Correction of the systematic error due to a slow moving V-408 PIMag Linear Stage in the scope of interferometry

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Abstract—**This paper presents the correction of the systematical error caused by a V-408 PIMag Linear stage used at speeds between 2nm.s^{-1} and 0.1nm.s^{-1} in an interferometry experiment. the presence of a position dependent pattern in the value of the wavelength is highlighted. An estimation of the correction required is made from a reference data set done with a green laser of known wavelength. This correction is applied to sets of data performed over the same range, with different sources and the stage moving at different speeds. Corrected sets of data present standard deviations on the mean wavelength between 1.6nm and 5.8nm with an average of 3.18nm. The average error on the wavelength due to the wrong position of the stage is calculated as $\pm$ 12.9nm. More specifically, the corrected data set for the Mercury light displays two clear peaks at 530.9$\pm$2.4nm and 577.5$\pm$3.5nm. The expected rays in the yellow region of mercury's spectrum have values 546.07nm, 576.96nm and 579.07nm [1]. The correction is not precise enough to distinguish the doublet in the second ray of the spectrum but the observed peak does include both theoretical values within its margin of error. The correction presented is highly relevant since these lines were not distinguishable from the original data set, which presented a single estimate of the wavelength at 580.3$\pm$17.3nm.**

I. Introduction and theory

INTERFEROMETRY is one of the most powerful techniques available to modern physicists. With a standard apparatus, this technique allows distance measurements of the order of a few nanometres [2]. When used with monochromatic high-frequency lasers and appropriate optical elements, the resolution achieved by an interferometer exceeds any other technique [3]. In 2014, a shrink in the space-time fabric of 10^{-18}m due to the collision of two black holes was made by the Laser Interferometer Gravitational-Wave Observatory (LIGO). This detection led to the discovery of gravitational waves [2]–[4].

However, any apparatus is only as good as its weakest component, whose quality is often limited by its price. In a teaching laboratory, for example, resources are limited and obviously LIGO's level of precision is completely out of reach for any laboratory with a university-sized budget. Yet, it is possible within a certain limit to correct for systematical errors induced by faulty equipment. This paper will investigate how to increase the precision of an interferometer without increasing the price by considering how the accuracy of its less reliable component -a V-408 PIMag Linear Stage- can be increased.

A. Interference

The experiment relies on the interference of two coherent beams of light. The analysis of the interference of the beams is made by varying the path difference between the beams by slowly moving one of the mirrors from which the light reflects. The intensity of the resulting beam as a function of the path difference of the beams is then collected and analysed.

If the path difference is an integer multiple of the wavelength then the interference is constructive and the intensity observed is maximal. For a path difference equal to an half-integer multiple of the wavelength, the interference is destructive and the light has minimal intensity [5]

For a given setup, it is very useful to define the null point. This corresponds to the position of the moving mirror such that the path difference is zero between the two beams [6]. Ideally the null point should lie at the centre of the stage, to allow the largest range of motion -and hence data collection- around the null point. When the moving mirror is at the null point, all wavelengths present in the source interfere constructively with one another. Examples of interference patterns can be seen in Fig.1.

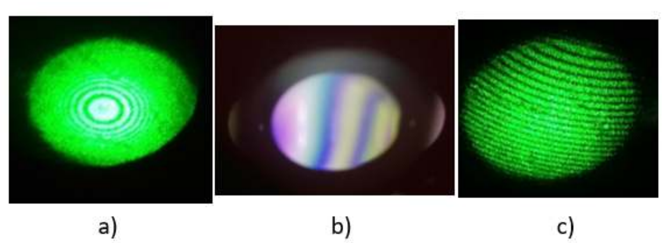

Fig. 1: a) Haidinger fringes obtained with a green laser. It is easy to see the bright regions of constructive interference and the darker regions of destructive interference b) Fizeau fringes obtained with a Tungsten lamp close to the null point. The different wavelengths composing the light interfere with one another, c) Fizeau fringes obtained with a green laser. Pictures a), b) and c) were obtained with the set-up shown in Fig.2.

