Homework 9

CS250 Discrete Structures I, Winter 2020

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Powerset

1. *Proof*:

   In my program, the set of bit strings of length n (variable: power_set_size) is used to expressed $\{0,1\}^n$. This list is used as positional reference for each element of the original set A in which we generating the power set for. The positional reference is a yes (1) or no (0) reference to the set of subset ($\mathcal{P}(A)$). To create a relationship between the bit string and A we need to create a bijective relationship.

   We first define $f : \{0,1\}^n \rightarrow \mathcal{P}(A)$ as $f((a_1, a_2, ..., a_n)) = \{i | a_i = 1\}$. What this function means is that bit strings map to the set of all positions 1's in the string. For example, n = 4, bit string is (1, 0, 1, 1), this would map to to $\{1, 3, 4\}$. To highlight bijectivity, we need to prove injection and surjection

2. Injective:

   Suppose that x and y both map to one subset $S \in \mathcal{P}(A)$. S defines the yes (1) or no (0) positional reference to this single subset. If $f(x) = f(y) = S$, x and y must have 1' in exactly the same position for the subset to exist (This is my special conditional statement in powerset.c: Line 63). n, x, and y are identical because bit string must only contain 0's and 1's. Therefore f is injective because at the most 1 bit string can map to a single subset

3. Surjective

   Examine a subset $S \in \mathcal{P}(A)$. Again, S only contains integers between 1 and n, inclusively speaking. The list $x = (x_1, x_2, ..., x_n)$ can be made such that all for all $i \in S$, $x_i =$, and all others are 0. f state that list x will map exactly to S. Thus, since all subsets S are mapped to by some list x representing a bit string of length n, f is surjective.

   With these two existing relationship of mappings, a bijective relationship exist between the set of bit strings of length n and the $\mathcal{P}(A)$ so these 2 sets have equal cardinality.