On the Performance Analysis of Digital Communications over Weibull-Gamma Channels

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***Abstract*—In this work, the performance analysis of digital communications over a composite Weibull-Gamma (WG) multipath-fading and shadowing channel is presented wherein WG distribution is appropriate for modeling fading environments when multipath is superimposed on shadowing. More specifically, in this work, exact closed-form expressions are derived for the probability density function, the cumulative distribution function, the moment generating function, and the moments of a composite WG channel. Capitalizing on these results, new exact closed-form expressions are offered for the outage probability, the higher-order amount of fading, the average error rate for binary and M-ary modulation schemes, and the ergodic capacity under various types of transmission policies, mostly in terms of Meijer's G functions. These new analytical results were also verified via computer-based Monte-Carlo simulation results.**

***Index Terms*—Weibull-Gamma, composite channels, fading, shadowing, moments, outage probability, binary modulation schemes, bit-error rate (BER), amount of fading (AF), ergodic capacity, Meijer's G function.**

I. Introduction

A. Background

Wireless communications are driven by a complicated phenomenon known as radio-wave propagation that is characterized by various effects such as fading, shadowing, and path-loss [1]. The statistical behavior of these effects is described by different models depending on the nature of the communication environment. As such, in many real life propagation scenarios, multipath fading and shadowing occur simultaneously leading to a composite fading environment [2]. Depending on the nature of the radio propagation environment, several distributions have been proposed in the literature for modeling multipath fading, including Rayleigh, Nakagami-m, and Weibull [2]. On the other hand, shadowing is modeled by the lognormal (LN) distribution and/or Gamma distribution [1]. Based on these distributions, various combinations have been suggested for obtaining the composite environment closely. These include, but are not limited to, Rayleigh-LN, Rician-LN, Nakagami-LN, Weibull-LN, Gamma-Gamma, and Rician-Gamma [1], and a relatively new one, the Weibull-Gamma (WG) [2], [3].

B. Motivation

A common difficulty in dealing with LN-based composite distributions is their mathematical complexity that further limits their potential applications. Hence, researchers resort to utilizing Gamma distribution in modeling the shadowing effects as Gamma-based composite channels are mathematically more convenient and indeed model the shadowing effects quite accurately [3]. The wide versatility, experimental validity, and analytical tractability of WG composite distribution makes it very convenient to be utilized in various communication systems of the current age. Using the Weibull multipath fading model that is versatile enough to model various multipath fading conditions ranging from severe fading to non-fading scenario, and the Gamma model for shadowing has led to the WG composite fading model [2]. WG distribution, recently being used in the context of wireless digital communications over fading channels, is one of the relatively new tractable models used to describe the statistical behavior of multipath fading and shadowing effects as compared to log-normal based models. The WG fading model is quite general model as it includes Rayleigh-Gamma and exponential-Gamma distributions as its special case and accurately approximates many other fading models [1].

C. Related Work

Various statistical characteristics such as the probability density function (PDF), the cumulative distribution function (CDF), the moment generating function (MGF), and the moments of the WG fading model were presented recently in [2]. Additionally, the authors in [3] have studied the WG composite channels in presence of interference. However, to the best of the authors knowledge, a thorough performance analysis of the WG fading model is not available in the open literature and thus is the subject of this work.

D. Contributions

Hence, in this work:

- Exact closed-form expressions [1] are derived for the PDF, the CDF, the MGF, and the moments of a composite WG channel.
- Capitalizing on these statistical characteristics, new exact closed-form expressions are offered for the higher-order

[1]The authors acknowledge the valuable work already done in [2] and would like to share that they are presenting these results again in this work only in terms of different parameters relative to the work presented in [2].

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amount of fading (AF) and the average error rate for binary modulation schemes.

- The diversity order and the coding gain are also derived and these results are, to the best of the authors knowledge, novel to the open literature.
- Finally, novel expressions for the ergodic capacity of a WG composite fading channel over various transmission adaptation policies are also derived.

The results are mostly in terms of the Meijer's G functions and these expressions clearly signify the performance of a single receiver operating over such multipath/shadowing channels.

E. Structure

The remainder of the paper is organized as follows. Section II describes the WG composite channel model followed by its statistical characteristics in Section III. Then, Section IV derives new exact closed-form expressions for various performance metrics applicable to the WG fading model. Finally, Section V shares some results and discussions followed by conclusion in Section VI.

II. Channel and System Models

A. Weibull Channel

Let us consider h_W representing the channel fading envelope and following the Weibull distribution with the PDF given by [1, Eq. (2.27)]

$$f_W\left(h_W\right) = c\left[\Gamma\left(1+2/c\right)/\Omega\right]^{c/2} h_W^{c-1} \times \exp\left\{-\left[h_W^2\,\Gamma\left(1+2/c\right)/\Omega\right]^{c/2}\right\}, \quad h_W \geq 0, \tag{1}$$

where c is the shaping parameter, Ω is the average fading power, and $\Gamma\left(.\right)$ is the Gamma function [4, Eq. (8.310)]. The subsequent power PDF for (1) is derived as [1, Eq. (2.29)]

$$f_W\left(\gamma_W\right) = c/2\left[\Gamma\left(1+2/c\right)/\overline{\gamma}_W\right]^{c/2} \gamma_W^{c/2-1} \times \exp\left\{-\left[\gamma_W\,\Gamma\left(1+2/c\right)/\overline{\gamma}_W\right]^{c/2}\right\}, \quad \gamma_W \geq 0, \tag{2}$$

where γ_W is the instantaneous signal-to-noise ratio (SNR) and $\overline{\gamma}_W$ is the average SNR of the Weibull fading distribution.

