Vi gick genom

1 – Polynom funktion $y = x^2 + 1$ $D_f = R$

2 – Rationella $y = \dfrac{x^2+1}{x^3 - 2x}$ $D_f = R - \{\text{rötterna till } N.\}$

3 exp $y = a^x$ $D_f = R$ $V_f = R^+$

4 logaritm $y = \log_a x$ $D_f = R^+$ $V_f = R$

5 Trigonometriska funktioner.

Def: $y = \sin x$, $y = \cos x$, $y = \tan x$

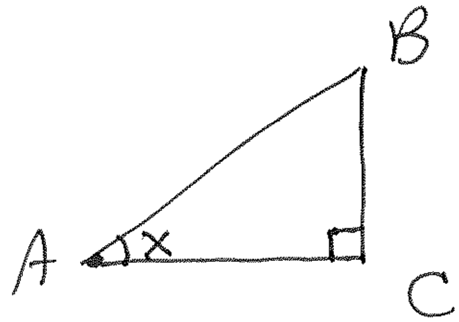
och $y = \cot x$ kallas för trigonometriska funktioner.

Definieras på två sätt

1. Genom rät vinklig triangel.
2. Genom enhet cirkel

 (en cirkel med radie = 1) i ett koord. system.

Genom Rät vinklig triangel.



$$\sin x = \frac{\text{motstående sida}}{\text{hypotenusan}} = \frac{BC}{AB}$$

$$\cos x = \frac{\text{närliggande katet}}{\text{hypotenusan}} = \frac{AC}{AB}$$

$$\tan x = \frac{\text{motstående}}{\text{närliggande}} = \frac{BC}{AC}$$

$$\cot x = \frac{\text{närliggande}}{\text{motstående}} = \frac{AC}{BC}$$

Några enkla trigonometriska formler.

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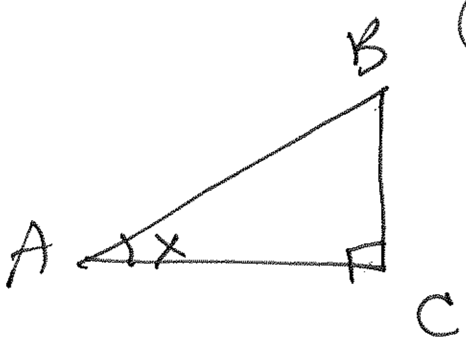
1. Trigonometriska ettan

$$\sin^2 x + \cos^2 x = 1$$



$$\sin x = \frac{BC}{AB}$$

$$\cos x = \frac{AC}{AB}$$

$$\sin^2 x + \cos^2 x = \frac{BC^2}{AB^2} + \frac{AC^2}{AB^2}$$

$$= \frac{BC^2 + AC^2}{AB^2} = \frac{AB^2}{AB^2} = 1$$

2) $$\boxed{\tan x = \frac{\sin x}{\cos x}}$$

Beviş: $$\frac{\sin x}{\cos x} = \frac{\frac{BC}{\cancel{AB}}}{\frac{AC}{\cancel{AB}}} = \frac{BC}{AC} = \tan x$$

3) $$\cot x = \frac{1}{\tan x}$$

4) $$\tan x \cdot \cot x = 1$$

5 $$1+\tan^2 x = \frac{1}{\cos^2 x}$$

$$VL = 1+\tan^2 x = \frac{1}{1} + \frac{\sin^2 x}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

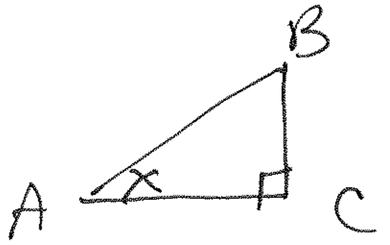
6 $$1+\cot^2 x = \frac{1}{\sin^2 x}$$

Beräkning av Sin, Cos, tan

Och Cot av

$x = 0°, 30°, 45°, 60°$

$90°$.



