Effect of Imperfect Channel Estimation on MRC Diversity in Fading Channels

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Abstract— **In this paper, we study the effect of imperfect channel estimation (ICE) on the performance of M-ary phase shift keying (M-PSK) with maximum ratio combining (MRC) in generalized Rician fading channels. Using decision variable (DV)-based and signal-to-noise ratio (SNR)-based moment generating function (MGF) approaches, error and outage probability formulas, and SNR statistics for the M-PSK MRC receiver are derived, taking into account the effects of ICE and all relevant system and channel parameters. We analytically quantify the average SNR (ASNR) loss of M-PSK caused by ICE, and provided a unified MGF expression for ICE and different M's. In addition, we point out a major approximation for a popular approach used in the literature to evaluate the adverse effect of Gaussian weighting errors.**

I. Introduction

Among the popular diversity formats maximum-ratio combining (MRC) gives the maximum output signal-to-noise ratio (SNR) and hence the optimum performance in the single-user communication scenario. However, the channel estimates cannot be perfect in fading channels, and thus the adverse effect of imperfect channel estimation (ICE) on the MRC performance must be taken into account in the system design. In the past, the effect of ICE on diversity reception has been studied in several papers. In [1], [2], pilot tone and pilot symbol assisted channel estimation schemes were studied. In [3], a probability density function (pdf) expression for the effective MRC output SNR taking into account Gaussian weighting errors was derived for an independent and identically distributed (i.i.d.) Rayleigh fading channel. Using the result in [3], the performances for MRC and generalized selection combining (GSC) schemes with ICE were studied in [4] and [5], respectively. The results in [3]–[5] are only applicable to an i.i.d. Rayleigh fading channel. Further, it has not been mentioned or identified that these results involve a major approximation and underestimate the adverse effect of ICE on diversity receivers.

In this paper, we provide a unified performance analysis of the M-PSK MRC receiver in generalized Rician fading channels, taking into account the effects of ICE and all the relevant system and channel parameters, e.g. correlated and non-identically distributed diversity branches. The result is applicable whenever the channel estimates and the estimation errors follow joint Gaussian distributions. Thus, pilot symbol assisted and decision feedback channel estimation (PSA-CE and DF-CE) schemes, and estimation filters based on maximum-likelihood (ML), minimum-mean-square-error (MMSE), and linear-interpolation algorithms, can all be included in this framework.

Further, we derive the MGF for the effective MRC output SNR for the ICE case for i.i.d. Rician fading channels. Also, we show that for the case where the estimation correlation coefficient ρ is real, a unified MGF expression for the MRC output SNR with ICE valid for different modulation formats can be obtained. Using the obtained SNR expression, we calculate the outage probability for the M-PSK MRC receiver. Our results also show that a phase mismatch in ρ (when ρ is complex) is very deleterious to the M-PSK performance.

Throughout this paper, we use superscripts $*$, T, H, -1 to represent the scalar conjugate, vector (or matrix) transpose, conjugate transpose, and matrix inversion, respectively. We use $\mathrm{Re}(x)$ and $E[\cdot]$ to denote the real part and the expected value of x, respectively; and $\det(\mathbf{A})$ is the determinant of matrix $\mathbf{A}$.

II. Signal Model

We denote the signals received in the ith symbol interval over all L diversity branches by the L-vector

$$\mathbf{r}(i) = \mathbf{c}(i)d(i) + \mathbf{n}(i) \tag{1}$$

where $d(i) \in \{e^{j2\pi n/M}, n = 0, 1, \ldots, M-1\}$ is an M-PSK symbol, and $\mathbf{c}(i) = [c_1(i), \ldots, c_L(i)]^\top$ is the channel-coefficient vector for the L branches. In Rician fading channels, $\mathbf{c}(i)$ can be written as $\mathbf{c}(i) = \boldsymbol{\mu}_c + \mathbf{c}_f(i)$, where $\boldsymbol{\mu}_c = E[\mathbf{c}(i)] = [\mu_{c,1}, \ldots, \mu_{c,L}]^\top$ is the line-of-sight (LOS) component of $\mathbf{c}(i)$, and $\mathbf{c}_f(i) = [c_{f,1}(i), \ldots, c_{f,L}(i)]^\top$ is the diffuse component of $\mathbf{c}(i)$. The additive background noise vector, $\mathbf{n}(i) = [n_1(i), \ldots, n_L(i)]^\top$, is a zero-mean circularly symmetric complex Gaussian process with average power $E[|n_k(i)|^2] = N_0$ for $k = 1, \ldots, L$. At the lth diversity branch, the Rician factor is defined as $K_l = |\mu_{c,l}|^2/\sigma_{c,l}^2$, where $\sigma_{c,l}^2 = E[|c_{f,l}(i)|^2]$ is the variance of the diffuse fading component at the lth branch. We define the noise correlation matrix as $\mathbf{R}_n = E[\mathbf{n}(i)\mathbf{n}^{\mathrm{H}}(i)]$.

