

# A Novel Simulation Method for Nakagami- $m$ Fading Channel

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## Abstract

The existing Nakagami channel simulation methods have drawbacks of the phase information lacking or bad performance between first-order and second-order channel characteristic. This paper presents a novel complex Nakagami- $m$  fading channel simulation scheme combining the brute force method and the sum-of-sinusoids method to achieve real channel envelope sequence and implementing phase information with rejection method. Simulation results show that the proposed method has good first order and second-order statistical channel properties.

## 1. Introduction

Nakagami- $m$  distribution can emulate fading occasions from mild, moderate to severe, which includes the distribution of Rayleigh and Rice fading process, and thus has a more extensive application in fading channel simulation. Main simulation method for Nakagami- $m$  fading channel simulation includes the brute force method [1], the sum-of-sinusoids method (SoS)[2], the inverse transform method[3], the rejection method[4-7], and the combination method[8]. These methods each have their own shortcomings, there is no one can simultaneously make the following properties consistent with theoretical values well: (1) a large traversing parameter  $m$ , (2) the first-order envelope and phase PDF(Probability Distribution Function), (3) the second order LCR(Level Cross Rate) and AFD(Average Fade Duration) characteristics. Literature [9] proposed an improved method combining the brute force and the SoS method. This scheme can achieve large traverse parameters  $m$ , and its envelope PDF and second-order characteristics consistent with theoretical value well. However, its phase PDF result is not so satisfactory. Based on literature [9], we propose a new complex Nakagami- $m$  fading channel simulation method to obtain better phase distribution information using uniform hats rejection method.

## 2. Nakagami- $m$ Fading Process Statistical Characteristic

The envelope and phase PDF of Nakagami- $m$  distribution are given as follows:

$$P(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{mr^2}{\Omega}}, r \geq 0 \quad (1)$$

$$P(\varphi) = \frac{\Gamma(m) |\sin 2\varphi|^{m-1}}{2^m \Gamma^2(\frac{m}{2})}, \varphi \in [0, 2\pi) \quad (2)$$

Where  $\Omega = E[r^2]$  is average power,  $m = \Omega^2 / E\{[r^2 - \Omega]^2\}$  is Nakagami distribution shape factor, and  $\Gamma(\cdot)$  is gamma function. Nakagami- $m$  fading LCR obey the following distribution:

$$N(r) = \frac{\sqrt{2\pi} f_d m^{m-0.5} r^{2m-1} e^{-mr^2}}{\Gamma(m)} \quad (3)$$

Where  $f_d$  is the maximum doppler frequency shift.

## 3. The Proposed Novel Nakagami- $m$ Fading Simulation Method

The proposed novel Nakagami- $m$  fading simulation model is shown in figure 1. Firstly, we get the envelope distribution simulation sequence  $R(t)$  by weighting the  $p+1$  rayleigh complex random process together with

doppler effect, and then obtain Nakagami phase distribution based on the rejection method, finally the complex channel simulation sequence  $Z(t)$  is got through equation (4).

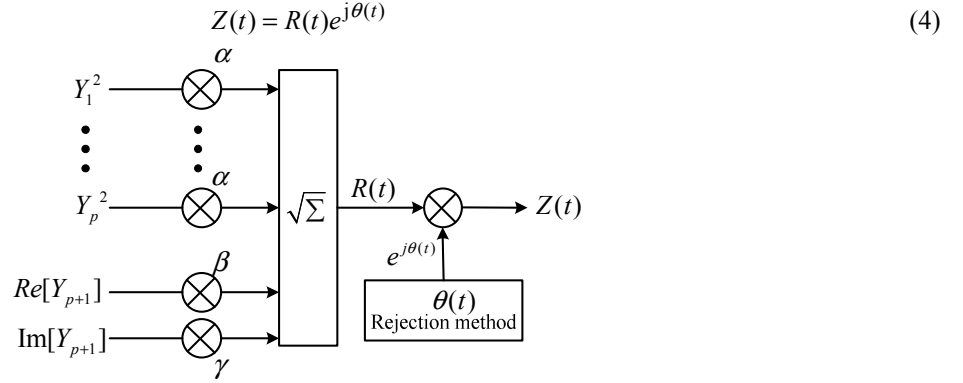


Figure 1 The proposed novel method schematic

### 3.1 The Envelope Simulation of Nakagami- $m$ Fading Process

Firstly, the Brute Force method is extended to an arbitrary integer  $m$  based on channel decomposition ideas in literature [10]. Secondly, Rayleigh sequence  $Y_k(t)$  is simulated using SoS method where Jakes power spectrum coefficients are calculated by precise doppler expansion method[11]. Thirdly,  $p+1$  Rayleigh sequence are weighted to compose a stochastic envelope sample sequence  $R(t)$  with the formula given as below:

$$R(t) = \sqrt{\alpha \sum_{k=1}^p Y_k(t)^2 + \beta \text{Re}[Y_{p+1}(t)]^2 + \gamma \text{Im}[Y_{p+1}(t)]^2} \quad (5)$$

