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Authors	Lei, Hongjiang; Ansari, Imran Shafique; pan, Gaofeng; Alomair, Basel; Alouini, Mohamed-Slim
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# Secrecy Capacity Analysis over $\alpha - \mu$ Fading Channels

Hongjiang Lei, *Member, IEEE*, Imran Shafique Ansari, *Member, IEEE*, Gaofeng Pan, *Member, IEEE*, Basel Alomair, *Member, IEEE*, and Mohamed-Slim Alouini, *Fellow, IEEE*

**Abstract**—In this work, we study the secrecy capacity of the classic Wyner’s model over the  $\alpha - \mu$  fading channels, where  $\alpha$  and  $\mu$  specify the nonlinearity and clustering of fading channels, respectively. The average secrecy capacity (ASC) is derived in closed-form by using the extended generalized bivariate Fox’s H-function (EGBFHF). Moreover, the asymptotic analysis of ASC in high signal-to-noise ratio (SNR) regime is conducted. The asymptotic results unveil that the ASC follows the scaling law of  $\Theta(\ln \rho)$ , where  $\rho$  stands for the ratio between the average powers of main channels and eavesdropping channels. Moreover, the ASC can be enhanced by increasing the transmit SNR, while there exists a ceiling of ASC as the SNRs at both sides are improved simultaneously. The accuracy of the analytical results is validated by Monte-Carlo simulations. The numerical results show that rigorous fading channels are beneficial to the secrecy performance, that is, serious nonlinearity (small  $\alpha$ ) and sparse clustering (small  $\mu$ ) will lead to the improvement of ASC.

**Index Terms**—Physical layer security, average secrecy capacity,  $\alpha - \mu$  fading channel, extended generalized bivariate Fox’s H-function.

## I. INTRODUCTION

RECENTLY, the secrecy performance analysis of digital systems over fading channels has become a research focus. Considering the effect of outdated channel state information (CSI), the secrecy performance of multiple-input multiple-output (MIMO) wiretap channels with multiple eavesdroppers over non-identical Nakagami- $m$  fading was investigated and the closed-form expressions for the exact and asymptotic

secrecy outage probability (SOP) and average secrecy capacity (ASC) were derived in [1]. The closed-form expressions for the probability of zero secrecy capacity with three different transmit antenna selection schemes were derived under Rayleigh fading environments in [2].

So far, most of the open literature on the secrecy performance analysis focused on particular fading channels, such as Rayleigh [3], [4], Nakagami- $m$  [1], [5], [6], Weibull [7], [8], or log-normal [9]. Few works focus on the secrecy performance over generalized fading channels, except [10], [11], [12]. The secrecy outage performance of single-input single-output (SISO) and single-input multiple-output (SIMO) system were investigated in [10], [11], respectively, in which the channels were assumed as generalized- $K$  fading channels.

The  $\alpha - \mu$  fading channel accounts for the nonlinearity of a propagation medium as well as clusters of multipath waves with two physical fading parameters  $\alpha$  and  $\mu$  reflecting the nonlinearity and clustering, respectively [13]. Most of the small-scale fading channels are special cases of  $\alpha - \mu$  distribution, such as exponential, Rayleigh, Nakagami- $m$ , Gamma, and Weibull. Furthermore, the probability density function (PDF) of some large-scale fading channels (such as log-normal channels) and composite fading channels (such as generalized- $K$  channels) also can be approximated by the PDF of an  $\alpha - \mu$  distribution with the methods proposed in [14] and [15], respectively.

Although the SOP over  $\alpha - \mu$  fading channels was studied in [12] and the expressions for the SOP bound and the strictly positive secrecy capacity (SPSC) were derived, notice that the SOP and the SPSC are fundamental metrics to evaluate the secrecy performance of passive eavesdropping scenarios [1], [5] wherein the CSI of the eavesdropper’s channel is not available at the source node. ASC is a metric to evaluate the secrecy performance of active eavesdropping scenarios wherein the transmitter is aware of the CSI of eavesdropper’s channel [1], [3], [5]. Technically speaking, it is much more challenging to obtain a closed-form expression for the ASC relative to that of the SOP or SPSC, especially applicable to generalized fading channels.

