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Analysis and Simulation of Nakagami Fading Channel with MATLAB*

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Abstract: The paper simulated and analyzed the statistical performances of the Nakagami fading channel in wireless communication with MATLAB, including the complex envelop of received signal, the Level Crossing Rates and Average Fade Durations on the Maximal-Ratio Combining diversity.

Key words: Multipath channel, Nakagami fading, MATLAB Simulation, Combining diversity

1. Introduction

The first work to researching and developing of digital mobile communication engineering is understanding mobile channel characteristics itself. In the propagation environment of digital mobile communication, the signal received by the mobile may consist of a large number of multipath component due to reflection, diffraction and scattering between the transmitter and the receiver. The randomly distributed phase, amplitudes, and angles of arrival of multipath component combine at the receiver give a resultant signal strength which can changes rapid over a small travel distance and be named fast fading. When there is no LOS component between mobile and BS, received signal consist of reflection and scattering wave from different directions and follow Rayleigh distribution. When there is a dominant stationary signal component present, the small-scale fading envelop distribution is Ricean. Rayleigh fading models are frequently utilized in simulating high frequency signals propogating in an ionospheric channel. However, Rayleigh fading falls short in describing long-distance fading effects with sufficient accuracy. This fact was first observed by Nakagami, who then formulated a parametric gamma distribution-based density function, to describe the experimental data he obtained. It was then showed by many researchers using real-life data that the model proposed by Nakagami provides a better explanation to less and more severe conditions than the Rayleigh and Ricean model and provides a better fit to the mobile communication channel data[1].

Because Nakagami fading could represent various fading condition in wireless channel, so simulation and analysis of the statistical characters of the Nakagami fading channel are very important. This paper made use of MATLAB to simulate and analyze the complex envelop of received signal, the Level Crossing Rates and Average Fade Durations on the Maximal-Ratio Combining diversity. The important issue in this paper is to develope a method to simulation fading channel using MATLAB and comprehend the statistical performances of Nakagami channel.

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2. Simulating the statistical characters of Nakagami fading channel model with MATLAB

2.1 Multipath Fading Channel Models

We first give the theoretic PDF of Rayleigh and Ricean fading and their relations to Nakagami probability distribution parameters in order to analyze and compare both states. Assuming that the receiver is moving relative to the transmitter, and there is no LOS component, then the received signal can be expressed as[2]

$$s_{ray}(t)=\sum_{i=1}^{N} a_i\cos(\omega_c t+\omega_{d_i}t+\phi_i) \qquad (1)$$

Where ω_c is carrier frequency, a_i and ϕ_i are the amplitude and phase of ith arrival path, phase ϕ_i are uniformly distributed over $[0,2\pi]$. $\omega_{d_i}=\frac{\omega_c v}{c}\cos\psi_i$ is the Doppler shift of ith reflected of scattering wave, where ψ_i is the angle relative to the direction of motion of the antenna and assumed ψ_i are uniformly distributed over $[0,2\pi]$,when N is large, the probability density function of the received signal envelope can be shown to be Rayleigh, when there exist a line-of-sight(LOS) component between the transmitter and the receiver, the received signal can be expressed as random multipath components superimposed on a dominant signal., i.e.[2].

$$s_{rice}(t)=\sum_{i=1}^{N-1} a_i\cos(\omega_c t+\omega_{d_i}t+\phi_i) + \rho\cos(\omega_c t+\omega_d t) \qquad (2)$$

where the constant ρ is the strength of the direct component, ω_d is the Doppler shift along the LOS path, the envelope, in this case, has a Rician density function. When there not exist a direct signal, (i.e. $\rho \to 0$), the Ricean distribution degenerates to a Rayleigh distribution. Contrary to the Ricean distribution, Nakagami distribution does not assume a LOS conditions, but use a parametric gamma distribution-based density function to describe the experimental data and get approximately distribution, So it is more universality, the PDF of Nakagami distribution is[4],

$$f(r) = \frac{2m^m r^{2m-1}}{\Omega^m \Gamma(m)} \exp\left\{-\frac{mr^2}{\Omega}\right\},$$

$$m \geq \frac{1}{2};\quad r \geq 0 \tag{3}$$

where m is Nakagami parameter, described the fading degree of propagation field due to scattering and multipath interference process, and Ω is the average power of multipath scatter field, $\Gamma(m)$ is the gamma function.