B. Fourier spectroscopy

It is possible to extract the different frequencies present in a light source from its interferogram. Assuming the law of superposition and that the electric field is a real quantity, it can be shown that this process involves a Fourier transform of the form [7]:

$$B(\sigma) = A \int_{-\infty}^{\infty} [I_R(\delta) - \frac{1}{2} I_R(0)] \cos(2\pi\sigma\delta) d\delta, \qquad (1)$$

where $B(\sigma)$ is the flux density corresponding to the wave number σ, $I_R(\delta)$ is the flux of light as a function of δ -the path difference between the two beams- and $I_R(0)$ is the flux of light measured when the moving mirror is at the null point. Applying equ.(1) to an interferogram reveals the wavelengths λ corresponding to wavenumbers $\sigma = \frac{2\pi}{\lambda}$ present in the light source [7].

C. Sampling theory

Upon collecting data, it is very important tomake the interference pattern visible. A few data points have to be recorded within a period of the wave to insure fidelity to the analog signal and to minimize sampling effects [6]. For a stage travelling at speed s and a detector collecting data at a frequency f, the distance travelled by the stage between two recorded points is :

$$d = \frac{s}{f}. \qquad (2)$$

According to the Nyquist theorem, a discrete signal contains all the information of an analog signal if the sample frequency f_s is greater than half of the highest frequency present in the analog signal. The corresponding equation is [8]:

$$f < 2F_{max} \qquad (3)$$

were F_{max} is the highest frequency present in the source's light. Noting that for a monochromatic electromagnetic wave the only frequency F_{max} present in the signal is related to the wavelength λ by $F_{max} = \frac{c}{\lambda}$. Rearranging eq.(3) one can find the maximal distance between consecutive points in the sample such that the discrete signal complies with Nyquist's sampling theorem. Combining this with eq.(2), we find the highest speed s_{max} at which the stage can travel so that the recorded signal is akin to the analogous signal using the following equation:

$$s_{max} = \frac{\lambda f}{2} \qquad (4)$$

were f is the sampling frequency and λ is the light's wavelength.

II. METHOD

A. Set-up

The set-up used corresponds to a Michelson interferometer and was based on ThorLab's EDU-MINT2 kit [2,6]. The sources used in this experiment were a green laser of wavelength 533 $\pm$ 1nm (experimentally verified) and a Mercury lamp with a yellow filter. The light was divided by the beamsplitter and reflected by two mirrors, one fixed and the another attached to a computer controlled moving stage, before being re-combined and measured by the detector. A picture of the set-up can be seen in Fig. 2.

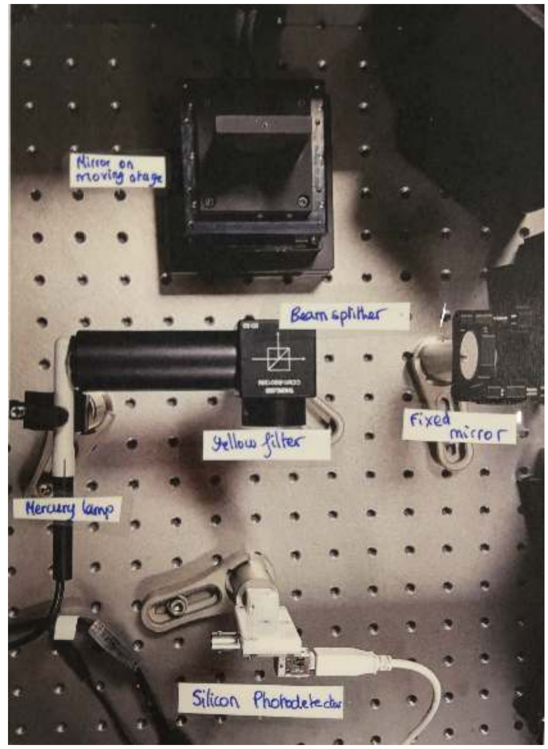

Fig. 2: Picture of the set-up used when analysing the Mercury light. When using the green laser, the yellow filter was removed and the Mercury lamp exchanged for the green laser fixed to a convex lens. The laser was directly fixed to the arm of the beamsplitter to ensure that the alignment and the amount of light received were optimal.

