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Effect of Channel Estimation Errors on MRC Diversity in Rician Fading Channels

Yao Ma, Robert Schober, and Subbarayan Pasupathy

***Abstract*—We study the effect of imperfect channel estimation (ICE) on the performance of M-ary phase shift keying (M-PSK) with maximum ratio combining (MRC) in generalized Rician fading channels. First, we derive the error probability formulas for M-PSK with MRC and ICE in arbitrary Rician fading channels. Furthermore, we derive the effective receiver output signal-to-noise ratio (SNR) statistics and the outage probability, and analytically quantify the average SNR loss of M-PSK caused by ICE, assuming independent diversity branches. Finally, we point out a major approximation in a popular approach used in the literature to evaluate the adverse effect of ICE.**

***Index Terms*—Correlated fading, imperfect channel estimation, maximum ratio combining (MRC), Rician fading channels.**

I. Introduction

Among the popular diversity formats maximum-ratio combining (MRC) gives the maximum output signal-to-noise ratio (SNR) and hence the optimum performance in the single-user communication scenario. However, in fading channels, channel estimation cannot be perfect, and thus the adverse effect of imperfect channel estimation (ICE) on the MRC performance must be taken into account in the system design. In the past, the effect of ICE on diversity reception has been studied in several papers. In [1] and [2], pilot tone and pilot symbol assisted channel estimation schemes were studied. In [3, Appendix C], some bit error probability (BEP) expressions for the M-ary phase shift keying (M-PSK) MRC receiver with ICE were derived for the Rayleigh and the nonfading channels, respectively, assuming independent and identically distributed (i.i.d.) diversity branches. In [4], a probability density function (pdf) expression for the effective MRC output SNR taking ICE into account was derived for an i.i.d. Rayleigh fading channel. Using the pdf result in [4], the performances of MRC and generalized selection combining (GSC) receivers with Gaussian weighting errors in an i.i.d. Rayleigh fading channel were analyzed in [5] and [6], respectively. Unfortunately, the obtained pdf expression in [4] is not valid for the performance analysis of digital modulation formats with ICE, as we will show in this letter. Thus, the results in [5] and [6] involve a major approximation and can only be regarded as loose performance bounds.

In this letter, we provide a unified performance analysis of M-PSK with MRC in generalized Rician fading channels, taking into account the effects of ICE and all the relevant system and channel parameters, e.g., correlated and non-identically distributed diversity branches. Our result is applicable whenever the channel estimates and the estimation errors follow joint Gaussian distributions. Thus, pilot symbol assisted modulation (PSAM) and decision-feedback channel estimation schemes, and estimation filters based on maximum-likelihood (ML), minimum-mean-square-error (MMSE), and linear-interpolation methods are all included in this framework. Further, we derive the moment generating function (MGF) for the effective MRC output SNR with ICE in the i.i.d. Rician fading channel, and calculate the outage probability. Also, for the case where the estimation correlation coefficient ρ (to be defined rigorously later) is real, a unified MGF expression for the receiver output SNR with ICE valid for different M is obtained. Our results also show that a phase mismatch in ρ is very deleterious to the M-PSK performance.

Throughout this letter, we use superscripts $*$, T, H, -1 to represent the scalar conjugate, vector (or matrix) transpose, conjugate transpose, and matrix inversion, respectively. We use $\Re(x)$, $\Im(x)$, and $E[x]$ to denote the real part, imaginary part, and the expected value of x, respectively; and $\det(\mathbf{A})$ is the determinant of matrix $\mathbf{A}$. $\mathrm{diag}[x_1,\ldots,x_L]$ is a diagonal matrix that is formed by setting the elements $x_1,\ldots,x_L$ on its main diagonal.

Manuscript received September 2, 2004; revised January 18, 2005. This paper was presented in part at IEEE International Conference on Communications (ICC), Paris, France, 2004. The review of this paper was coordinated by Prof. G. Saulnier.

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Digital Object Identifier 10.1109/TVT.2005.853454

II. Signal Model

We denote the signals received in the ith symbol interval over all L diversity branches by

$$\mathbf{r}(i) = \mathbf{c}(i)d(i) + \mathbf{n}(i) \tag{1}$$

where $d(i) \in \{e^{j2\pi n/M}, n = 0,1,\ldots,M-1\}$ is an M-PSK symbol, and $\mathbf{c}(i) = [c_1(i),\ldots,c_L(i)]^\top$ is the channel-coefficient vector for the L branches. In Rician fading channels, $\mathbf{c}(i)$ can be written as $\mathbf{c}(i) = \boldsymbol{\mu}_c(i) + \mathbf{c}_f(i)$, where $\boldsymbol{\mu}_c(i) = E[\mathbf{c}(i)] = [\mu_{c,1}(i),\ldots,\mu_{c,L}(i)]^\top$ and $\mathbf{c}_f(i) = [c_{f,1}(i),\ldots,c_{f,L}(i)]^\top$ are the line-of-sight (LOS) and the diffuse components of $\mathbf{c}(i)$, respectively. At the lth branch, $\mu_{c,l}(i) = |\mu_{c,l}(i)|e^{j[2\pi(f_{d_l}+f_{o_l})iT_s+\phi_l]}$, where f_{d_l} is the Doppler shift of the LOS component, f_{o_l} is the frequency offset, ϕ_l is a constant phase, and T_s is a symbol duration. The Rician factor is defined as $K_l = |\mu_{c,l}(i)|^2/\sigma_{c,l}^2$, where $\sigma_{c,l}^2 = E[|c_{f,l}(i)|^2]$ is the variance of the diffuse fading component at the lth branch. The additive background noise vector, $\mathbf{n}(i) = [n_1(i),\ldots,n_L(i)]^\top$, is a zero-mean circularly symmetric complex Gaussian process with average power $E[|n_k(i)|^2] = N_0$ for $k = 1,\ldots,L$. We define the noise correlation matrix as $\mathbf{R}_n = E[\mathbf{n}(i)\mathbf{n}^{\mathrm{H}}(i)]$.