Based on the open literature and to the best of the authors’ knowledge, it is still an open area to study the secrecy capacity over  $\alpha - \mu$  fading channels. The main contributions of our work are listed as follows:

- 1) The secrecy capacity of the classic Wyner’s model over  $\alpha - \mu$  fading channels is derived in closed-form, which enables the performance evaluation. Specifically, the numerical results unveil more secrecy capacity can be

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H. Lei is with Chongqing Key Lab of Mobile Communications Technology, Chongqing University of Posts and Communications, Chongqing 400065, China. H. Lei is also with Computer, Electrical, and Mathematical Science and Engineering (CEMSE) Division, King Abdullah University of Science and Technology (KAUST), Thuwal 23955-6900, Kingdom of Saudi Arabia (e-mail: leihj@cqupt.edu.cn).

I. S. Ansari is with the Department of Electrical and Computer Engineering (ECEN), Texas A&M University at Qatar (TAMUQ), Education City, Doha, Qatar (e-mail: imran.ansari@qatar.tamu.edu).

G. Pan is with Chongqing Key Laboratory of Nonlinear Circuits and Intelligent Information Processing, Southwest University, Chongqing, 400715, China (e-mail: gfp@swu.edu.cn).

B. Alomair is with the National Center for Cybersecurity Technology (C4C), King Abdulaziz City for Science and Technology (KACST), Riyadh 11442, Saudi Arabia (e-mail: alomair@kacst.edu.sa).

M.-S. Alouini is with Computer, Electrical, and Mathematical Sciences and Engineering (CEMSE) Division, King Abdullah University of Science and Technology (KAUST), Thuwal 23955-6900, Kingdom of Saudi Arabia (e-mail: slim.alouini@kaust.edu.sa).

achieved under rigorous fading channels, that is, serious nonlinearity (small  $\alpha$ ) and sparse clustering (small  $\mu$ ) will lead to the improvement of ASC.

- 2) The asymptotic analysis of ASC in high signal-to-noise ratio (SNR) regime is further performed to extract more insightful results. It is found that the ASC follows the scaling law of  $\Theta(\ln \rho)$ , where  $\rho$  is the ratio of the average power gains of the main channel to that of the eavesdropping channel. In addition, although ASC can be enhanced by increasing the transmit SNR, the asymptotic analysis proves that ASC is upper bounded as the transmit SNR approaches to infinity.

## II. SYSTEM MODEL

We consider the classic Wyner's wiretap model, as assumed in [9], [10], [12]. It is assumed that all the channels experience independent  $\alpha$ - $\mu$  fading and the CSI of all the channels is available at  $S$  since active eavesdropping is considered in our work.

The PDF of SNR for  $\alpha$ - $\mu$  fading channels is expressed as [12],

$$f_k(\gamma) = \eta_k \gamma^{\frac{\alpha_k \mu_k}{2} - 1} e^{-\lambda_k \gamma^{\frac{\alpha_k}{2}}}, \quad (1)$$

where  $k \in \{D, E\}$ ,  $\eta_k = \frac{\alpha_k \mu_k}{2\Gamma(\mu_k)} \bar{\gamma}_k^{-\frac{\alpha_k \mu_k}{2}}$ ,  $\lambda_k = \mu_k \bar{\gamma}_k^{-\frac{\alpha_k}{2}}$ ,  $\alpha_k$  and  $\mu_k$  are the two physical fading parameters that reflect the nonlinearity and clustering.  $\Gamma(c) = \int_0^\infty t^{c-1} e^{-t} dt$  is the well-known Gamma function,  $\bar{\gamma}_k = \gamma_t \bar{Y}_k$  are the average SNRs,  $\gamma_t$  are transmit SNRs, and  $\bar{Y}_k$  are the average power gains. This model is one of the most general fading model and can span a wide range of multipath fading models, such as Rayleigh ( $\alpha_k = 2, \mu_k = 1$ ), Nakagami- $m$  ( $\alpha_k = 2, \mu_k = m$ ), and Weibull ( $\mu_k = 1$ ) fading [12], [13].

