

Homework 1

CS250 Discrete Structures I, Winter 2020

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Due: April 5, 2020

Problem 1 Figure out how to typeset the following mathematical statements.

1. $f(x) = \mathcal{O}(n \log n)$ (Big ‘O’ notation)
2. $\neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B)$ (De Morgan’s law in propositional logic notation)
3. $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (De Morgan’s law in set theory notation)
4. $f(x) = \log_2 x^2$ (Subscripts and superscripts)
5. $A = \frac{\pi d^2}{4}$ (Fraction and special symbols)
6. $S = \{a, b, c, d\}$ (A set definition)
7. (Truth Table)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

8. (A summation statement)

$$\sum_{k=1}^n n$$

Problem 2 Read chapter 0.1 and 0.2 of the textbook and write up solutions to the following exercises (page 17–23 in the pdf version)

Exercises: 1, 3, 10, 12, 16, and 17.

1. For each sentence below, decide it is an atomic statement, a molecular statment, or not a statement at all.
 - (a) Customers must wear shoes
 - (b) The customers wore shoes
 - (c) The customers wore shoes and they wore socks
3. Suppose P and Q are the statements: P : Jack passed math. Q : Jill passed math.
 - (a) Translate "Jack and Jill both passed math" into symbols
 - (b) Translate "If Jack passed math, then Jill did not" into symbols

- (c) Translate " $P \vee Q$ " into English
 - (d) Translate " $\neg(P \wedge Q) \rightarrow Q$ " into English
 - (e) Suppose you know that if Jack passed math, then so did Jill. What can you conclude if you know that:
 - i. Jill passed math?
 - ii. Jill did not pass math?
10. Write each of the following statements in the form, "if . . . , then" Careful, some of the statements might be false (which is alright for the purposes of this question).
- (a) To lose weight, you must exercise.
 - (b) To lose weight, all you need to do is exercise.
 - (c) Every American is patriotic.
 - (d) You are patriotic only if you are American.
 - (e) The set of rational numbers is a subset of the real numbers
 - (f) A number is prime if it is not even.
 - (g) Either the Broncos will win the Super Bowl, or they won't play in the Super Bowl.
12. Let $P(x)$ be the predicate, " $3x + 1$ is even"
- (a) is $P(5)$ true or false?
 - (b) What, if anything, can you conclude about $\exists x P(x)$ from the truth value of $P(5)$?
 - (c) What, if anything, can you conclude about $\forall x P(x)$ from the truth value of $P(5)$?
16. Translate into symbols. Use $E(x)$ for " x is even" and $O(x)$ for " x is odd."
- (a) No number is both even and odd.
 - (b) One more than any even number is an odd number.
 - (c) There is prime number that is even.
 - (d) Between any two numbers there is a third number.
 - (e) There is no number between a number and one more than that number.
17. Translate into English:
- (a) $\forall x(E(x) \rightarrow E(x + 2))$
 - (b) $\forall x \exists y(\sin(x) = y)$
 - (c) $\forall y \exists x(\sin(x) = y)$
 - (d) $\forall x \forall y(x^3 = y^3 \rightarrow x = y)$