B. Gamma Channel

Now, when the multipath fading is superimposed on the shadowing, $\overline{\gamma}_W$ slowly varies and hence it can be considered as a random variable following the Gamma distribution with the PDF given by [1, Eq. (2.21)]

$$f_G\left(\gamma_G\right) = \frac{m^m\,\gamma_G^{m-1}}{\Gamma\left(m\right)\overline{\gamma}_G^m}\exp\left\{-\frac{m}{\overline{\gamma}_G}\gamma_G\right\}, \quad \gamma_G \geq 0, \tag{3}$$

where $m > 0$ and $\overline{\gamma}_G > 0$ are known as fading figure representing the diversity order of the fading environment and the mean of the local power, respectively. In more details, the parameter m quantifies the severity of multipath fading, in the sense that small values of m indicates severe multipath fading and vice versa.

C. Composite Weibull-Gamma Channel

Under such a scenario, the PDF in (2) is conditioned on $\overline{\gamma}_W$ and in order to remove this conditioning, (2) is averaged over (3) as

$$f_{WG}\left(\gamma_{WG}\right) = \int_0^\infty f_W\left(\gamma_W|\overline{\gamma}_W\right)\, f_G\left(\gamma_G\right)\, d\overline{\gamma}_W. \tag{4}$$

On substituting (2) and (3) appropriately in (4), replacing the exponentials with appropriate Meijer's G function representation via utilizing $\exp\left(-x\right) = \mathrm{G}_{0,1}^{1,0}\left[x\,\middle|\,{-\atop 0}\right]$ [5, Eq. (11)], where G[.] is the Meijer's G function as defined in [4, Eq. (9.301)], applying [4, Eq. (9.31.2)], and finally utilizing [5, Eq. (21)], the composite WG PDF is obtained as

$$f_{WG}\left(\gamma_{WG}\right) = \frac{c^{m-\frac{c-1}{2}}\,m^{\frac{c}{2}}}{\sqrt{2}\,\sqrt{2\pi}^{\,c}\,\Gamma\left(m\right)}\left[\Gamma\left(1+\frac{2}{c}\right)\right]^{\frac{c}{2}}\frac{\gamma_{WG}^{\frac{c}{2}-1}}{\overline{\gamma}_{WG}^{\frac{c}{2}}} \times \mathrm{G}_{0,c+2}^{c+2,0}\left[\frac{1}{4}\left[\frac{m\,\Gamma\left(1+\frac{2}{c}\right)}{c\,\overline{\gamma}_{WG}}\right]^c \gamma_{WG}^c\,\middle|\,{- \atop \frac{1}{2},0,\Delta\left(c,m-\frac{c}{2}\right)}\right], \tag{5}$$

where $\Delta\left(a,b\right) = b/a,\ldots,\left(b+a-1\right)/a$. Some special cases of (5) are given as follows. When $m \to \infty$, (5) leads to the Weibull distribution and when $m \to \infty$ and $c \to \infty$, (5) approaches the additive white Gaussian noise (AWGN) channel [2]. The result in (5) is equivalent to the one derived in [2, Eq. (4)].

III. Closed-Form Statistical Characteristics

This section will derive various other statistical characteristics by capitalizing on the PDF derived above in (5).

A. Cumulative Distribution Function

Utilizing [6, Eq. (07.34.21.0084.01)] and performing some simple algebraic manipulations, the CDF for the WG fading channel can be shown to be given by

$$F_{WG}\left(\gamma_{WG}\right) = \frac{c^{m-\frac{c+1}{2}}\,m^{\frac{c}{2}}}{\sqrt{2}\,\sqrt{2\pi}^{\,c}\,\Gamma\left(m\right)}\left[\Gamma\left(1+\frac{2}{c}\right)\right]^{\frac{c}{2}}\frac{\gamma_{WG}^{\frac{c}{2}}}{\overline{\gamma}_{WG}^{\frac{c}{2}}} \times \mathrm{G}_{1,c+3}^{c+2,1}\left[\frac{1}{4}\left[\frac{m\,\Gamma\left(1+\frac{2}{c}\right)}{c\,\overline{\gamma}_{WG}}\right]^c \gamma_{WG}^c\,\middle|\,{\frac{1}{2} \atop \frac{1}{2},0,\Delta\left(c,m-\frac{c}{2}\right),-\frac{1}{2}}\right]. \tag{6}$$

This result[2] in (6) is equivalent to the one derived in [2, Eq. (10)].

B. Moment Generating Function

The MGF is defined as

$$\mathcal{M}_{WG}(s) \triangleq \mathbb{E}\left[e^{-\gamma_{WG}\,s}\right] = \int_0^\infty e^{-\gamma_{WG}\,s}\, f_{WG}(\gamma_{WG})\, d\gamma_{WG}. \tag{7}$$

Utilizing [6, Eq. (07.34.21.0088.01)] and performing some simple algebraic manipulations, the MGF for the WG fading

[2]This result in (6) can be easily expressed in terms of simple elementary functions via utilizing (33) from the Appendix.

channel can be shown to be given by

$$\mathcal{M}_{WG}(s) = \frac{c^m\, m^{\frac{c}{2}}}{\sqrt{2}\, (2\pi)^{c-\frac{1}{2}}\, \Gamma(m)} \left[\Gamma\left(1+\frac{2}{c}\right)\right]^{\frac{c}{2}} \frac{1}{(s\,\overline{\gamma}_{WG})^{\frac{c}{2}}} \times \mathrm{G}_{c,c+2}^{c+2,c}\left[\frac{1}{4}\left[\frac{m\,\Gamma\left(1+\frac{2}{c}\right)}{s\,\overline{\gamma}_{WG}}\right]^c \middle| \begin{matrix} \Delta\left(c, 1-\frac{c}{2}\right) \\ \frac{1}{2}, 0, \Delta\left(c, m-\frac{c}{2}\right) \end{matrix}\right]. \tag{8}$$

This result [3] in (8) is equivalent to the one derived in [2, Eq. (11)].