## III. AVERAGE SECRECY CAPACITY ANALYSIS

In active eavesdropping scenarios,  $S$  can transmit confidential messages to guarantee perfect secrecy at an achievable rate since the CSI of the eavesdropper's channel is available, which is a fundamental assumption in physical layer security [1], [3], [5], [10]. The secrecy capacity, which is the maximum achievable secrecy rate, is essentially a fundamental secrecy performance metric. According to [3], the instantaneous secrecy capacity is defined as  $C_s = [\ln(1 + \gamma_D) - \ln(1 + \gamma_E)]^+$ , where  $\ln(1 + \gamma_D)$  and  $\ln(1 + \gamma_E)$  are the capacity of the main and eavesdropper channels, respectively, and  $[x]^+ = \max\{x, 0\}$ .

According to [10], ASC can be given as  $\bar{C}_s = I_1 + I_2 - I_3$ , where  $I_1 = \int_0^\infty \ln(1 + \gamma_D) f_D(\gamma_D) F_E(\gamma_D) d\gamma_D$ ,  $I_2 = \int_0^\infty \ln(1 + \gamma_E) f_E(\gamma_E) F_D(\gamma_E) d\gamma_E$ ,  $I_3 = \int_0^\infty \ln(1 + \gamma_E) f_E(\gamma_E) d\gamma_E$ ,  $F_k(\gamma) = \frac{1}{\Gamma(\mu_k)} \Upsilon\left(\mu_k, \lambda_k \gamma^{\frac{\alpha_k}{2}}\right)$  ( $k \in \{D, E\}$ ) is the cumulative distribution function (CDF) of the SNR, and  $\Upsilon(\alpha, x) = \int_0^x e^{-t} t^{\alpha-1} dt$  is the lower incomplete Gamma function (defined by eq. (8.350.1) of [16]).

Substituting (1) and  $F_E(\gamma)$  into  $I_1$ , and making use of (8) and (9) of [12], (11) of [17], we obtain

$$I_1 = \frac{\eta_D}{\Gamma(\mu_E)} \int_0^\infty \gamma_D^{\frac{\alpha_D \mu_D}{2} - 1} G_{2,2}^{1,2} \left[ \gamma_D \left| \begin{matrix} 1, 1 \\ 1, 0 \end{matrix} \right. \right] \times G_{0,1}^{1,0} \left[ \lambda_D \gamma_D^{\frac{\alpha_D}{2}} \middle| 0 \right] G_{1,2}^{1,1} \left[ \lambda_E \gamma_D^{\frac{\alpha_E}{2}} \middle| \begin{matrix} 1 \\ \mu_E, 0 \end{matrix} \right] d\gamma_D, \quad (2)$$

where  $G_{p,q}^{m,n}[\cdot]$  is Meijer's  $G$ -function, as defined by eq. (9.301) of [16].

By making use of (6.2.8) of [18] and (2.3) of [19], the closed-form of  $I_1$  is obtained as (3), shown at the top of next page, where  $H_{p,q}^{m,n}[\cdot]$  is the Fox's  $H$ -function, as defined by eq. (1.2) of [20] and  $H_{p_1, q_1: p_2, q_2: p_3, q_3}^{m_1, n_1: m_2, n_2: m_3, n_3}[\cdot]$  is the EGBFHF function<sup>1</sup>, as defined by eq. (2.57) of [20].

Similar to  $I_1$ , the closed-form expression of  $I_2$  is obtained as (4), shown at the top of next page.

Now, on utilizing eqs. (11) and (21) of [17],  $I_3$  is obtained as

$$I_3 = \frac{\eta_E 2^{0.5} \alpha_E^{-1}}{(2\pi)^{\alpha_E - 0.5}} G_{2\alpha_E, 2+2\alpha_E}^{2+2\alpha_E, \alpha_E} \left[ \frac{\lambda_E^2}{4} \left| \begin{matrix} \chi, \Delta(\alpha_E, 1 - \frac{\alpha_E \mu_E}{2}) \\ \Delta(2, 0), \chi, \chi \end{matrix} \right. \right], \quad (5)$$

where  $\chi = \Delta(\alpha_E, -\frac{\alpha_E \mu_E}{2})$  and  $\Delta(m, a) = \frac{a}{m}, \frac{a+1}{m}, \dots, \frac{a+m-1}{m}$ .

