$

$= \dfrac{\cancel{e^x} \cdot e}{\cancel{e^{x-1+1}}} = e$

$f'(x) = 0$

f) $f(x) = x^{-2,7}$ $\quad x > 0$

$f'(x) = -2,7\, x^{-2,7-1}$

$= -2,7\, x^{-3,7}$

6.4

a) $f(x) = \sin x^2$

$f'(x) = 2x \cdot \cos x^2$ (Kedje-regeln)

b) $f(x) = \sin(\arcsin x) = x$

$f'(x) = 1$

c) $f(x) = \dfrac{\cos^2 x}{\tan x}$

$f'(x) = \dfrac{-2\cos x \cdot \sin x \cdot \overset{\frac{\sin x}{\cos x}}{\tan x} - \frac{1}{\cos^2 x} \cdot \cos^2 x}{\tan^2 x}$

$f'(x) = \dfrac{-2\sin^2 x - 1}{\tan^2 x} = -\dfrac{2\sin^2 x + 1}{\tan^2 x}$

d) $$f(x) = \frac{(\sin x^2)^{n+1}}{x(n+1)}$$

glöm inte kedjeregeln !

$(\sin x^2)^{n+1}$ deriveras till :

$$(n+1)(\sin x^2)^n \cdot \underbrace{(2x\cos x^2)}_{\text{inre derivatan}}$$

$$f'(x) = \left[(n+1)(\sin x^2)^n (2x\cos x^2) \cdot x(n+1) - (n+1)(\sin x^2)^{n+1}\right] \Big/ x^2(n+1)^2$$

$$f'(x) = \cancel{(n+1)}(\sin x^2)^n \left[\begin{array}{l} 2x^2\cos x^2 (n+1) \\ - 1 \cdot \sin x^2 \end{array}\right] \Big/ x^2(n+1)^{\cancel{2}}$$

$$f'(x) = \frac{(\sin x^2)^n \left(2x^2(n+1)\cos x^2 - \sin x^2\right)}{x^2(n+1)}$$

$$= 2(\sin x^2)^n \cos x^2 - \frac{(\sin x^2)^{n+1}}{x^2(n+1)}$$

$\blacksquare$

6.5

a) $f(x) = \arctan x \cdot \ln\left(\frac{1}{1+x^2}\right)$

produktregeln ska användas här

$$f'(x) = \frac{1}{1+x^2} \cdot \ln\left(\frac{1}{1+x^2}\right)$$

$$+ \arctan x \cdot \frac{-2x}{(1+x^2)}$$

$$f'(x) = \frac{1}{1+x^2}\left(\ln \frac{1}{1+x^2} - 2x \arctan x\right)$$

$$f'(1) = \frac{1}{2}\left(\ln \frac{1}{2} - 2 \arctan 1\right)$$

$$= \frac{1}{2} \ln \frac{1}{2} - \frac{2}{2} \frac{\pi}{4}$$

$$= \frac{1}{2} \ln 1 - \frac{1}{2} \ln 2 - \frac{\pi}{4}$$

$$= 0 - \frac{1}{2} \ln 2 - \pi/4$$

$$= -\frac{1}{2} \ln 2 - \frac{\pi}{4}$$

b) $f(x) = \frac{x \cdot \ln x}{x^2+1}$

$f'(x) = [(1 \cdot \ln x + x \cdot \frac{1}{x})(x^2+1) - 2x(x \cdot \ln x)] / (x^2+1)^2$

$= \frac{(\ln x + 1)(x^2+1) - 2x^2 \ln x}{(x^2+1)^2}$

$f'(1) = \frac{(\cancel{\ln 1} + 1)(1+1) - \cancel{2 \cdot 1 \cdot \ln 1}}{(1+1)^2}$

$f'(1) = \frac{2}{2^2} = 2^{-1} = 1/2$

c) $f(x) = \frac{x^2+3}{(x^3-3)^2}$

$f'(x) = \frac{2x(x^3-3)^{\cancel{2}} - 2 \cdot 3x^2 \cancel{(x^3-3)}(x^2+3)}{(x^3-3)^{\cancel{4}\,3}}$

