Homework 1

CS250 Discrete Structures I, Winter 2020

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Due: April 5, 2020

Problem 1 Figure out how to typeset the following mathematical statements.

1. $f(x) = \mathcal{O}(n \log n)$ (Big 'O' notation)
2. $\neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B)$ (De Morgan's law in propositional logic notation)
3. $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (De Morgan's law in set theory notation)
4. $f(x) = \log_2 x^2$ (Subscripts and superscripts)
5. $A = \frac{\pi d^2}{4}$ (Fraction and special symbols)
6. $S = \{a, b, c, d\}$ (A set definition)
7. (Truth Table)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

8. (A summation statement)

$$\sum_{k=1}^{n} n$$

Problem 2 Read chapter 0.1 and 0.2 of the textbook and write up solutions to the following exercises (page 17–23 in the pdf version).

1. For each sentence below, deicide it is an atomic statement, a molecular statment, or not a statement at all.
   (a) Customers must wear shoes
   (b) The customers wore shoes
   (c) The customers wore shoes and they wore socks
3. Suppose P and Q are the statements: P: Jack passed math. Q: Jill passed math.
   (a) Translate "Jack and Jill both passed math" into symbols
   (b) Translate "If Jack passed math, then Jill did not" into symbols
   (c) Translate "$P \vee Q$" into English
   (d) Translate "$\neg(P \wedge Q) \rightarrow Q$" into English

(e) Suppose you know that if Jack passed math, then so did Jill. What can you conclude if you know that:

i. Jill passed math?

ii. JIll did not pass math?

10. Write each of the following statements in the form, "if . . . , then . . . ." Careful, some of the statements might be false (which is alright for the purposes of this question).

(a) To lose weight, you must exercise.

(b) To lose weight, all you need to do is exercise.

(c) Every American is patriotic.

(d) You are patriotic only if you are American.

(e) The set of rational numbers is a subset of the real numbers

(f) A number is prime if it is not even.

(g) Either the Broncos will win the Super Bowl, or they won't play in the Super Bowl.

12. Let $P(x)$ be the predicate, "$3x + 1$ is even"

(a) is $P(5)$ true or false?

(b) What, if anything, can you conclude about $\exists x P(x)$ from the truth value of $P(5)$?

(c) What, if anything, can you conclude about $\forall x P(x)$ from the truth value of $P(5)$?

16. Translate into symbols. Use $E(x)$ for " x is even" and $O(x)$ for "x is odd."

(a) No number is both even and odd.

(b) One more than any even number is an odd number.

(c) There is prime number that is even.

(d) Between any two numbers there is a third number.

(e) There is no number between a number and one more than that number.

17. Translate into English:

(a) $\forall x(E(x) \to E(x + 2)$

(b) $\forall x \exists y(\sin(x) = y)$

(c) $\forall y \exists x(\sin(x) = y)$

(d) $\forall x \forall y(x^3 = y^3 \to x = y)$