How to calculate P(k) and X(z)

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How to calculate $P(k)$

See parameter file given by Anthony in $/Document/Pullen/parameter\_file.ini$ and input them on web https://lambda.gsfc.nasa.gov/toolbox/tb$_{c}amb_{f}orm.cfm//$

$$\left(\frac{r_0}{r}\right)^{\gamma} = \frac{1}{2\pi}\int dkk^2 P(k)\frac{sin(kr)}{kr} \tag{1}$$

Here $\gamma \sim 1.8$ and $r_0 \sim 5(h^{-1}Mpc)$. We can see the unit of distance is basically Mpc/h, and unit of power spectrum $P(k)$ should be $(Mpc/h)^3$.

Keep in mind that h is a unitless constant (Hubble constant), representing the uncertainty of Hubble rate H. Anthony is using $h = 0.676$, while in MODERN COSMOLOGY $h = 0.72 \pm 0.08$ (This book is tens years old).

When plotting power spectrum, always plot $log_{10}{-}log_{10}$ graphs (Not natural log, so use np.log()/np.log(10) always). It looks like Figure 1.

How to calculate $X(z)$

On MODERN COSMOLOGY, $\chi(z)$ is called "comoving distance between a distant emitter and us". In a time dt, light travels a comoving distance:

$$dx = \frac{cdt}{a} = \frac{dt}{a} \tag{2}$$

(setting $c = 1$, but the unit of c–km/s is kept. a is unitless.).

$$H(a) = \frac{da/dt}{a}, a = \frac{1}{1+z} \tag{3}$$

H is Hubble rate, z is redshift.

$$\chi(z) = \int_{t(a)}^{t_0} \frac{dt'}{a(t')} = \int_{t(a)}^{t_0} \frac{(dt'/da')da'}{a'} = \int_{t(a)}^{t_0} \frac{da'}{a'^2 \frac{da'/dt'}{da'}} = \int_a^1 \frac{da'}{a'^2 H(a')} = -\int_z^0 \frac{dz'}{H(z')} = \int_0^z \frac{dz'}{H(z')} \tag{4}$$

According to wikipedia "Hubble's law":

$$H^2(z) = H_0^2(\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda) \tag{5}$$

Here $\Omega_i = \frac{\rho_i}{\rho_{cr}}a^3 = \frac{\rho_i}{\rho_{cr}}$ (Ω is unitless. All density in this formula are density today). M means matter, Λ means cosmological constant or vacuum density (dark energy– constant). Ω_k is spatial curvature density.

Anthony said let's assume the universe is flat (no curvature density), and radiation density is neglegible ($\Omega_M + \Omega_\Lambda = 1$):

$$H = H_0 \sqrt{\Omega_M (1+z)^3 + 1 - \Omega_M} \tag{6}$$

According to parameters Anthony is using: $\Omega_b h^2 = \Omega_{baryon} h^2 = 0.0226$,$\Omega_c h^2 = \Omega_{cdm} h^2 = 0.112$, $\Omega_M = \Omega_b + \Omega_c = \dfrac{0.0226 + 0.112}{h^2}$, $H_0 = 100[h/s/Mpc]$. Unit of H is unit of H_0 is $[h/s/Mpc]$. Now recall in Eq (2) we set $c = 1$. Time the integration result of $\chi(z)$ with $c = 3 \times 10^5 [km/s]$, the unit becomes $[Mpc/h]$, which is the unit of distance. Data points are in Fig 2.

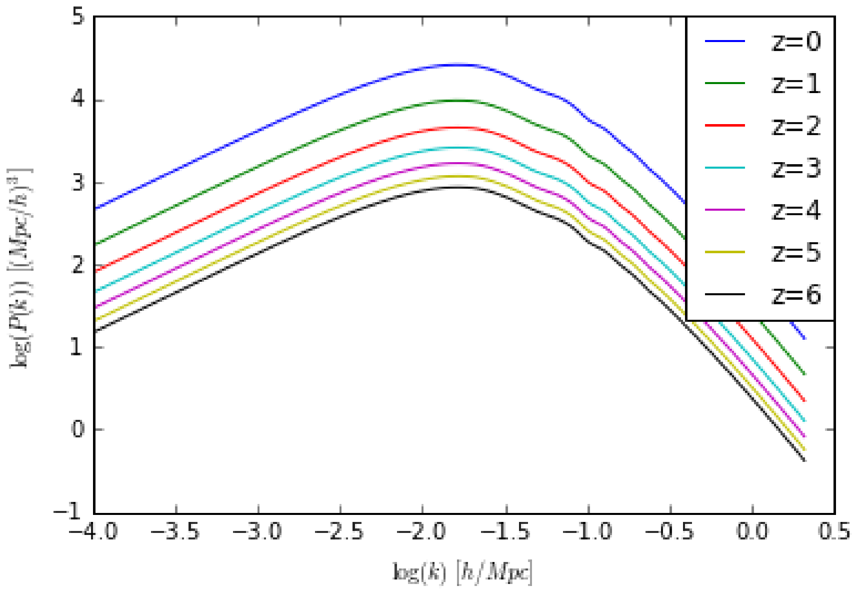

Figure 1: log(k)-log(P(k))

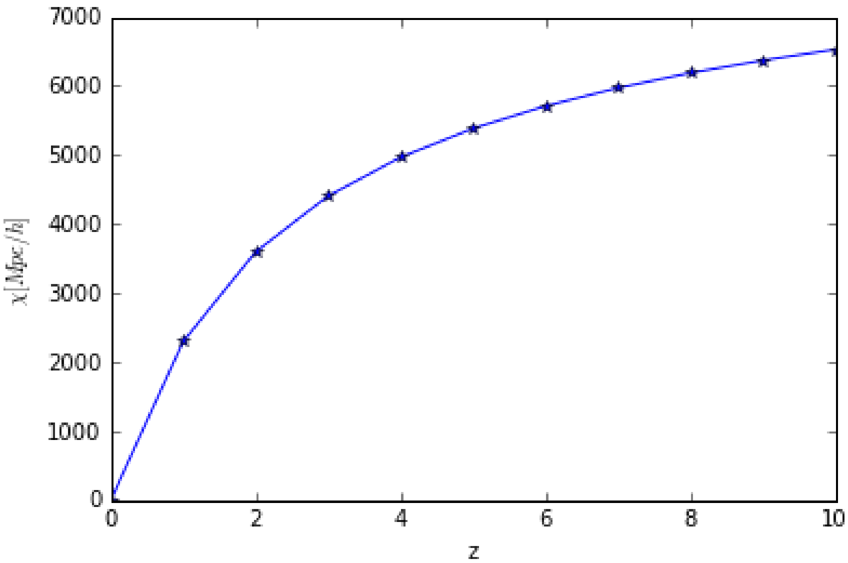

Figure 2: $z - \chi(z)$