$x = 0^\circ \Rightarrow$

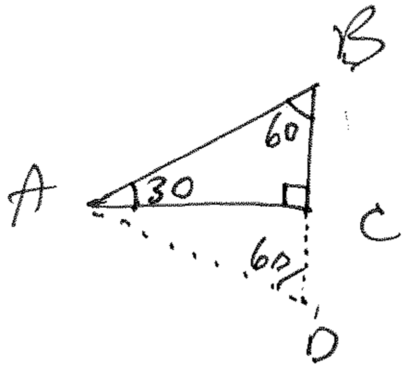
$$\sin 0^\circ = \frac{0}{AB} = 0$$

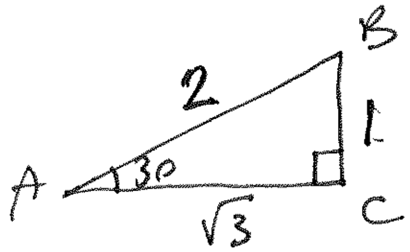
$$\cos 0^\circ = 1 \qquad \tan 0^\circ = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$$

$$\cot 0 = \frac{1}{0} = \infty$$

$\underline{x = 30^\circ}$

$$BC = \frac{1}{2} AC$$





$$\sin 30 = \frac{1}{2}$$

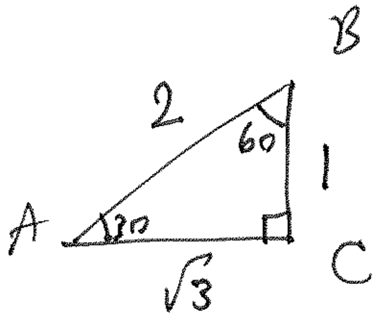
$$\cos 30 = \frac{\sqrt{3}}{2}$$

$$\tan 30 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cot 30 = \sqrt{3}$$

6

$x = 60^\circ$



$\sin 60^\circ = \frac{\sqrt{3}}{2}$ $\tan 60 = \sqrt{3}$

$\cos 60^\circ = 1/2$ $\cot 60^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

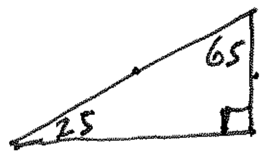
Ann 30 och 60

$\sin 30 = \cos 60$

$\cos 30 = \sin 60$

$\tan 30 = \cot 60$

$\cot 30 = \tan 60$



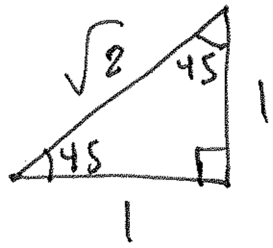
för

Ann 29 och 61
28 " 62

Summan = 90

får vi samma som 30 och 60

$x = 45°$



$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$\cos 45° = 1/\sqrt{2}$ $\quad$ $\tan 45 = 1$

$\cot 45 = 1$

$x = 90°$

$$0 + 90° = 90°$$

$$\begin{cases} \sin 90 = \cos 0 = 1 \\ \cos 90 = \sin 0 = 0 \\ \tan 90 = \cot 0 = \infty \\ \cot 90 = \tan 0 = 0 \end{cases}$$

Anm nackdel med rätvinklig

är att Sin Cos tan Cot

definieras för bara vinklar

mellan 0 och 90°

Vad blir Sin, Cos, tan ...

för vinklar > 90 eller

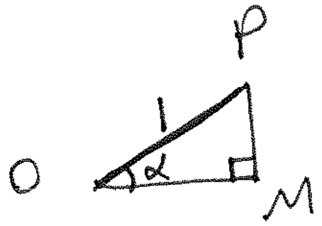
negativa vinklar. ?

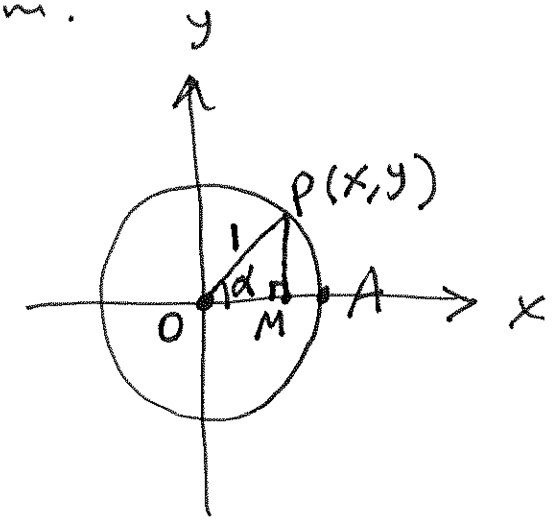
2. Definition av Sin Cos tan Cot

genom enhet Cirkel.

Def <u>Enhet Cirkel.</u>

är den cirkel som har radie $= 1$ och centrum i <u>origo</u> i ett koordinat system.