Substituting (3), (4), and (5) into  $\bar{C}_s$ , the closed-form expression of ASC is obtained.

## IV. ASYMPTOTIC AVERAGE SECRECY CAPACITY ANALYSIS

Although we have obtained the exact closed-form expression for ASC in section III, it is difficult to achieve more insights from the results. In order to get more insights, we analysis the secrecy capacity performance in the high transmit SNR regime, which implies  $\gamma_t \rightarrow \infty$  in this section. The asymptotic expression ASC is given in Lemma 1.

**Lemma 1:** When  $\gamma_t \rightarrow \infty$ , we have  $\bar{C}_s \approx \frac{1}{\Gamma(\mu_D)\Gamma(\mu_E)} \sum_{i=1}^N \omega_i g(t_i)$ , where  $\omega_i$  and  $t_i$  are the abscissas and weight factors for the Gaussian-Laguerre integration [21, eq. (25.4.45)],  $\rho = \bar{Y}_D/\bar{Y}_E$ ,  $g(t) = t^{\mu_D-1} \ln\left((t/\mu_D)^{2/\alpha_D} \bar{Y}_D\right) \Upsilon\left(\mu_E, \mu_E \rho^{\frac{\alpha_E}{2}} \left(\frac{t}{\mu_D}\right)^{\alpha_E/\alpha_D}\right) - t^{\mu_E-1} \ln\left(\bar{Y}_E(t/\mu_E)^{2/\alpha_E}\right) \Gamma\left(\mu_D, \mu_D \rho^{-\alpha_D/2} (t/\mu_E)^{\alpha_D/\alpha_E}\right)$ , and  $\Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1} dt$  is the upper incomplete Gamma function (defined by eq. (8.350.2) of [16]).

**Proof:** Based on the definition of ASC, we have

$$\bar{C}_s = \int_0^\infty f_E(\gamma_E) d\gamma_E \int_{\gamma_E}^\infty \ln\left(\frac{1 + \gamma_D}{1 + \gamma_E}\right) f_D(\gamma_D) d\gamma_D. \quad (6)$$

Substituting  $\gamma_D = \gamma_t Y_D = \gamma_t \bar{Y}_D x$ ,  $\gamma_E = \gamma_t Y_E = \gamma_t \bar{Y}_E y$ , and (1) into (6), and after some algebra, we obtain

$$\begin{aligned} \bar{C}_s &= \Xi \int_0^\infty \int_{y/\rho}^\infty \ln\left(1 + \frac{\bar{Y}_D x - \bar{Y}_E y}{\frac{1}{\gamma_t} + \bar{Y}_E y}\right) \varphi(x, y) dx dy \\ &\stackrel{\gamma_t \rightarrow \infty}{\approx} \Xi \int_0^\infty \int_{y/\rho}^\infty \ln\left(\frac{x \bar{Y}_D}{y \bar{Y}_E}\right) \varphi(x, y) dx dy \\ &= \Xi (H_1 - H_2). \end{aligned} \quad (7)$$

where  $\varphi(x, y) = x^{\frac{\alpha_D \mu_D}{2} - 1} y^{\frac{\alpha_E \mu_E}{2} - 1} e^{-\mu_D x^{\frac{\alpha_D}{2}}} e^{-\mu_E y^{\frac{\alpha_E}{2}}}$ ,  $\Xi = \frac{\alpha_D \alpha_E \mu_D^{\frac{\alpha_D}{2}} \mu_E^{\frac{\alpha_E}{2}}}{4\Gamma(\mu_D)\Gamma(\mu_E)}$ ,  $H_1 = \int_0^\infty \int_{y/\rho}^\infty \ln(x \bar{Y}_D) \varphi(x, y) dx dy$ , and  $H_2 = \int_0^\infty \int_{y/\rho}^\infty \ln(y \bar{Y}_E) \varphi(x, y) dx dy$ .