C. Moments

Since both the Weibull and the Gamma are independent random variables, the moments can be defined as

$$\mathbb{E}\left[\gamma_{WG}^n\right] = \mathbb{E}\left[\gamma_W^n\right]\mathbb{E}\left[\gamma_G^n\right]. \tag{9}$$

Based on (9), the moments of the Weibull distribution can be easily derived as $\mathbb{E}\left[\gamma_W^n\right] = \Gamma\left(1+\frac{2n}{c}\right)/\Gamma\left(1+\frac{2}{c}\right)^n$ and the moments of the Gamma distribution are well known to be $\mathbb{E}\left[\gamma_G^n\right] = \Gamma(m+n)/\left[m^n\Gamma(m)\right]$ [1, Eq. (2.23)]. Utilizing these, the moments for the WG composite fading channel are given as

$$\mathbb{E}\left[\gamma_{WG}^n\right] = \frac{\Gamma\left(1+\frac{2n}{c}\right)\Gamma(m+n)}{\left[m\,\Gamma\left(1+\frac{2}{c}\right)\right]^n\Gamma(m)}\,\overline{\gamma}_{WG}^n. \tag{10}$$

This result in (10) is equivalent to the one derived in [2, Eq. (7)].

IV. Applications

This section will capitalize on the statistical characteristics derived in Section III to derive various performance metrics such as the OP, the higher-order AF, the average bit error rate (BER), the average symbol error rate (SER), and the ergodic capacity under various transmission policies.

A. Outage Probability

When the instantaneous output SNR γ_{WG} falls below a given threshold $\gamma_{WG_{\mathrm{th}}}$, a situation labeled as outage is encountered and it is an important feature to study the OP of a system. Hence, another important fact worth stating here is that the expression derived in the (6) also serves the purpose for the expression of the OP of a WG composite fading channel or in other words, when the desired user is subject to a WG composite fading channel, the probability that the SNR falls below a predetermined protection ratio $\gamma_{WG_{\mathrm{th}}}$ can be simply expressed by replacing γ_{WG} with $\gamma_{WG_{\mathrm{th}}}$ in (6) as

$$P_{\mathrm{out}}(\gamma_{WG_{\mathrm{th}}}) = F_\gamma(\gamma_{WG_{\mathrm{th}}}). \tag{11}$$

[3]This result in (8) can be easily expressed in terms of simple elementary functions via utilizing (33) from the Appendix.

TABLE I
Conditional Bit Error Probability (BER) Parameters of Binary Modulations

Modulation	p	q
Coherent Binary Frequency Shift Keying (CBFSK)	0.5	0.5
Coherent Binary Phase Shift Keying (CBPSK)	0.5	1
Non-Coherent Binary Frequency Shift Keying (NBFSK)	1	0.5
Differential Binary Phase Shift Keying (DBPSK)	1	1

B. Higher-Order Amount of Fading

The AF is an important measure for the performance of a wireless communication system as it can be utilized to parameterize the distribution of the SNR of the received signal [7]. In particular, the n^{th}-order AF for the instantaneous SNR γ_{WG} is defined as [7]

$$AF_{\gamma_{WG}}^{(n)} = \frac{\mathbb{E}\left[\gamma_{WG}^n\right]}{\mathbb{E}\left[\gamma_{WG}\right]^n} - 1. \tag{12}$$

Now, substituting (10) into (12), the n^{th}-order AF is obtained as

$$AF_{\gamma_{WG}}^{(n)} = \frac{\Gamma\left(1+\frac{2n}{c}\right)\Gamma(m+n)\,\Gamma(m)^{n-1}}{\Gamma\left(1+\frac{2}{c}\right)^n\Gamma(m+1)^n} - 1. \tag{13}$$

For $n=2$, as a special case, the classical AF [8] is obtained as

$$AF = AF_{\gamma_{WG}}^{(2)} = \frac{\Gamma\left(1+\frac{4}{c}\right)\Gamma(m+2)\,\Gamma(m)}{\Gamma\left(1+\frac{2}{c}\right)^2\Gamma(m+1)^2} - 1. \tag{14}$$

C. Average BER

Utilizing the expression derived in [9, Eq. (12)] by substituting (6) into it and utilizing [6, Eq. (07.34.21.0088.01)], a new exact closed-form for the average BER of a variety of binary modulations is obtained in terms of the Meijer's G function as

$$\overline{P}_{b_{WG}} = \frac{q^{-\frac{c}{2}}\, c^{m+p-1}\, m^{\frac{c}{2}}}{2\sqrt{2}\,(2\pi)^{c-\frac{1}{2}}\,\Gamma(p)\,\Gamma(m)} \left[\Gamma\left(1+\frac{2}{c}\right)\right]^{\frac{c}{2}} \frac{1}{\overline{\gamma}_{WG}^{\frac{c}{2}}} \times \mathrm{G}_{c+1,c+3}^{c+2,c+1}\left[\frac{1}{4}\left[\frac{m\,\Gamma\left(1+\frac{2}{c}\right)}{q\,\overline{\gamma}_{WG}}\right]^c \middle| \begin{matrix} \Delta\left(c, 1-p-\frac{c}{2}\right), \frac{1}{2} \\ \frac{1}{2}, 0, \Delta\left(c, m-\frac{c}{2}\right), -\frac{1}{2} \end{matrix}\right], \tag{15}$$

where the parameters p and q account for different modulation schemes [4]. For an extensive list of modulation schemes represented by these parameters, one may look into [10] or refer to Table I.