The estimated channel vector for $\mathbf{c}(i)$ is given by $\hat{\mathbf{c}}(i) = [\hat{c}_1(i),\ldots,\hat{c}_L(i)]^\top$. $\hat{\mathbf{c}}(i)$ can be expressed as $\hat{\mathbf{c}}(i) = \hat{\boldsymbol{\mu}}_c(i) + \hat{\mathbf{c}}_f(i)$, where $\hat{\boldsymbol{\mu}}_c(i)$ and $\hat{\mathbf{c}}_f(i)$ are the estimates for the LOS and diffuse components of $\mathbf{c}(i)$, respectively. We assume that $\hat{\boldsymbol{\mu}}_c(i)$ and $\hat{\mathbf{c}}_f(i)$ may both be imperfect. Using the estimated channel vector $\hat{\mathbf{c}}(i)$ to detect $d(i)$, the complex decision variable (DV) is given by

$$\tilde{D} = \hat{\mathbf{c}}^{\mathrm{H}}(i)\mathbf{r}(i) = \sum_{l=1}^{L} \hat{c}_l^*(i) r_l(i). \tag{2}$$

The symbol is estimated as $\hat{d}(i) = e^{j2\pi\hat{n}/M}$, where $\hat{n} = \mathrm{argmax}_n \Re(\tilde{D}e^{-j2\pi n/M})$.

III. DV-Based Error Probability Analysis

To evaluate the symbol and bit error probabilities (SEPs and BEPs) for M-PSK, we define a new decision variable $D(\beta) = \Re(2\tilde{D}e^{-j\beta})$, where β is a phase angle introduced to facilitate a half-plane decision method, for which the error rate is given by the probability that the decision variable falls into a half plane specified by β in the decision space [7], [8]. For binary and quaternary PSK (BPSK and QPSK), $\beta = \alpha$ and $\beta = \alpha \pm \pi/4$, respectively, where α is the transmitted phase with $d(i) = e^{j\alpha}$ [7], [8]. Without loss of generality, we assume $\alpha = 0$ (i.e., $d(i) = 1$) and further we define a cumulative distribution function (cdf) expression $F_D(x \mid \beta) = \Pr\{D(\beta) < x | d(i) = 1\}$, where $\Pr\{A \mid B\}$ is the probability of event A conditioned on event B. By approximating the symbol-error event of M-PSK by two half-planes (with a slight overlap causing the approximation of the result) [7]–[9], the SEP of M-PSK can be calculated by $P_{s,\mathrm{MP}} \simeq \sum_{\beta=\pm(\pi/2-\pi/M)} F_D(0 \mid \beta)$. Assuming Gray mapping, the BEP for M-PSK is given by

$$P_{b,\mathrm{MP}} \simeq \frac{1}{\log_2 M} \sum_{\beta=\pm(\pi/2-\pi/M)} F_D(0 \mid \beta). \tag{3}$$

Equation (3) gives the *exact* BEP for BPSK and QPSK, as shown by $P_{b,\mathrm{BP}} = F_D(0 \mid 0)$ and $P_{b,\mathrm{QP}} = (1)/(2)[F_D(0 \mid \pi/4) + F_D(0 \mid -\pi/4)]$. Using a result in [7], [10], an approximate BEP formula for 8-PSK tighter than (3) for low SNRs is given by $P_{b,8\mathrm{P}} \simeq (1)/(3)[F_D(0 \mid 3\pi/8) + F_D(0 \mid -\pi/8) F_D(0 \mid -5\pi/8) + F_D(0 \mid -3\pi/8) + F_D(0 \mid \pi/8) F_D(0 \mid 5\pi/8)]$.

We use the DV-based MGF approach to evaluate the cdf $F_D(0 \mid \beta)$. In detail, we derive the MGF for the DV $D(\beta)$, and then use the inverse Laplace transform (ILT) to obtain the cdf $F_D(0 \mid \beta)$, which in conjunction with (3) gives the BEP for M-PSK with ICE.

A. General Correlated-Branch Diversity in Rician Fading

We express $D(\beta)$ as a Gaussian quadratic form (assuming $\alpha = 0$)

$$D(\beta) = \Re(2\hat{\mathbf{c}}^{\mathrm{H}}(i)\mathbf{r}(i)e^{-j\beta}) = \mathbf{v}^{\mathrm{H}}(\beta)\mathbf{Q}_{2L}\mathbf{v}(\beta) \tag{4}$$

where

$$\mathbf{v}(\beta) = \begin{bmatrix} \hat{\mathbf{c}}(i) \\ (\mathbf{c}(i) + \mathbf{n}(i))e^{-j\beta} \end{bmatrix}$$

and

$$\mathbf{Q}_{2L} = \begin{bmatrix} \mathbf{0}_{L\times L} & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{0}_{L\times L} \end{bmatrix}$$

$\mathbf{0}_{L\times L}$ and $\mathbf{I}_L$ represent the $L \times L$ all-zero and identity matrices, respectively.

The covariance matrix of $\mathbf{v}(\beta), \mathbf{P}_v(\beta)$, can be expressed as

$$\mathbf{P}_v(\beta) = \begin{bmatrix} \boldsymbol{\Sigma}_{\hat{c}} & \boldsymbol{\Sigma}_{\hat{c},c} e^{j\beta} \\ \boldsymbol{\Sigma}_{\hat{c},c}^{\mathrm{H}} e^{-j\beta} & \boldsymbol{\Sigma}_c + \mathbf{R}_n \end{bmatrix}$$

where $\boldsymbol{\Sigma}_{\hat{c},c} = E[\hat{\mathbf{c}}_f(i)\mathbf{c}_f^{\mathrm{H}}(i)]$ is the covariance matrix between $\hat{\mathbf{c}}(i)$ and $\mathbf{c}(i), \boldsymbol{\Sigma}_{\hat{c}} = E[\hat{\mathbf{c}}_f(i)\hat{\mathbf{c}}_f^{\mathrm{H}}(i)]$, and $\boldsymbol{\Sigma}_c = E[\mathbf{c}_f(i)\mathbf{c}_f^{\mathrm{H}}(i)]$. Note that $\boldsymbol{\Sigma}_{\hat{c}}, \boldsymbol{\Sigma}_{\hat{c},c}$, and $\boldsymbol{\Sigma}_c$ completely model the effects of imperfect (noisy) channel estimation for the diffuse channel components, and the signal correlation between different branches. Further, an estimation error for the LOS components is modelled by $\hat{\boldsymbol{\mu}}_c(i) \neq \boldsymbol{\mu}_c(i)$. The mean of $\mathbf{v}(\beta)$ is given by $\bar{\mathbf{v}}(\beta) = [\hat{\boldsymbol{\mu}}_c^{\top}(i), \boldsymbol{\mu}_c^{\top}(i)e^{-j\beta}]^{\top}$. Using a property of Gaussian quadratic forms [11], we obtain the MGF of $D(\beta)$ as

$$\Phi_D(s \mid \beta) = \frac{\exp\left[\bar{\mathbf{v}}^{\mathrm{H}}(\beta)\left[\mathbf{Q}_{2L}^{-1}s^{-1} - \mathbf{P}_v(\beta)\right]^{-1}\bar{\mathbf{v}}(\beta)\right]}{\det(\mathbf{I}_{2L} - s\mathbf{P}_v(\beta)\mathbf{Q}_{2L})}. \tag{5}$$