$\sin\alpha = PM$

$\cos\alpha = OM$

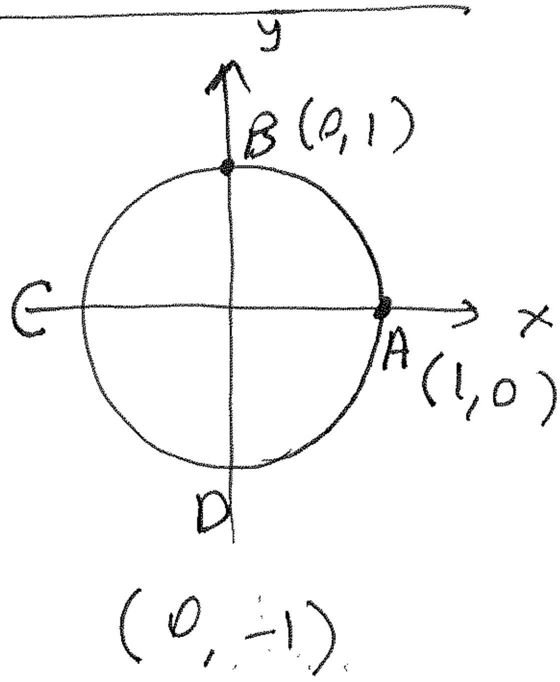
Å andra sidan vet vi att

$OM = x$ och $PM = y$

Viktig

$$\boxed{\begin{array}{l} x \text{ koordinat} = \cos\alpha \\ y \text{ koordinat} = \sin\alpha \end{array}}$$

Vinklar $0^\circ, 90^\circ, 180^\circ, 270^\circ$



A 0°

$\sin 0 = 0$

$\cos 0 = 1$

B 90°

$\cos 90^\circ = 0$

$\sin 90^\circ = 1$

C 180°

$\cos 180 = -1$

$\sin 180 = 0$

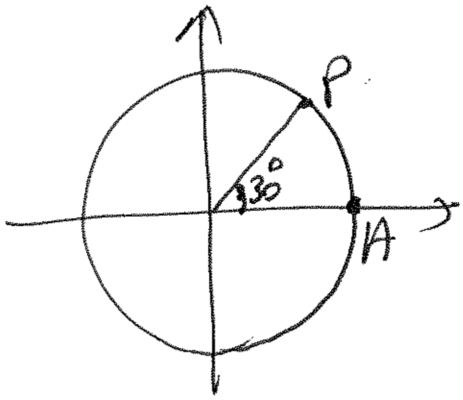
D 270°

$\cos 270^\circ = 0$

$\sin 270^\circ = -1$

Anm

Vinkel 30° om vinkel t.ex 390°
båda sammanfaller på punkt P



$\sin(390^\circ) = \sin 30^\circ$

$\cos(390^\circ) = \cos 30^\circ$

$\tan(390^\circ) = \tan 30^\circ$

$\cancel{\tan}$ cot $(390^\circ) = \cot 30$

I allmänt

$\sin(360n + \alpha) = \sin\alpha$

$\sin(750^\circ) = \sin(2 \cdot 360 + 30)$

$= \sin 30 = 1/2$

Anm

$360^\circ \longleftrightarrow 2\pi$

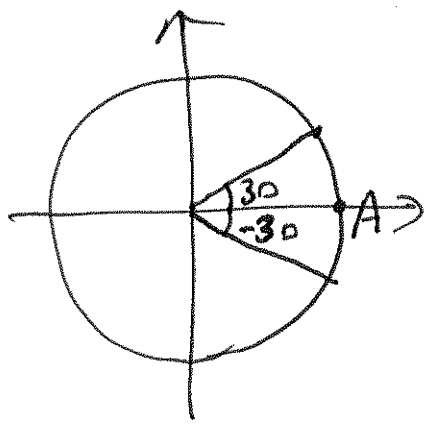
grad radian

$30^\circ = \pi/6$ $90 = \pi/2$

$60^\circ = \pi/3$ $180 = \pi$

$45^\circ = \pi/4$

Anm medurs räknas som negativa



Tecken för sin, cos, tan och cot

i olika kvadrant.

y

$x > 0$
$y > 0$

första kvadrant

andra
$x < 0$
$y > 0$

x

tredje
$x < 0$
$y < 0$

fjärde
$x > 0$
$y < 0$

$\boxed{\begin{aligned} x &= \cos \\ y &= \sin \end{aligned}}$

Detta med för att

cos −
sin +
tan −
cot −

cos +
sin +
tan +
cot +

x

cos −
sin −
tan +
cot +

cos +
sin −
tan −
cot −

t.ex.