1) Diversity Order and Coding Gain: With the help of average BER, one may also obtain the coding gain and the diversity order via utilizing $\overline{P}_b \approx (G_c\,\overline{\gamma}_{WG})^{-G_d}$ [11, Eq. (1)]. To express the derived expression of $\overline{P}_b$ in (15) in an appropriate form to be able to deduce the coding gain and the diversity order, (15) is first algebraically manipulated via utilizing [12, Eq. (6.4.2)] and [4, Eq. (9.31.2)]. Then

[4]This result in (15) has been easily expressed in (16) in terms of simple elementary functions via utilizing (33) from the Appendix.

the resulting expression is expressed in terms of a simple summation, utilizing the Meijer's G function expansion given in the Appendix (33) and with the help of some simple algebraic manipulations, as

$$\begin{aligned}\overline{P}_b \underset{\overline{\gamma}_{WG}>>1}{\approxeq} & \frac{c^{m+p-1}}{\sqrt{2}\,(2\,\pi)^{c-\frac{1}{2}}\,\Gamma(p)\,\Gamma(m)} \\ & \times \sum_{k=1}^{c+2}\left[4\left(\frac{q}{m\,\Gamma\left(1+\frac{2}{c}\right)}\right)^{c}\overline{\gamma}_{WG}^{\,c}\right]^{a_k-1} \\ & \times \frac{\prod_{l=1;\,l\neq k}^{c+2}\Gamma(a_k-a_l)\prod_{l=1}^{c+1}\Gamma(1+b_l-a_k)}{\Gamma(2-a_k)},\end{aligned} \tag{16}$$

where a_k represents the k-th element of $\{0,1/2,\Delta(c,-m+(c+1)/2)\}$ comprising of $c+2$ terms, a_l represents the l-th element of $\{0,1/2,\Delta(c,-m+(c+1)/2),1\}$ comprising of $c+3$ terms, and b_l represents the l-th element of $\{\Delta(c,p+(c-1)/2),0\}$ comprising of $c+1$ terms. The dominant term can be easily deduced to result in $\min(0,1/2,\Delta(c,-m+(c+1)/2))$. Based on the dominant term, $\overline{P}_b$ can be further reduced to a single term without the summation leading to the resulting diversity order to be given as $G_d = c\,(1-\min(0,\Delta(c,-m+(c+1)/2)))$ and subsequently the resulting coding gain to be given as

$$\begin{aligned}G_c = & \frac{4^{\frac{1}{c}}\,q}{m\,\Gamma\left(1+\frac{2}{c}\right)}\left[\frac{c^{m+p-1}}{\sqrt{2}\,(2\,\pi)^{c-\frac{1}{2}}\,\Gamma(p)\,\Gamma(m)}\right. \\ & \left.\times\frac{\prod_{l=1;\,l\neq k}^{c+2}\Gamma(a_k-a_l)\prod_{l=1}^{c+1}\Gamma(1+b_l-a_k)}{\Gamma(2-a_k)}\right]^{\frac{1}{c\,(a_k-1)}}.\end{aligned} \tag{17}$$

D. Average SER

In [13], the conditional SER has been presented in a desirable form and utilized to obtain the average SER of M-AM, M-PSK, and M-QAM. For example, for M-PSK the average SER $\overline{P}_s$ over generalized fading channels is given by [13, Eq. (41)]. Similarly, for M-AM and M-QAM, the average SER $\overline{P}_s$ over generalized fading channels is given by [13, Eq. (45)] and [13, Eq. (48)], respectively. On substituting (8) into [13, Eq. (41)], [13, Eq. (45)], and [13, Eq. (48)], the SER of M-PSK, M-AM, and M-QAM, respectively, can be derived easily. The analytical SER performance expressions obtained via the above substitutions are exact and can be easily estimated accurately by utilizing the Gauss-Chebyshev Quadrature (GCQ) formula [14, Eq. (25.4.39)] that converges rapidly, requiring only few terms for an accurate result [15].

E. Ergodic Capacity

The ergodic capacity of a fading channel is defined as the maximum achievable transmission rate under which the errors are recoverable. Depending on the knowledge of the channel state information (CSI) at the transmitter and/or receiver, various channel adaptive transmission policies are available in the literature such as the optimal rate adaptation (ORA), the optimal power and rate adaptation (OPRA), the channel inversion with fixed rate (CIFR), and the truncated channel inversion with fixed rate (TIFR).

1) Optimal Rate Adaptation: The ORA policy requires that the transmitted power is contant and CSI is available only at the receiver end. The ergodic channel capacity under the ORA transmission policy is given by

$$\overline{C}_{ORA_{WG}} = \frac{1}{\ln(2)}\int_0^\infty \ln(1+\gamma_{WG})\, f_{\gamma_{WG}}(\gamma_{WG})\, d\gamma_{WG}. \tag{18}$$

Now, on substituting (5) into (18), replacing the logarithmic function in this definition with appropriate Meijer's G function representation via utilizing $\ln(1+x) = \mathrm{G}_{2,2}^{1,2}\left[x\,\middle|\,\begin{smallmatrix}1,1\\1,0\end{smallmatrix}\right]$ [5, Eq. (11)], and finally applying [5, Eq. (21)], the new exact closed-form expression for the ergodic capacity of the WG composite fading channel is obtained (in Nats/sec/Hz) as given in (19)[5].

