Analysis of Finite Blocklength Communication under Imperfect CSI and Rayleigh Fading

Nikolaos I. Miridakis, George C. Alexandropoulos *Senior Member, IEEE*, and Theodoros A. Tsiftsis, *Senior Member, IEEE*

***Abstract*—Point-to-point wireless communication systems operating in the finite blocklength transmission regime and performing practical Channel State Information (CSI) estimation are considered. We particularly adopt Minimum Mean Squared Error (MMSE) channel estimation and assume that received signals undergo Rayleigh channel fading conditions. Novel analytical and closed-form expressions for key performance metrics, namely the outage probability, mean transmission delay, effective throughput, and level crossing rate are presented. Our representative numerically evaluated results verified by equivalent simulations corroborating the effectiveness of the proposed analysis, while providing useful insights on the joint impact of imperfect MMSE channel estimation and finite blocklength communication.**

***Index Terms*—Channel estimation, finite blocklength, MMSE, low latency, performance analysis, ultra reliable communication.**

I. Introduction

Massive machine type and ultra reliable low latency communications are already indispensable use cases of fifth generation (5G) networks [1], [2]. Ultra-high reliability of even less than 99.9999% and low latency in the order of less that 1ms represent two of the cornerstone prerequisites of the latter cases with applications ranging from remote surgery to virtual reality. Unlike conventional communication realized with large packets approaching Shannon's infinite blocklength assumption, the 5G low latency applications can only be realized in the finite blocklength regime (few hundreds of channel uses, e.g., 100-300 for vehicle-to-vehicle communication [3]).

In the finite blocklength regime, a significant performance loss may occur due to the fact that the unavoidable transmission errors cannot be effectively compensated by conventional channel coding schemes. These schemes perform well under in the very large blocklength paradigm, however, further innovation in channel coding is necessary for short packet communication. Additionally, Channel State Information (CSI) estimation mechanisms are commonly implemented in practical wireless systems so as to allow the receiver to effectively decode the transmitted signal. Nevertheless, perfect CSI is a rather overoptimistic condition in realistic scenarios, mainly due to the unexpected user mobility, vast channel fading variations, and lack of feedback signaling.

N. I. Miridakis and T. A. Tsiftsis are with the School of Electrical and Information Engineering and the Institute of Physical Internet, Jinan University, Zhuhai 519070, China. N. I. Miridakis ia also with the Department of Computer Systems Engineering, Piraeus University of Applied Sciences, Aegaleo 12244, Greece (e-mails: nikozm@unipi.gr, theo_tsiftsis@jnu.edu.cn).

G. C. Alexandropoulos is with the Mathematical and Algorithmic Sciences Lab, Paris Research Center, Huawei Technologies France SASU, Boulogne-Billancourt 92100, France (e-mail: george.alexandropoulos@huawei.com).

The performance of finite blocklength communications has lately attracted significant research interest (see [1]–[4] and references therein). However, most of the works assume only perfect CSI conditions. For example, [3], [4] studied the average packet error with finite blocklength transmission for the case where the receiver knows CSI perfectly. Only recently, imperfect CSI conditions were considered. In [5], a double integral form for the system outage probability is presented, which is, however, quite involved and cannot be used for further elaboration. Considering a well known CSI imperfection model, [6] studied the delay performance of finite blocklength communication by proposing an upper bound for the system outage performance.

Motivated by the lack of analytical investigation on the impact of practical CSI estimation schemes in the performance of finite blocklength communication, we present an analytical study of such point-to-point systems when adopting the linear Minimum Mean-Squared Error (MMSE) channel estimation technique [7]. This estimation scheme is realized on a per block basis using a training phase prior to the data communication phase, and provides an efficient tradeoff between performance and complexity. As such, it is suitable for delay critical and/or ultra reliable applications. We present novel simple closed-form expressions of some key system performance metrics; namely, the outage probability, mean transmission delay, effective throughput, and level crossing rate (LCR). Along with numerically evaluated performance results, some useful engineering insights are provided.