The mirrors were carefully aligned facing the appropriate side of the beamsplitter so that the two paths were perpendicular. The set up was constructed so that the light hit each mirror as close to its centre as possible, to reduce any error due to potential curvature of these mirrors.

The moving stage used was the V-408 PIMag Linear Stage. According to the data sheet, its minimum incremental motion is 20nm [9]. The detector used was a Silicon PIN Photodiode model SFH 2200 which is most efficient between wavelength of 400nm and 1000 nm [10]. The sampling frequency of the detector used was 50Hz which implies, by eq.(4) that all measurements had to be done with a stage moving slower than 13.3μm.s^{-1}. For higher accuracy, the speed of the stage was set between 2μm.s^{-1} and 0.1 μm.s^{-1}. Because of the sampling frequency, this corresponds distances between consecutive points of 2nm to 40nm. This lower bound was chosen to investigate how much the stage movement was affected by its speed being slower than the recommended limit. A second factor for choosing this range of speed was the time constraint.

III. MOTIVATION

A. Preliminary results

One of the aims of this set-up was to estimate the spectrum of light emitted by various sources. An easy way to estimate the mean wavelength of a signal is to manually measure the distance between the nodes in the interferogram. The mean distance between the nodes is equal to half of the mean wavelength [11].

Measuring the mean distance between nodes in the interferogram for Mercury light with a yellow filter yields wavelength of 583.5 ± 31.0nm. For accuracy, the periods measured were taken roughly uniformly along the interferogram. This method is quite inaccurate due to subjectivity in the location of the nodes and it is expected that the computed calculation of the wavelength presented in Fig. 3. should be more precise and more accurate.

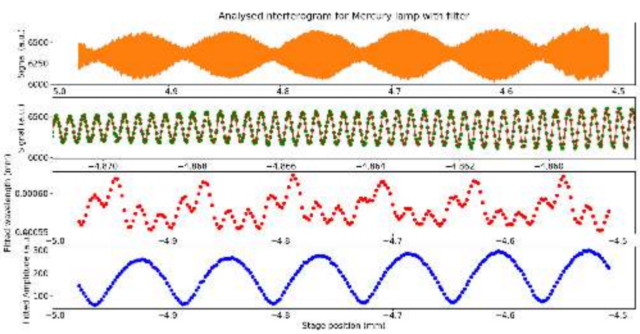

Fig. 3: From top to bottom: a) interferogram of the mercury lamp with yellow filter, b) the same interferogram fitted with a sequence of sinusoidal functions (in red), c) the wavelength of the fitted functions against the mean of the region fitted, d) amplitude of the signal against position showing a clear beating pattern, proof of interference between the lights of different wavelength composing Mercury's emission spectrum.

The spread in estimated wavelength for this preliminary data is approximately 40nm and its mean value is 580.33 ± 17.25nm. This value is well in agreement with the manually measured value and is much more precise. However the 2.97% error on this measurement is large for a high precision interferometer and a good proportion of lines in atomic spectra cannot be correctly discerned with such a high error [12].

The pattern that arises in Fig.3c. could imply a error due to the position of the stage. Theoretically, the value of the calculated wavelength should be indeed independent of the stage's position if the latter moves uniformly, and return correctly the value of its position. The estimated wavelength should also be independent of the stage's speed, assuming this obeys eq.(4).

