



$$T_{01} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} c_2 & 0 & -s_2 & L_1 \\ 0 & 1 & 0 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{23} = \begin{bmatrix} c_3 & 0 & -s_3 & L_2 \\ 0 & 1 & 0 & 0 \\ s_3 & 0 & c_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{3ee} = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now, $T_{0ee} = T_{01} T_{12} T_{23} T_{3ee}$

$$T_{0ee} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & -s_2 & L_1 \\ 0 & 1 & 0 & 0 \\ s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & 0 & -s_3 & L_2 \\ 0 & 1 & 0 & 0 \\ s_3 & 0 & c_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The last column gives us
 x, y, z

$$X = L_3 C_1 C_2 C_3 - L_3 C_1 S_2 S_3 + L_2 C_1 C_2 + L_1 C_1$$

$$Y = L_3 S_1 S_2 C_3 - L_3 S_1 S_2 S_3 + L_2 S_1 C_2 + L_1 S_1$$

$$Z = L_3 S_2 C_3 + L_2 C_2 S_3 + L_2 S_2$$

$$X = L_3 C_1 [C_2 C_3 - S_2 S_3] + C_1 [L_2 C_2 + L_1]$$

$$X = C_1 [L_3 \cos(\beta + \gamma) + L_2 C_2 + L_1]$$

Similarly $Y = S_1 [L_3 \cos(\beta + \gamma) + L_2 C_2 + L_1]$

$$Z = L_3 S_2 S_3 + L_2 S_3 + L_2 S_2$$

So, $X = \cos \alpha [15 \cos(\beta + \gamma) + 10 \cos \beta + 5]$

$$Y = \sin \alpha [15 \cos(\beta + \gamma) + 10 \cos \beta + 5]$$

$$Z = 15 \sin(\beta + \gamma) + 10 \sin \beta$$

$$\frac{X}{Y} = \frac{1}{\tan \alpha}$$

$$\alpha = \tan^{-1} \left(\frac{Y}{X} \right)$$

We use Jacobin to calc. β, γ