Notation: The conjugate and absolute value of scalar x are denoted by $x^\star$ and $|x|$, respectively. $\mathbb{E}[\cdot]$ is the expectation operator and symbol $\overset{\text{d}}{=}$ means equality in distribution. $f_X(\cdot)$ and $F_X(\cdot)$ represent the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) of a Random Variable (RV) X, respectively. $X \overset{\text{d}}{=} \mathcal{CN}(\mu,\sigma^2)$ means that X is a complex-valued Gaussian RV with mean μ and variance σ^2, and $X \overset{\text{d}}{=} \exp(\lambda)$ denotes that X is exponentially distributed with mean λ. $\cosh(\cdot)$ and $\sinh(\cdot)$ are the hyperbolic cosine [8, Eq. (1.311.4)] and hyperbolic sine [8, Eq. (1.311.2)] functions, respectively, $Q(\cdot)$ is the Gaussian Q-function, $\Gamma(\cdot,\cdot)$ denotes the upper incomplete Gamma function [8, Eq. (8.350.2)], and $\mathrm{Ei}(\cdot)$ is the exponential integral Ei [8, Eq. (8.211)].

II. System and Channel Models

Consider a point-to-point wireless data link subject to Rayleigh fading channel conditions. The information streams are transmitted in discrete blocks of certain finite blocklength

L in terms of channel uses. We denote the transmission coding rate (i.e., the ratio of the information amount over the total channel uses) as R. Supposing that p denotes the transmit signal-to-noise ratio (SNR), the received signal reads as

$$r = \sqrt{p}hs + n, \tag{1}$$

where h denotes the Rayleigh channel coefficient, s is the transmitted symbol chosen from a unit-power signal constellation (i.e., $\mathbb{E}[ss^{\star}] = 1$), and $n \overset{\mathrm{d}}{=} \mathcal{CN}(0,1)$ represents the additive white Gaussian noise at the receiver side. In practice, the receiver relies on channel estimation to learn h as closer as possible to its exact value. We hereinafter adopt the linear, yet efficient, MMSE channel estimation [7], which is realized with training signals spanning m consecutive channel uses. According to this technique, the exact h can be expressed as

$$h \triangleq \hat{h} + \tilde{h}, \tag{2}$$

where $\hat{h}$ and $\tilde{h}$ stand for the channel estimate and channel estimation error, respectively. One of the key statistical properties of MMSE channel estimation is the *mutual independence* between RVs $\hat{h}$ and $\tilde{h}$, for which holds $\hat{h} \overset{\mathrm{d}}{=} \mathcal{CN}(0, pm/(1+pm))$ and $\tilde{h} \overset{\mathrm{d}}{=} \mathcal{CN}(0, 1/(1+pm))$ yielding $h \overset{\mathrm{d}}{=} \mathcal{CN}(0,1)$. For asymptotically high m and/or p, the estimation error vanishes and $\hat{h} \to h$. Nonetheless, in practice, the latter ideal condition is rather overoptimistic and cannot be established (e.g., due to the finite blocklength limitations). Hence, imperfect CSI is always present and may reflect detrimental effects onto the system performance. Such effects may play a critical role to various practical networking setups, which can be much more emphatic in delay-critical and/or ultra-reliable applications.

Capitalizing on the adopted MMSE channel estimation to design the equalizing filter $w \triangleq \hat{h}^{\star}/|\hat{h}|^2$ at the receiver, the detected signal is given by

$$\hat{r} \triangleq wr = \sqrt{p}w(\hat{h} + \tilde{h})s + wn, \tag{3}$$

which results in the following expression for the instantaneous received SNR after equalization:

$$\gamma = \frac{p}{p|w\tilde{h}|^2 + |w|^2} = \frac{p|\hat{h}|^2}{p\frac{|w\tilde{h}|^2}{|w|^2} + 1} \overset{\mathrm{d}}{=} \frac{X}{Y+1}. \tag{4}$$

where $X \triangleq p|\hat{h}|^2$ and $Y \triangleq p|\tilde{h}|^2$. We next present closed-form expressions for some key performance metrics of the considered system operating at the finite blocklength regime and adopting MMSE channel estimation.