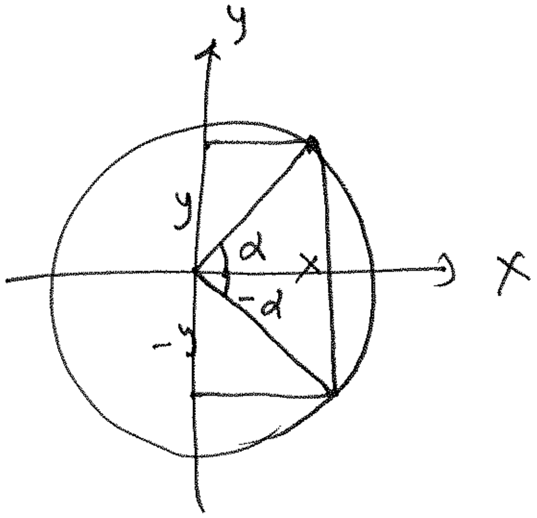
$x = 125^\circ$ ligger i andra

$90 \leq 125^\circ \leq 180$

$\sin 125^\circ = +$

$\cos 125^\circ = -$

Sambandet mellan $\underline{\alpha}$ och $\underline{-\alpha}$



α och $-\alpha$
har samma x
alltså samma $\cos$

$\cos(-\alpha) = \cos\alpha$

$\sin(-\alpha) = -\sin\alpha$

$\tan(-\alpha) = \dfrac{\sin(-\alpha)}{\cos(-\alpha)} = \dfrac{-\sin\alpha}{\cos\alpha} = -\tan\alpha$

$\cot(-\alpha) = -\cot\alpha$

Mer trigonometriska formler

① $\sin(a+b) = \sin a \cdot \cos b + \cos a \cdot \sin b$

② $\sin(a-b) = \sin a \cdot \cos b - \cos a \cdot \sin b$

③ $\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$

④ $\cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b$

tillämpning $\quad 75^\circ \quad 15^\circ$

$$\sin(75^\circ) = \sin(45^\circ + 30^\circ)$$

$$= \sin 45^\circ \cdot \cos 30 + \cos 45^\circ \cdot \sin 30$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$\sin(15^\circ) = \sin(45^\circ - 30)$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Formeln 1 ger

$\sin(a+b) = \sin a \cdot \cos b + \cos a \cdot \sin b$

$a = b = x.$

$\sin(x+x) = \sin x \cdot \cos x + \cos x \cdot \sin x$

5 $\boxed{\sin(2x) = 2\sin x \cdot \cos x}$

Formeln 3

$\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$

$a = b = x$

6 $\boxed{\cos(2x) = \cos^2 x - \sin^2 x}$

$\cos 80 = \cos^2 40 - \sin^2 40$

Formeln 6 dvs.

$$\cos(2x) = \cos^2 x - \sin^2 x$$

ger två viktiga andra formler.

$$\sin^2 x + \cos^2 x = 1$$

$$\cos 2x = \underbrace{\cos^2 x}_{1-\sin^2 x} - \sin^2 x$$

$$\cos(2x) = 1 - 2\sin^2 x$$

7 $$\boxed{\sin^2 x = \frac{1 - \cos 2x}{2}}$$

$$\cos(2x) = \cos^2 x - \underbrace{\sin^2 x}_{(1-\cos^2 x)}$$

$$\cos(2x) = 2\cos^2 x - 1$$

8 $$\boxed{\cos^2 x = \frac{1+\cos 2x}{2}}$$

Anm Formeln 7 och 8 kallar vi

för integral formler

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin^2 x \, dx = \int \frac{1-\cos(2x)}{2} \, dx$$

$$= \frac{1}{2} \int 1 \, dx - \frac{1}{2} \int \cos(2x) \, dx$$

$$\frac{1}{2} x - \frac{1}{2} \cdot \frac{1}{2} \sin 2x + C$$

$$\int \sin^2 x = \frac{1}{2} x - \frac{1}{4} \sin(2x) + C$$

En annan tillämpning av 7

Sin 22.5

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$x = 22.5$

$2x = 45$

$$\sin^2(22.5) = \frac{1-\cos 45}{2}$$

$$= \frac{1-1/\sqrt{2}}{2} = \frac{\sqrt{2}-1}{2\sqrt{2}}$$

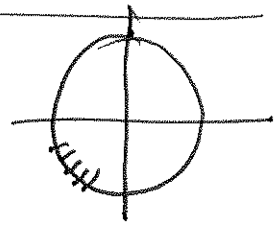
$$\boxed{\sin(22.5) = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}}$$

Anm Given en av sin, cos, tan eller cot av en vinkel kan man beräkna de andra.

