Quantum Computing: An Applied Approach

Chapter 5 Problems: Building a Quantum Computer

1. In this chapter, we outline eight approaches to building a quantum computer. In theory, any quantum-mechanical system can be used for quantum computation. Suggest three additional physical phenomena that could be used for quantum computing. What are the pros and cons of such systems compared with some of approaches detailed in this chapter?

2. Which of the different ways of building a quantum computer shares common technology with the primary approach to quantum communication?

Literature questions

1. In *A quantum engineer's guide to superconducting qubits* (https://arxiv.org/pdf/1904.06560.pdf), the authors present a detailed introduction and overview of superconducting qubits. In this problem, we consider how gates are implemented in transmon qubits.

   The Hamiltonian for a transmon qubit can be modeled as

   $$H = -\frac{\omega_q}{2}\sigma_z + \Omega V_d(t)\sigma_y \tag{1}$$

   where ω_q (the qubit frequency) and Ω are determined from details of the transmon circuit—such as capacitance and inductance—and $V_d(t)$ is the controllable voltage applied to the circuit. (This is said to *drive* the circuit or qubitand is the origin of the subscript d.) The terms σ_z and σ_y are the usual Pauli Z and Y operators.

(a) Define the Hamiltonians

$$H_0 := -\frac{\omega_q}{2}\sigma_z \tag{2}$$

$$H_d := \Omega V_d(t)\sigma_y \tag{3}$$

so that H above can be written $H = H_0 + H_d$. Using the paper reference as a guide (see Section V.D), show that the *drive Hamiltonian* H_d in the *rotating frame* can be written

$$\tilde{H}_d = \Omega V_d(t)\left(\cos(\omega_q t) - \sin(\omega_q t)\sigma_x\right). \tag{4}$$

(b) By letting $V_d(t) = V_0 v(t)$ where $v(t)$ has the general form of a driven exponential

$$v(t) = s(t)\sin(\omega_d t + \phi)s(t)\left(\cos(\phi)\sin(\omega_d t) + \sin(\phi)\cos(\omega t)\right), \tag{5}$$

and defining $I = \cos(\phi)$ and $Q = \sin(\phi)$, show that

$$\tilde{H}_d = \Omega V_0 s(t)(I\sin(\omega_d t) - Q\cos(\omega_d t))(\cos(\omega_q t)\sigma_y - \sin(\omega_q t)\sigma_x) \tag{6}$$

(c) Using the rotating wave approximation to drop fast-rotating terms—i.e., terms with $\omega_q + \omega_d$—show that

$$\tilde{H}_d = -\frac{1}{2}\Omega V_0 s(t)\begin{bmatrix} 0 & e^{i\delta_\omega t+\phi} \\ e^{-i\delta_\omega t+\phi} & 0 \end{bmatrix} \tag{7}$$

where $\delta_\omega := \omega_q - \omega_d$.

(d) The above equation is a powerful tool for understanding single-qubit gates in transmon (and generally in superconducting) qubits. Supposing that $\delta_\omega = 0$, show that

$$\tilde{H}_d = -\frac{1}{2}\Omega V_0 s(t)(I\sigma_x + Q\sigma_y). \tag{8}$$

(e) Show that setting $\phi = 0$ in the definition of I and Q corresponds to a rotation around the x-axis. This type of pulse is called an *in-phase* pulse.

(f) Show that setting $\phi = \pi/2$ in the definition of I and Q corresponds to a rotation around the y-axis. This type of phase is called an *out-of-phase* pulse.

(g) By exponentiating the Hamiltonian (i.e., solving Schrödinger's equation), we can write the unitary evolution for an in-phase pulse as

$$U(t) = \exp\left(\left[\frac{i}{2}\Omega V_0 \int_0^t s(t')\,dt'\right]\sigma_x\right) \tag{9}$$

Note that this has the general form of an x-rotation gate

$$R_x(\theta) := \exp\left(-i\theta\sigma_x/2\right) \tag{10}$$

where the angle is given by

$$\theta(t) := -\Omega V_0 \int_0^t s(t')\,dt'. \tag{11}$$

What angle should $\theta(t)$ be in order to implement a NOT gate?

(h) Suppose that you have an experiment setup where $\Omega = V_0 = 1$ and the waveform envelope $s(t)$ is given by $s(t) = t$. How long should you apply the in-phase pulse in order to flip the state of a qubit?