$f'(x) = \frac{2x(x^3-3) - 6x^2(x^2+3)}{(x^3-3)^3}$

$f'(1) = \frac{2(-2) - 6(4)}{(-2)^3}$

$= \frac{-4-24}{-8} = \frac{28}{8} = \frac{14}{4} = 7/2$

d) $f(x) = \ln|\arctan x|$

$$f'(x) = \frac{\frac{1}{1+x^2}}{\arctan x}$$

$$f'(1) = \frac{\frac{1}{1+1}}{\arctan 1} = \frac{1}{2} \cdot \frac{1}{\frac{\pi}{4}}$$

$$= \frac{1}{2} \cdot \frac{4}{\pi} = \frac{2}{\pi}$$ ■

6.6 a) $f(x) = \sin^4 x - 2\sin^2 x$

$$f'(x) = 4\sin^3 x \cos x - 4 \sin x \cos x$$

$f'(x) = 0 \implies$

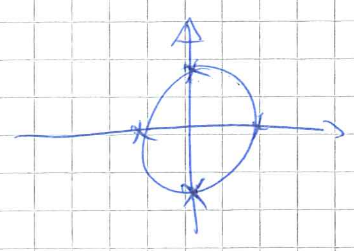
$$4 \sin x \cos x (\sin^2 x - 1) = 0$$

$\sin x = 0 \implies x = n\pi$

eller $\cos x = 0 \implies x = \pm \pi/2 + 2\pi n$

eller $\sin x = 1 \implies x = \pi/2 + 2\pi n$

eller $\sin x = -1 \implies x = -\pi/2 + 2\pi n$



$$x = n\frac{\pi}{2}$$ ■

b) $f(x) = \ln |x \cdot e^{2x}|$

$$f'(x) = \frac{1 \cdot e^{2x} + x \cdot 2e^{2x}}{x \cdot e^{2x}}$$

$$= \frac{\cancel{e^{2x}}(1+2x)}{\cancel{e^{2x}} \cdot x}$$

$f'(x) = 0 \implies 1 + 2x = 0$

$2x = -1$ $\quad x = -1/2$

c) $f(x) = \dfrac{2x}{1+x^2}$

$$f'(x) = \frac{2(1+x^2) - 2x(2x)}{(1+x^2)^2}$$

$f'(x) = 0 \implies$

$2 + 2x^2 - 4x^2 = 0$

$2 - 2x^2 = 0$

$2x^2 = 2 \quad x^2 = 1 \quad x = \pm 1$

d) $f(x) = \arctan(1-x^2)$

$$f'(x) = \frac{1 \cdot (-2x)}{1 + (1-x^2)^2} = \frac{-2x}{1+(1-x^2)^2}$$

$f'(x) = 0$ dvs $-2x = 0$

$x = 0$

6.7

$f(x) = 3x^3 - 3x^2 - 25x + 2$

tangentens ekvation:

$$y = f'(x_0)(x - x_0) + y_0$$

i punkten $x_0 = 2$

$y_0 = f(x_0) = f(2) = 3 \cdot 8 - 3 \cdot 4 - 25 \cdot 2 + 2$

$y_0 = 24 - 12 - 50 + 2 = -36$

$f'(x) = 9x^2 - 6x - 25$

$f'(2) = 9 \cdot 4 - 6 \cdot 2 - 25 = -1$

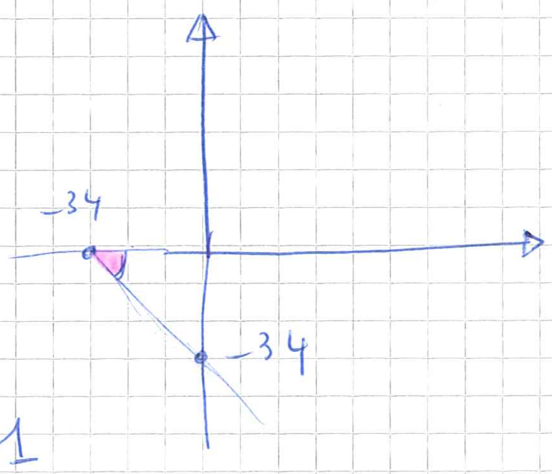
tangentens ekvation:

