Performance Analysis of Selection Combining in Decode-and-Forward Cooperative Diversity Networks over Weibull Fading Channel

Yuejun Lei[1, 2], Zhimin Zeng
[1]School of Information and Communication Engineering
Beijing University of Posts and Telecommunications
Beijing, P.R. China
lyj1978@gmail.com, zengzm@bupt.edu.cn

Weijun Cheng
[2]School of Information Engineering
Minzu University of China
Beijing, P.R. China
wjcheng@pku.edu.cn

***Abstract*—In this paper, we investigate decode-and-forward (DF) cooperative diversity over independent and identically distributed (i.i.d) Weibull fading channel using selection combining (SC) technique, derive a closed form expression for the probability density function (PDF) of the signal-to-noise ratio (SNR) for the Weibull fading distribution, and obtain closed form expressions for the average symbol error probability (ASEP), the outage probability, the average channel capacity and the moment of average SNR. Computer simulations are also performed to verify the analytical results.**

Keywords- Cooperative diversity; decode-and-forward; Weibull fading; error probability; outage probability.

I. INTRODUCTION

Recently years, very much attention is paid to cooperative wireless communications due to the fact that these systems can substantially enhance system performance with respect to much less power consumption, higher system capacity, smaller packet loss rate, and higher network resilience. Cooperative diversity technology using relay nodes cooperating with direct node to transmit data to destination node is one of important technologies in cooperative wireless communications. The performance analysis of cooperative diversity over Rayleigh and Nakagami fading channel has been widely studied in the past few years, the authors in [1] proposed a variety of low complexity cooperative protocols which are amplify-and-forward (non-regenerative) relaying and decode-and-forward (regenerative) relaying and also analyzed the outage probability over Rayleigh fading channel. In [2]-[5], the authors present the performance analysis of Rayleigh fading channel. In [6]-[8], the authors present the performance analysis of Nakagami fading channel.

Most papers are focus on Rayleigh fading channel and Nakagami fading channel, while there are few papers investigating about the performance of cooperative diversity over Weibull fading channel which fits well with experimental fading channel measurements, for both indoor and outdoor terrestrial radio propagation environments [9]-[10], Dual Hop Relaying over Weibull fading channel has been presented in [11-12], In [13] the author present the performance analysis of amplify-and-forward cooperative diversity over Weibull fading channel with SC.

This paper is focus on the performance analysis of decode-and-forward cooperative diversity over i.i.d Weibull fading channel with SC, The remainder of this paper is organized as follows. In the next section, we derived closed-form expressions for PDF and the cumulative distribution function (CDF) of the SNR at the destination in decode-and-forward cooperative diversity Networks over Weibull fading channel with SC. In section 3, the ASEP, the outage probability, the nth SNR moments, and the channel capacity at the destination are obtained. Section 4 presents some numerical simulations. Finally the conclusions are given in Section 5.

II. SYSTEM MODEL

A. System Model

We consider a system with a source node (S) communicating with a destination node (D) and a relay node (R) which retransmits the messages received from the source node to the destination node using decode-and-forward scheme over flat Weibull fading channel, as shown in Fig.1. We also assume that the additive white Gaussian noise (AWGN) terms of all links have zero mean and equal variance (N_0). The instantaneous SNR γ_0 between S and D, the instantaneous SNR γ_1 between S and R, and the instantaneous SNR γ_2 between R and D are given by $\gamma_i = h_i^2 E_s / N_0$ (i=0, 1, 2), the PDF of γ_i is given by [14]

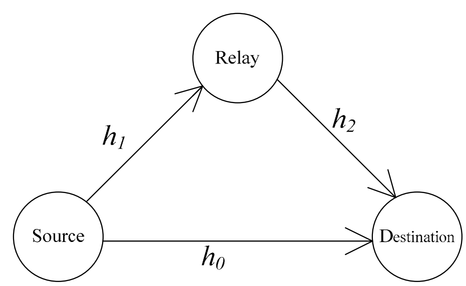

Fig. 1. Illustration of cooperative diversity network

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$$f_{\gamma i}\left(\gamma\right)=\frac{c/2}{\beta_i}\gamma^{c/2-1}\exp\left(-\frac{\gamma^{c/2}}{\beta_i}\right) \quad (1)$$

where $\beta_i=\left(\overline{\gamma_i}/\Gamma\left(1+\frac{2}{c}\right)\right)^{c/2}$, $\overline{\gamma_i}=E\left(h_i^2\right)\frac{E_s}{N_0}$, i=0, 1, 2, h_0, h_1 and h_2 which are mutually independent and identical are the Weibull fading coefficients between source and destination, between source and relay, between relay and destination, $\Gamma(\ )$ is the Gamma function and c is the fading parameter expressing the severity of fading.

