Homework 1

CS250 Discrete Structures I, Winter 2020

Armant Touche

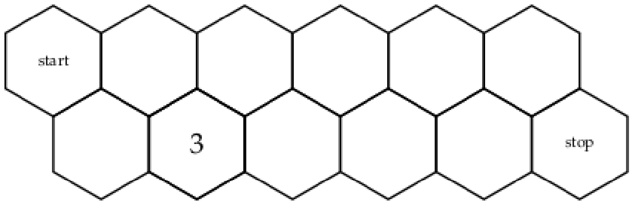
Due: April 26, 2020

Homework Exercises Chapter 2: Sequences

Problem 1 2.1 Describing Sequences (pg. 135-147)

From section 2.1 in the textbook, complete exercises 2, 3, 7, 10, 13, 18

2. For each sequence given below, find a closed formula for a_n , the nth term of the sequence (assume the first terms are a_0) by relating it to another sequence for which you already know the formula. In each case, briefly say how you got your answers.
   (a) 4, 5, 7, 11, 19, 35, . . .
   (b) 0, 3, 8, 15, 24, 35,. . .
   (c) 6, 12, 20, 30, 42, . . .
   (d) 0, 2, 7, 15, 26, 40, 57, . . . (Cryptic Hint: these might be called "house numbers")
3. Write out the first 5 terms (starting with a_0) of each of the sequences described below. Then give either a closed formula or a recursive definition for the sequence (whichever is NOT given in the problem).
   (a) $a_n = \frac{1}{2}(n^2 + n)$
   (b) $a_n = 2a_{n-1} - a_{n-2}$ with $a_0 = 0$ and $a_1 = 1$
   (c) $a_n = na_{n-1}$ with $a_0 = 1$
7. Write out the first few terms of the sequence given by $a_1 = 3$; $a_n = 2a_{n-1} + 4$. Then find a recursive definition for the sequence 10, 24, 52, 108, . . .
10. Show that $a_n = 2^n - 5^n$ is also a solution to the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$. What would the initial conditions need to be for this to be the closed formula for the sequence?
13. Use summation (Σ) or product (Π) notation to rewrite the following
    (a) $2 + 4 + 6 + 8 + ... + 2n$
    (b) $1 + 5 + 9 + 13 + ... + 425$
    (c) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{50}$
    (d) $2 \cdot 4 \cdot 6 \cdot ... \cdot 2n$
    (e) $(\frac{1}{2})(\frac{2}{3})(\frac{3}{4})...(\frac{100}{101})$
18. When bees play chess, they use a hexagonal board like the one shown below. The queen bee can move one space at a time either directly to the right or angled up-right or down-right (but can never move leftwards). How many different paths can the queen take from the top left hexagon to the bottom right hexagon? Explain your answer, and this relates to the previous question. (As an example, there are three paths to get to the second hexagon on the bottom row.)



Problem 2 2.2 Arithmetic and Geometric Sequences (pg. 148-159)

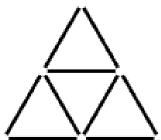
From section 2.2 in the textbook, complete exercises 1, 3, 4, 13, 15

1. Consider the sequence 5, 9, 13, 17, 21, . . . with $a_1 = 5$
   (a) Give a recursive definition for the sequence.
   (b) Give a closed formula for the nth term of the sequence.
   (c) Is 2013 a term in the sequence? Explain.
   (d) How many terms does the sequence 5, 9, 13, 17, 21, . . . , 533 have?
   (e) Find the sum: $5 + 9 + 13 + 17 + 21 + \cdots + 533$. Show your work.
   (f) Use what you found above to find b_n , the nth term of 1, 6, 15, 28, 45, . . ., where $b_0 = 1$

3. Consider the sum $4 + 11 + 18 + 25 + \cdots + 249$.
   (a) How many terms (summands) are in the sum?
   (b) Compute the sum using a technique discussed in this section.

4. Consider the sequence 1, 7, 13, 19, . . . , $6n + 7$.
   (a) How many terms are there in the sequence? Your answer will be in terms of n.
   (b) What is the second-to-last term?
   (c) Find the sum of all the terms in the sequence, in terms of n.

13. If you have enough toothpicks, you can make a large triangular grid. Below, are the triangular grids of size 1 and of size 2. The size 1 grid requires 3 toothpicks, the size 2 grid requires 9 toothpicks.



   (a) Let t_n be the number of toothpicks required to make a size n triangular grid. Write out the first 5 terms of the sequence $t_1, t_2, ...$
   (b) Find a recursive definition for the sequence. Explain why you are correct
   (c) Is the sequence arithmetic or geometric? If not, is it the sequence of partial sums of an arithmetic or geometric sequence? Explain why your answer is correct.
   (d) Use your results from part (c) to find a closed formula for the sequence. Show your work.

15. Here is a surprising use of sequences to answer a counting question: How many license plates consist of 6 symbols, using only the three numerals 1, 2, and 3 and the four letters a, b, c, and d, so that no numeral appears after any letter? For example, "31ddac" and "12321" are acceptable license plates, but "13ba2c" is not.
   (a) First answer this question by considering different cases: how many of the license plates contain no numerals? How many contain one numeral, etc.
   (b) Now use the techniques of this section to show why the answer is $4^7 - 3^7$.