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The estimated channel vector for $\mathbf{c}(i)$ is given by $\hat{\mathbf{c}}(i) = [\hat{c}_1(i), \ldots, \hat{c}_L(i)]^\top$. $\hat{\mathbf{c}}(i)$ can be expressed as $\hat{\mathbf{c}}(i) = \hat{\boldsymbol{\mu}}_c + \hat{\mathbf{c}}_f(i)$, where $\hat{\boldsymbol{\mu}}_c$ and $\hat{\mathbf{c}}_f(i)$ are the estimates for the LOS and diffuse components of $\mathbf{c}(i)$, respectively. The presence of a static estimation bias (error) may be manifested by $\hat{\boldsymbol{\mu}}_c \neq \boldsymbol{\mu}_c$. Using the estimated channel vector $\hat{\mathbf{c}}(i)$ to detect $d(i)$, the complex decision variable (DV) is given by

$$\tilde{D} = \hat{\mathbf{c}}^{\mathrm{H}}(i)\mathbf{r}(i) = \sum_{l=1}^{L} \hat{c}_l^*(i) r_l(i). \tag{2}$$

The symbol is estimated as $\hat{d}(i) = e^{j2\pi\hat{n}/M}$, where $\hat{n} = \mathrm{argmax}_n \,\mathrm{Re}(\tilde{D}e^{-j2\pi n/M})$.

III. DV-Based Error Probability Analysis

To evaluate the symbol and bit error probabilities (SEP and BEP) for M-PSK, we define a new decision variable $D(\beta) = \mathrm{Re}(\tilde{D}e^{-j\beta})$, where β is a phase angle related to the transmitted phase $\alpha = \arg(d(i))$ [6], [7]. Assuming $\alpha = 0$ (i.e. $d(i) = 1$) and without loss of generality, we define a cumulative distribution function (cdf) expression $F_D(x|\beta) = \Pr\{D(\beta) < x | d(i) = 1\}$, where $\Pr\{A|B\}$ is the probability of event A conditioned on event B. The SEP for M-PSK can be calculated by

$$P_{s,\mathrm{MP}} \simeq \sum_{\beta = \pm(\pi/2 - \pi/M)} F_D(0|\beta) \tag{3}$$

Assuming Gray-coded bit-mapping, the BEP for M-PSK is given by

$$P_{b,\mathrm{MP}} \simeq \frac{1}{\log_2 M} \sum_{\beta = \pm(\pi/2 - \pi/M)} F_D(0|\beta) \tag{4}$$

Equation (4) gives the exact BEP for binary and quaternary PSK (BPSK and QPSK), as shown by $P_{\mathrm{BP}} = F_D(0|0)$ and $P_{\mathrm{QP}} = \frac{1}{2}[F_D(0|\pi/4) + F_D(0|-\pi/4)]$.

We use the DV-based MGF approach to evaluate the cdf $F_D(0|\beta)$. In detail, we derive the MGF for the DV $D(\beta)$, and then use the inverse Laplace transform (ILT) to obtain the cdf $F_D(0|\beta)$, which in conjunction with (3)–(4) gives the exact or approximate SEP and BEP for M-PSK.

A. General Correlated-Branch Diversity in Rician Fading

We express $D(\beta)$ as a Gaussian quadratic form

$$D(\beta) = \mathrm{Re}(\hat{\mathbf{c}}^{\mathrm{H}}(i)\mathbf{r}(i)e^{-j\beta}) = \mathbf{v}^{\mathrm{H}}(\beta)\mathbf{Q}_{2L}\mathbf{v}(\beta) \tag{5}$$

where $\mathbf{v}(\beta) = \begin{bmatrix} \hat{\mathbf{c}}(i) \\ (\mathbf{c}(i) + \mathbf{n}(i))e^{-j\beta} \end{bmatrix}$ and $\mathbf{Q}_{2L} = \begin{bmatrix} \mathbf{0}_L & 0.5\mathbf{I}_L \\ 0.5\mathbf{I}_L & \mathbf{0}_L \end{bmatrix}$, where $\mathbf{0}_L$ and $\mathbf{I}_L$ represent the $L \times L$ all-zero and identity matrices, respectively.