Asymptotic Analysis: The ergodic capacity under the ORA transmission policy can also be analyzed asymptotically in terms of simple elementary functions via utilizing the moments derived earlier. A ***high SNR*** asymptotic analysis can be done by utilizing the moments as [7, Eqs. (8) and (9)]

$$\overline{C}_{ORA_{WG}} \underset{\overline{\gamma}_{WG}>>1}{\approxeq} \log(\overline{\gamma}_{WG}) + \zeta, \tag{20}$$

where

$$\begin{aligned}\zeta &= \partial/\partial n\left(\mathbb{E}\left[\gamma_{WG}^n\right]/\mathbb{E}\left[\gamma_{WG}\right]^n - 1\right)\Big|_{n=0} \\ &= \partial/\partial n\, AF_{\gamma_{WG}}^{(n)}\Big|_{n=0}.\end{aligned} \tag{21}$$

The expression in (20) can be simplified to

$$\begin{aligned}\overline{C}_{ORA_{WG}} &\underset{\overline{\gamma}_{WG}>>1}{\approxeq} \log(\overline{\gamma}_{WG}) + \frac{\partial}{\partial n}\left(\frac{\mathbb{E}\left[\gamma_{WG}^n\right]}{\mathbb{E}\left[\gamma_{WG}\right]^n} - 1\right)\Bigg|_{n=0} \\ &= \frac{\partial}{\partial n}\,\mathbb{E}\left[\gamma_{WG}^n\right]\Big|_{n=0}.\end{aligned} \tag{22}$$

Hence, the first derivative of the moments in (10) is required to be evaluated at $n=0$ for high SNR asymptotic approximation to the ergodic capacity. The first derivative of the moments is derived as

$$\begin{aligned}\frac{\partial}{\partial n}\mathbb{E}\left[\gamma_{WG}^n\right] = & \frac{\Gamma\left(1+\frac{2\,n}{c}\right)\Gamma(m+n)}{\left[m\,\Gamma\left(1+\frac{2}{c}\right)\right]^n\Gamma(m)}\left\{-\ln[m]\right. \\ & -\ln\left[\Gamma(1+2/c)\right] + \psi(0,m+n) - 2\,\psi(0,1+2\,n/c)/c \\ & \left.+\ln\left[\overline{\gamma}_{WG}\right]\right\}\overline{\gamma}_{WG}^{\,n},\end{aligned} \tag{23}$$

where $\psi(.)$ is the digamma (psi) function [14, Eq. (6.3.1)], [4, Eq. (8.360.1)]. Evaluating (23) at $n=0$, following is obtained

$$\begin{aligned}\overline{C}_{ORA_{WG}} \underset{\overline{\gamma}_{WG}>>1}{\approxeq} & -\ln[m] - \ln\left[\Gamma\left(1+\frac{2}{c}\right)\right] - \frac{2}{c}E \\ & + \psi(0,m) + \ln\left[\overline{\gamma}_{WG}\right],\end{aligned} \tag{24}$$

where $E \approxeq 0.577216$ denotes the Euler-Mascheroni constant/Euler's Gamma/Euler's constant. Hence, (24) gives the required expression for the ergodic capacity $\overline{C}_{ORA_{WG}}$ at high SNR in terms of simple elementary functions. Furthermore, at high SNR, the ergodic capacity for the ORA policy and the OPRA policy perform similarly. Therefore, the result in (24) is applicable to both the ergodic capacity policies (i.e. ORA as well as OPRA).

[5]This result in (19) can be easily expressed in terms of simple elementary functions via utilizing (33) from the Appendix.

$$\overline{C}_{ORA_{WG}} = \frac{c^{m-\frac{c+1}{2}}\, m^{\frac{c}{2}}}{\sqrt{2}\,(2\pi)^{\frac{3\,c}{2}-1}\,\Gamma(m)} \left[\Gamma\left(1+\frac{2}{c}\right)\right]^{\frac{c}{2}} \frac{1}{\overline{\gamma}_{WG}^{\frac{c}{2}}}\, \mathrm{G}^{2\,c+3,c}_{c+1,2\,c+3}\left[\frac{1}{4}\left[\frac{m\,\Gamma\left(1+\frac{2}{c}\right)}{c\,\overline{\gamma}_{WG}}\right]^{c} \middle| \begin{matrix} \Delta\left(c,-\frac{c}{2}\right),\frac{1}{2} \\ \frac{1}{2},0,\Delta\left(c,m-\frac{c}{2}\right),\Delta\left(c,-\frac{c}{2}\right),-\frac{1}{2}\end{matrix}\right] \tag{19}$$

2) Optimal Power and Rate Adaptation: The OPRA transmission policy requires, along side an average transmitted power constraint, that the CSI is available both at the transmitter and the receiver ends so that the transmitter can adapt both its transmit power and rate according to the variations of the fading channels [16]. In such a way, the transmitter can allocate high power levels and rates when the end-to-end SNR γ_{WG} is high and vice versa. Hence, the ergodic capacity under OPRA policy is given as [16, Eq. (7)]

$$\overline{C}_{OPRA_{WG}} = \frac{1}{\ln(2)} \int_{\gamma_c}^{\infty} \ln\left(\frac{\gamma_{WG}}{\gamma_c}\right) f_{\gamma_{WG}}(\gamma_{WG})\, d\gamma_{WG}, \tag{25}$$

where γ_c is the optimal cut-off SNR below which the data transmission is suspended and must satisfy the following equality [16, Eq. (6)]

$$\int_{\gamma_c}^{\infty} \left(\frac{1}{\gamma_c} - \frac{1}{\gamma}\right) f_{WG}(\gamma_{WG})\, d\gamma_{WG} = 1. \tag{26}$$

Now, substituting (5) into (25), replacing the logarithmic function in this definition with appropriate Meijer's G function representation via utilizing $\ln(x)\, H(x-1) = \mathrm{G}^{0,2}_{2,2}\left[x \middle| \begin{smallmatrix}1,1\\0,0\end{smallmatrix}\right]$ [17, vol. 3, Eq. (8.4.6.2)], and finally applying [5, Eq. (21)], the new exact closed-form expression for the ergodic capacity of the WG composite fading channel under the OPRA transmission policy is obtained (in Nats/sec/Hz) as given in (27) [6].