2.2 Simulate the first-order statistics of Nakagami fading channel

Based on the multipath fading channel models described above, the Nakagami fading channel was simulated using MATLAB to generated the small-scale fast fading envelop, the relationship between the parameter m and the fading envelop are evaluated, and compare the result with Rayleigh and Ricean distribution, this helps to better understand the statistics of Nakagami fading channel.

Because Nakagami distribution encompasses both Rayleigh and Ricean distribution, in the case when m are lager, the channel fading characterize can simulating by (1) plus a dominant signal, so the Nakagami distribution can simulating by combined the Rayleigh and Ricean distribution, we adopted (1)(2) to generated a Nakagami fading envelop, if the number of path(NLOS) N is to small then result will be not accurate, here N=50.

Expressing the equation(1) in inphase and quadrature form,

$$s_{ray}(t) = \sum_{i=1}^{N} a_i \cos(\omega_{d_i} t + \phi_i)\cos\omega_c t$$

$$-\sum_{i=1}^{N} a_i \sin(\omega_{d_i} t + \phi_i)\sin\omega_c t$$

$$= I(t)\cos\omega_c t - Q(t)\sin\omega_c t \tag{4}$$

The envelop is given by:

$$R(t) = \sqrt{[I(t)]^2 + [Q(t)]^2} \tag{5}$$

The simulation condition is: carrier frequency used was 900MHz. The number of path(no LOS)N was 50,the velocity of mobile was 80km/h, sampling rate is quadruple of carrier frequency, simulation was carried out for a time interval corresponding 1.25s, the detail is,

1. Generating the received signal using (1). From the Statistics Toolbox of MATLAB, we can easily obtained the distribution of path amplitudes a_i, phase ϕ_i and the ψ_i of Doppler shift, where a_i taken to be *Weibull*-distributed random variables. both phases ϕ_i and ψ_i were taken to be uniform in $[0,2\pi]$ and were generated using the function *unifrnd* from the statistics toolbox, it is reasonable in most case, and the Doppler shift generate in this case is symmetrical.

2. Using the command *demod* from the Signal Processing Toolbox demodulated the signal to get the inphase and quadrature components, subsequently, the envelop was calculated using(5).

3. Do step1 and step2 for Rayleigh and Ricean process, from (2), to simulate the Ricean fading envelop, a term without any random phase must be added to the signal generated in the case of Rayleigh fading. The value of ρ of LOS component reflect the fading condition, here ρ=4, both Rayleigh and Ricean fading envelop are superimposed according to (5) to reflected the change of Nakagami parameter m to the impact of signal envelop.

$$R_{Nakagami} = R_{ray}e^{1-m} + R_{rice}\left(1 - e^{1-m}\right) \tag{6}$$

figure 1 to figure 3 is the simulation results of small-scale fast fading effect of received signal, from the figure it can be see the rapid changes in signal strength over a small travel distance of time interval and the variations with different m. In the same time we consider the varying Doppler shifts on different multipath signals, $m=1,1.5,10$ corresponding to Rayleigh distribution(severe shadow fading environment),Ricean distribution and light shadow fading environment. It is show that

(1) when m=1, there is no single line-of-sight path between MS and BS, the signal received by the mobile at any point in space may consist of a large number of plane waves having randomly distributed amplitudes, phase, and angles of arrival, and can cause the signal received by the mobile to severe distort or fade. When $m=1.5$, there exist a LOS path between MS and BS, fading envelop approximately Ricean distribution, it can be seen from fig.2 whether frequency of fading or depth of fading, it is greatly better than fig.1, while $m=10$ is still some improvement relative to fig.2, but less than that of fig.2 to fig.1.

(2) no matter what case it is, it is inevitably to appear deep fading, which can be overcame by diversity.