$y = -1 \cdot (x - 2) + (-36)$

$y = -x - 34$

$x = 0 \Rightarrow y = -34$

$y = 0 \Rightarrow x = -34$



$\tan v = \frac{-34}{-34} = 1$

$v = \pi/4$

6.8

$$y = \frac{2x-1}{x^2+2}$$

$$y'(x) = \frac{2(x^2+2) - 2x(2x-1)}{(x^2+2)^2}$$

$$y' = \frac{2x^2+4-4x^2+2x}{(x^2+2)^2}$$

$$y' = \frac{-2x^2+2x+4}{(x^2+2)^2}$$

Tangenten parallell med x-axeln innebär att $y'(x) = 0$

$\Longrightarrow$

$$-2x^2+2x+4=0$$

$$x^2-x-2=0$$

Jag ser att $x_1 = -1$ $\quad (1+1-2)$

$x_2 = 2$

eller

$$x = \frac{1}{2} \pm \sqrt{\frac{1}{4}+\frac{4 \cdot 2}{4}}$$

$$x = \frac{1}{2} \pm \sqrt{9/4}$$

$$x = \frac{1}{2} \pm \frac{3}{2} \qquad x_1 = \frac{4}{2} = 2$$

$$x_2 = \frac{-2}{2} = -1$$

∎

6.9

$$f(x) = \arctan(x^2 - 4)$$

$$f(2) = \arctan(4-4) = 0$$

$$f'(x) = \frac{2x \cdot 1}{1 + (x^2-4)^2}$$

$$f'(2) = \frac{2 \cdot 2 \cdot 1}{1 + (4-4)^2}$$

$$f'(2) = \frac{4}{1+0} = 4$$

$$y = f'(2)(x-2) + f(2)$$

$$y = 4(x-2) + 0$$

$$y = 4x - 8$$

6.10

$$f(x) = 3x^2 - 2x$$

$$f'(x) = 6x - 2$$

$$f'(1) = 6 - 2 = 4$$

$$f'(-1) = -6 - 2 = -8$$

tangent i $(1,1)$

$$y = 4(x-1) + 1 = 4x - 4 + 1 = 4x - 3$$

tangent i $(-1;5)$

$$y = -8(x+1) + 5 = -8x - 8 + 5 = -8x - 3$$

skärningspunkten : $4x - 3 = -8x - 3$

$$12x = 0 \quad x = 0$$

$x = 0 \quad y = -3$

$(0; -3)$

6.11 $T = 12 + 6\sin\left(\frac{\pi t - 8\pi}{12}\right)$

$T'(t) = 6 \cdot \frac{\pi}{12} \cos\left(\frac{\pi(t-8)}{12}\right) = \frac{\pi}{2}\cos\left(\frac{\pi}{12}(t-8)\right)$

$T'(4) = \frac{\pi}{2}\cos\left(\frac{\pi}{12}(4-8)\right) = \frac{\pi}{2}\cos\left(-\frac{\pi}{3}\right) = \frac{\pi}{4}$

$T'(8) = \frac{\pi}{2}\cos\left(\frac{\pi}{12}(8-8)\right) = \frac{\pi}{2}\cos(0) = \frac{\pi}{2}$

$T'(16) = \frac{\pi}{2}\cos\left(\frac{\pi}{12}(16-8)\right) = \frac{\pi}{2}\cos\left(\frac{4\pi}{3}\right) = -\frac{\pi}{4}$