III. Performance Metrics

We commence by deriving closed-form CDF and PDF expressions of the received SNR γ, which are required for analyzing the targeted performance metrics. The CDF of γ is given by

$$\begin{aligned} F_{\gamma}(z) &\triangleq \int_0^{+\infty} F_X(z(y+1)) f_Y(y) dy \\ &= 1 - \int_0^{+\infty} \exp\left(-\frac{z(y+1)}{\kappa(p,m)pm}\right) \frac{\exp\left(-\frac{y}{\kappa(p,m)}\right)}{\kappa(p,m)} dy \\ &= 1 - \frac{\exp\left(-\frac{z}{\kappa(p,m)pm}\right)}{1 + z/(pm)}, \end{aligned} \tag{5}$$

where $\kappa(p,m) \triangleq p/(pm+1)$. Taking the first derivative of (5), the PDF of γ stems as

$$f_{\gamma}(z) = \frac{pm \exp\left(-\frac{z}{\kappa(p,m)pm}\right)}{(pm+z)^2} + \frac{\exp\left(-\frac{z}{\kappa(p,m)pm}\right)}{\kappa(p,m)(pm+z)}. \tag{6}$$

A. Packet Outage Probability

Based consecutively on the tight approximations [9, eq. (59)] and [3, eq. (14)], the packet outage probability conditioned on the received SNR γ is given by

$$\begin{aligned} P_{\mathrm{out}}(\gamma) &\approx Q\left(\frac{\sqrt{L}(\ln(1+\gamma) - R)}{(1-(1+\gamma)^{-2})^{\frac{1}{2}}}\right) \\ &\approx \begin{cases} 1, & \gamma \leq C \\ 1/2 - A(\gamma - B), & C < \gamma < D, \\ 0, & D \leq \gamma \end{cases} \end{aligned} \tag{7}$$

where the latter expression is given by taking the first-order Taylor expansion of the former one. In addition, $A \triangleq \sqrt{L/[2\pi(\exp(2R)-1)]}$, $B \triangleq \exp(R) - 1$, $C \triangleq \cosh(R) + \sinh(R) - 1 - \sqrt{\pi \mathrm{sinh(R)} \exp(R)/L}$, and $D \triangleq \cosh(R) + \sinh(R) - 1 + \sqrt{\pi \mathrm{sinh(R)} \exp(R)/L}$. Consequently, the unconditional packet outage probability P_{out} can be obtained as

$$\begin{aligned} P_{\mathrm{out}} &\triangleq \int_0^{\infty} P_{\mathrm{out}}(z) f_{\gamma}(z) dz \\ &= F_{\gamma}(C) + \int_C^D \left[\frac{1}{2} - A(z-B)\right] f_{\gamma}(z) dz. \end{aligned} \tag{8}$$

To extract a closed-form solution for P_{out} from (8), we introduce the following auxiliary expressions:

$$\Phi_1(\rho;a;b;u;v) \triangleq \int_u^v \frac{\exp(-\rho x)}{(ax+b)} dx = \frac{\exp\left(\frac{\rho b}{a}\right)}{a}\left[\Gamma\left(0, \rho u + \frac{\rho b}{a}\right) - \Gamma\left(0, \rho v + \frac{\rho b}{a}\right)\right], \tag{9}$$

$$\Phi_2(\rho;a;b;u;v) \triangleq \int_u^v \frac{\exp(-\rho x)}{(ax+b)^2} dx = \frac{\rho \exp\left(\frac{\rho b}{a}\right)}{a^2}\left[\Gamma\left(-1, \rho u + \frac{\rho b}{a}\right) - \Gamma\left(-1, \rho v + \frac{\rho b}{a}\right)\right], \tag{10}$$

$$\begin{aligned} \Phi_3(\rho;a;b;u;v) &\triangleq \int_u^v \frac{x \exp(-\rho x)}{(ax+b)} dx = \frac{\exp(-\rho(u+v))}{a^2 \rho} \Bigg\{ a(\exp(\rho v) - \exp(\rho u)) \\ &+ b\rho \exp\left(\rho\left(u+v+\frac{b}{a}\right)\right)\left[\mathrm{Ei}\left(-\frac{\rho(b+au)}{a}\right) - \mathrm{Ei}\left(-\frac{\rho(b+av)}{a}\right)\right]\Bigg\}, \end{aligned} \tag{11}$$

$$\begin{aligned} \Phi_4(\rho;a;b;u;v) &\triangleq \int_u^v \frac{x \exp(-\rho x)}{(ax+b)^2} dx = \frac{b \exp(-\rho(u+v))}{a^2} \\ &\times \left[\frac{\exp(\rho u)}{av+b} - \frac{\exp(\rho v)}{au+b}\right] - \frac{(a+b\rho)\exp\left(\frac{b\rho}{a}\right)}{a^3}\left[\mathrm{Ei}\left(-\frac{\rho(b+au)}{a}\right) - \mathrm{Ei}\left(-\frac{\rho(b+av)}{a}\right)\right]. \end{aligned} \tag{12}$$