(3) from fig1.-fig.3 it can be seen that the signal

level in three case are approximately -10 dB、0 dB、10 dB, that is, the signal quality improved when m increased. It is evident due to the improvement of the propagation environment.

3. Simulation of second-order statistics of Nakagami fading channel under diversity reception

Due to the influence of multipath fading, it is very difficult to maintaining good communication. Antenna space diversity can prevent deep fading nulls, by exploits the random nature of radio propagation by finding independent signal paths for communication, both the instantaneous and average SNRs at the receiver may be improved, often by as much as 20dB to 30dB. In this section, the level-crossing rates (LCRs) and average fade durations (AFDs) of Nakagami fading channel under maximal-ration combining (MRC) are given, and some analysis using MATLAB are performed.

3.1 LCR and AFD for diversity in Nakagami fading channel

the level-crossing rates (LCRs) and average fade durations (AFDs) are two quantities which statistically characterize fading channel, these quantities reflect not only scattering environment but also velocity of mobile, and thus the second-order statistics of a fading channel. The LCR $N_R(r)$ is defined as the number of times per second that the envelop of fading channel crosses a specified level R in a positive-going direction ($\dot{r} > 0$), the AFD $\tau_R(r)$ is defined as the average period of time for which the received signal is below a specified level R. Let r be the sampled value of the diversity combined envelope $R(t)$ of a fading channel. The LCR $N_R(r)$ and ADF $\tau_R(r)$ are defined as a function of r by [6]

$$N_R(r) = \int_0^\infty \dot{r} p_{R,\dot{R}}(r,\dot{r}) d\dot{r} \tag{7}$$

$$\tau_R(r) = F_R(r)/N_R(r) \tag{8}$$

where $\dot{r}$ is the time derivative of r, $F_R(r) = \int_0^r p_R(\alpha) d\alpha$ is the CDF of the fading channel, and $p_R(r)$ is the corresponding PDF. The LCR can be rewritten in terms of $p_R(r)$ and the conditional distribution $p_{\dot{R}}(\dot{r} \mid r)$ as

$$N_R(r) = \int_0^\infty \dot{r} p_{\dot{R}}(\dot{r} \mid r) p_R(r) d\dot{r} \tag{9}$$

In the case of Nakagami fading, the PDF of r_l is express by equation(3) and can rewritten as

$$p_{R_l}(r_l) = \frac{2 m_l^{m_l} r_l^{2m_l - 1}}{\Omega_l^{m_l} \Gamma(m_l)} \exp\left\{ -\frac{m_l r_l^2}{\Omega_l} \right\}$$
$$l = 1,2,\Lambda, L \qquad r_l \geq 0 \tag{10}$$

where Ω_l and m_l are the average power and the parameter m of the lth channel, respectively, the corresponding CDF is given by

$$F_{R_l}(r_l) = \frac{\gamma(m_l, \frac{m_l}{\Omega_l} r_l^2)}{\Gamma(m_l)} \tag{11}$$

where $\gamma(.)$ is the incomplete gamma function of the first kind.

The output envelop of a MRC diversity system is given by

$$r = \left[\sum_{l=1}^{L} r_l^2 \right]^{\frac{1}{2}} \tag{12}$$

and its derivative is

$$\dot{r} = \frac{\sum_{l=1}^{L} r_l \dot{r}_l}{r} \tag{13}$$

suppose that the diversity channels are identically distributed, using equation (9)(10)(12)(13) leads to the following result[7]

$$N_R(r) = \frac{\sqrt{2\pi} f_m}{\Gamma(m_T)} \left(\frac{m_T}{\Omega_T} r^2 \right)^{m_T - \frac{1}{2}} e^{-\frac{m_T}{\Omega_T} r^2} \tag{14}$$

where $f_m = v/\lambda$ is the maximum Doppler shift for a vehicle speed v and carrier wavelength λ. Based on the definition of AFD of equation (8), substituting (11) and (14) yields the AFD