Subsequently, the optimal cut-off SNR is also derived by solving (26) for γ_c. This is done as follows. At first, (5) is substituted into (26) followed by utilizing $x^v\, H(x-1) = \mathrm{G}^{0,1}_{1,1}\left[x \middle| \begin{smallmatrix}v+1\\v\end{smallmatrix}\right]$ [17, vol. 3, Eq. (8.4.2.2)] and finally applying [5, Eq. (21)] to obtain the optimal cut-off SNR as given in (28). Eq. (28) can be further simplified via numerical methods.

3) Channel Inversion with Fixed Rate: The CIFR adaptation policy is also referred to as zero outage capacity. Herein, the transmitter exploits the CSI to maintain a constant SNR at the receiver i.e. it inverts the channel fading [16]. Hence, the ergodic capacity for CIFR transmission policy is expressed as [16, Eq. (9)]

$$\overline{C}_{CIFR_{WG}} = \frac{1}{\ln(2)} \ln\left(1 + \frac{1}{\int_0^{\infty} \gamma_{WG}^{-1}\, f_{\gamma_{WG}}(\gamma_{WG})\, d\gamma_{WG}}\right), \tag{29}$$

where simply $\int_0^{\infty} \gamma_{WG}^{-1}\, f_{\gamma_{WG}}(\gamma_{WG})\, d\gamma_{WG} = \mathbb{E}\left[1/\gamma_{WG}\right]$. Now, substituting (5) into (29), applying change of variables and finally utilizing [4, Eq. (7.811.4)], the new exact closed-form expression for the ergodic capacity of the WG composite fading channel under the CIFR transmission policy is obtained (in Nats/sec/Hz) as given in (30).

[6]This result in (27) can be easily expressed in terms of simple elementary functions via utilizing (33) from the Appendix.

4) Truncated Channel Inversion with Fixed Rate: Since CIFR policy suffers with a large capacity penalty due to the large amount of transmitted power required to compensate for the deep channel fading, a better approach was introduced that is well known as the truncated CIFR or also referred to as TIFR [16]. Herein, the channel fading is inverted above a fixed cut-off fade depth, γ_0. The TIFR policy improves the channel capacity but at the expense of the OP. Hence, the ergodic capacity with the TIFR adaptation policy is expressed as [16, Eq. (12)]

$$\begin{aligned}\overline{C}_{TIFR_{WG}} = {} & \frac{1}{\ln(2)} \max_{\gamma_0 > 0} \left(1 - P_{out}(\gamma_0)\right) \\ & \times \ln\left(1 + \left(\int_{\gamma_0}^{\infty} \frac{1}{\gamma_{WG}}\, f_{\gamma_{WG}}(\gamma_{WG})\, d\gamma_{WG}\right)^{-1}\right),\end{aligned} \tag{31}$$

where γ_0 is the optimal cut-off SNR that can obtained by numerically solving $\partial C_{TIFR_{WG}}/\partial\gamma_0 = 0$. Now, substituting (5) and (6) for $\gamma_{WG} = \gamma_0$ into (31), utilizing [17, vol. 3, Eq. (8.4.2.2)], and finally applying [5, Eq. (21)], the new exact closed-form expression for the ergodic capacity of the WG composite fading channel under the TIFR transmission policy is obtained (in Nats/sec/Hz) as given in (32) [7]. Subsequently, the optimal cut-off SNR γ_0 can be obtained based on the derivative given right after (31) via numerical methods, similar to the $\overline{C}_{OPRA_{WG}}$ scenario in (28).

V. Results and Discussion

As an illustration of the mathematical formalism, numerical and simulation results for different performance metrics of a WG composite fading channel are shown. The link is modeled as WG fading channel with the effects of fading as ($c = 2, 3, 4$) and ($m = 1, 5$) unless specified otherwise.[8]

The OP is presented in Fig. 1 for the WG composite fading channel across the normalized average SNR. It can be observed from Fig. 1 that the simulation results provide a perfect match to the analytical results obtained in this work. Additionally, it can be observed that as the effect of the parameter c increases, the performance improves. Also, the results are as expected i.e. the OP decreases as the SNR increases.

Additionally, the average BER performance of coherent binary phase shift keying modulation scheme is presented in Fig. 2. The results are similar to the OP scenario discussed above in Fig. 1. In addition, different digital modulation

[7]This result in (32) can be easily expressed in terms of simple elementary functions via utilizing (33) from the Appendix.

[8]It is important to note here that these values for the parameters were selected from [2] subject to the standards to prove the validity of the obtained results and hence other specific values can be used to obtain the required results by design communication engineers before deployment. Also, for all cases, 10^6 realizations of the random variable were generated to perform the Monte-Carlo simulations in MATLAB.