Grouping (9)-(12) and inserting them into (8) yields

$$\begin{aligned} P_{\mathrm{out}} = F_{\gamma}(C) + \left(\tfrac{1}{2} + AB\right)\Bigg[&\Phi_1\left(\tfrac{1}{\kappa(p,m)pm}; \kappa(p,m); \kappa(p,m)pm; C; D\right) \\ &+ pm\, \Phi_2\left(\tfrac{1}{\kappa(p,m)pm}; 1; pm; C; D\right)\Bigg] \end{aligned}$$

$$-A\left[\Phi_3\Big(\tfrac{1}{\kappa(p,m)pm};\kappa(p,m);\kappa(p,m)pm;C;D\Big)+\Phi_4\Big(\tfrac{1}{\kappa(p,m)pm};1;pm;C;D\Big)\right]. \tag{13}$$

The latter formula includes a mixture of simple elementary functions and the special functions $\Gamma(\cdot,\cdot)$ and $\mathrm{Ei}(\cdot)$, which are embedded in the most popular mathematical software packages. Hence, (13) can be numerically evaluated quite easily. Moreover, we can easily show that (9)-(12) tend to zero for asymptotically high SNR, i.e., when $p \to +\infty$. In this case, it holds that

$$P_{\mathrm{out}}^{(p\to+\infty)} \approx F_\gamma(C) \approx 1-\frac{1}{1+\frac{C}{pm}}, \tag{14}$$

which indicates a proportionally linear decay of the packet outage probability with respect to an increased transmit SNR p and/or the number of training channel uses m. Notably, by comparing (13) and (14), the packet outage probability in the finite blocklength regime is directly related to the conventional outage probability $F_\gamma(\cdot)$ (i.e., in the infinite blocklength regime) when $p \to +\infty$; yet, relying on a different outage threshold. In fact, the considered (14) uses the C threshold, which is a scaled version of the typical outage threshold $\gamma_{\mathrm{th}} \triangleq 2^R - 1$, whereas the blocklength L plays a crucial role on such a scaling. Whenever $L \to +\infty$ (as in the infinite blocklength regime), then $C \to \gamma_{\mathrm{th}}$ and, hence, these two outage probabilities converge.

B. Mean Transmission Delay and Goodput

Let τ denote the duration of each transmission time slot. The mean transmission delay is defined as the time required for a successful data transmission, and is mathematically given by $\overline{D} \triangleq \tau/(1-P_{\mathrm{out}})$ [10, eq. (16)], which can be straightforwardly evaluated using (13). In addition, the goodput (in bps/Hz), which represents the effective throughput, reads as

$$G \triangleq \left(1-\frac{m}{L}\right) R(1-P_{\mathrm{out}}), \tag{15}$$

and can be directly computed via (13).

In the finite blocklength regime, it is of paramount importance to find the optimal number of channel uses $m^\star$ yielding acceptable channel estimation performance, while using the remaining $L-m^\star$ channel uses for data transmission. The packet outage probability is a monotonically decreasing function of m, which means that the more training signals used the lower this probability is. Hence, given a maximum allowable training window, say $m_{\max}$ with $1 \le m \le m_{\max}$, it is obvious that the minimum mean transmission delay is obtained by setting $m = m_{\max}$. However, the latter strategy may not always be the most efficient one with respect to the achievable goodput performance. There exist certain applications that require to satisfy a target mean delay $\overline{D}_{\mathrm{T}}$, whereby the achievable goodput can be further enhanced. It is straightforward to show that goodput expression (15) represents a concave function with respect to m. In turn, the achievable goodput can always be optimized within a given range $[1, m_{\max}]$ as long as $\overline{D}_{\mathrm{T}}$ is always satisfied. To illustrate this behavior, we sketch the mean transmission delay $\overline{D}$ and goodput G in Fig. 1 versus