The cdf $F_D(0 \mid \beta)$ can be evaluated by the ILT

$$F_D(0 \mid \beta) = \frac{1}{2\pi}\Re\left(\int_{c-j\infty}^{c+j\infty} \frac{\Phi_D(-s \mid \beta)}{js} ds\right) \tag{6}$$

where c is a real constant in the convergence region. Equation (6) can be evaluated by a Gauss–Chebyshev quadrature (GCQ) formula [12], $F_D(0 \mid \beta) = (1)/(2N) \sum_{n=1}^{N} \hat{\Phi}(((2n-1))/(2N)\pi \mid \beta) + \hat{R}_N$, where $\hat{\Phi}(\theta|\beta) = \Phi_D(-c - jc\tan(\theta/2)|\beta)[1 - j\tan(\theta/2)]$, and $\hat{R}_N$ is a residual term which vanishes for $N \to \infty$. The effect of ICE on the performance of M-PSK may be manifested, for example, in the fact that even when $\mathbf{R}_n \to \mathbf{0}_{L\times L}$ matrix $\mathbf{P}_v\mathbf{Q}_{2L}$ has negative eigenvalues,[1] which causes the error floor when the input SNR goes to infinity.

B. Independent But NonIdentically Distributed (I.N.D) Branches

To explicitly illustrate the effect of ICE, we study the case of i.n.d. diversity branches below. Let $\sigma_{c,l}^2 = E[|c_{f,l}(i)|^2]$ and $\sigma_{\hat{c},l}^2 = E[|\hat{c}_{f,l}(i)|^2]$ be the average powers of the true and estimated diffuse components at the lth branch, respectively. We define the normalized estimation correlation coefficient ρ_l as

$$\rho_l = E[\hat{c}_{f,l}^*(i)c_{f,l}(i)]/[\sigma_{c,l}\sigma_{\hat{c},l}]. \tag{7}$$

Note that the definition of ρ_l in (7), when simplified to an i.i.d. Rayleigh channel case, is equivalent to that used in [4] and [5].[2] In general, ρ_l is a function of the average SNR (ASNR), the fading power spectrum, and the Doppler fading bandwidth, etc. For the ideal MMSE channel estimator (MMSE-CE), ρ_l is real-valued. For non-MMSE channel estimators (e.g., sinc- and Gaussian-interpolators [13]), in the presence of a frequency offset, ρ_l may be complex-valued. Thus, to generalize our results, we assume ρ_l to be complex below. Let $\rho_l = |\rho_l|e^{j\Delta\theta_l} = \rho_{c,l} + j\rho_{s,l}$, where $\Delta\theta_l = \tan^{-1}(\rho_{s,l}/\rho_{c,l})$ denotes the phase offset (or mismatch) of ρ_l. The effect of ICE at the lth branch may be manifested by the fact that $|\rho_l| < 1$ and $\Delta\theta_l \neq 0$. Note that even when $|\rho_l| = 1$, $\Delta\theta_l \neq 0$ causes a phase mismatch. Also, the presence of a static estimation bias (error) may be manifested by $\hat{\mu}_{c,l}(i) \neq \mu_{c,l}(i)$.

With the assumption of i.n.d. branches, we get $\boldsymbol{\Sigma}_{\hat{c}} = \mathrm{diag}(\sigma_{\hat{c},1}^2, \ldots, \sigma_{\hat{c},L}^2)$, $\boldsymbol{\Sigma}_c = \mathrm{diag}(\sigma_{c,1}^2, \ldots, \sigma_{c,L}^2)$, and $\boldsymbol{\Sigma}_{\hat{c},c} = \mathrm{diag}(\rho_1^*\sigma_{c,1}\sigma_{\hat{c},1}, \ldots, \rho_L^*\sigma_{c,L}\sigma_{\hat{c},L})$, and obtain the equation shown at the bottom of the page.

For a Hermitian matrix $\boldsymbol{\Sigma} = \begin{bmatrix} a & b \\ b^* & c \end{bmatrix}$ and matrix $\mathbf{Q}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (which can be obtained by setting $L = 1$ in $\mathbf{Q}_{2L}$), the two eigenvalues of matrix $\boldsymbol{\Sigma}\mathbf{Q}_2$ are, respectively, given by

$$\begin{aligned} \lambda^+ &= \Re(b) + \sqrt{ac - \Im(b)^2} \\ \lambda^- &= \Re(b) - \sqrt{ac - \Im(b)^2} \end{aligned} \tag{8}$$

[1] When $\mathbf{P}_v$ is a positive definite Hermitian matrix and $\mathbf{Q}_{2L}$ is as defined in the paper, $\mathbf{P}_v\mathbf{Q}_{2L}$ has $2L$ real eigenvalues, with half of them being positive and the remaining half of them being negative, respectively.

[2] In [4], the normalized correlation coefficient between the true and estimated channel, $c_l(i)$ and $\hat{c}_l(i)$, when using the notations of this paper and dropping the branch index l, can be written as $\rho_c/[\sigma_c\sigma_{\hat{c}}] + j\rho_s/[\sigma_c\sigma_{\hat{c}}]$ (see [4, (12)–(16)]), which is equivalent to the definition of ρ in (7) in this paper. However, due to a major approximation involved in the SNR definition in [4], ρ in the pdf result of [4] is equivalent to $|\rho|$ in this paper, cf. Section V.

$$\mathbf{P}_v(\beta) = \begin{bmatrix} \mathrm{diag}\left(\sigma_{\hat{c},1}^2, \ldots, \sigma_{\hat{c},L}^2\right) & \mathrm{diag}(\rho_1^*\sigma_{c,1}\sigma_{\hat{c},1}, \ldots, \rho_L^*\sigma_{c,L}\sigma_{\hat{c},L})e^{j\beta} \\ \mathrm{diag}(\rho_1\sigma_{c,1}\sigma_{\hat{c},1}, \ldots, \rho_L\sigma_{c,L}\sigma_{\hat{c},L})e^{-j\beta} & \mathrm{diag}\left(\sigma_{c,1}^2 + N_0, \ldots, \sigma_{c,L}^2 + N_0\right) \end{bmatrix}$$