Medelhastigheten

$T(6) = 12 + 6\sin\left(\frac{6\pi - 8\pi}{12}\right) = 12 - 3 = 9$

$T(18) = 12 + 6\sin\left(\frac{18\pi - 8\pi}{12}\right) = 12 + 3 = 15$

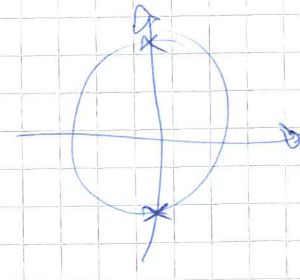
det är 12 timmar mellan 6 och 18

Medelhastighet : $\frac{T(18) - T(6)}{12} = \frac{15-9}{12} = \frac{1}{2}$

$0{,}5 \ °C/h$

Förändringshastighet är noll dvs

$T'(t) = 0$



$0 = \frac{\pi}{2}\cos\left(\frac{\pi}{12}(t-8)\right)$

$\frac{\pi}{12}(t-8) = \pm\frac{\pi}{2} + 2\pi n$

$\frac{1}{12}(t-8) = \pm\frac{1}{2} + 2n$

$t - 8 = \pm 6 + 24n$ $\qquad t = \pm 6 + 8$

$t_1 = 14$ dvs. kl. 2 på

$t_2 = 2$ morgonen och på e.m

6.12

$$N(t) = 750 + 10t^2$$

$$N'(t) = 20t$$

$$N_{rb} = \frac{N'(t)}{N} = \frac{20t}{750 + 10t^2}$$

$$N_{rb} = \frac{\cancel{10}(2t)}{\cancel{10}(75 + t^2)} = \frac{2t}{75 + t^2}$$

6.13

$$s(t) = 3000 \cdot \sin \frac{\pi t}{20}$$

$$s'(t) = 3000 \cdot \frac{\pi}{20} \cos \frac{\pi t}{20}$$

$$s'(0) = \frac{3000 \cdot \pi}{20} = 150\pi \approx 471 \text{ m/min}$$

fulfart

$$s'(5) = \frac{3000\pi}{20} \cdot \cos \frac{\pi \cdot 5}{20} = \boxed{\frac{150\pi \cdot \sqrt{2}}{2}}$$

$$= 333 \text{ m/min}$$

Vid tid $t = 5$ har hjulen hastigheten 333 m/min

ett varv = omkretsen = πd

$$= \boxed{\pi \cdot 0{,}76 \text{ m}}$$

ett varv = $\boxed{0{,}76\pi}$ m

hur många varv är $\boxed{\frac{150\pi\sqrt{2}}{2}}$

$$\frac{150\cancel{\pi} \cdot \frac{\sqrt{2}}{2}}{0{,}76\cancel{\pi}} = 139{,}56 \text{ varv/min}$$

hastigheten är ca 140 varv/min

6.14

$$M = 90 h^3$$

$$\frac{dM}{dh} = 90 \cdot 3 \cdot h^2$$

$$h \frac{dM}{dh} = 3 \cdot 90 h^3$$

$$h \frac{dM}{dh} = 3 \cdot M$$

6.15

$$F = -\frac{dV}{dr} \quad \text{med } V = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

$$F = -\frac{dV}{dr} = -4\varepsilon\left[\sigma^{12} \cdot (-12) r^{-13} - \sigma^{6} (-6) r^{-7}\right]$$

$$F = -4\varepsilon\left[-\frac{12\sigma^{12}}{r^{13}} + \frac{6\sigma^{6}}{r^{7}}\right]$$

6.16

$$H = \frac{t^2}{4} + t \qquad t \in \{4, 5, 6\}$$

$$H(6) = \frac{36}{4} + 6$$

$$H(4) = \frac{16}{4} + 4$$

$$\frac{H(6) - H(4)}{6 - 4} = \frac{15 - 8}{2}$$

$$= \frac{7}{2} = 3{,}5 \text{ mm/vecka.}$$

$$H'(t) = \frac{2t}{4} + 1$$

$$H'(5) = \frac{2 \cdot 5}{4} + 1 = \frac{10}{4} + 1 = 2{,}5 + 1 = 3{,}5 \text{ mm/vecka}$$