<sup>1</sup>The EGBFHF function can be easily realized by utilizing the MATLAB® (see [22]) or with the MATHEMATICA® (see Table 1).

$$I_1 = \frac{\eta_D \lambda_D^{-\mu_D}}{\alpha_D \Gamma(\mu_D)} H_{1,0:2;2:1,2}^{1,0:1,2:1,1} \left[ - \left( 1 - \mu_D \frac{2}{\alpha_D}, \frac{\alpha_D}{\alpha_D} \right) \left| \begin{matrix} (1,1), (1,1) \\ (1,1), (0,1) \end{matrix} \right| \begin{matrix} (1,1) \\ (\mu_E, 1), (0,1) \end{matrix} \right] \lambda_D^{-\frac{2}{\alpha_D}}, \lambda_E \lambda_D^{-\frac{\alpha_E}{\alpha_D}} \right], \quad (3)$$

$$I_2 = \frac{\eta_E \lambda_E^{-\mu_E}}{\alpha_E \Gamma(\mu_D)} H_{1,0:2;2:1,2}^{1,0:1,2:1,1} \left[ - \left( 1 - \mu_E \frac{2}{\alpha_E}, \frac{\alpha_D}{\alpha_E} \right) \left| \begin{matrix} (1,1), (1,1) \\ (1,1), (0,1) \end{matrix} \right| \begin{matrix} (1,1) \\ (\mu_D, 1), (0,1) \end{matrix} \right] \lambda_E^{-\frac{2}{\alpha_E}}, \lambda_D \lambda_E^{-\frac{\alpha_D}{\alpha_E}} \right]. \quad (4)$$

Now altering the order of integration, making use of (3.351.1) of [16], and with some algebraic manipulations, we obtain  $H_1 = \frac{4\mu_D^{-\mu_D} \mu_E^{-\mu_E}}{\alpha_D \alpha_E} \sum_{i=1}^N \omega_i g_1(t_i)$ , where  $\omega_i$  and  $t_i$  are the abscissas and weight factors for the Gaussian-Laguerre integration [21, eq. (25.4.45)], and  $g_1(t) = \Upsilon(\mu_E, \mu_E \rho^{\alpha_E/2} (t/\mu_D)^{\alpha_E/\alpha_D}) t^{\mu_D-1} \ln((t/\mu_D)^{2/\alpha_D} \bar{Y}_D)$ .

Similarly, making use of (3.351.2) of [16], we have  $H_2 = \frac{4\mu_D^{-\mu_D} \mu_E^{-\mu_E}}{\alpha_D \alpha_E} \sum_{i=1}^N \omega_i g_2(t_i)$ , where  $g_2(t) = \Gamma(\mu_D, \mu_D \rho^{-\alpha_D/2} (\frac{t}{\mu_E})^{\alpha_D/\alpha_E}) t^{\mu_E-1} \ln((t/\mu_E)^{2/\alpha_E} \bar{Y}_E)$ . Substituting  $H_1$  and  $H_2$  into (7), the upper bound expression for ASC is derived. ■

*Remark 1:* Obviously, one can find that the ASC will be improved as  $\gamma_t$  increases as indicated by (7).

*Remark 2:* We rewrite eq. (7) as follows

$$\bar{C}_s = \int_0^\infty \int_{y/\rho}^\infty \ln(x/y) \Xi \varphi(x, y) dx dy + \ln \rho \int_0^\infty \int_{y/\rho}^\infty \Xi \varphi(x, y) dx dy. \quad (8)$$

Note that both integrals in (8) are consistent, so it can be concluded that the ASC follows the scaling law of  $\Theta(\ln \rho)$  in high SNR regime.

In the Section V, one will find that the ASC can be improved with  $\gamma_t$ , which means the transmitting power at  $S$  increases. From *Lemma 1*, it can be found that there exists a ceiling for the ASC when in the high  $\gamma_t$  region since the SNR at both  $D$  and  $E$  are improved simultaneously.