Comparing results with other groups and other sets of data indicated that this pattern arose in any set of data and that this error could potentially affect all the motored stages. It was therefore decided to investigate this phenomenon, and the green laser was chosen over the other sources for quantitative analysis because its light is close to monochromatic and has a known wavelength.

B. A recurring pattern

As mentioned in the previous section, a recognizable pattern appears when fitting the wavelength. as shown in Fig.3c, the estimation of the wavelength seems to be dependant on position of the stage. Figure 4 shows the superposition of these pattern for runs done over the same range but at different speed with the green laser.

Qualitatively, it is quite straightforward to see that the pattern is independent of the speed of the stage. It is important to note that runs done at 2μm.s^{-1}, corresponding to a distance between samples of 40nm, displayed the same pattern as sets of data done with sampling distances of 15nm, 6nm and 2nm. The hypothesis that the stage's limiting distance between samples is 20nm (as stated on its data sheet) can therefore be seriously doubted. For reference, the wavelength of this pattern was measured as $79.4\pm10.7\mu$m.

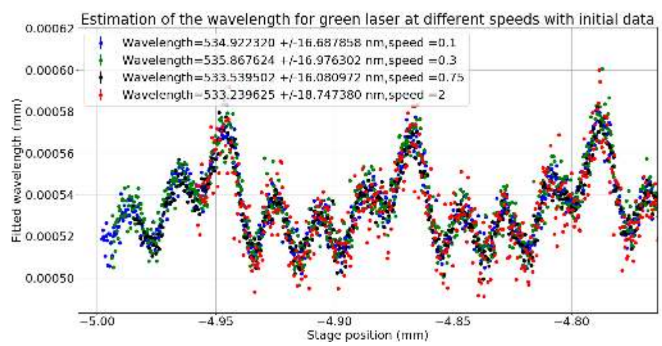

Fig. 4: Comparison of the calculation of the light's wavelength as a function of the stage position for runs made with the green laser at different speeds. Speeds are given is μm.s^{-1}. It appears from this graph that the pattern is independent of the stage's speed.

Secondly, it was important to ensure that the observed pattern was only due to the stage and not the source used. Figure 5 presents the estimation of the source's mean wavelength as a function of the stage's position for the green laser and the mercury lamp with a yellow filter.

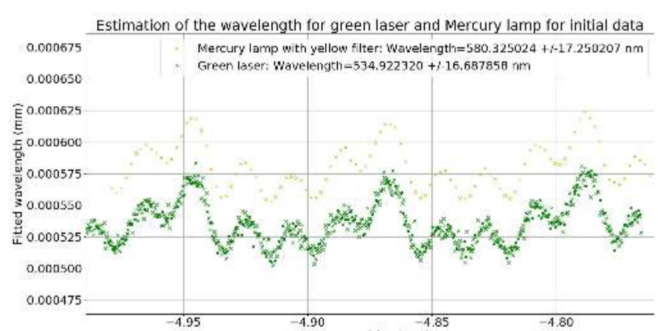

Fig. 5: Comparison of the calculation of the light's wavelength as a function of the stage position for runs made with the green laser and the mercury lamp with a yellow filter at the same speed of 1μm.s^{-1}. It appears from this graph that the pattern is identical and only shifted upwards for sources with different mean wavelengths.

It should be noticed that the pattern is present in both data sets but only shifted vertically. This is completely expected since the green laser has a mean wavelength of 533nm whereas the yellow filter used with the Mercury lamp only transmits light of wavelength between 567.3nm and 592.7nm [13].

Figure 5 hints that the fault in the stage affects the precision of the results but not their accuracy. Calculating the mean wavelength and standard deviation of each set of wavelength gives 534.92 ± 16.68nm for the green laser and 580.33 ± 17.25nm for the Mercury lamp.

Finally, the data sets presented in Fig.4 and 5 were collected over the period of a few days during which the stage had been removed and put back into the system. The stage's error cannot therefore be explained by a faulty fixing of the stage to its support.