The covariance matrix of $\mathbf{v}(\beta)$, $\mathbf{P}_v(\beta)$, can be expressed as $\mathbf{P}_v(\beta) = \begin{bmatrix} \boldsymbol{\Sigma}_{\hat{c}} & \boldsymbol{\Sigma}_{\hat{c},c}e^{j\beta} \\ \boldsymbol{\Sigma}_{\hat{c},c}^{\mathrm{H}}e^{-j\beta} & \boldsymbol{\Sigma}_c + \mathbf{R}_n \end{bmatrix}$, where $\boldsymbol{\Sigma}_{\hat{c},c} = E[\hat{\mathbf{c}}_f(i)\mathbf{c}_f^{\mathrm{H}}(i)]$ is the covariance matrix between $\hat{\mathbf{c}}(i)$ and $\mathbf{c}(i)$, $\boldsymbol{\Sigma}_{\hat{c}} = E[\hat{\mathbf{c}}_f(i)\hat{\mathbf{c}}_f^{\mathrm{H}}(i)]$, and $\boldsymbol{\Sigma}_c = E[\mathbf{c}_f(i)\mathbf{c}_f^{\mathrm{H}}(i)]$. Note that $\boldsymbol{\Sigma}_{\hat{c}}$, $\boldsymbol{\Sigma}_{\hat{c},c}$, and $\boldsymbol{\Sigma}_c$ completely model the effects of imperfect (noisy) channel estimation for the diffuse channel components, and the signal correlation between different branches. The mean of $\mathbf{v}(\beta)$ is given by $\bar{\mathbf{v}}(\beta) = [\hat{\boldsymbol{\mu}}_c^\top, \boldsymbol{\mu}_c^\top e^{-j\beta}]^\top$. Using a property of Gaussian quadratic forms [8], we obtain the MGF of $D(\beta)$ as

$$\Phi_D(s) = \frac{\exp(\bar{\mathbf{v}}^{\mathrm{H}}[\mathbf{Q}_{2L}^{-1}s^{-1} - \mathbf{P}_v]^{-1}\bar{\mathbf{v}})}{\det(\mathbf{I}_{2L} - s\mathbf{P}_v\mathbf{Q}_{2L})}. \tag{6}$$

The cdf $F_D(0|\beta)$ can be evaluated by the ILT,

$$F_D(0|\beta) = \frac{1}{2\pi}\mathrm{Re}\left(\int_{c-j\infty}^{c+j\infty} \frac{\Phi_D(-s)}{js} ds\right) \tag{7}$$

where c is a small positive real constant in the convergence region. Equation (7) can be evaluated by a Gauss-Chebyshev quadrature (GCQ) formula [9],

$$F_D(0|\beta) \;=\; \frac{1}{2N}\sum_{n=1}^{N} \hat{\Phi}\left(\frac{(2n-1)}{2N}\pi\right) + \hat{R}_N \tag{8}$$

where $\hat{\Phi}(\theta) = \Phi_D(-c - jc\tan(\theta/2))(1 - j\tan(\theta/2))$, and $\hat{R}_N$ is a residual term which vanishes for $N \to \infty$. Equation (8) with (3)–(4) gives the accurate error probability results (or tight approximations) for M-PSK in arbitrary Rician fading channels taking into account ICE, and correlated and non-identically distributed diversity branches.

B. I.I.D. Diversity in Rician Fading

In Rician fading, $\mathbf{c}(i) = \boldsymbol{\mu}_c + \mathbf{c}_f(i)$ and $\hat{\mathbf{c}}(i) = \hat{\boldsymbol{\mu}}_c + \hat{\mathbf{c}}_f(i)$. Let $\sigma_{c_l}^2 = E[|c_{f,l}(i)|^2]$ and $\sigma_{\hat{c}_l}^2 = E[|\hat{c}_{f,l}(i)|^2]$ be the average powers of the true and estimated diffuse components at the lth branch, respectively. We assume both the estimates for the LOS components and the diffuse components are imperfect. To explicitly model the effect of ICE, we define the normalized complex correlation coefficient ρ_l between $c_{f,l}(i)$ and $\hat{c}_{f,l}(i)$ as

$$\rho_l = E[\hat{c}_{f,l}^*(i)c_{f,l}(i)]/[\sigma_{c_l}\sigma_{\hat{c}_l}]. \tag{9}$$

From here on we call ρ_l the normalized estimation correlation coefficient, and we drop the symbol index i when no confusion arises. In practical systems, ρ_l may be complex due to the frequency offset and residual complex interference in the received signal used for channel estimation. Let $\rho_l = |\rho_l|e^{j\Delta\theta_l} = \rho_{c,l} + j\rho_{s,l}$, where $\Delta\theta_l = \tan^{-1}(\rho_{s,l}/\rho_{c,l})$ denotes the phase offset (or mismatch) of ρ_l. The effect of ICE at the lth branch is manifested by the fact that $|\rho_l| < 1$ and $\Delta\theta_l \neq 0$ (i.e. non-zero phase offset). For the i.i.d. channel, we make the following assumptions:

- $\hat{\boldsymbol{\mu}}_c = \hat{\mu}_c\mathbf{1}_{L\times 1}$, and $\boldsymbol{\mu}_c = \mu_c\mathbf{1}_{L\times 1}$, where $\mathbf{1}_{L\times 1}$ denotes an $L \times 1$ all-one vector, and $|\hat{\mu}_c|^2 = |\mu_c|^2$.
- $\sigma_{c,l}^2 = \sigma_{\hat{c},l}^2 = \sigma_c^2$, also $\rho_l = \rho$ for all l.
- The noise is white and $\mathbf{R}_n = N_0\mathbf{I}_L$.