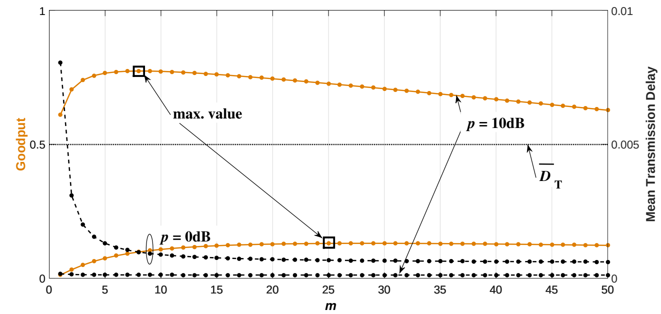

Fig. 1. Goodput G in bps/Hz and mean transmission $\overline{D}$ in μsec as function of the m values for the channel uses for $L=200$, $m_{\max}=L/4=50$, $R=1$, $\tau=100\mu$sec, $p=\{0,10\}$dB, and target mean transmission delay set to $\overline{D}_T = 0.005\mu$sec. Analytical and simulation results are illustrated by curves and mark signs, respectively.

different m values of the channel uses for channel estimation. Clearly and as expected, increasing m and/or p improves the mean transmission delay. However, goodput is maximized for a specific m value that depends on p. The higher p is, the smaller is the m value maximizing goodput performance. It is also shown that curves from both analytical expressions coincide with equivalent results from computer simulations, thus corroborating the presented analysis.

C. Level Crossing Rate (LCR)

In delay critical applications, the duration of an outage may be more crucial than the outage probability itself. This is even more pronounced when CSI is imperfectly known [11] at the transceivers. LCR is a key performance metric unveiling the wireless link resilience, which is directly associated with the duration of outage. This metric can be further utilized to derive others relevant to delay critical communication, like the mean time of reliability, as well as survivability and availability of an operational wireless link (see, e.g., [10, subsection III.C]). The average LCR of the SNR is defined as the rate at which the received SNR γ crosses a certain SNR level Γ_{T} in the positive or negative direction. In mathematical terms, LCR as a function of Γ_{T} is given by [11, eq. (2)] $\mathrm{LCR}(\Gamma_{\mathrm{T}}) \triangleq \int_0^\infty \dot{\gamma} f_{\gamma,\dot{\gamma}}(\Gamma_{\mathrm{T}},\dot{\gamma})d\dot{\gamma}$, where $\dot{\gamma} \triangleq \partial\gamma/\partial t$ denotes the time derivative of γ and $f_{\gamma,\dot{\gamma}}(\cdot,\cdot)$ represents the bivariate PDF of γ and $\dot{\gamma}$. It follows from (4) that γ is a mixture of mutually independent exponential RVs; hence, LCR can be derived in closed-form using [11, Eq. (30)] as the following function of Γ_{T}, m, and p:

$$\mathrm{LCR}(\Gamma_{\mathrm{T}},m,p) = \frac{f_{\mathrm{d}}\sqrt{2\pi pm\Gamma_{\mathrm{T}}}\exp\left(\frac{1}{\kappa(p,m)}\right)\Gamma\left(\frac{3}{2},\frac{1}{\kappa(p,m)}\right)}{\exp\left(\frac{\Gamma_{\mathrm{T}}}{\kappa(p,m)pm}\right)(pm+\Gamma_{\mathrm{T}})}, \tag{16}$$

where f_{d} is the maximum Doppler frequency of the Rayleigh process reflecting the level of fading variability over time.

For the considered finite blocklength communication, the threshold Γ_{T} needs to be appropriately designed so as to constrain the packet outage probability $P_{\mathrm{out}}(\gamma)$ in (7) to an acceptable level. Suppose that ϵ denotes the maximum allowable prescribed outage probability level. Then, the effective SNR threshold $\Gamma_{\mathrm{T}}^{\star}$ can be obtained from the following solution:

$$Q\left(\frac{\sqrt{L}(\ln(1+\Gamma_{\mathrm{T}}^{\star})-R)}{(1-(1+\Gamma_{\mathrm{T}}^{\star})^{-2})^{\frac{1}{2}}}\right)=\epsilon. \tag{17}$$

Note that the computed $\Gamma_{\mathrm{T}}^{\star}$ depends only on ϵ, R, and L, and thus needs to be recomputed only when one of these parameters changes values. Moreover, noticing that $Q(\cdot)$ is a decreasing function while neglecting the denominator of the left-hand side of (17), an upper bound of $\Gamma_{\mathrm{T}}^{\star}$ is given by $\Gamma_{\mathrm{T}}^{\star}\leq\exp(Q^{-1}(\epsilon)/\sqrt{L}+R)-1$, where $Q^{-1}(\cdot)$ denotes the inverse function of $Q(\cdot)$. Plugging the computed delay critical SNR level $\Gamma_{\mathrm{T}}^{\star}$ from the solution of (17) into (16) results in $\mathrm{LCR}(\Gamma_{\mathrm{T}}^{\star},m,p)$ (we shortly write as $\mathrm{LCR}(m,p)$ hereinafter) that depends only on the channel estimation parameter m and the transmit SNR level p.

