# Homework 1

CS250 Discrete Structures I, Winter 2020

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Due: April 5, 2020

**Problem 1** Figure out how to typeset the following mathematical statements.

1. $f(x) = \mathcal{O}(n \log n)$ (Big 'O' notation)
2. $\neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B)$ (De Morgan's law in propositional logic notation)
3. $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (De Morgan's law in set theory notation)
4. $f(x) = \log_2 x^2$ (Subscripts and superscripts)
5. $A = \frac{\pi d^2}{4}$ (Fraction and special symbols)
6. $S = \{a, b, c, d\}$ (A set definition)
7. (Truth Table)

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ |
| $F$ | $F$ | $F$ |

8. (A summation statement)

$$\sum_{k=1}^{n} n$$

**Problem 2** Read chapter 0.1 and 0.2 of the textbook and write up solutions to the following exercises (page 17–23 in the pdf version).

1. For each sentence below, deicide it is an atomic statement, a molecular statment, or not a statement at all.
   (a) Customers must wear shoes
   Not a statement
   (b) The customers wore shoes
   Atomic statement
   (c) The customers wore shoes and they wore socks
   Molecular statement
3. Suppose $P$ and $Q$ are the statements: $P$: Jack passed math. $Q$: Jill passed math.
   (a) Translate "Jack and Jill both passed math" into symbols
   $P \wedge Q$

(b) Translate "If Jack passed math, then Jill did not" into symbols
$P \wedge \neg Q$

(c) Translate "$P \vee Q$" into English
Jack or Jill passed math (or both).

(d) Translate "$\neg(P \wedge Q) \rightarrow Q$" into English
If Jack and Jill both did not pass math, then Jill passed math.

(e) Suppose you know that if Jack passed math, then so did Jill. What can you conclude if you know that:

i. Jill passed math?
Cannot conclude anything.

ii. Jill did not pass math?
Jack did not pass math.

10. Write each of the following statements in the form, "if . . . , then . . . ." Careful, some of the statements might be false (which is alright for the purposes of this question).

(a) To lose weight, you must exercise.
If you want to lose weight, then you must exercise.

(b) To lose weight, all you need to do is exercise.
If you exercise, then you will lose weight

(c) Every American is patriotic.
If you are American, then you are patriotic.

(d) You are patriotic only if you are American.
If you are patriotic, then you are American.

(e) The set of rational numbers is a subset of the real numbers
If there is a set of rational numbers, then that set of numbers is a subset of real numbers.

(f) A number is prime if it is not even.
If the number is not even, then the number is prime.

(g) Either the Broncos will win the Super Bowl, or they won't play in the Super Bowl.
If the Broncos play, then they win the Super Bowl but otherwise, they do not play the Super Bowl.

12. Let $P(x)$ be the predicate, "$3x + 1$ is even"

(a) Is $P(5)$ true or false?
True

(b) What, if anything, can you conclude about $\exists x P(x)$ from the truth value of $P(5)$?
Since P(5) is true, $\exists x P(x)$ is true because 5 is such an $x$.

(c) What, if anything, can you conclude about $\forall x P(x)$ from the truth value of $P(5)$?
Since there is not a known domain, we cannot verify if $\forall x P(x)$ to be true since we do not all the truth values. $P(4)$ is false for example.

16. Translate into symbols. Use $E(x)$ for "$x$ is even" and $O(x)$ for "$x$ is odd."

(a) No number is both even and odd.
$\neg\exists x(E(x) \wedge O(x))$

(b) One more than any even number is an odd number.
$\forall x(E(x) \rightarrow O(x + 1))$

(c) There is prime number that is even.
$\exists x(E(x) \wedge P(x))$ (where $P(x)$ means "$x$ is prime)

(d) Between any two numbers there is a third number.
$\forall x \forall y \exists z (x < z < y \vee y < z < x)$

(e) There is no number between a number and one more than that number.
$\forall x \neg \exists y (x < y < x + 1)$

17. Translate into English:

(a) $\forall x (E(x) \rightarrow E(x + 2)$
Any even number plus 2 is an even number

(b) $\forall x \exists y (\sin(x) = y)$
For any $x$ there exists $y$ such that $sin(x) = y$

(c) $\forall y \exists x (\sin(x) = y)$
For any $y$ there exists $x$ such that $sin(x) = y$

(d) $\forall x \forall y (x^3 = y^3 \rightarrow x = y)$
For any $x$ or $y$, if cubes of those two numbers are equal, then $x = y$