As in [3], let $c_{f,l} = \rho\hat{c}_{f,l} + z_{f,l}$ for the signal at the lth branch ($l = 1, \ldots, L$), where ρ is defined in (9) and is assumed to be identical for all the L branches, and $z_{f,l}$ is the channel estimation error term for the diffuse component and is assumed to be independent of $\hat{c}_l$. $z_{f,l}$ is circularly symmetric and follows a complex Gaussian distribution with zero mean and variance $(1-|\rho|^2)\sigma_c^2$. Further, we assume the LOS component and its estimate follow the same relationship as the diffuse component, i.e. $\mu_l = \rho\hat{\mu}_l + z_{\mu,l}$, where $z_{\mu,l}$ is the channel estimation error term for the LOS component and follows a Gaussian distribution with zero mean and variance $(1-|\rho|^2)|\mu_c|^2$. The mismatch of the LOS components may be used to model the case of the non-ideal carrier phase recovery for a large class of nonlinear phase estimation techniques, see [10, eq. (3.39)].

With these assumptions, we get the decomposition for the Rician channel coefficients as $c_l = \rho\hat{c}_l + z_{f,l} + z_{\mu,l}$, for $l = 1,\ldots,L$. Based on the assumptions above, we have

$$\mathbf{P}_v(\beta) = \begin{bmatrix} \sigma_c^2\mathbf{I}_L & \rho^*\sigma_c^2 e^{j\beta}\mathbf{I}_L \\ \rho\sigma_c^2 e^{-j\beta}\mathbf{I}_L & [\sigma_c^2 + N_0 + (1-|\rho|^2)|\mu_c|^2]\mathbf{I}_L \end{bmatrix} \quad (10)$$

$$\bar{\mathbf{v}}(\beta) = \begin{bmatrix} \mu_c \mathbf{1}_{L\times 1} \\ \rho\mu_c e^{-j\beta}\mathbf{1}_{L\times 1} \end{bmatrix} \quad (11)$$

The MGF for the DV $D(\beta)$ can be evaluated by substituting (10) and (11) into (6), and then the cdf $F(0|\beta)$ can be evaluated to yield the BEPs for M-PSK.

C. I.N.D. Diversity in Rayleigh Fading

For Rayleigh fading, the MGF for the DV $D(\beta)$ is given by $\Phi_D(s) = \det(\mathbf{I}_{2L} - s\mathbf{P}_v\mathbf{Q}_{2L})^{-1}$. Further for independent diversity channels, $\Phi_D(s) = \prod_{l=1}^{L}(\mathbf{I}_2 - s\mathbf{P}_l\mathbf{Q}_2)^{-1}$, where $\mathbf{P}_l = \begin{bmatrix} \sigma_{\hat{c}_l}^2 & \rho_l^*\sigma_{\hat{c}_l}\sigma_{c_l}e^{j\beta} \\ \rho_l\sigma_{\hat{c}_l}\sigma_{c_l}e^{-j\beta} & \sigma_{c_l}^2 + N_0 \end{bmatrix}$. Let the instantaneous and the average SNRs at the lth branch be defined as $\gamma_l = |c_l|^2/N_0$ and $\bar{\gamma}_l = E[|c_l|^2]/N_0 = \sigma_{c_l}^2/N_0$, respectively. Also, let the SNR of the estimated channel coefficient at the lth branch be defined as $\hat{\gamma}_l = |\hat{c}_l|^2/N_0$, and the average SNR be defined as $\bar{\hat{\gamma}}_l = E[|\hat{c}_l|^2]/N_0 = \sigma_{\hat{c}_l}^2/N_0$.