Using decode-and-forward scheme, the destination SNR with SC can be expressed as

$$\gamma_{sc}=\begin{cases}\max\left(\gamma_0,\gamma_2\right) & \text{If relay decode correctly}\\ \gamma_0 & \text{If relay decode uncorrectly}\end{cases} \quad (2)$$

Due to the difficulty of finding the PDF of γ_{sc} given in (2), the decode-and-forward cooperative diversity network in Fig. 1 can be visualized as a system that has effective 2 paths between the source and the destination[3]-[4]. Let 1th path represent the S → D direct link and 2th path represent the S → R → D indirect (relaying) link. The 2th path equivalent output SNR at the destination can be represented as a random variable ξ that takes account of both the source to the relay link and the relay to destination link. Therefore, ξ has PDF as [8]

$$\begin{aligned}f_\xi\left(\gamma\right)=&f_{\xi|R\text{ Decodes Incorrectly}}\left(\gamma\right)\Pr\left(R\text{ Decodes Incorrectly}\right)\\&+f_{\xi|R\text{ Decodes Correctly}}\left(\gamma\right)\Pr\left(R\text{ Decodes Correctly}\right)\end{aligned} \quad (3)$$

When relay node decodes incorrectly, the received SNR at the destination caused by Relay will be zero, therefore $f_{\xi|R\text{ Decodes Incorrectly}}\left(\gamma\right)=\delta\left(\gamma\right)$, when relay node decodes correctly, therefore

$$f_{\xi|R\text{ Decodes Correctly}}\left(\gamma\right)=\frac{c/2}{\beta_2}\gamma^{c/2-1}\exp\left(-\frac{\gamma^{c/2}}{\beta_2}\right) \quad (4)$$

The probability of relay node decodes incorrectly for coherent modulation (M-PSK) is given by

$$\text{Z}=\Pr\left(R\text{ Decodes Incorrectly}\right)=\int_0^\infty P_{SE}\left(e|\gamma\right)f_{\gamma 1}\left(\gamma\right)d\gamma \quad (5)$$

where $P_{SE}\left(e|\gamma\right)$ for coherent modulation is $A\mathrm{erfc}(\sqrt{B\gamma})$ in which *erfc*( ) is the complementary error function, the values of A and B depend on the modulation schemes [14], for binary phase shift keying (BPSK) the value of A is 1/2 and the value of B is 1. With the help of Meijer's G-function [15, (9.3)] and [16, (21)], (5) can be rewritten as follows.

$$\text{Z}=\frac{Ac\sqrt{k}l^{\frac{c}{2}-1}B^{-\frac{c}{2}}}{2\sqrt{\pi}\beta_1\left(\sqrt{2\pi}\right)^{k+l-2}}G_{2l,k+l}^{k,2l}\left[\frac{l^l}{B^l\beta_1^k k^k}\left|\begin{matrix}\Delta\left(l,1-\frac{c}{2}\right),\Delta\left(l,\frac{1}{2}-\frac{c}{2}\right)\\ \Delta\left(k,0\right),\Delta\left(l,-\frac{c}{2}\right)\end{matrix}\right.\right] \quad (6)$$

where $\Delta(p,q)=q/p,q+1/p,\ldots\ ,q+p-1/q$,in which p is a positive integer, q is a real constant, G( ) is the Meijer's G-function, l and k are the smallest positive integers of equation $l/k=c/2$, for example if c=2.5,then k= 5, l=4.

Therefore, the PDF of ξ can be expressed as

$$f_\xi\left(\gamma\right)=Z\delta\left(\gamma\right)+\left(1-Z\right)\frac{c/2}{\beta_2}\gamma^{c/2-1}\exp\left(-\frac{\gamma^{c/2}}{\beta_2}\right) \quad (7)$$

Using SC at the destination node, the total SNR at the destination node can be written as follows

$$\gamma_{sc}=\max\left(\xi,\gamma_0\right) \quad (8)$$

B. Probability Density Function (PDF)

Assuming $\overline{\gamma_0}=\overline{\gamma_1}=\overline{\gamma_2}=\overline{\gamma}$, then $\beta_0=\beta_1=\beta_2=\beta$, the PDF of the total SNR at the destination node is given by

$$\begin{aligned}f_{\gamma_{sc}}(\gamma)=&(ZU\left(\gamma\right)+2\left(1-Z\right))\frac{c/2}{\beta}\gamma^{c/2-1}\exp\left(-\frac{\gamma^{c/2}}{\beta}\right)\\&-\left(1-Z\right)\frac{c/2}{\beta/2}\gamma^{c/2-1}\exp\left(-\frac{\gamma^{c/2}}{\beta/2}\right)\\&+Z\delta\left(\gamma\right)\left(1-\exp\left(-\frac{\gamma^{c/2}}{\beta}\right)\right)\end{aligned} \quad (9)$$