$$\overline{C}_{OPRA_{WG}} = \frac{c^{m-\frac{c+3}{2}} \left(m\,\gamma_c\right)^{\frac{c}{2}}}{\sqrt{2}\,\sqrt{2\pi}^{\,c}\,\Gamma\left(m\right)} \left[\Gamma\left(1+\frac{2}{c}\right)\right]^{\frac{c}{2}} \frac{1}{\overline{\gamma}_{WG}^{\frac{c}{2}}} \mathrm{G}_{1,c+3}^{c+3,0}\left[\frac{1}{4}\left[\frac{m\,\gamma_c\,\Gamma\left(1+\frac{2}{c}\right)}{c\,\overline{\gamma}_{WG}}\right]^{c} \middle| \begin{matrix} \frac{1}{2} \\ 0, \Delta\left(c, m-\frac{c}{2}\right), -\frac{1}{2}, -\frac{1}{2} \end{matrix}\right] \tag{27}$$

$$\begin{aligned} \gamma_c = {} & \frac{c^{m-\frac{c+1}{2}}\, m^{\frac{c}{2}}}{\sqrt{2}\,\sqrt{2\pi}^{\,c}\,\Gamma\left(m\right)} \left[\Gamma\left(1+\frac{2}{c}\right)\right]^{\frac{c}{2}} \frac{1}{\overline{\gamma}_{WG}^{\frac{c}{2}}} \left(\mathrm{G}_{1,c+3}^{c+3,0}\left[\frac{1}{4}\left[\frac{m\,\gamma_c\,\Gamma\left(1+\frac{2}{c}\right)}{c\,\overline{\gamma}_{WG}}\right]^{c} \middle| \begin{matrix} -\frac{1}{2} \\ \frac{1}{2}, 0, \Delta\left(c, m-\frac{c}{2}\right), \frac{1}{2} \end{matrix}\right]\right. \\ & \left. -\mathrm{G}_{1,c+3}^{c+3,0}\left[\frac{1}{4}\left[\frac{m\,\gamma_c\,\Gamma\left(1+\frac{2}{c}\right)}{c\,\overline{\gamma}_{WG}}\right]^{c} \middle| \begin{matrix} \frac{1}{c}+\frac{1}{2} \\ \frac{1}{2}, 0, \Delta\left(c, m-\frac{c}{2}\right), \frac{1}{c}-\frac{1}{2} \end{matrix}\right]\right) \end{aligned} \tag{28}$$

$$\overline{C}_{CIFR_{WG}} = \ln\left(1 + \frac{4^{\frac{1}{c}}\,\sqrt{2\pi}^{\,c}\,\Gamma\left(m\right)\,\overline{\gamma}_{WG}}{\sqrt{2}\,c^{m-\frac{3}{2}}\,m\,\Gamma\left(1+\frac{2}{c}\right)\,\Gamma\left(1-\frac{1}{c}\right)\,\Gamma\left(\frac{1}{2}-\frac{1}{c}\right)\,\Gamma\left(\frac{m-1}{c}\right)\ldots\Gamma\left(\frac{m-2}{c}+1\right)}\right) \tag{30}$$

$$\begin{aligned} \overline{C}_{TIFR_{WG}} = {} & \ln\left(1 + \frac{\sqrt{2}\,\sqrt{2\pi}^{\,c}\,\Gamma\left(m\right)}{c^{m-\frac{c+1}{2}}\,m^{\frac{c}{2}}\,\gamma_0^{\frac{c}{2}-1}} \left[\Gamma\left(1+\frac{2}{c}\right)\right]^{-\frac{c}{2}} \overline{\gamma}_{WG}^{\frac{c}{2}}\, \mathrm{G}_{1,c+3}^{c+3,0}\left[\frac{1}{4}\left[\frac{m\,\gamma_0\,\Gamma\left(1+\frac{2}{c}\right)}{c\,\overline{\gamma}_{WG}}\right]^{c} \middle| \begin{matrix} \frac{1}{c}+\frac{1}{2} \\ \frac{1}{2}, 0, \Delta\left(c, m-\frac{c}{2}\right), \frac{1}{c}-\frac{1}{2} \end{matrix}\right]^{-1}\right) \\ & \times\left(1 - \frac{c^{m-\frac{c+1}{2}}\,m^{\frac{c}{2}}}{\sqrt{2}\,\sqrt{2\pi}^{\,c}\,\Gamma\left(m\right)} \left[\Gamma\left(1+\frac{2}{c}\right)\right]^{\frac{c}{2}} \frac{\gamma_0^{\frac{c}{2}}}{\overline{\gamma}_{WG}^{\frac{c}{2}}} \mathrm{G}_{1,c+3}^{c+2,1}\left[\frac{1}{4}\left[\frac{m\,\gamma_0\,\Gamma\left(1+\frac{2}{c}\right)}{c\,\overline{\gamma}_{WG}}\right]^{c} \middle| \begin{matrix} \frac{1}{2} \\ \frac{1}{2}, 0, \Delta\left(c, m-\frac{c}{2}\right), -\frac{1}{2} \end{matrix}\right]\right) \end{aligned} \tag{32}$$

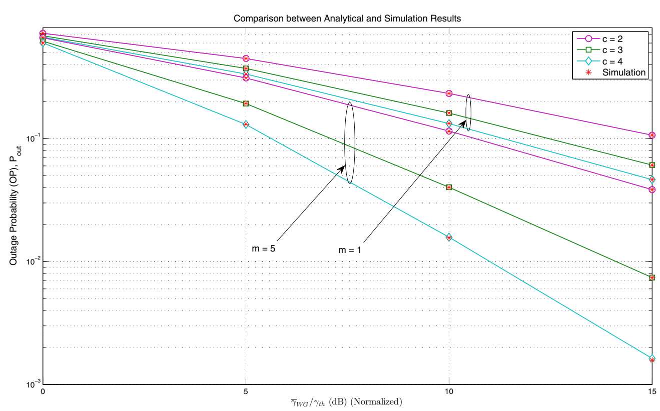

Fig. 1. OP showing the performance of a WG composite fading channel under different fading conditions (i.e. with varying effects of c and m).

