Acoustoelectric Effect

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1 Theory

1.1 EM

distribution of the lead field.

The potential ϕ of the lead fields ($\vec{E} = -\nabla\phi$) is a solution to:

electric field is related to the electric potential by the gradient relationship.

$$\nabla\sigma\nabla\phi = 0, \tag{1}$$

where $\sigma(x)$ is the electrical conductivity distribution. We assume (but that can easily be generalized) that we have Dirichlet boundary conditions at the electrodes and zero Neumann (insulating) boundary conditions at the other boundaries.

We assume that we have a small perturbation of $\sigma(x)$, named $d\sigma(x)$. The following equation needs to be fulfilled:

$$\nabla(\sigma + d\sigma)\nabla(\phi + d\phi) = 0, \tag{2}$$

where $d\phi$ is the resulting change of the potential.

do all these equations assume a constant conductivity over time?

It follows that $\nabla\sigma\nabla d\phi + \nabla d\sigma\nabla\phi + \nabla d\sigma\nabla d\phi = 0$. Assuming that the last term on the left is negligible in a first order approximation, we obtain:

$$\nabla d\sigma\nabla\phi = -\nabla\sigma\nabla d\phi. \tag{3}$$

The boundary conditions for $d\phi$ are zero Dirichlet at the electrodes and zero Neumann elsewhere.

1.1.1 σ constant in the region of the perturbation

this assumes a homogenous medium-> conductivity constant

spatially

Assuming that $\sigma(x)$ is constant in the region (support) of the perturbation, such that $\nabla\sigma\nabla\phi = \sigma\Delta\phi = 0$, we can rewrite the left-hand side of Eq. 3 ($\nabla d\sigma\nabla\phi$) as $\nabla d\sigma \cdot \nabla\phi + d\sigma\Delta\phi = -\nabla d\sigma \cdot \vec{E}$. Similarly, we can rewrite the right-hand side of Eq. 3 ($-\nabla\sigma\nabla d\phi$) as $-\sigma\Delta d\phi$.

We thus obtain the equation:

this is just saying that the change in field at the focal point is equal to the lead field*laplace of conductivity over condtivity of medium

$$\Delta d\phi = \frac{\vec{E} \cdot \nabla d\sigma}{\sigma}, \tag{4}$$

which is a typical diffusion equation with a source term that can be solved using a wide range of solvers (including the Ohmic-current dominated EQS solver from Sim4Life).

To do: read over what a source term is in diffusion equations, and generally read up on diffusion equations…

why is it the divergence = to the change of charge multiplied by conductivity?

1.1.2 Heterogeneous σ

this has added complexity in every step…
start with a petri dish style example.

xxx todo xxx

1.2 Acoustics

For acoustic propagation in tissue at the relevant intensities, it is common to write that the change in local density $d\rho = \rho_0 \beta_0 p$, where p is the pressure and $\beta_0 = \frac{1}{\rho_0}\left[\frac{d\rho}{dp}\right]_{\text{adiab.}}$ is the adiabatic compressibility. As the speed of sound $c = \frac{1}{\sqrt{\rho_0 \beta_0}}$, we can obtain $\beta_0 = \frac{1}{\rho_0 c^2}$ from readily available material parameters.

Using the assumption[1] from Song et al. [?] that $\frac{d\sigma}{\sigma} = \frac{dV}{V}$ (where dV is the change in volume V), together with the first order approximation $\frac{d\sigma}{\sigma} = -\frac{d\rho}{\rho_0}$ (readily obtained using $\rho_0 = m/V$, where m is the constant mass), we obtain $\frac{dV}{V} = -\frac{d\rho}{\rho_0}$. Therefore:

$$d\sigma = -\sigma \frac{d\rho}{\rho_0} = -\sigma \beta_0 p. \tag{5}$$

density value is negative as density decreases as volume increases

2 Practically speaking

The workflow thus is the following:

1. perform an acoustic simulation to determine p

   This is the time varying pressure field variation caused by the focused transducer.

2. perform an EM simulation to determine E

   This is the lead field distribution alone.

3. compute the source term $S(x) = \frac{\vec{E}\cdot\nabla d\sigma}{\sigma} = -\frac{\vec{E}\cdot\nabla(\sigma\beta_0 p)}{\sigma} = -\vec{E}\cdot\nabla(\beta_0 p)$

   This needs to be calculated over time to see any mixing effects. Ignore this on first pass.

4. solve the diffusion equation $\Delta d\phi = S(x)$ to obtain the induced field $\vec{E}_I = -\nabla d\phi$

   This is the time varying electric field due to the acoustoelectric effect.

Note: Keep in mind that we have to treat p as a phasor. I.e., do not compute $\nabla(\beta_0 p)$ based on the absolute value of p!

5. Does this new E1 field then just take the place of one of the fields in the TI quasi-static simulation? The quasi-static approximation assumes a constant conductivity when it's computed. Is this still a fair assumption with this added acoustoelectric conductivity modulation?

Later:
How would the temperature component introduced by ultrasound, be introduced as an additional change in conductivity?

[1] check validity xxx