Where  $p = \lfloor m \rfloor$  represents the integer part of the value  $m$ ;  $\alpha, \beta, \gamma$  are the weighting factors, and the following conditions must be satisfied

$$\begin{cases} \alpha = 0(p=0) & \alpha = 1(p>0) \\ \beta = m - p + \sqrt{(p-m)(m-p-1)} \\ \gamma = m - p - \sqrt{(p-m)(m-p-1)} \end{cases} \quad (6)$$

### 3.2 The Phase Simulation of Nakagami- $m$ Fading Process

The uniform distribution hat function of rejection method is used to generate Nakagami phase sequence. The uniform hat function  $t(x)$  is

$$t(x) = b \quad [0, a) \quad (7)$$

Where  $a$  is the horizontal boundary of uniform distribution hat function, i.e.  $2\pi$ ;  $b$  is the longitudinal boundary of hat function corresponding to the maximum Nakagami phase probability value. The following algorithm is used to produce phase random variables  $X$  that obey Nakagami distribution.

- (1) Generate a random variable  $Y$  with probability density function:

$$h(x) = \frac{t(x)}{\int_{-\infty}^{\infty} t(x) dx} \quad (8)$$

- (2) Generate a uniform random variable  $U [0, 1]$ ;
- (3) If  $U \leq P(\varphi) / t(Y)$ , make  $X=Y$ , otherwise, returns to step (1);

## 4. Simulation Results Analysis

The envelop/phase PDF and LCR of the proposed Nakagami channel simulation result comparison with theoretical values are given in figure 2 and figure 3 respectively. It can be seen that the simulation results agree with theoretical value very well under distinguished  $m$ . Since the channel envelope simulation is based on SoS method and

the channel phase simulation is based on rejection method, simulation accuracy and simulation speed can be adjusted though it is an inverse relationship between the two.

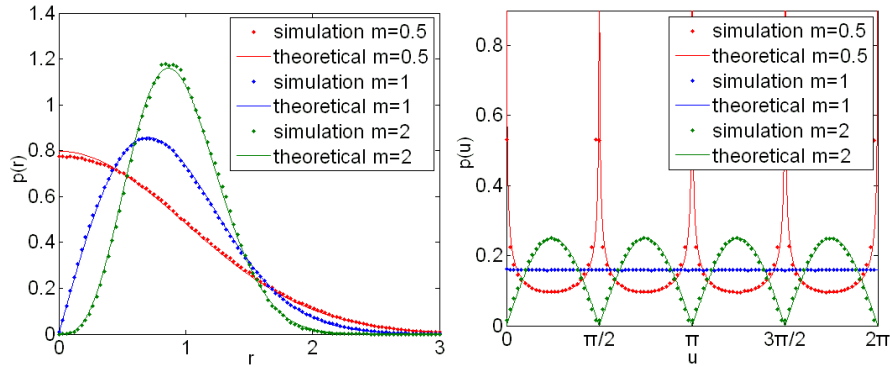


Figure 2 The envelope and phase PDF of the improved Nakagami- $m$  simulation method

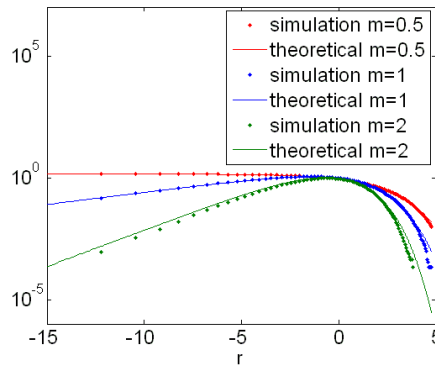


Figure 3 LCR of the improved Nakagami- $m$  simulation method

## 5. Conclusion

This paper propose a new Nakagami- $m$  complex fading channel simulation method using channel decomposition technology with rejection method to achieve accurate phase distribution characteristics based on the existing Brute Force and SoS combination method, Simulation results show that the statistical properties of the first order and second order make good agreement with theoretical values. In addition, this paper considers the doppler spread enabling fast fading realization, which makes the proposed complex Nakagami- $m$  simulation channel a greater scope of application.

## 6. Acknowledgments

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