$$\tau_R(r) = \frac{\gamma\left(m_T, \frac{m_T}{\Omega_T} r^2 \right) e^{\frac{m_T}{\Omega_T} r^2}}{\sqrt{2\pi} f_m \left(\frac{m_T}{\Omega_T} r^2 \right)^{m_T - \frac{1}{2}}} \tag{15}$$

where $m_T = mL$, $\Omega_T = \Omega L$

3.2. Simulation and discussion with MATLAB

Fig.4-9 presented the analysis of the average fade durations and the level crossing rates based on the maximal-ratio combining diversity in the Nakagami fading channels. They are plotted in logarithmic scale, f_m be normalized by $N_R(r)$, and selected $r_n = r/\sqrt{\Omega}$ as x-coordinate in order to reach the maximum value at zero in these figures. Supposed each branch is in the case of independent identically distributed. We can obtain the LCR and AFD under different condition by change the diversity branchs L and the parameter of Nakagami m. Fig.4 compare the LCR for the diversity techniques presented above with L=1,2,3,4,10, for m=0.6 corresponds to severe fading(worse than Reyleigh). Fig.5 compare the LCR above with L=1,2,3,4,10, for m=1(Reyleigh). In the Fig.6, $m=1.5$(approximate to Rician distribution), L=1,2,3,4,10.

Fig4-6 show that for the different value of m, using diversity can reduce the influence of fading effectively, as expected, diversity mitigates the effect of fading, the average LCR becomes smaller with diversity order increase. when m increased, owe to the improvement of the propagation environment, the LCR gradually decreased. If m is lager and the diversity branches $L=10$, the improvement of LCR is not great as that of $L=1,2,3,4$. i.e. using more diversity branches will not bring much improvement in terms of LCR when m is lager. The LCR can be explained as the times of the signal envelop below a threshold in a unit time, with the diversity branchs L and m increasing, the quality of received signal will be improved obviously. At the same time, we notice that the normalized LCR is independent of the diversity at about $r_n = 0dB$.

Fig.7 shows the normalized AFD as the function of the normalized envelop level r_n ($r_n = r/\sqrt{\Omega}$), when the threshold value r_n is small($r_n < 0dB$), the AFD is direct ratio to r_n. From the figure, we can draw a conclusion that the AFD in low signal level will decrease with the increasing of the diversity order L when the value of m is fixed, in other words, the deep fading only occurs accidently and the duration is not too longer, on the other hand, the AFD in high signal level will increase with the increasing of the diversity L, if r_n is the receive threshold level, the average duration of the fading signal level lower than threshold level r_n will decrease, which means the outage probabilities reduce. From Fig.7-9 it can be seen that with the increasing of m, if the diversity also increased, the effect of the anti-fading is more obvious.

4. Conclusion

This paper simulated and analysed the statistical characteristics of Nakagami fading channels using MATLAB, as a strong software for mathematic calculation, MATLAB appears to be a simple and straightforward tool to simulate and analyse the characteristics of channels, by changes each parameters, a little change in the fading channel can be observed, it is useful for us to understand the basic conception of radio channel. The simulation in this paper can be fit to not only Namagami fading channel, but also other fading channels as well. The second-order statistics dicussed in the last section is very important in the modeling and designing of radio communication system, for example, it can be used to calculate the outage, to select the right threshold level of the receiver, to estimate the length of burst errors and so on. the paper is realistic to these engeerings.

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Appendix

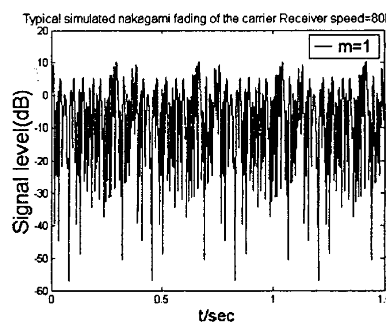

fig.1 Simulating received signal envelop when m=1

(corresponding to Rayleigh and severe shadow fading)

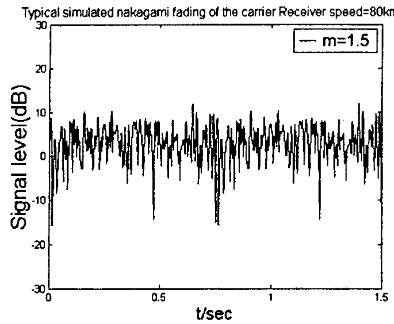

fig.2 Simulating received signal envelop when m=1.5 (corresponding to Rician distribution)