IV. Results

A. Correcting the motion of the stage

To account for the error caused by the stage, it was chosen to base the correction on a single set of reference data collected with the green laser. The position was then adjusted for the calculated wavelength to match the accepted value as accurately and as precisely as possible. The parameters of this correction were then used for correcting any set of overlapping data as the error was dependent only on the position of the stage.

The hypothesis chosen for the fitting was that the recorded positions lagged behind the true position by a scaling factor. This hypothesis is in line with the data taken but its physical explanation is still unclear. To correct this, the program functioned with a small section of the sample corresponding to a few periods of the interferogram. A sinusoidal function was then fitted to this data and the optimization parameters were saved. Because the positions were recorded after the corresponding data, the wavelength of the signal was modified. The correction of the positions followed a scaling equation where x', the corrected position of the signal, was given by:

$$x' = x'_{p,corr} + (x - x_0)\frac{\lambda_{laser}}{\lambda_{fitted}} \tag{5}$$

where x is the uncorrected position, λ_{laser} is the known value of wavelength of the test run, λ_{fitted} is the wavelength calculated for this small section of data points, $x'_{p,corr}$ is the first uncorrected point of this section of data and x_0 is the last corrected point of the previous section of points.

As the parameters $x'_{p,corr}$, λ_{fitted} and x_0 were saved, it was then simple to correct any point by applying the correction given in eq.(5) with the fitting parameters corresponding to the region of space in which this specific point lied. Results of these fits are presented in the next section.

B. Validity of the model

Multiple tests were undertaken to verify the validity of the calculated correction.

1) Tests with the green laser: Firstly, sets of data taken with the green laser as the source over the same range of distances but at different speeds were corrected using the fitting parameters calculated from another set of data, taken again over the same range and in the same direction. Plots showing the calculated wavelength for the same set of data, corrected and uncorrected sets are presented in Fig 6.

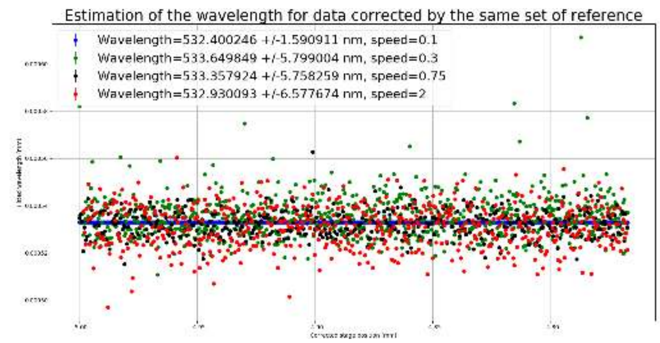

Fig. 6: Estimation of the wavelength for sets of data corrected by the set with speed 0.1μm.s^{-1}. All sets corresponds to the green laser with different stage positions given in μm.s^{-1}. This figure should be compared to Fig.4 to that the calculated fit is efficient as it presents the same sets of data. The precision is improved and the accuracy unchanged.

From this plot, it is clear that:

- the beating in the plot of wavelength against the position is removed.
- the mean error in wavelength is 4.94nm which is lower than the mean error for the unfitted data sets which was 16.73nm in average.
- the mean of the wavelength is approximately equal to that in the unfitted case.

From these observations, we can infer that the fit is efficient as it improves precision but conserves the accuracy. It is useful to mention that the team suspects the quality of the sample taken as reference influences the quality of the corrected data.

2) Tests with the Mercury lamp: It was verified that the systematic error was independent of the source in section III. It is therefore acceptable to infer that the correction calculated from the green laser's data should be applicable to any set of data acquired over the same range. It is especially useful to increase the resolution of the spectrum when working with sources have emission rays separated by only a few nanometres.