6.17 $\dfrac{dT}{dt} = -k(T - 20)$ $\quad k > 0$

6.18 $T \cdot V^{0,4} = k$

$T = k \cdot V^{-0,4}$

$\dfrac{dT}{dV} = k \cdot (-0,4) \cdot V^{-1,4}$

$\dfrac{dT}{dV} = k\,(-0,4) \cdot V^{-0,4} \cdot V^{-1}$

multipli med $(\cdot V)$

$V \cdot \dfrac{dT}{dV} = k \cdot V^{-0,4} \cdot (-0,4)$

$V \cdot \dfrac{dT}{dV} = T\,(-0,4)$

dela med T

$\dfrac{V}{T} \cdot \dfrac{dT}{dV} = -0,4$

6.19 $f(x) = 3 - 2x^5$ $\qquad 5 = 3 - 2x^5 \quad x = -1$

$(f^{-1})'(y) = \dfrac{1}{f'(x)} = \dfrac{1}{-10x}$

$(f^{-1})'(5) = \dfrac{1}{-10 \cdot (1)}$

$(f^{-1})'(5) = \dfrac{-1}{10}$

6.20 $(f^{-1})'(3)$ om $f(x) = \dfrac{x+4}{x-2}$

$$(f^{-1})'(y) = \frac{1}{f'(x)}$$

$$\frac{x+4}{x-2} = 3 \implies x+4 = 3x-6$$

$$2x = 10$$

$$x = 5$$

$$(f^{-1})'(3) = \frac{1}{f'(5)}$$

$$f'(x) = \frac{\cancel{x}-2-\cancel{x}-4}{(x-2)^2}$$

$$f'(x) = \frac{-6}{(x-2)^2}$$

$$= \frac{1}{-2/3}$$

$$f'(5) = \frac{-6}{(5-2)^2}$$

$$(f^{-1})'(3) = \frac{-3}{2}$$

$$= \frac{-6}{9} = -\frac{2}{3}$$

6.21 $f(x) = \arctan x$

$$\frac{\pi}{4} = \arctan x \qquad x = 1$$

$$f'(x) = \frac{1}{1+x^2} \qquad f'(1) = \frac{1}{1+1} = \frac{1}{2}$$

$$(f^{-1})'\left(\frac{\pi}{4}\right) = \frac{1}{f'(1)} = \frac{1}{\frac{1}{2}} = 2$$

Kom ihåg:

$$(f^{-1})'(y) = \frac{1}{f'(x)}$$

6.22

a) $$f(x) = (1-x^2)\arcsin x$$

$$f'(x) = -2x \cdot \arcsin x + (1-x^2)\frac{1}{\sqrt{1-x^2}}$$

$$f'(x) = -2x \arcsin x + \sqrt{1-x^2}$$ $\blacksquare$

b) $$f(x) = \ln(1+e^x)$$

$$f'(x) = \frac{e^x}{1+e^x} \qquad \left(\frac{u'}{u}\right)$$ $\blacksquare$

c) $$f(x) = \frac{\sqrt{1-x}}{\sqrt{1+x}} = \left(\frac{1-x}{1+x}\right)^{1/2}$$

$$f'(x) = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2} \cdot \underbrace{\qquad}_{}$$ une dérivation

$$f'(x) = \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}} \cdot \frac{-(1+x)-1(1-x)}{(1+x)^2}$$

$$f'(x) = \frac{1}{2}\frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{-1\cancel{-x}-1\cancel{+x}}{(1+x)^2}$$

$$f'(x) = \frac{1}{\cancel{2}}\frac{\cancel{\sqrt{1+x}}}{\sqrt{1-x}}(-\cancel{2}) \cdot \frac{1}{(1+x)\sqrt{1+x}\cancel{\sqrt{1+x}}}$$

$$f'(x) = \frac{-1}{\sqrt{1-x}\sqrt{1+x}(1+x)} = \frac{-1}{\sqrt{1-x}(1+x)^{3/2}}$$

ellen $$= \frac{-1}{\sqrt{1-x^2}(1+x)}$$ $\blacksquare$

6.23

$$x^2 + y^2 = r^2$$

$y \geq 0$

$y = \pm\sqrt{}$
men $y \geq 0$

$$2x + 2yy' = 0$$

$$2yy' = -2x$$

$$y' = -\frac{x}{y}$$

Men $y'(x)$
ska innehålla
bara x och
konstanter

$$y' = \frac{-x}{\sqrt{r^2 - x^2}}$$

6.24

$$x^2 - y^2 = 8 \qquad x^2 - 8 = y^2$$

implicit derivering ger

$$2x - 2yy' = 0 \qquad 2x = 2yy'$$

$$y' = \frac{x}{y} = \frac{x}{\sqrt{x^2 - 8}}$$

$(3;1)$

$$y = y'(3)(x-3) + 1$$

$$y = \frac{3}{\sqrt{9-8}}(x-3) + 1$$

$$y = 3x - 9 + 1 \qquad y = 3x - 8$$

(y är positiv
$y = 1$)

6.27 Kom ihåg

$$D\left(\ln|f(x)|\right) = \frac{f'(x)}{f(x)} \quad \left(\text{eller } \frac{U'}{U}\right)$$

dvs

$$f'(x) = f(x) \cdot D\left(\ln|f(x)|\right) \quad (*)(*)$$

a) $f(x) = (x+1)^3 (x^2-1)^2 e^{x^2}$

$$\ln|f(x)| = \ln(x+1)^3 + \ln(x^2-1)^2 + \ln e^{x^2}$$

$$= 3\ln(x+1) + 2\ln(x^2-1) + x^2$$

$$D\left(\ln|f(x)|\right) = \frac{3 \cdot 1}{x+1} + 2 \cdot \frac{2x}{x^2-1} + 2x$$

$$= \frac{3}{x+1} + \frac{4x}{x^2-1} + 2x$$

$$f'(x) = (x+1)^3 (x^2-1)^2 e^{x^2} \left(\frac{3}{x+1} + \frac{4x}{x^2-1} + 2x\right)$$

enligt (*)(*)