genom rät vinklig triangel

eller

formler



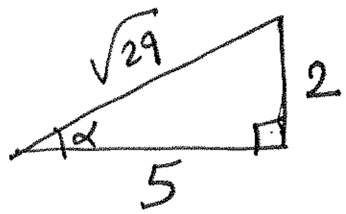
$\underline{\text{Ex}}$ Man vet att α ligger i 3:e kvadrand och $\tan\alpha = 2/5$

beräkna $\sin\alpha$, $\cos\alpha$, och $\cot\alpha$

$$\sin = \frac{2}{\sqrt{29}}$$

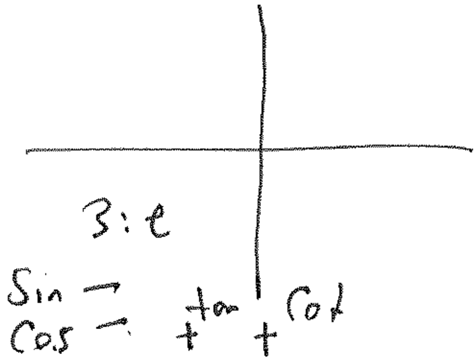
$$\cos = \frac{5}{\sqrt{29}}$$

$$\cot = 5/2$$



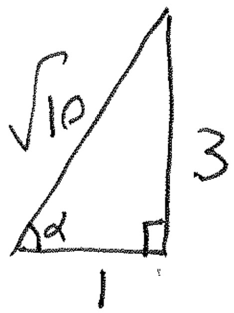
$$\sin\alpha = -\frac{2}{\sqrt{29}} \qquad \cot\alpha = 2.5$$

$$\cos\alpha = -\frac{5}{\sqrt{29}}$$



Ans

$$\tan\alpha = 3 = \frac{3}{1}$$



$$\sin(90-\alpha) = \cos\alpha$$

$$\cos(90-\alpha) = \sin\alpha$$

$$\tan(90-\alpha) = \cot\alpha$$

$$\cot(90-\alpha) = \tan\alpha$$

$$\sin(90-\alpha) = \overset{1}{\cancel{\sin 90}} \cdot \cos\alpha - \underset{0}{\cancel{\cos 90}} \cdot \sin\alpha$$

$$= \cos\alpha$$

$$\sin(180-\alpha) = \sin\alpha$$

$$\cos(180-\alpha) = -\cos\alpha$$

$$\tan(180-\alpha) = -\tan\alpha$$

$$\cot(180-\alpha) = -\cot\alpha$$

$$\sin(180-\alpha) = \underbrace{\cancel{\sin 180}}_{0} \cdot \cos\alpha - \underbrace{\cancel{\cos 180}}_{-1} \cdot \sin\alpha$$

$$= \sin\alpha$$

$$\sin(270+\alpha) = -\cos\alpha$$

$$\cos(270+\alpha) = \sin\alpha$$

logaritm.

Om basen för logaritm är e

betecknas log med Ln

$$\log_e x = \ln x$$

Om $\log_{10} x = \lg x$

lös för x

$$\ln(5-2x) = -3$$

$$5-2x = e^{-3}$$

$$5-e^{-3} = 2x$$

$$\boxed{x = \frac{5-e^{-3}}{2}}$$

$$\ln x + \ln(x-1) = 1$$

$$\ln[x(x-1)] = 1$$

$$x(x-1) = e^1$$

$$x^2 - x - e = 0$$

$$x = \frac{1 \pm \sqrt{1+4e}}{2}$$

$$\boxed{x = \frac{1+\sqrt{1+4e}}{2}} \quad \text{OK}$$

lös

$$\ln(\ln x) = 1 \Leftrightarrow \ln x = e^1$$

$$\Leftrightarrow \boxed{x = e^e}$$

$$2^{x-5} = 3$$

App. ln till bå ~~ta~~ led.

$$\ln 2^{(x-5)} = \ln 3$$

$$(x-5)\ln 2 = \ln 3$$

$$x(\ln 2) - 5\ln 2 = \ln 3$$

$$x\cancel{(\ln 2)} = \frac{5\ln 2 + \ln 3}{\ln 2}$$

$$\boxed{x = 5 + \frac{\ln 3}{\ln 2}}$$

$$2^{x-5} = 3 \qquad \log_2$$

$$\log_2 2^{(x-5)} = \log_2 3 \qquad (x-5)\cancel{\log_2 2} = \log_2 3$$

$$\boxed{x = 5 + \log_2 3}$$