For M-PSK modulation, without loss of generality, we can re-scale each entry of $\mathbf{P}_l$ by a factor of $1/N_0$ and obtain $\mathbf{P}_l = \begin{bmatrix} \bar{\hat{\gamma}}_l & \rho_l^*\sqrt{\bar{\gamma}_l\bar{\hat{\gamma}}_l}e^{j\beta} \\ \rho_l\sqrt{\bar{\gamma}_l\bar{\hat{\gamma}}_l}e^{-j\beta} & \bar{\gamma}_l + 1 \end{bmatrix}$. Thus, the two eigenvalues of $\mathbf{P}_l\mathbf{Q}_2$ are given by

$$\lambda_l^{\pm}(\beta) = |\rho_l|\sqrt{\bar{\gamma}_l\bar{\hat{\gamma}}_l}\cos(\beta - \Delta\theta_l) \pm\sqrt{\bar{\gamma}_l\bar{\hat{\gamma}}_l[1-|\rho_l|^2\sin^2(\beta-\Delta\theta_l)] + \bar{\hat{\gamma}}_l}. \quad (12)$$

Since the signals in the L branches are independent, all the poles of $\Phi_D(s)$ are given by $\{1/\lambda_l^-(\beta), 1/\lambda_l^+(\beta)\}_{l=1}^L$, where $\{\lambda_l^{\pm}(\beta)\}_{l=1}^L$ are given by (12). For convenience we sort the $2L$ eigenvalues in ascending order so that $\{\lambda_l\}_{l=1}^L$ are negative, and $\{\lambda_l\}_{l=L+1}^{2L}$ are positive. Assuming all the negative eigenvalues of $\mathbf{P}_v\mathbf{Q}_{2L}$ are distinct, the cdf $F_D(0|\beta)$ is given by

$$F_D(0|\beta) = \sum_{l=1}^{L}\prod_{m=1,m\neq l}^{2L}\frac{\lambda_l(\beta)}{\lambda_l(\beta)-\lambda_m(\beta)} \quad (13)$$

where for $1 \le l \le L$, $\lambda_l(\beta) = \lambda_l^-(\beta)$ and $\lambda_{l+L}(\beta) = \lambda_l^+(\beta)$. The SEP and BEP for M-PSK can be obtained by substituting (13) into (3) – (4), respectively.

D. I.I.D. Diversity in Rayleigh Fading

Without loss of generality, we assume again $\sigma_{\hat{c}_l}^2 = \sigma_{c_l}^2 = \sigma_c^2$, so that $\bar{\gamma}_l = \bar{\hat{\gamma}}_l = \bar{\gamma}$ for $l = 1,\ldots,L$. Also, we assume $\rho_l = \rho$ for all l. In this case, $\mathbf{P}_v\mathbf{Q}_{2L}$ has only two distinct eigenvalues with each of them being repeated L times. They can be denoted as $\lambda^{\pm}(\beta) = |\rho|\bar{\gamma}\cos(\beta - \Delta\theta) \pm \sqrt{\bar{\gamma}^2[1-|\rho|^2\sin^2(\beta-\Delta\theta)] + \bar{\gamma}}$. Using a result in [11], $F_D(0|\beta)$ can be obtained as

$$F_D(0|\beta) = [A(\beta)]^L\sum_{i=0}^{L-1}\binom{L+i-1}{i}[1-A(\beta)]^i \quad (14)$$

$$A(\beta) = \frac{\lambda^-(\beta)}{\lambda^-(\beta)-\lambda^+(\beta)} = \frac{1}{2}\left[1-\sqrt{\frac{\bar{\gamma}|\rho|^2\cos^2(\beta-\Delta\theta)}{\bar{\gamma}[1-|\rho|^2\sin^2(\beta-\Delta\theta)]+1}}\right]. \quad (15)$$

IV. MGF FOR THE MRC OUTPUT SNR WITH ICE

A. I.I.D. Rician fading

Below, we derive the MGF for the MRC output SNR with ICE in an i.i.d. Rician fading channel, assuming the same channel model as in Section III-B.

The relevant real part of the output signal for binary modulation at the lth branch can be expressed as $D_l = \mathrm{Re}(\hat{c}_l^*[c_l + n_l])$. Using $\rho = \rho_c + j\rho_s$, D_l can be expressed as the summation of two terms $D_l = |\hat{c}_l|^2\rho_c + \mathrm{Re}(\hat{c}_l^*(z_{f,l} + z_{\mu,l} + n_l))$, where the first term denotes the desired signal, and the second one is related to the noise and the channel estimation error. After some manipulations, we obtain the effective SNR for the BPSK MRC output signal with ICE in Rician fading as

$$\gamma_{\text{ICE,BP}}^{\text{MRC}} = \frac{\rho_c^2\sum_{l=1}^{L}|\hat{c}_l|^2}{(1-|\rho|^2)[\sigma_c^2+|\mu_c|^2]+N_0} = \frac{\rho_c^2\sum_{l=1}^{L}\hat{\gamma}_l}{(1-|\rho|^2)\bar{\gamma}+1}$$