## V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, numerical and Monte-carlo simulations results are presented to verify our analytical results. Furthermore, the impact of both the fading parameters and the average SNRs on the secrecy performance are demonstrated. To the authors' best knowledge, the EGBFHF is not available in standard mathematical packages and MATLAB® library. As the implementation in [22] is not a common case to evaluate EGBFHF function, an efficient Mathematica® implementation of this function is offered for any general case (hence it also fits for the numerical evaluation of eqs. (3) and (4)), as shown in Table 1<sup>2</sup>. The main parameters used in simulations and analysis are set as  $\alpha_D = \alpha_E = \alpha$  and  $\mu_D = \mu_E = \mu$ . The curves are examined for various  $\alpha$  and  $\mu$  for comparison purposes while varying  $\gamma_t$ .

<sup>2</sup>The EGBMGF code given in [23] is a special case of this.

TABLE I  
MATHEMATICA® IMPLEMENTATION OF EXTENDED GENERALIZED BIVARIATE BIVARIATE FOX'S H-FUNCTION

```
ClearAll; Clear[H];
H:=InconsistentCoeffs="Inconsistent coefficients!";
H[{ast_, bst_}, {as_, bs_}, {at_, bt_}, {zs_, zt_}]:=Module[{},
  Pas =
    Function[u, Product[Gamma[1-as[[1, n, 1]]-uas[[1, n, 2]]], {n, 1, Length[as[[1]]]}];
  Qas = Function[u, Product[Gamma[as[[2, n, 1]]+uas[[2, n, 2]]], {n, 1, Length[as[[2]]]}];
  Pbs = Function[u, Product[Gamma[bs[[1, n, 1]]+ubs[[1, n, 2]]], {n, 1, Length[bs[[1]]]}];
  Qbs = Function[u, Product[Gamma[1-bs[[2, n, 1]]-ubs[[2, n, 2]]], {n, 1, Length[bs[[2]]]}];
  Ms = Function[u, Pas[u] Pbs[u] / (Qas[u] Qbs[u]);
  Pat =
    Function[u, Product[Gamma[1-at[[1, n, 1]]-uat[[1, n, 2]]], {n, 1, Length[at[[1]]]}];
  Qat = Function[u, Product[Gamma[at[[2, n, 1]]+uat[[2, n, 2]]], {n, 1, Length[at[[2]]]}];
  Pbt = Function[u, Product[Gamma[bt[[1, n, 1]]+ubt[[1, n, 2]]], {n, 1, Length[bt[[1]]]}];
  Qbt = Function[u, Product[Gamma[1-bt[[2, n, 1]]-ubt[[2, n, 2]]], {n, 1, Length[bt[[2]]]}];
  Mt = Function[u, Pat[u] Pbt[u] / (Qat[u] Qbt[u]);
  Past = Function[{u, v}, Product[Gamma[
    1-ast[[1, n, 1]]-uas[[1, n, 2]]-vast[[1, n, 3]]], {n, 1, Length[ast[[1]]]}];
  Qast = Function[{u, v}, Product[Gamma[ast[[2, n, 1]]+uas[[2, n, 2]]+vast[[2, n, 3]]], {n, 1, Length[ast[[2]]]}];
  Qbst = Function[{u, v}, Product[Gamma[1-bst[[2, n, 1]]-ubst[[2, n, 2]]-
    vbst[[2, n, 3]]], {n, 1, Length[bs[[2]]]}];
  Mst = Function[{u, v}, Past[u, v] / (Qast[u, v] Qbst[u, v]);
  MT = Function[{u, v}, Ms[u] Mt[v] Mst[u, v]];
  Rs = -1/2; Rt = 1/2; Zs = zs; Zt = zt; W = 10;
  value = 1/(2*pi*I)^2 NIntegrate[MT[s, t] Zs^-s Zt^-t, {s, Rs-IW, Rs+IW}, {t, Rt-IW, Rt+IW}];
  Return[value];];
```

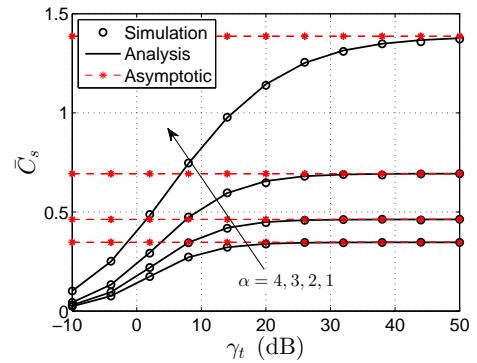


Fig. 1. ASC for various  $\alpha$ .