C. Cumulative Distribution Function (CDF)

The Cumulative Distribution Function is given by

$$F_{\gamma sc}(\gamma)=\left(2-Z\right)\left(1-\exp\left(-\frac{\gamma^{c/2}}{\beta}\right)\right)-\left(1-Z\right)\left(1-\exp\left(-\frac{\gamma^{c/2}}{\beta/2}\right)\right) \quad (10)$$

III. PERFORMANCE ANALYSIS

A. The Moments

The nth order moment of the destination SNR is given by

$$u_{sc}=E_{sc}\left(\gamma^n\right) \quad (11)$$

With the help of [15, (3.326)], (11) can be rewritten as

$$u_{sc}=\left(\left(2-Z\right)\beta^{\frac{n}{c/2}}-\left(1-Z\right)\left(\beta/2\right)^{\frac{n}{c/2}}\right)\Gamma\left(\frac{n+c/2}{c/2}\right) \quad (12)$$

When n=1, the average SNR is obtained.

B. Error Performance

The ASEP for coherent modulation (M-PSK) can be written as

$$P_{SE} = \int_0^\infty P_{SE}\left(e|\gamma\right) f_{\gamma sc}\left(\gamma\right) d\gamma \tag{13}$$

With the help of Meijer's G-function [15, (9.3)] and [16, (21)], (13) can be rewritten as follows.

$$\begin{aligned} P_{SE} = & \frac{(2-Z) Ac\sqrt{k} l^{\frac{c}{2}-1} B^{-\frac{c}{2}}}{2\sqrt{\pi}\beta\left(\sqrt{2\pi}\right)^{k+l-2}} G_{2l,k+l}^{k,2l}\left[\frac{l^l}{B^l \beta^k k^k}\middle|\begin{matrix}\Delta\left(l,1-\frac{c}{2}\right),\Delta\left(l,\frac{1}{2}-\frac{c}{2}\right)\\ \Delta(k,0),\Delta\left(l,-\frac{c}{2}\right)\end{matrix}\right] \\ & -\frac{(1-Z) Ac\sqrt{k} l^{\frac{c}{2}-1} B^{-\frac{c}{2}}}{\sqrt{\pi}\beta\left(\sqrt{2\pi}\right)^{k+l-2}} G_{2l,k+l}^{k,2l}\left[\frac{l^l}{B^l (\beta/2)^k k^k}\middle|\begin{matrix}\Delta\left(l,1-\frac{c}{2}\right),\Delta\left(l,\frac{1}{2}-\frac{c}{2}\right)\\ \Delta(k,0),\Delta\left(l,-\frac{c}{2}\right)\end{matrix}\right] \end{aligned} \tag{14}$$

where $\Delta(p,q)=q/p,q+1/p,\ldots\ ,q+p-1/q$, in which p is a positive integer, q is a real constant, G( ) is the Meijer's G-function, l and k are the smallest positive integers of equation $l/k=c/2$, for example if c=2.5, then k= 5, l=4.

C. Outage Probability

The Outage Probability is given by

$$P_{out} = F_{\gamma sc}(\gamma_{th}) = (2-Z)\left(1-\exp\left(-\frac{\gamma_{th}^{c/2}}{\beta}\right)\right) - (1-Z)\left(1-\exp\left(-\frac{\gamma_{th}^{c/2}}{\beta/2}\right)\right) \tag{15}$$

D. Channel Capacity

Considering a signal's transmission of bandwidth BW, the average channel capacity of two branch cooperative diversity can be obtained as

$$\overline{C} = \frac{BW}{2}\int_0^\infty \log_2\left(1+\gamma\right) f_{\gamma sc}\left(\gamma\right) d\gamma \tag{16}$$

With the help of Meijer's G-function [15, (9.3)] and [16, (21)], (16) can be rewritten as

$$\begin{aligned} \overline{C} = & \frac{(2-Z) BW\sqrt{k}}{4\ln 2\beta l\left(\sqrt{2\pi}\right)^{k+2l-3}} G_{2l,k+2l}^{k+2l,2l}\left[\frac{1}{(\beta)^k k^k}\middle|\begin{matrix}\Delta\left(l,-\frac{c}{2}\right),\Delta\left(l,1-\frac{c}{2}\right)\\ \Delta(k,0),\Delta\left(l,-\frac{c}{2}\right),\Delta\left(l,-\frac{c}{2}\right)\end{matrix}\right] \\ & -\frac{(1-Z) BW\sqrt{k}}{2\ln 2\beta l\left(\sqrt{2\pi}\right)^{k+2l-3}} G_{2l,k+2l}^{k+2l,2l}\left[\frac{1}{(\beta/2)^k k^k}\middle|\begin{matrix}\Delta\left(l,-\frac{c}{2}\right),\Delta\left(l,1-\frac{c}{2}\right)\\ \Delta(k,0),\Delta\left(l,-\frac{c}{2}\right),\Delta\left(l,-\frac{c}{2}\right)\end{matrix}\right] \end{aligned} \tag{17}$$

IV. NUMERICAL RESULTS

In this section, assuming that BPSK signaling is considered and using the previous analytical results, (14), (15) and (17) have been numerically evaluated over i.i.d flat Weibull fading channel, and the results are shown in Figs. 2, Figs. 3 and Figs.4.