where $\bar{\gamma} = [\sigma_c^2 + |\mu_c|^2]/N_0$ is the average SNR per branch, and $\hat{\gamma}_l = |\hat{c}_l|^2/N_0$. The MGF of $\gamma_{\text{ICE,BP}}^{\text{MRC}}$ can be obtained as $\Phi_{\gamma_{\text{ICE,BP}}^{\text{MRC}}}(s) = E[e^{s\gamma_{\text{ICE,BP}}^{\text{MRC}}}] =$

$$\left(\frac{1+K}{1+K-s\bar{\gamma}_{\text{ICE,BP}}}\right)^L \exp\left(\frac{LKs\bar{\gamma}_{\text{ICE,BP}}}{1+K-s\bar{\gamma}_{\text{ICE,BP}}}\right) \quad (16)$$

where $\bar{\gamma}_{\text{ICE,BP}} = \rho_c^2\bar{\gamma}/[(1-|\rho|^2)\bar{\gamma}+1]$, and we assume $K_l = K$ for all l. Also, using a similar procedure, the MGF for the effective SNR for M-PSK can be obtained as $\Phi_{\gamma_{\text{ICE,MP}}^{\text{MRC}}}(s|\beta) =$

$$\left(\frac{1+K}{1+K-s\bar{\gamma}_{\text{ICE,MP}}(\beta)}\right)^L \exp\left(\frac{LKs\bar{\gamma}_{\text{ICE,MP}}(\beta)}{1+K-s\bar{\gamma}_{\text{ICE,MP}}(\beta)}\right) \quad (17)$$

where $\bar{\gamma}_{\text{ICE,MP}}(\beta) = \left[\frac{|\rho|^2\cos^2(\beta-\Delta\theta)\bar{\gamma}}{(1-|\rho|^2)\bar{\gamma}+1}\right]$, and $\beta = \pm(\pi/2 - \pi/M)$ for M-PSK. Note that (17) includes 2-, 4- and 8-PSK as special cases.

For verification purpose, by using (17), the cdf $F_D(0|\beta)$ for M-PSK can be calculated from

$$F_D(0|\beta) = \frac{1}{\pi}\int_0^{\pi/2} \Phi_{\gamma^{\text{MRC}}_{\text{ICE,MP}}}(-1/\sin^2(\theta)|\beta)d\theta \quad (18)$$

The equivalence of (18) and the cdf result obtained using (10) and (11) has been confirmed by numerical computations. When we set $K = 0$ in (17), the results for Rayleigh fading can be obtained.

B. Loss of Average SNR Caused by ICE

For the case of a real ρ, $\bar{\gamma}_{\text{ICE,MP}}(\beta) = \left[\frac{|\rho|^2 \sin^2(\pi/M)\bar{\gamma}}{(1-|\rho|^2)\bar{\gamma}+1}\right]$ is true, and $\bar{\gamma}_{\text{CSI,MP}}(\beta) = \sin^2(\pi/M)\bar{\gamma}$. To analytically quantify the loss of average SNR (ASNR) due to ICE with respect to channel state information (CSI), i.e., perfect channel estimates, we define the ASNR loss for M-PSK as a factor $\kappa_{\text{MP}}(\bar{\gamma})$ such that the following equality holds

$$\Phi_{\gamma^{\text{MRC}}_{\text{ICE,MP}}}(s\kappa_{\text{MP}}(\bar{\gamma})) = \Phi_{\gamma^{\text{MRC}}_{\text{CSI,MP}}}(s) \quad (19)$$

We can obtain $\kappa_{\text{MP}}(\bar{\gamma}) = \frac{(1-|\rho|^2)\bar{\gamma}+1}{|\rho|^2}$. Equation (19) nicely relates the MGFs of the effective M-PSK MRC output SNRs for the CSI and ICE cases independent of M, and facilitates a unified performance analysis for ICE.

V. Outage Probability

Using (3) the outage for the MRC effective SNR of M-PSK in Rician and Rayleigh fading channels is given by $P_{\text{out,MP}}(\gamma_{\text{th}}) \simeq$

$$\frac{1}{2\pi j}\sum_{\beta=\pm(\pi/2-\pi/M)} \int_{c-j\infty}^{c+j\infty} \frac{e^{-s\gamma_{\text{th}}}\Phi_{\gamma^{\text{MRC}}_{\text{ICE,MP}}}(s|\beta)}{s} ds \quad (20)$$

where c is the saddle point of $e^{-s\gamma_{\text{th}}}\Phi_{\gamma^{\text{MRC}}_{\text{ICE,MP}}}(s)/s$. An efficient formula for calculating (20) can be obtained using a result in [12]. More details are given in [13].