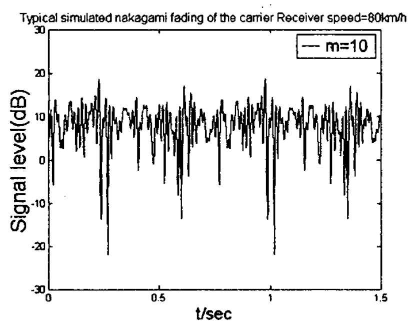

fig.3 Simulated received signal envelop when m=10 (light shadow fading)

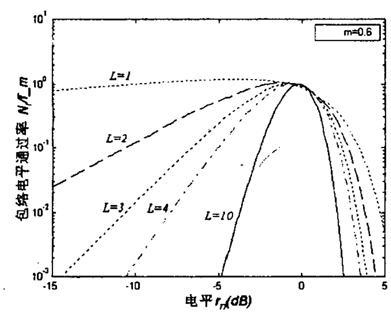

fig.4 Normalized LCR for m=0.6 with diversity orders L=1,2,3,4,10

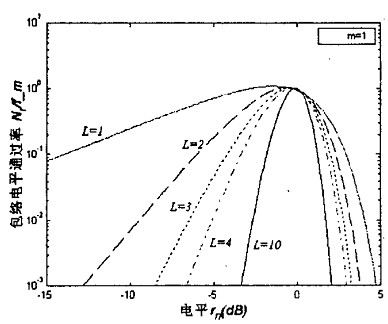

fig.5 Normalized LCR for m=1with diversity orders L=1,2,3,4,10

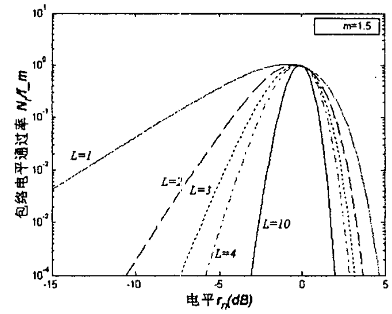

fig.6 Normalized LCR for m=0.6（m=1.5, with diversity orders L=1,2,3,4,10

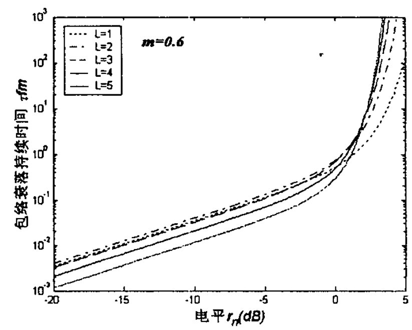

fig.7 Normalized AFD for m=0.6 with diversiy orders L=1,2,3,4,5

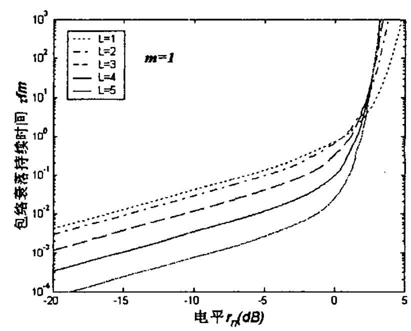

fig.8 Normalized AFD for m=1 with diversiy orders L=1,2,3,4,5

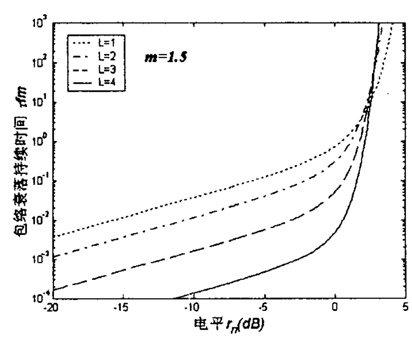

fig.9 Normalized AFD for m=1.5 with diversity orders L=1,2,3,4