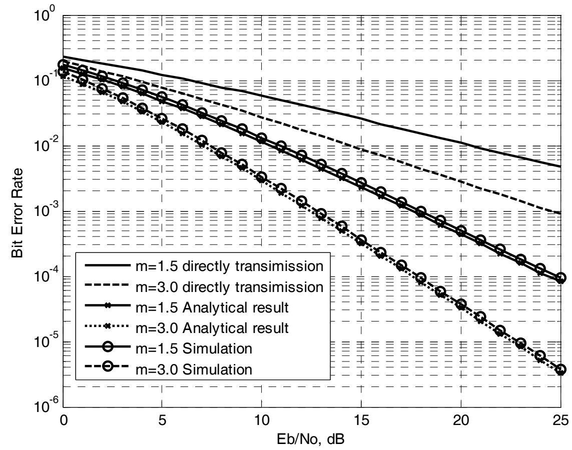

Fig. 2. Error probability for different values of c.

Computer simulations are also performed to verify the analytical results.

Fig.2 depicts bit error rate (BER), obtained analytically and via simulation for direct transmission and cooperative transmission of different value of c over Weibull fading channel. It is shown that BER decreases as c increases from 1.5 to 3, and BER for cooperative transmission is smaller than that for direct transmission.

Fig.3 shows the analytical and simulated outage probability for direct transmission and cooperative transmission of different value of c over Weibull fading channel. It is shown that outage probability becomes smaller as c becomes larger. The outage probability for cooperative transmission is also better than that for direct transmission.

In Fig.4, analytical and simulated average capacity for different values of c over Weibull fading channel is plotted. Average capacity is improved as c increased, while average capacity for cooperative transmission is worse than that for direct transmission.

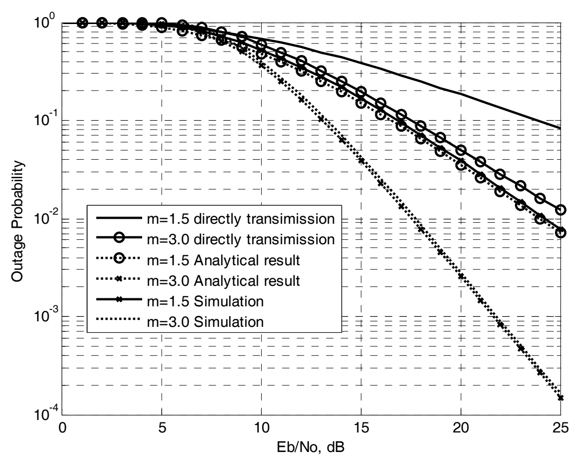

Fig. 3. Outage probability for different values of c, γ_{th}=10 db.

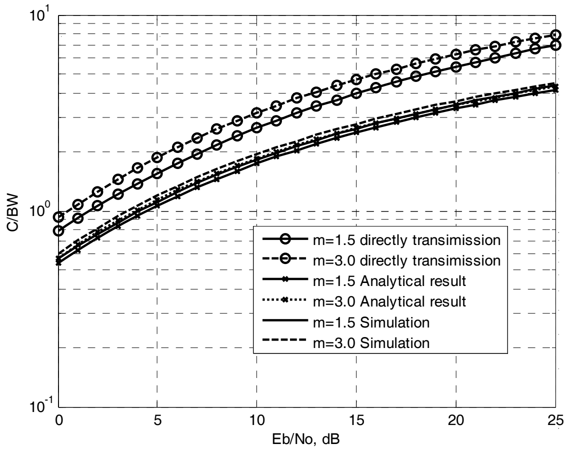

Fig. 4. Average capacity for different values of *c*.

V. CONCLUSION

Performance analysis of decode-and-forward cooperative diversity networks over flat Weibull fading channel is presented in this paper. Closed-form expressions of ASEP, outage probability, the nth moment of the destination SNR and channel average capacity are obtained, Analytical and simulated results are also presented to illustrate the cooperative diversity with SC can decrease ASEP, the outage probability and the average capacity compared to direct transmission over Weibull fading channel.

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