VI. Discussion

We compare our results for the MRC output SNR for ICE to that obtained by Gans [3] for an i.i.d Rayleigh fading channel. In [3] the MRC output SNR with ICE was defined as (see [3, Eqs.(4),(5),(10),(19)])

$$\tilde{\gamma}_{\text{ICE}} = \frac{S_0}{N_0} = \frac{|\sum_{l=1}^L \hat{c}_l^* c_l|^2}{|\sum_{l=1}^L \hat{c}_l^* n_l|^2} \quad (21)$$

where $S_0 = |\sum_{l=1}^L \hat{c}_l^* c_l|^2$ was defined as the power of the desired signal, and $N_0 = |\sum_{l=1}^L \hat{c}_l^* n_l|^2$ was the power of noise.

We have to point out that the definition for SNR in (21) involves a major approximation. The probability density function (pdf) result in [3], when applied to the performance analysis of digital communication in fading channels with ICE, will result in loose performance upper bounds. We explain the approximation involved in (21) below. Assume $\hat{c}_l = jc_l$, i.e., the channel estimate has a $\pi/2$ phase mismatch error. Using (21) the resulting SNR for this ICE case is the same as the CSI case (i.e. $\hat{c}_l = c_l$). Note that in practice a $\pi/2$ phase error would render the coherent PSK receiver completely useless. To illustrate further, let us rewrite c_l as $c_l = (\rho_c + j\rho_s)\hat{c}_l + z_{f,l}$, and decompose the signal term S_0 to $S_0 = E[|\sum_{l=1}^L \hat{c}_l^* c_l|^2] = E[|\sum_{l=1}^L [|\hat{c}_l|^2\rho_c + j|\hat{c}_l|^2\rho_s + \hat{c}_l^* z_{f,l}]|^2]$. Since $|\hat{c}_l|^2\rho_c$ is the only desired signal, the definition of the desired signal in (21) improperly includes[1] the term $\sum_{l=1}^L [j|\hat{c}_l|^2\rho_s + \hat{c}_l^* z_{f,l}]$, which actually corresponds the phase offset of ρ (i.e. ρ_s) and the channel mismatch error $z_{f,l}$. Only for the case of CSI, the definition of $|\sum_{l=1}^L \hat{c}_l^* c_l|^2$ as the desired signal is strict.

Due to the problem with the SNR definition, some misleading conclusions result. For example, when $|\rho| = 0$ or $L = 1$, the result in [3] leads to the false conclusions that single channel reception is immune to ICE, and that multi-channel reception with completely noisy references (i.e. $|\rho| = 0$) will result in a single channel receiver with CSI [4], [5].

VII. Numerical Examples

In order to better illustrate the effect of the normalized estimation correlation coefficients $\{\rho_l\}_{l=1}^L$, we assume i.i.d. diversity branches, and that the $\{\rho_l\}_{l=1}^L = \rho$ are identical in the different branches. In Figure 1, the BEPs for the BPSK and QPSK MRC receivers with CSI and ICE are presented, where we assume $L = 2$, $K = [3, 7]$ dB, $|\rho| = 0.995$, and $\Delta\theta = \pi/20$, and that the estimates of both $\boldsymbol{\mu}_c$ and the diffuse component are imperfect, and the channel estimation error follows the distribution described in Section III-B. It is notable that with CSI the BEP performances for BPSK and QPSK are identical. However, with ICE the performance of QPSK is considerably worse than that of BPSK, which is in agreement with our analytical result that with ICE the effective SNRs for BPSK and QPSK are different.

Next, the outage probability for BPSK MRC with CSI and ICE is given in Fig. 2. We assume the same channel and system parameters as in Fig. 1. The higher K factors result in lower outage probabilities, as expected. For all K the outage probabilities in case of ICE are significantly worse than for CSI, and outage floors appear when the input normalized SNR exceeds 25 dB.

In [4], the BEP for binary differential PSK (BDPSK) with MRC and ICE is given by [4, Eq. (16)], which was obtained using Gans' pdf formula. To show the approximation involved in this result, by using (16) and setting $K = 0$ therein an exact BEP expression for BDPSK with ICE and MRC is obtained as

$$P_{\text{BDP}} = 0.5 \cdot \left(1 - \frac{\rho_c^2\bar{\gamma}}{[(1-|\rho|^2)\bar{\gamma}+1]}\right)^{-L}. \quad (22)$$

A comparison between [4, Eq. (16)] and (22) is given in Fig. 3, where we assume $L = 1, 2, 3$, and $\rho = 0.995e^{j\pi/20}$.

[1] Specifically, in [3, eq. (19)], only the term X is the desired signal, and x, y and Y are actually terms representing noisy components but were also improperly defined as the desired signal.