For example, Mercury has a singlet line at 546.074 nm and a doublet at 576.959 nm and 579.065 nm [1]. Since it was shown that the wavelength could vary by more than 40nm for the unfitted data set and since it is expected that the periodicity in the stage's error alters the signal, it seemed impossible to resolve the lines of Mercury mentioned above without a proper correction of the systematical error. Figure 7 presents the distribution of the wavelength of the mercury light with a yellow filter in the fitted and unfitted cases. It is very straightforward to see that the fitted data is not only more precise but also more accurate than the unfitted data.

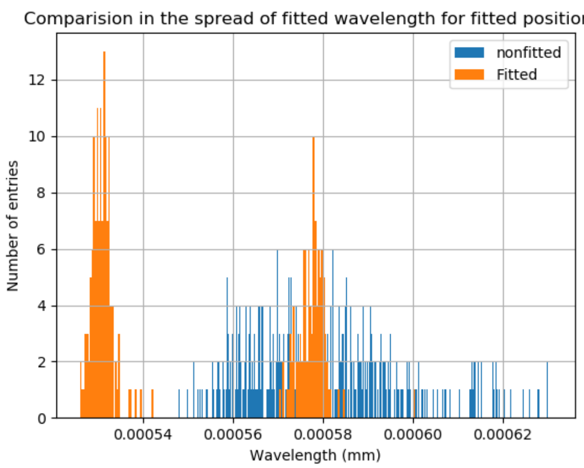

Fig. 7: Comparison of the distribution of the calculated wavelength for the Mercury light with a yellow filter. The set of data used as reference for fit was a run made at 1μm.s^{-1} with the green laser over the same range. The two lines in the fitted data have a wavelength of 530.9nm and 577.5nm and have a standard deviation of 2.4nm and 3.5nm respectively.

Figure 7 shows that the correction greatly improves the accuracy and precision of the signal. The unfitted data spreads from 630nm to 544nm. The fitted data, on the other hand presents two clear peaks in the wavelength at 530.19 ± 2.4nm and 577.5 ± 3.5nm. It should be mentioned that both the original and corrected data sets present wavelengths smaller than 567.3nm which should not be the case if the yellow filter was properly used. This anomaly is outside the scope of this study but could be investigated to estimate the overall accuracy of the set-up and analysis methods used.

If the first line in the corrected data is quite far from the expected value at 546.07nm [12], the second line corresponds nearly exactly to the mean between the wavelenghs of Mercury's doublet at 576.96nm and 579.07nm. This second line contains both wavelengths within its margin of error.

C. Summer 2019 Edit

Further work have been undertaken on this topic posterior to the submission of this report. With comprehensiveness in mind, this later result is added as an edit to the present paper. Nothing of the original paper has been modified.

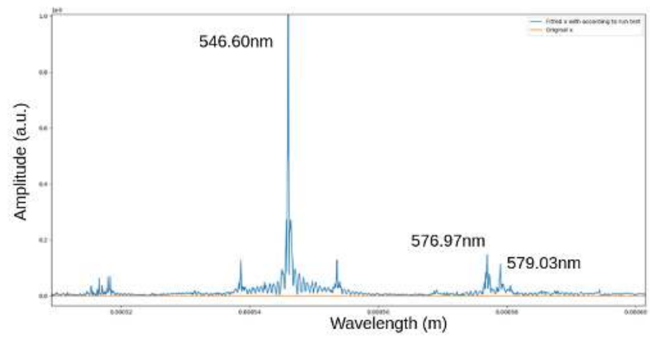

Fig. 8: Distribution of the mercury wavelengths after final corrections and bug removal. The three largest peaks in wavelength are presented and correspond to the peaks expected at 546.07nm, 576.96nm and 579.07nm [1]. For comparison, the results obtained without the calibration presented in this work are indicated in orange clearly showing the accuracy gain.

D. Errors

This experiment was affected by many sources of error. We have already mentioned the faults in the stage's position recording which is the biggest source of systematic error. However, this experiment is also affected by random errors and other systematical errors. Table I summarises the different sources of error, their magnitudes and the means by which the latter can be reduced. Unless stated otherwise, the magnitude of the errors are given as their contribution to the wavelength's standard deviation for the green laser.