and we use $\lambda^{\pm}$ as a short-hand for $\{\lambda^+, \lambda^-\}$ below. A sufficient condition for $\lambda^- < 0$ is given by $|b|^2 - ac < 0$. Letting $b = |\rho_l|\sigma_{c,l}\sigma_{\hat{c},l}e^{j(\beta-\Delta\theta_l)}$, $a = \sigma^2_{\hat{c},l}$, and $c = \sigma^2_{c,l} + N_0$ in (8), and using the fact that the eigenvalue pairs at different branches can be decoupled for independent diversity, we can express all the $2L$ eigenvalues of $\mathbf{P}_v(\beta)\mathbf{Q}_{2L}$ as $\{\lambda_l^+(\beta), \lambda_l^-(\beta)\}_{l=1}^L$, where $\lambda_l^{\pm}(\beta) = |\rho_l|\sigma_{c,l}\sigma_{\hat{c},l}\cos(\beta - \Delta\theta_l) \pm \sigma_{\hat{c},l}\sqrt{\sigma^2_{c,l}[1 - |\rho_l|^2\sin^2(\beta - \Delta\theta)] + N_0}$. Therefore, a sufficient condition for $\lambda_l^-(\beta) < 0$ is given by $|\rho_l|^2\sigma^2_{c,l} < (\sigma^2_{c,l} + N_0)$, and when $|\rho_l| < 1$ (for $l = 1, \ldots, L$), $\lambda_l^-(\beta) < 0$ holds true even when $N_0 = 0$, so that $\mathbf{P}_v(\beta)\mathbf{Q}_{2L}$ has both positive and negative eigenvalues. Using (5), we observe that in this case the MGF $\Phi_D(s)$ has both positive and negative poles in the s-plane. Further, by using (6) and the residue theorem we can conclude that at high SNRs an error floor for M-PSK occurs in Rician fading channels if $|\rho_l| < 1$ at any diversity branch.

IV. Results for I.I.D. Channel Estimation Errors

While the results in Section III are valid for arbitrary linear channel estimators and the corresponding channel estimation errors, here we provide new results for a popular channel error model in i.i.d. Rician channels, and analytically illustrate the performance penalty caused by ICE.

A. A Popular Estimation Error Model

Assuming a popular channel estimation error model for PSAM and MMSE-CE [2], [14], for the signal at the lth branch $(l = 1, \ldots, L)$, we define[3]

$$c_{f,l} = \hat{c}_{f,l} + z_{f,l} \tag{9}$$

where $z_{f,l}$ is the channel estimation error term for the diffuse component and is assumed to be independent of $\hat{c}_{f,l}$. $z_{f,l}$ is zero-mean circularly symmetric and follows a complex Gaussian distribution $z_{f,l} \sim \mathrm{CN}(0, (1 - |\rho|^2)\sigma_c^2)$, and ρ is defined in (7). Furthermore, we assume the LOS component and its estimate follow the same relationship as the diffuse components, i.e., $\mu_{c,l} = \hat{\mu}_{c,l} + z_{\mu,l}$, where $z_{\mu,l}$ is the channel estimation error term[4] for the LOS component and is Gaussian distributed with $z_{\mu,l} \sim \mathrm{CN}(0, (1 - |\rho|^2)|\mu_c|^2)$. With these assumptions, we obtain $c_l = \hat{c}_l + z_l$, for $l = 1, \ldots, L$, where $\hat{c}_l = \hat{\mu}_{c,l} + \hat{c}_{f,l}, z_l = z_{f,l} + z_{\mu,l} \sim \mathrm{CN}(0, (1 - |\rho|^2)[\sigma_c^2 + |\mu_c|^2])$,[5] and that

$$\mathbf{P}_v(\beta) = \begin{bmatrix} |\rho|^2\sigma_c^2\mathbf{I}_L & |\rho|\rho^*\sigma_c^2 e^{j\beta}\mathbf{I}_L \\ |\rho|\rho\sigma_c^2 e^{-j\beta}\mathbf{I}_L & [\sigma_c^2 + N_0 + (1 - |\rho|^2)|\mu_c|^2]\,\mathbf{I}_L \end{bmatrix}$$

and

$$\bar{\mathbf{v}}(\beta) = \begin{bmatrix} |\rho|\mu_c\mathbf{1}_{L\times 1} \\ \rho\mu_c e^{-j\beta}\mathbf{1}_{L\times 1} \end{bmatrix}. \tag{10}$$

The MGF for the DV $D(\beta)$ can be obtained by substituting (10) into (5), and then the cdf $F(0\,|\,\beta)$ can be evaluated to yield the BEP for M-PSK.

Note that for deriving (10) we assumed that both the first- and second-order statistics (mean and variance) of the channel estimates are imperfect. On the other hand, for the ideal case that the mean of the channel estimate is known perfectly, i.e., $\hat{\mu}_c = \mu_c$, we obtain

$$\mathbf{P}_v(\beta) = \begin{bmatrix} |\rho|^2\sigma_c^2\mathbf{I}_L & |\rho|\rho^*\sigma_c^2 e^{j\beta}\mathbf{I}_L \\ |\rho|\rho\sigma_c^2 e^{-j\beta}\mathbf{I}_L & [\sigma_c^2 + N_0]\,\mathbf{I}_L \end{bmatrix}$$

and

$$\bar{\mathbf{v}}(\beta) = \begin{bmatrix} \mu_c\mathbf{1}_{L\times 1} \\ \mu_c e^{-j\beta}\mathbf{1}_{L\times 1} \end{bmatrix}. \tag{11}$$

The BEP of M-PSK can be obtained by substituting (11) into (5).

To facilitate a comparison with our results in case of Rayleigh fading with those in [4]–[6], we scale the estimated channel $\hat{c}_l(i)$ by a factor $1/|\rho|$, and still denote it by $\hat{c}_l(i)$, for all l. Thus, we can obtain an alternative estimation error model

$$c_{f,l}(i) = \rho\hat{c}_{f,l}(i) + z_{f,l} \tag{12}$$

where $\hat{c}_{f,l}(i)$ now has a variance identical to that of $c_{f,l}(i)$, denoted by $\sigma^2_{\hat{c}} = \sigma^2_c$. We can verify that z_l still has a variance $(1 - |\rho|^2)\sigma_c^2$, and the correlation coefficient between $c_{f,l}(i)$ and $\hat{c}_{f,l}(i)$ is still ρ. We underscore that for non-constant modulation formats, such as quadrature amplitude modulation (QAM), only the model given in (9) is valid. For M-PSK with ICE the two definitions in (9) and (12) lead to identical performance evaluation results, because the scaling of the amplitudes of the channel estimates is unimportant.[6] Furthermore, in [4]–[6] the estimation error model equivalent to (12) was used, where the assumption $\sigma^2_{\hat{c}} = \sigma^2_c$ was made. Therefore, for comparison purpose we will use model (12) in the next subsection.