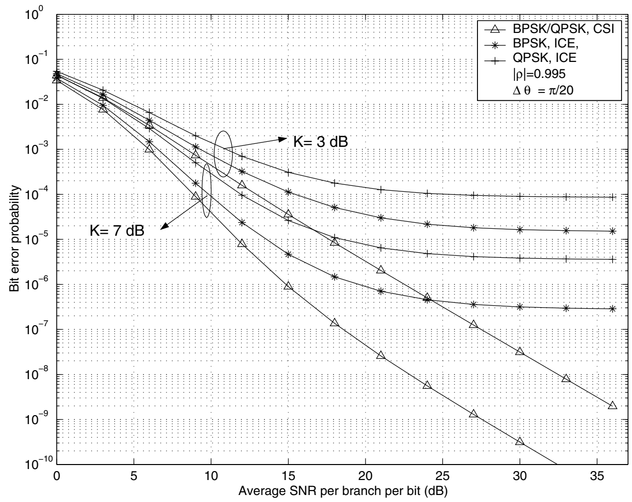

Fig. 1. BEP versus the average SNR per branch per bit for the BPSK/QPSK MRC receivers with CSI and ICE in i.i.d. Rician fading channels. $L = 2$, $K = [3, 7]$ dB, $|\rho| = 0.995$, and $\Delta\theta = \pi/20$.

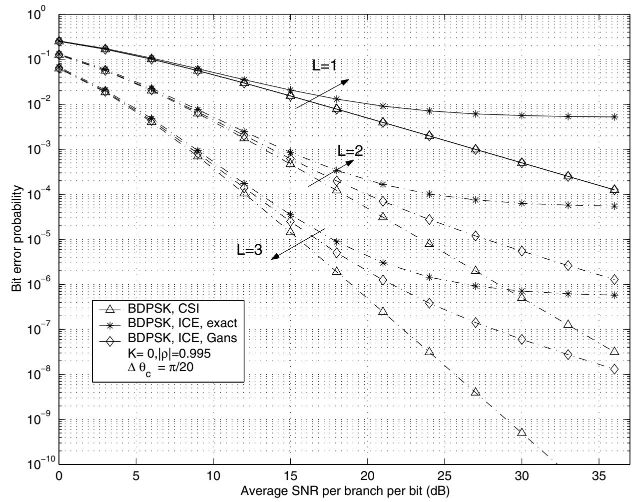

Fig. 3. BEP versus the average SNR per branch for the BDPSK MRC receiver with CSI and ICE in an i.i.d. Rayleigh fading channel, with $L = 1, 2, 3$, $|\rho| = 0.995$, and $\Delta\theta = \pi/20$.

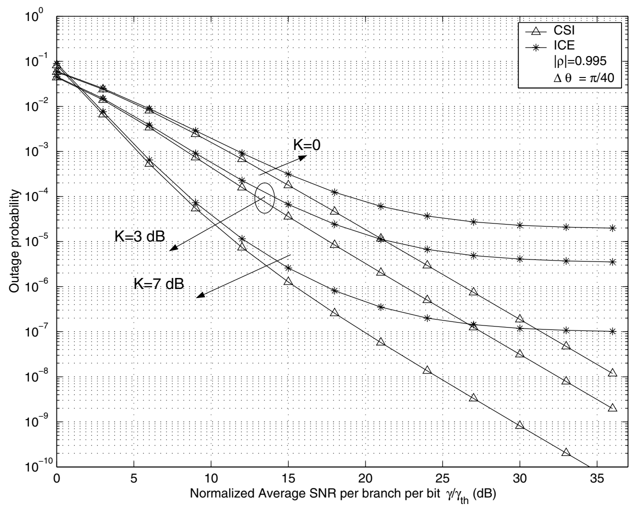

Fig. 2. Outage probability versus the normalized average SNR for BPSK MRC with CSI and ICE in i.i.d. Rician ($K = [3, 7]$ dB) and Rayleigh ($K = 0$) fading channels. $L = 2$, $|\rho| = 0.995$, and $\Delta\theta = \pi/20$.

For all L, [4, Eq. (16)] significantly underestimates the adverse effect of ICE on the detection performance. This is because, as we pointed out in Section VI, Gans' pdf result involves a major approximation and cannot properly take into account the ICE. For high SNRs, error floors exist for all L and both Rician and Rayleigh fading channels given that $|\rho| < 1$. However, the curves based on [4, Eq. (16)] fail to predict the error floors, and in contrast show that a diversity order of one is achieved at high SNRs even with ICE (see e.g. [4]).

VIII. Conclusions

In this paper, by using DV-based and SNR-based MGF approaches, respectively, error and outage probability formulas for the M-PSK MRC receiver with ICE have been derived. When ρ is real, we analytically quantify the ASNR loss of M-PSK caused by ICE, and provided a unified MGF expression for ICE valid for different M's. The proposed approaches for ICE are also applicable to other two-dimensional modulations, and to more general communication scenarios, e.g. coded communication and space-time communication.

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