Efficient Generation of Geographically Accurate Transit Maps

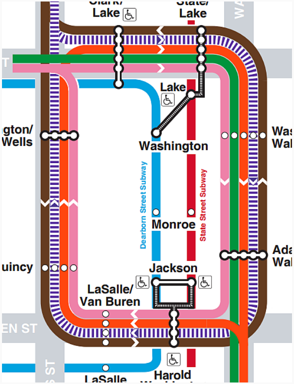
Hannah Bast[1], Patrick Brosi[1] and Sabine Storandt[2]

[1] University of Freiburg
[2] LMU Würzburg

26th ACM SIGSPATIAL - Seattle, Washington, USA

Motivation

Official CTA map



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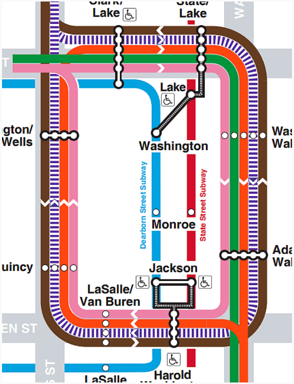
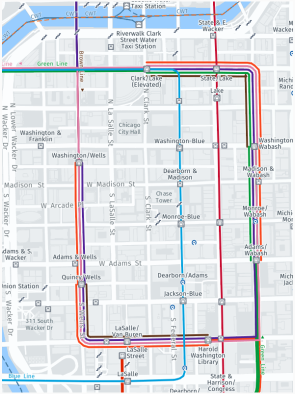


HERE



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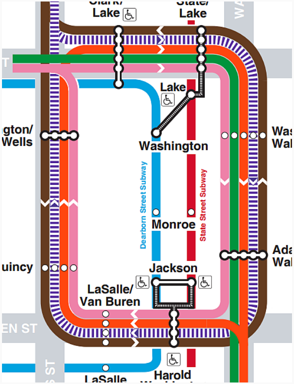

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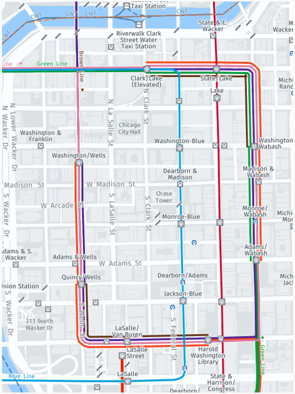
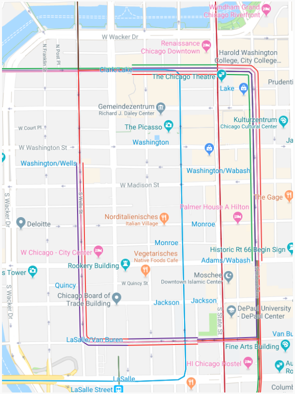


Google



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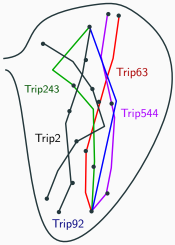

”Bag of trips”
(GTFS)

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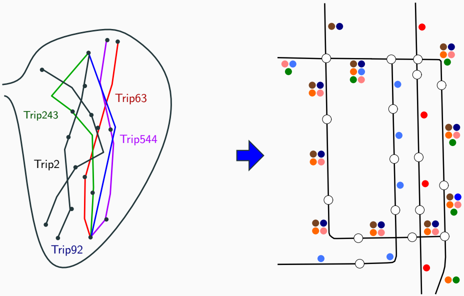

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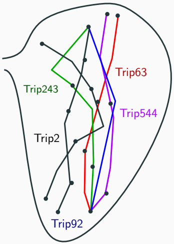

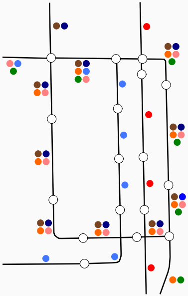

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Line graph

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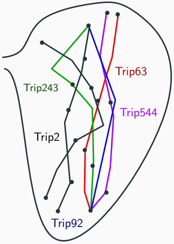