B. MGF of the Effective Output SNR

Below, we derive the MGF for the MRC effective output SNR with ICE in an i.i.d. Rician fading channel, assuming the channel estimation error model in the previous subsection. The relevant real part of the output signal for binary modulation at the lth branch can be expressed as $D_l = \Re(\hat{c}_l^*[c_l + n_l])$. Using $\rho = \rho_c + j\rho_s$, D_l can be expressed as the summation of two terms $D_l = |\hat{c}_l|^2\rho_c + \Re(\hat{c}_l^*(z_{f,l} + z_{\mu,l} + n_l))$, where the first term denotes the desired signal, and the second one is related to the noise and the channel estimation error. After some manipulations, we obtain the effective SNR for the BPSK MRC output signal with ICE in Rician fading as

$$\gamma^{\mathrm{MRC}}_{\mathrm{ICE,BP}} = \frac{\rho_c^2\sum_{l=1}^L|\hat{c}_l|^2}{(1 - |\rho|^2)\,[\sigma_c^2 + |\mu_c|^2] + N_0} = \frac{\rho_c^2\sum_{l=1}^L\hat{\gamma}_l}{(1 - |\rho|^2)\bar{\gamma} + 1} \tag{13}$$

where $\bar{\gamma} = [\sigma_c^2 + |\mu_c|^2]/N_0$ is the average SNR per branch, and $\hat{\gamma}_l = |\hat{c}_l|^2/N_0$. Assuming $\hat{\gamma}_l$ and γ_l have identical statistics, the MGF of $\gamma^{\mathrm{MRC}}_{\mathrm{ICE,BP}}$ can be obtained as

$$\Phi_{\gamma^{\mathrm{MRC}}_{\mathrm{ICE,BP}}}(s) = E\left[e^{s\gamma^{\mathrm{MRC}}_{\mathrm{ICE,BP}}}\right] = \left(\frac{1+K}{1+K-s\bar{\gamma}_{\mathrm{ICE,BP}}}\right)^L \times \exp\left(\frac{LKs\bar{\gamma}_{\mathrm{ICE,BP}}}{1+K-s\bar{\gamma}_{\mathrm{ICE,BP}}}\right) \tag{14}$$

where $\bar{\gamma}_{\mathrm{ICE,BP}} = \rho_c^2\bar{\gamma}/[(1 - |\rho|^2)\bar{\gamma} + 1]$, and we assume $K_l = K$ for all l. Also, using a similar procedure, the MGF for the effective SNR

[3] Below we drop the symbol index i when no confusion arises.

[4] The mismatch of the LOS components may be used to model the case of non-ideal carrier phase recovery for a large class of nonlinear phase estimation techniques, see ([15], (3.39)).

[5] We assume that $\hat{\boldsymbol{\mu}}_c(i) = \hat{\mu}_c(i)\mathbf{1}_{L\times 1}$ and $\boldsymbol{\mu}_c(i) = \mu_c(i)\mathbf{1}_{L\times 1}$, where $\mathbf{1}_{L\times 1}$ is an $L \times 1$ all-one vector.

[6] In [16], based on the model (12) we provided a numerically equivalent formula to (10) but in a slightly different form, see [16, eqs. (10), (11)].

for M-PSK can be obtained as

$$\Phi_{\gamma_{\mathrm{ICE,MP}}^{\mathrm{MRC}}}(s\,|\,\beta) = \left(\frac{1+K}{1+K-s\bar{\gamma}_{\mathrm{ICE,MP}}(\beta)}\right)^{L} \times \exp\left(\frac{LKs\bar{\gamma}_{\mathrm{ICE,MP}}(\beta)}{1+K-s\bar{\gamma}_{\mathrm{ICE,MP}}(\beta)}\right) \quad (15)$$

where $\bar{\gamma}_{\mathrm{ICE,MP}}(\beta) = [(|\rho|^2\cos^2(\beta-\Delta\theta)\bar{\gamma})/((1-|\rho|^2)\bar{\gamma}+1)]$, and $\beta = \pm(\pi/2-\pi/M)$ for M-PSK. Note that (15) includes 2-, 4-, and 8-PSK as special cases.

For verification purpose, for the special case considered in this section, by using (15), the cdf $F_D(0|\beta)$ for M-PSK can be obtained from

$$F_D(0|\beta) = \frac{1}{\pi}\int_0^{\pi/2}\Phi_{\gamma_{\mathrm{ICE,MP}}^{\mathrm{MRC}}}(-1/\sin^2(\theta)|\beta)d\theta. \quad (16)$$

The equivalence of (16) and the cdf result obtained using (10) has been confirmed by numerical computations. By setting $K=0$ in (15), the result for Rayleigh fading can be obtained [16].

C. Loss of Average SNR Caused by ICE

For the case of a real $\rho, \bar{\gamma}_{\mathrm{ICE,MP}}(\beta) = [(|\rho|^2\sin^2(\pi/M)\bar{\gamma})/((1-|\rho|^2)\bar{\gamma}+1)]$ is true. Since $\Delta\theta = 0$ in this case, $\beta = (\pi/2-\pi/M)$ and $-(\pi/2-\pi/M)$ will result in identical conditional error probabilities. Thus, without loss of generality, we may suppress β below when no confusion arises. For perfect channel state information (CSI), the effective SNR $\bar{\gamma}_{\mathrm{ICE,MP}}$ reduces to $\bar{\gamma}_{\mathrm{CSI,MP}} = \sin^2(\pi/M)\bar{\gamma}$. To analytically quantify the loss of ASNR due to ICE with respect to the CSI case, we define the ASNR loss for M-PSK as a factor $\kappa_{\mathrm{MP}}(\bar{\gamma})$ such that the following equality holds:

$$\Phi_{\gamma_{\mathrm{ICE,MP}}^{\mathrm{MRC}}}(s\kappa_{\mathrm{MP}}(\bar{\gamma})) = \Phi_{\gamma_{\mathrm{CSI,MP}}^{\mathrm{MRC}}}(s). \quad (17)$$

We can obtain that $\kappa_{\mathrm{MP}}(\bar{\gamma}) = ((1-|\rho|^2)\bar{\gamma}+1)/|\rho|^2$. Equation (17) nicely relates the MGFs of the effective M-PSK MRC output SNRs for the CSI and ICE cases independent of M, and facilitates a unified performance analysis for ICE. A close inspection of $\kappa_{\mathrm{MP}}(\bar{\gamma})$ reveals that the ASNR loss becomes larger for increasing M given the same ρ and bit ASNR (note that $\bar{\gamma}$ stands for symbol ASNR).