In Figs. 1-3, simulation and analytical results are compared for the ASC over  $\alpha$ - $\mu$  fading channels. It is clear that our analytical results have been verified by the simulation results. Figs. 1 and 2 present the ASC vs.  $\gamma_t$  for various  $\alpha$  or  $\mu$ . It can be observed from Figs. 1 and 2 that the ASC for a lower  $\alpha$  or  $\mu$  outperforms the one for a higher  $\alpha$  or  $\mu$ , reflecting the nonlinearity and clustering, respectively. This is because low  $\alpha$  and  $\mu$  means serious nonlinearity and sparse clustering (i.e., worse channel conditions). It justifies that the inherent nonlinearity and clustering of fading channels can be exploited to prevent the information being overheard by the

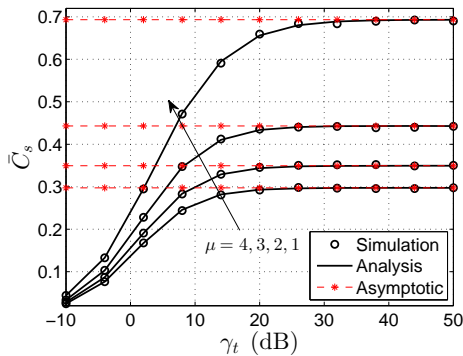


Fig. 2. ASC for various  $\mu$ .

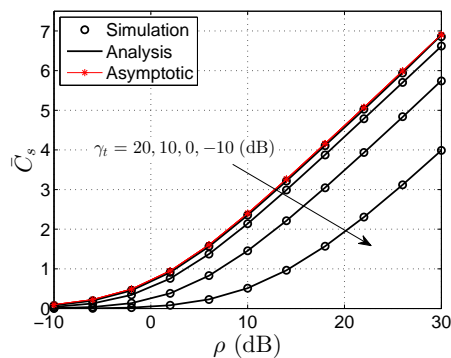


Fig. 3. ASC for various  $\gamma_t$ .

eavesdropper in physical layer security scheme. Furthermore, it can be observed that the ASC can be improved while increasing  $\gamma_t$ , which means improving the transmit SNR at  $S$  can enhance the security performance. This is because the appropriate code scheme can be utilized to widen the SNR gap between the main and the wiretap channels when the perfect CSI of the eavesdropping channel is available at  $S$ . When  $\alpha_D = \alpha_E = 2, \mu_D = \mu_E = 1$ , our results match the results in [3]. When  $\alpha_D = \alpha_E = 2$ , our results consistent with the results in [6].

Fig. 3 presents the ASC vs.  $\rho$  for various  $\gamma_t$ , while  $\mu = 1$  and  $\alpha = 2$ . One can observe that ASC can be improved with increasing  $\rho$  due to a higher  $\rho$  representing the quality of the main channel superior than the eavesdropping channel. One also can find from Fig. 3 that the ASC gradually approaches to the upper bound as  $\gamma_t$  increases since the SNR at both  $D$  and  $E$  are improved simultaneously. Furthermore, we can find that there is a linear relationship between the growth rate of ASC and the  $\rho$  in the high  $\rho$  regime.

## VI. CONCLUSION

In this work, the exact and asymptotic closed-form expressions for the average secrecy capacity over  $\alpha$ - $\mu$  fading channel were derived and validated through simulations. The proposed models can be used to analyze the secrecy performance over small-scale fading channels (such as exponential, Rayleigh, Gamma, Nakagami- $m$ , Weibull), large-scale fading channels

(such as log-normal), and composite fading channels (such as generalized- $K$ ).

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