V. Further investigations

This study was quite limited in time and there is room for many adjustments and further precision. The following are considered of interest by the team and ordered by importance:

1) Write the code that would permit computation of the Fourier transform of the corrected signal. Attempts were made using a non-uniform Fourier transform program, but remained unsatisfactory. The construction of a uniform set of data with the proper wavelength was also attempted, but the difference in fitting parameters between the different sections of data introduced discontinuity in the signal.
2) Check the ranges of speeds over which the pattern arises and the data can be corrected. Compare with Nyquist's limiting frequency to determine the quality of the stage.
3) Check if the correction can be applied to any stage of the same model, or if it stage dependent.
4) Check if the correction is dependant of the direction of the motor or only on its position on the stage.
5) Modify the code created for this study to be self explanatory and for it to automatically perform corrections on any set of data if provided with a set of reference data.
6) Investigate the relationship between the size of the section taken for fitting and the quality of the corrected data. Optimize the current code accordingly.
7) Use a laser whose wavelength is unique and experimentally verified to construct the data of reference.

TABLE I: Summary of the sources of error and their relative magnitude as well as methods used to reduce them

[H]

Source of error	**Magnitude**	**Methods to reduce the relative impact**
Inaccurate position recording by the motor	± 12.9nm	Correcting the position using a test run with a laser of known wavelength as presented above
Vibrations	More data is required but assumed small because of large number of measurements	Do the experiment in an empty laboratory, screw the optical components more tightly to the optical bench
Align each mirror so that light reflects upon their exact centre. Line broadening by Doppler effect	$\approx$ 3nm [6], [14], [15]	Switch off the mercury lamp regularly to allow cool down.
Curvature of the mirror modifying the path difference	$\pm\frac{\lambda}{10}$ in path difference [16]	Systematical error only shifting the null point
Line broadening by Doppler effect	$\approx$ 5nm [6], [14], [15]	Switch off the mercury lamp regularly to allow it to cool down.
Curvature of the beam-splitter modifying the path difference	$\pm\frac{\lambda}{10}$ in path difference [17]	Systematical error only shifting the null point
Computer lags due to other actions performing at the same time and not recording some chunks of the data	$\geq 10nm$	Dedicate a computer for data taking and use another one for analysis. Discard the data sets taken with a faulty computer
Shift in signal due to silicon detector's limited detection range	Negligible when working with the right wavelength	Work only with between 400nm and 1100nm [10] and/or when work with monochromatic light
Vertical and horizontal misalignment	More data needed	Use a spirit level during the alignment process

8) Investigate the relationship between the quality of the data used as reference and the quality of the corrected data.

VI. Conclusion

This paper proves the presence of a systematical error due to an inaccurate recording of the moving stage position in an interferometry experiment. It presents a method to correct for this error by computing a periodic scaling of the position for a set of data taken over a long range with a laser of know wavelength. The validity of this approach is then investigated by correcting various set of data with different test runs.

This correction is very efficient as it reduces the broadening of the spectral line from tens of nanometre to a few namometres whilst keeping the accuracy of the initial set of data. More precisely, in the case of the Mercury lamp, the correction made it possible to distinguish two separate peaks at 530.91 ± 0.21nm and 577.51 ± 0.34nm when the non-corrected data presented a band of wavelengths between 630nm and 544nm approximately. If this first corrected wavelength is shifted significantly from the accepted value of 546.07nm [1], the second ray is close to the middle of the two wavelengths that compose the expected doublet at 576.959 nm and 579.065 nm [1].

The correction presented in this paper negated the V-408 PIMag Linear Stage's physical limitation, allowing confident use of this set-up at low speeds. However, this work is ongoing and the code to perform the Fourier transform of the corrected signal is still to be completed.

Acknowledgments

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