Unfortunately, for complex ρ, $\kappa_{\mathrm{MP}}(\bar{\gamma})$ in (17) is a function of s, and therefore for different modulation formats, $\kappa_{\mathrm{MP}}(\bar{\gamma})$ has to be found using a numerical method.

D. Outage Probability

The outage probability for the MRC effective symbol SNR of M-PSK in i.i.d. Rician and Rayleigh fading channels can be obtained as

$$P_{\mathrm{out,MP}}(\gamma_{\mathrm{th}}) \simeq \frac{1}{2\pi j}\sum_{\beta=\pm(\pi/2-\pi/M)}\int_{c-j\infty}^{c+j\infty}\frac{e^{-s\gamma_{\mathrm{th}}}\Phi_{\gamma_{\mathrm{ICE,MP}}^{\mathrm{MRC}}}(s|\beta)}{s}ds \quad (18)$$

where γ_{th} is the prescribed SNR threshold, and c is the saddle point of $e^{-s\gamma_{\mathrm{th}}}\Phi_{\gamma_{\mathrm{ICE,MP}}^{\mathrm{MRC}}}(s)/s$. An efficient formula for evaluating $P_{\mathrm{out,MP}}(\gamma_{\mathrm{th}})$ can be obtained using a result in [15].

For an i.i.d Rayleigh fading channel closed-form expressions for $P_{\mathrm{out,BP}}(\gamma_{\mathrm{th}})$ and $P_{\mathrm{out,MP}}(\gamma_{\mathrm{th}})$ can be derived. For BPSK, by using the residue theorem, the pdf of $\gamma_{\mathrm{ICE,BP}}^{\mathrm{MRC}}$ can be obtained as $f_{\gamma_{\mathrm{ICE,BP}}^{\mathrm{MRC}}}(\gamma) = (1/(L-1)!)(\gamma^{L-1}/[\bar{\gamma}_{\mathrm{ICE,BP}}]^L)e^{-\gamma/\bar{\gamma}_{\mathrm{ICE,BP}}}$. Then, the outage probability for BPSK with ICE and MRC is given by $P_{\mathrm{out,BP}}(\gamma_{\mathrm{th}}) = \int_0^{\gamma_{\mathrm{th}}} f_{\gamma_{\mathrm{ICE,BP}}^{\mathrm{MRC}}}(\gamma)d\gamma = (1/(L-1)!)\gamma(L,\gamma_{\mathrm{th}}/\bar{\gamma}_{\mathrm{ICE,BP}})$, where $\gamma(L,x) = \int_0^x e^{-t}t^{L-1}dt$ is the incomplete Gamma function [12]. Similarly, the outage probability for M-PSK is given by $P_{\mathrm{out,MP}}(\gamma_{\mathrm{th}}) = (1/(L-1)!)\sum_{\beta=\pm(\pi/2-\pi/M)}\gamma(L,\gamma_{\mathrm{th}}/[\bar{\gamma}_{\mathrm{ICE,MP}}(\beta)])$.

V. Discussion

We compare our results for the MRC output SNR for ICE with those obtained by Gans [4] for an i.i.d Rayleigh fading channel. In [4], the MRC output SNR with ICE was defined as (see[4], (4), (5), (10), (19))

$$\tilde{\gamma}_{\mathrm{ICE}} = \frac{S_0}{N_0} = \frac{\left|\sum_{l=1}^{L}\hat{c}_l^* c_l\right|^2}{\left|\sum_{l=1}^{L}\hat{c}_l^* n_l\right|^2} \quad (19)$$

where $S_0 = |\sum_{l=1}^{L}\hat{c}_l^* c_l|^2$ was defined as the power of the desired signal, and $N_0 = |\sum_{l=1}^{L}\hat{c}_l^* n_l|^2$ was the power of the noise.

We have to point out that the definition for SNR in (19) involves a major approximation. The pdf result in [4], when applied to the performance analysis of digital communication in fading channels with ICE, will result in loose performance upper bounds. We explain the approximation involved in (19) . Assume $\hat{c}_l = jc_l$, i.e., the channel estimate has a $\pi/2$ phase mismatch error. Using (19) the resulting SNR for this ICE case is the same as that for the perfect CSI case (i.e., $\hat{c}_l = c_l$). Note that in practice a $\pi/2$ phase error would render the coherent PSK receiver completely useless. To illustrate further, let us consider the i.i.d. Rayleigh fading ICE model studied in [4]–[6]. We can rewrite c_l as $c_l = (\rho_c + j\rho_s)\hat{c}_l + z_{f,l}$, and decompose the signal term S_0 to $S_0 = |\sum_{l=1}^{L}\hat{c}_l^* c_l|^2 = |\sum_{l=1}^{L}[|\hat{c}_l|^2\rho_c + j|\hat{c}_l|^2\rho_s + \hat{c}_l^* z_{f,l}]|^2$. Since $|\hat{c}_l|^2\rho_c$ is the only desired signal, the definition of the desired signal in (19) improperly includes [7] the term $\sum_{l=1}^{L}[j|\hat{c}_l|^2\rho_s + \hat{c}_l^* z_{f,l}]$, which actually corresponds to the phase offset of ρ (i.e., ρ_s) and the channel mismatch error $z_{f,l}$. Even for the PSAM model $c_l = \hat{c}_l + z_{f,l}$ (with $\sigma_{\hat{c}}^2 = |\rho|^2\sigma_c^2$), we obtain $S_0 = |\sum_{l=1}^{L}\hat{c}_l^* c_l|^2 = |\sum_{l=1}^{L}[|\hat{c}_l|^2 + \hat{c}_l^* z_{f,l}]|^2$. The definition in (19) still improperly includes the term $\sum_{l=1}^{L}[\hat{c}_l^* z_{f,l}]$. In summary, only for the case of CSI, the definition of $|\sum_{l=1}^{L}\hat{c}_l^* c_l|^2$ as the signal power is correct.

Due to the problem with the SNR definition, some misleading conclusions result. For example, when $L=1$ or $|\rho|=0$, the result in [4] leads to the false conclusions that single channel reception is immune to ICE, and that multi-channel reception with completely noisy references (i.e., $|\rho|=0$) is equivalent to a single-channel receiver with perfect CSI [5], [6]. Further, when $\bar{\gamma}\to\infty$ and $|\rho|<1$, the BEP predicted by [5] follows the nondiversity case (i.e., with a diversity order of one). However, our results prove that a detection error floor always exists if $|\rho|<1$, and thus the diversity order approaches zero as $\bar{\gamma}\to\infty$. In [4], due to the major approximation involved, the resulting pdf expression is a function of $|\rho|$ (the magnitude of ρ) only, see e.g., [4, (47)], [5, (5), (6)], [6, (7), (60)], but not of $\Delta\theta$ (the phase of ρ). Accordingly, the effect of $\Delta\theta$ is not included in the analysis given in [4]–[6].