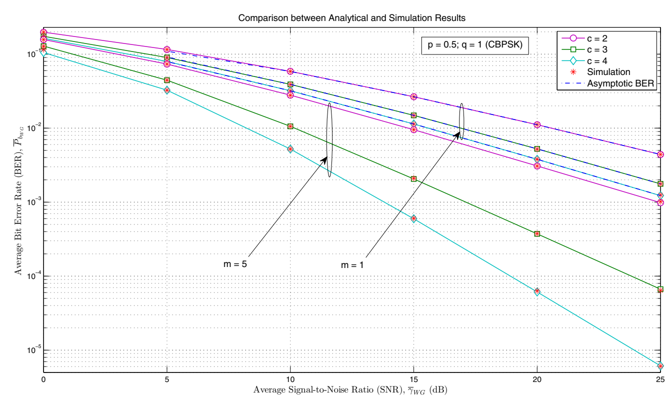

Fig. 2. Average BER of CBPSK modulation scheme for a WG composite fading channel with varying effects of c and m.

schemes are represented based on the values of p and q where $p = 0.5$ and $q = 0.5$ represents coherent binary frequency shift keying (CBFSK), $p = 0.5$ and $q = 1$ represents coherent binary phase shift keying (CBPSK), non-coherent binary frequency shift keying (NBFSK) is represented by $p = 1$ and $q = 0.5$, and differential binary phase shift keying (DBPSK) is represented by $p = 1$ and $q = 1$. All these possibilities are plotted in Fig. 3 conforming the fact that coherent binary modulation schemes out-perform their non-coherent counterparts.

Finally, in Fig. 4, it can be observed that as the fading conditions get severe, the ergodic capacity starts decreasing i.e. the higher the values of c and m, the higher will be the ergodic capacity. Additionally, from Fig. 4, one may observe the tightness of the asymptotic results derived in (24).

VI. Concluding Remarks

Exact closed-form expressions are derived for the PDF, the CDF, the MGF, and the moments of a WG composite fading channel. Further, novel exact closed-form analytical expressions are derived for various performance metrics of a WG composite fading channel including the OP, the higher-order AF, the error rate of a variety of modulation schemes, and the ergodic capacity applicable for various adaptation policies. Additionally, this work also derived tight asymptotic results for the ORA ergodic capacity at high SNR regime. Finally, this work also presented numerical and simulation examples to validate and illustrate the mathematical formulation developed in this work and to show the effect of the fading conditions severity and unbalance on the system performance.

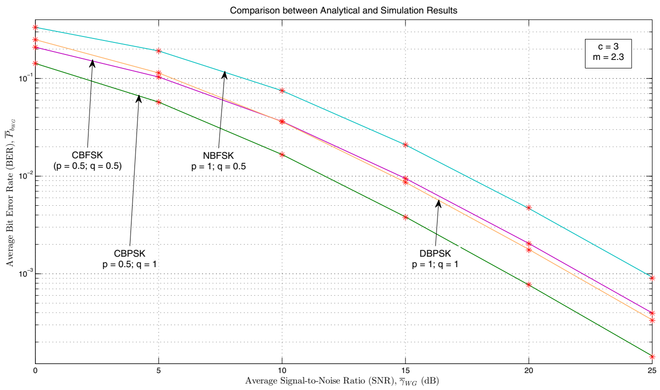

Fig. 3. Average BER of various binary modulation schemes for a WG composite fading channel with constant effect of $c = 3$ and $m = 2.3$.

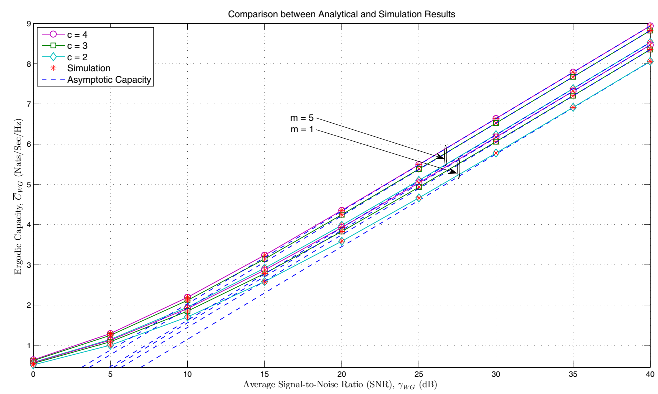

Fig. 4. Ergodic capacity under ORA policy of a WG composite fading channel with varying fading parameters c's and m's along with their respective high SNR asymptotes.

Appendix: Meijer's G Function Expansion

The Meijer's G function can be expressed, at a very high value of its argument, in terms of basic elementary functions via utilizing Meijer's G function expansion in [18, Theorem 1.4.2, Eq. (1.4.13)] and $\lim_{x\to 0^+} {}_cF_d\left[e; f; x\right] = 1$ [19] as

$$\lim_{z\to\infty^+} \mathrm{G}_{p,q}^{m,n}\left[z \left| \begin{matrix} a_1, \ldots, a_n, \ldots, a_p \\ b_1, \ldots, b_m, \ldots, a_q \end{matrix} \right.\right] = \sum_{k=1}^{n} z^{a_k - 1} \times \frac{\prod_{l=1;\, l\neq k}^{n} \Gamma(a_k - a_l) \prod_{l=1}^{m} \Gamma(1 + b_l - a_k)}{\prod_{l=n+1}^{p} \Gamma(1 + a_l - a_k) \prod_{l=m+1}^{q} \Gamma(a_k - b_l)}, \tag{33}$$

where $a_k - a_l \neq 0, \pm 1, \pm 2, \ldots; (k, l = 1, \ldots, n; k \neq l)$ and $a_k - b_l \neq 1, 2, 3, \ldots; (k = 1, \ldots, n; l = 1, \ldots, m)$.

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