(*) Kom ihåg: logaritm lagarna!

$$\log(A \cdot B) = \log A + \log B$$

$$\log(A/B) = \log A - \log B$$

$$\log a^X = X \cdot \log a.$$

b) $f(x) = \dfrac{\sin^2 x \cos^3 x}{\sin 3x \cos^2 2x}$

$$\ln|f(x)| = \ln\left|\frac{\sin^2 x \cos^3 x}{\sin 3x \cdot \cos^2 2x}\right|$$

$$= \ln|\sin^2 x \cos^3 x| - \ln|\sin 3x \cos^2 2x|$$

$$= \ln|\sin^2 x| + \ln|\cos^3 x| - \ln|\sin 3x| - \ln|\cos^2 2x|$$

$$= 2\ln|\sin x| + 3\ln|\cos x| - \ln|\sin 3x| - 2\ln|\cos 2x|$$

$$D(\ln|f(x)|) = 2\frac{\cos x}{\sin x} + 3\frac{-\sin x}{\cos x}$$

$$\frac{3\cos 3x}{\sin 3x} - 2\frac{-2\sin 2x}{\cos 2x}$$

$$D(\ln|f(x)|) = 2\frac{\cos x}{\sin x} - 3\frac{\sin x}{\cos x} - \frac{3\cos 3x}{\sin 3x} + \frac{4\sin 2x}{\cos 2x}$$

$$= \frac{2}{\tan x} - 3\tan x - \frac{3}{\tan 3x} + 4\tan 2x$$

$$f'(x) = \left(\frac{2}{\tan x} - 3\tan x - \frac{3}{\tan 3x} + 4\tan 2x\right) \cdot \left(\frac{\sin^2 x \cos^3 x}{\sin 3x \cos^2 2x}\right)$$

c) $f(x) = x^3 \cdot e^{\sin x}$

$\ln|f(x)| = \ln|x^3| + \ln|e^{\sin x}|$

$= 3\ln|x| + \sin x$

$D(\ln|f(x)|) = \frac{3}{x} + \cos x$

$f'(x) = \left(\frac{3}{x} + \cos x\right) x^3 e^{\sin x}$ ■

d) $f(x) = x^{\cos x}$

$\ln|f(x)| = \ln|x^{\cos x}|$

$= \cos x \cdot \ln x$

$D(\ln|f(x)|) = \frac{1}{x}\cos x - \sin x \ln x$

(produktregeln här)

och

$f'(x) = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \cdot \ln x\right)$

■

6.28 a) $f(x) = \cos 3x$

$f'(x) = -3 \sin 3x$

$f''(x) = -9 \cos 3x$

$f'''(x) = 27 \sin 3x$ ■

b) $f(x) = \ln x$

$f'(x) = \frac{1}{x}$ $\quad (x^{-1})$

$f''(x) = \frac{-1}{x^2}$ $\quad (-x^{-2})$

$f'''(x) = \frac{2}{x^3}$ ■

c) $f(x) = x^2 - x$

$f'(x) = 2x - 1$

$f''(x) = 2$

$f'''(x) = 0$ ■

6.29 $S = 3000 \cdot \sin \frac{\pi t}{20}$

$S' = 3000 \frac{\pi}{20} \cos \frac{\pi t}{20}$

$S'' = 3000 \frac{\pi}{20} \left(\frac{-\pi}{20}\right) \sin \frac{\pi t}{20}$

$S''(5) = \frac{-30\cancel{00} \cdot \pi^2}{4\cancel{00}} \cdot \sin \frac{5\pi}{20} = -7{,}5 \cdot \pi^2 \cdot \sin \frac{\pi}{4}$

$= -\frac{30\pi^2 \sqrt{2}}{4 \cdot 2} = -7{,}5 \cdot \pi^2 \cdot \frac{\sqrt{2}}{2} \approx -52{,}3$ ■