VI. Numerical Examples

For all the figures in this section, we keep ρ constant for all SNRs in order to better illustrate the effect of ρ, and to facilitate a comparison

[7] Specifically, in [4, (19)], only the term X is the desired signal, while x, y, and Y are actually terms representing noisy components but were also improperly included in the desired signal.

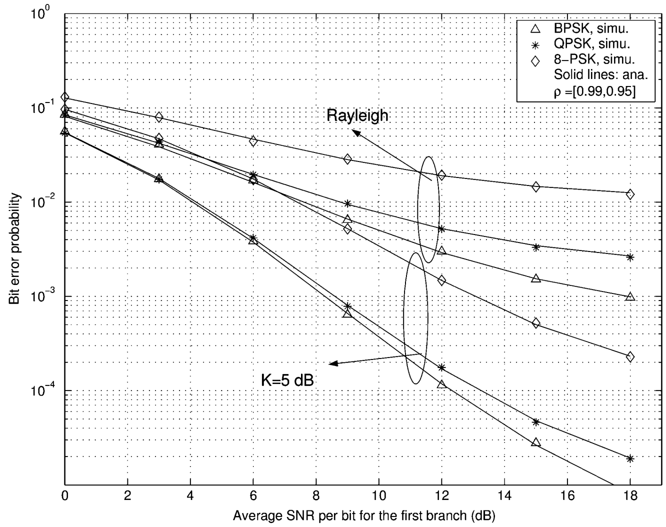

Fig. 1. BEP versus the average SNR per bit for the first branch for the M-PSK MRC receiver with ICE in i.n.d. Rayleigh and Rician ($K = 5$ dB) fading channels, respectively, with $L = 2$, $\boldsymbol{\rho} = [0.99, 0.95]$, and the average SNRs differ by 2 dB between the two branches.

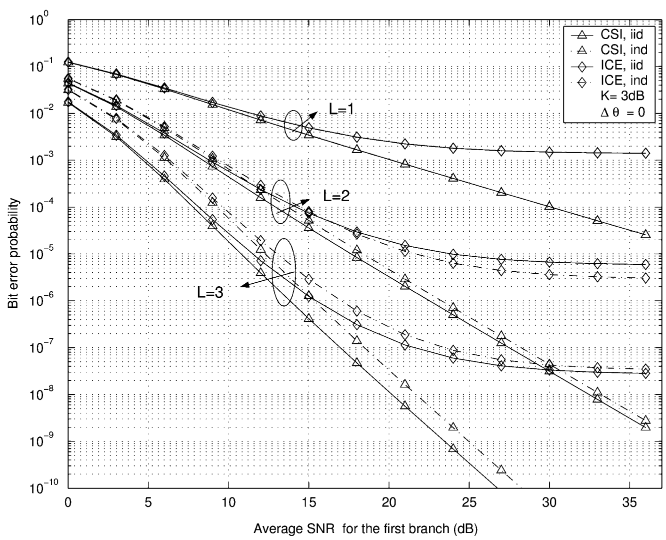

Fig. 2. BEP versus the average SNR per bit for the first branch for the QPSK MRC receiver with CSI and ICE, respectively, in i.i.d. and i.n.d. Rician fading channels with $L = 1, 2, 3$, $K = 3$ dB, and $\Delta\theta = 0$.

with the results in [4] and [5]. It should be understood that in practice $|\rho|$ may increase as the ASNR increases and then be upper-bounded by a constant smaller than (e.g., due to the Doppler effect) or equal to one.

In Fig. 1, we present the simulated and analytical BEPs for the M-PSK MRC receiver with ICE in i.n.d. Rayleigh and Rician ($K = 5$ dB) fading channels, respectively, with $L = 2, \boldsymbol{\rho} = [0.99, 0.95]$ (where $\boldsymbol{\rho} = [\rho_1, \rho_2]$), and the ASNR in the first branch is 2 dB larger than that in the second branch. For the case of Rician fading with ICE we assume that the estimates of the LOS components are perfect, but those of the diffuse components are imperfect. The results verify the accuracy of our analytical formulas and show the error floors for ICE for high SNRs.

In Fig. 2, we compare the BEPs for QPSK with MRC in i.i.d. and i.n.d. Rician fading channels ($K = 3$ dB) with CSI and ICE, respectively. For the i.i.d. case with ICE, $\rho = 0.99$ for all the branches; for the

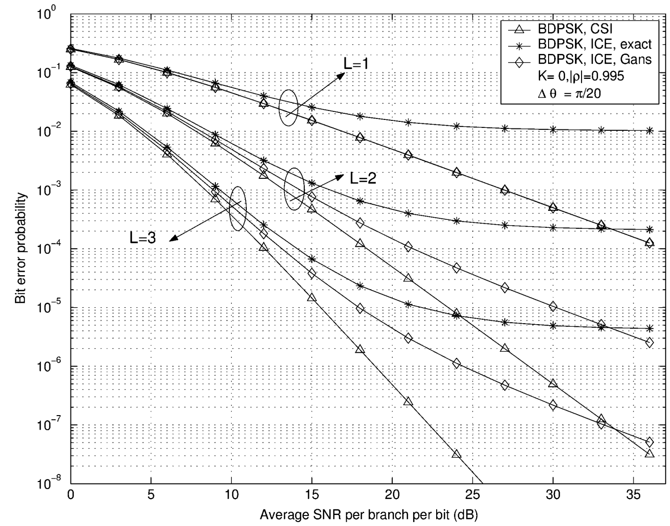

Fig. 3. BEP versus the average SNR per branch for the BDPSK MRC receiver with CSI and ICE, respectively, in an i.i.d. Rayleigh fading channel with $L = 1, 2, 3$, $|\rho| = 0.995$, and $\Delta\theta = \pi/20$.