The level crossing rate for the ideal scenario of perfect CSI is obtained from $\mathrm{LCR}(m,p)$ for $m\rightarrow+\infty$. In this asymptotic case, it holds that $f_{\gamma|m\rightarrow+\infty}(z)=\exp(-z/p)/p$, which implies that the instantaneous received SNR γ is exponentially distributed. In addition, $\lim_{\beta\rightarrow+\infty}\Gamma(3/2,\beta)\rightarrow\exp(-\beta)\sqrt{\beta}$ [8, Eq. (8.357.1)]. Putting the latter two expressions together leads to the reduction of $\mathrm{LCR}(m,p)$ for the perfect CSI to the closed-form expression $\mathrm{LCR}^{(\text{perfect CSI})}(p)=f_{\mathrm{d}}\sqrt{2\pi\Gamma_{\mathrm{T}}^{\star}/p}\exp(-\Gamma_{\mathrm{T}}^{\star}/p)$, which is in accordance to the known expression [12, eq. (10)] for the case where $\Gamma_{\mathrm{T}}^{\star}$ is obtained from (17).

In Fig. 2, we have numerically evaluated the performance of $\mathrm{LCR}(m,p)/f_{\mathrm{d}}$ as a function of p for the $\Gamma_{\mathrm{T}}^{\star}$ values computed from (17) for the cases where $L=200$, $\epsilon=10^{-5}$ (ultra reliable communication), and $R=\{0.5,1\}$. In the same figure we include equivalent computer simulations prepared using the sum-of-cisoids approach presented in [13]. This approach is adequate for modeling the time variability of the considered Rayleigh fading process. Evidently, the curves from the presented LCR analytical expression match perfectly the equivalent results from computer simulations, thus verifying our analysis. As expected, LCR gets its largest values for small-to-moderate SNR p levels, where the specific p level range depends on R. Larger R results in shifting the p level range maximizing the LCR to higher values. This behavior reveals that stronger codes are needed for reliable communication with finite blocklength transmissions. In addition, Fig. 2 showcases the role of the channel estimation parameter m on LCR. The lower m and p become, the more LCR curves are shifted to the right. This means that as we reduce the quality of channel estimation (via smaller m and p values) LCR degrades. Combinedly, channel estimation and finite blocklength may severely impact the operational efficiency of wireless links. To further emphasize on the latter observation, the conventional (i.e., very large) blocklength transmission is also presented in Fig. 2, which may serve as a performance benchmark. Obviously, the impact on the size of L plays a key role to the overall system performance.

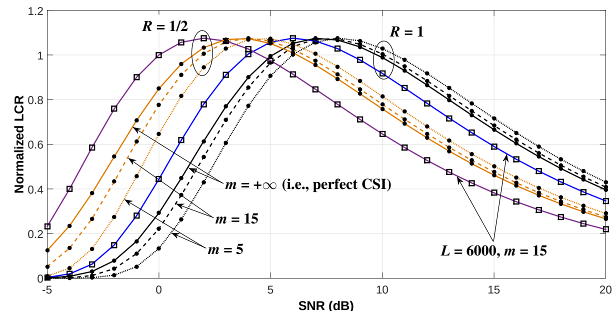

Fig. 2. Normalized level crossing rate $\mathrm{LCR}(m,p)/f_{\mathrm{d}}$ vs. the transmit SNR p for $L=200$ (unless otherwise specified), $\epsilon=10^{-5}$, and different values for R and m. Analytical and simulation results are illustrated by curves and mark signs, respectively. The special case that approaches the conventional infinite blocklength ($L=6000$) is marked with square-signs.

IV. Conclusion

The performance of point-to-point wireless communication with finite blocklength and MMSE channel estimation was analyzed over Rayleigh fading channels. We have particularly presented novel closed-form expressions for the outage probability, mean transmission delay, effective throughput, and level crossing rate. Our performance evaluation results corroborated the validity of the proposed analysis and demonstrated the joint impact of finite blocklength, channel training size, and ultra-reliable constraints on the targeted performance metrics.

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