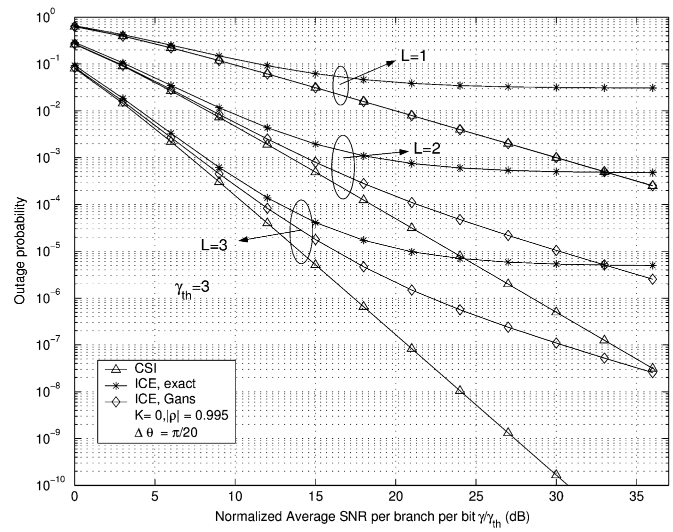

Fig. 4. Outage Probability versus the normalized average SNR per branch for the BPSK MRC receiver with CSI and ICE, respectively, in an i.i.d. Rayleigh fading channel with $L = 1, 2, 3$, $|\rho| = 0.995$, $\gamma_{\rm th} = 3$, and $\Delta\theta = \pi/20$.

i.n.d. case with ICE, $\rho = 0.99$ for $L = 1$, $\boldsymbol{\rho} = [0.999, 0.99]$ for $L = 2$, and $\boldsymbol{\rho} = [0.999, 0.99, 0.98]$ for $L = 3$; and the average SNRs differ by 1.5 dB from the strongest branch (the first branch) to the weakest branch (the Lth branch). The results in Fig. 2 show that with CSI the QPSK MRC receiver performs better in the i.i.d. channels than in the i.n.d. channels for $L = 2, 3$. However, for the case of ICE in the considered example, due to the nonuniform $\boldsymbol{\rho}$ (with $\boldsymbol{\rho} = [0.999, 0.99, 0.98]$) in different branches, the i.n.d. case may result in a better performance than the i.i.d. case for high SNRs (cf. $L = 2$). In [5], the BEP for binary differential PSK (BDPSK) with MRC and ICE is given by [5, (16)], which was obtained using Gans' pdf formula. To show the approximation involved in this result, by using (14) and setting $K = 0$ therein an exact BEP expression for BDPSK with ICE and MRC is obtained as $P_{\rm BDP} = 0.5 \cdot (1 - (\rho_c^2 \bar{\gamma}/[(1 - |\rho|^2)\bar{\gamma} + 1]))^{-L}$. Also, using Gans' pdf result an approximate outage probability expression is given by [4, (48)]. The accurate outage probability can be computed using the

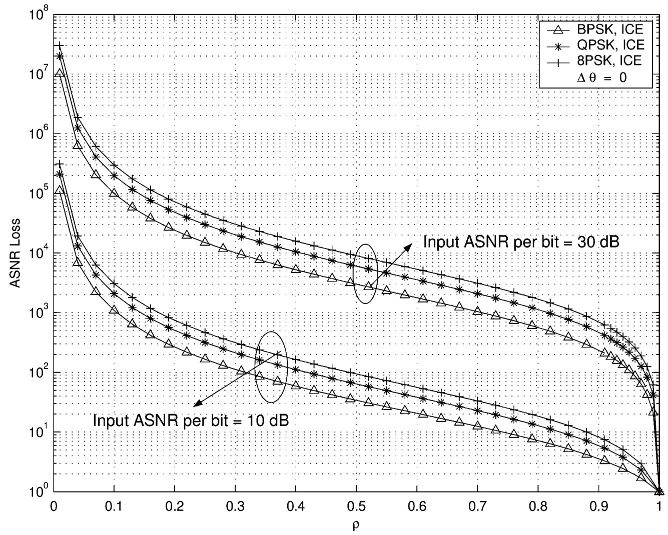

Fig. 5. ASNR loss versus the normalized estimation correlation coefficient ρ for M-PSK with ICE in i.i.d. Rician and Rayleigh fading channels.

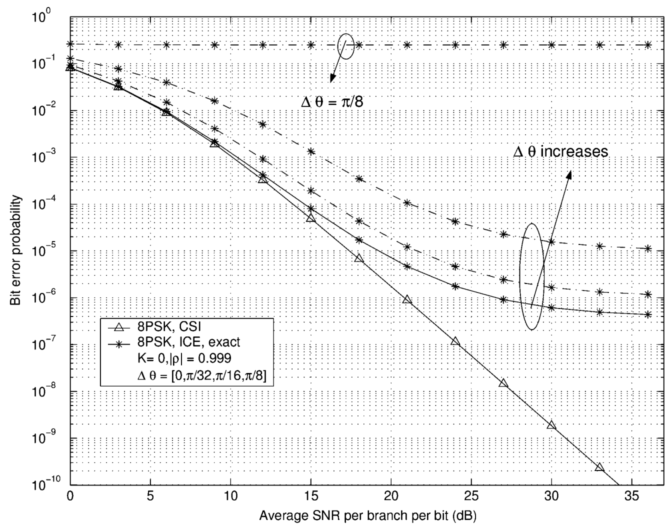

Fig. 6. BEP versus the average SNR per branch per bit for the 8-PSK MRC receiver with CSI and ICE, respectively, in an i.i.d. Rayleigh fading channel, with $L = 3, |\rho| = 0.999$, and $\Delta\theta = \{0, \pi/32, \pi/16, \pi/8\}$, respectively.

results in Section IV-D. The BEP and outage probability results are presented in Figs. 3 and 4, respectively. For all considered L, the BEP results based on [5, (16)] significantly underestimate the adverse effect of ICE on the detection performance, and fail to predict the error floors. Also, for $L = 1, 2, 3$, [4, (48)] gives lower outage probabilities than the exact ones.

Next, we study the effect of ICE in i.i.d. Rician and Rayleigh fading channels, and we assume the estimation error model in Section IV-A. We present the ASNR loss for M-PSK with ICE versus ρ in Fig. 5. The ASNR loss increases as ρ decreases and as the ASNR or M increases, which suggests that ICE severely limits the performance of the M-PSK MRC receiver for high SNRs.

Finally, the effect of the phase offset $\Delta\theta$ on the 8-PSK MRC performance is studied in Fig. 6. The results show that the performance of M-PSK degrades very rapidly as $\Delta\theta$ increases, and thus demonstrates that the adverse effect of the phase mismatch $\Delta\theta$ is significant.

VII. Conclusion

We have derived the exact error probability formulas for the M-PSK MRC receiver with ICE in general Rician fading channels. We further obtained the MGF for the effective receiver output SNR, the outage probability, and the ASNR loss in i.i.d. Rician fading channels. For the i.i.d. Rayleigh fading case, we have compared our result with a popular approach used in the literature to evaluate the adverse effect of ICE, and analytically and numerically showed a major approximation involved therein. Our proposed method for the analysis of ICE can also be applied to other diversity and modulation formats.

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