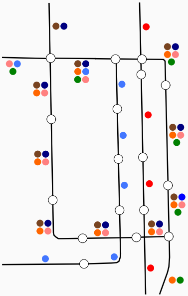

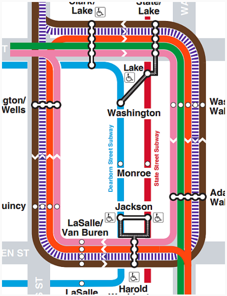

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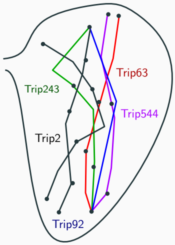
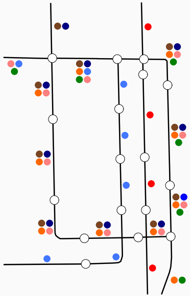
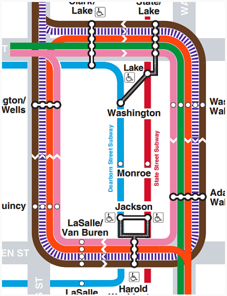






"Bag of trips" (GTFS)

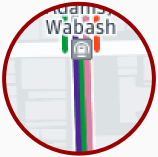
Line graph

Final map

Challenges

Official

HERE



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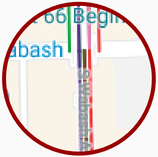
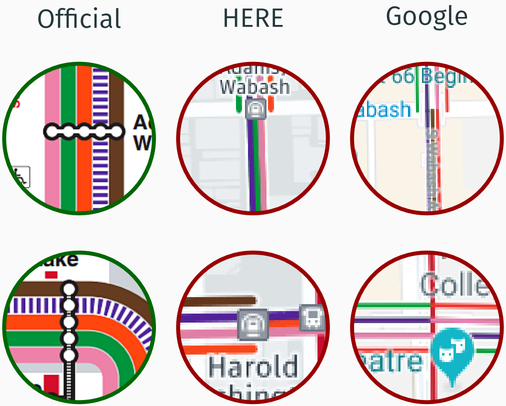


I. Avoid line overlaps

Challenges

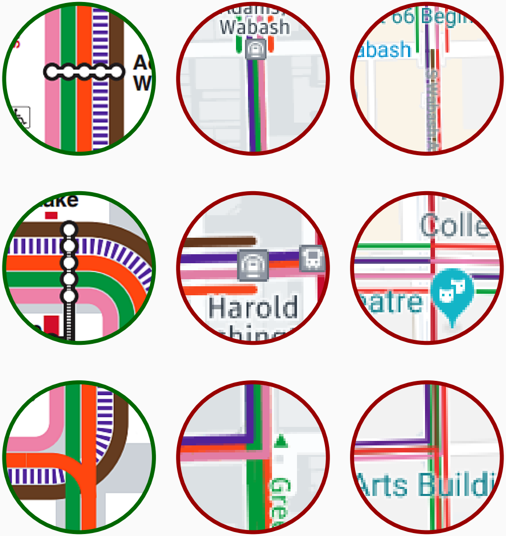


I. Avoid line overlaps

II. Match line orderings

Challenges

Official HERE Google



I. Avoid line overlaps

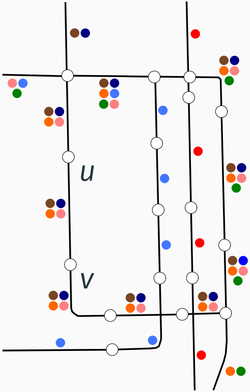
II. Match line orderings

III. Clearly indicate line continuations

Line graph construction

Line graph:

- Undirected labeled graph $G = (V, E, L)$



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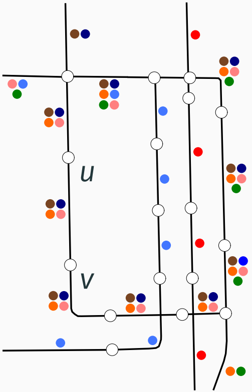

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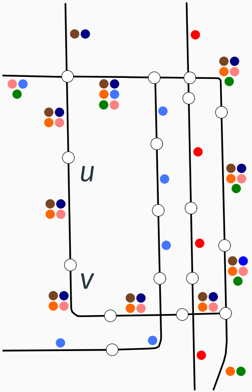

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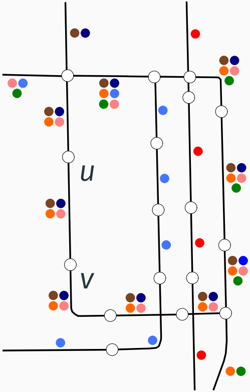
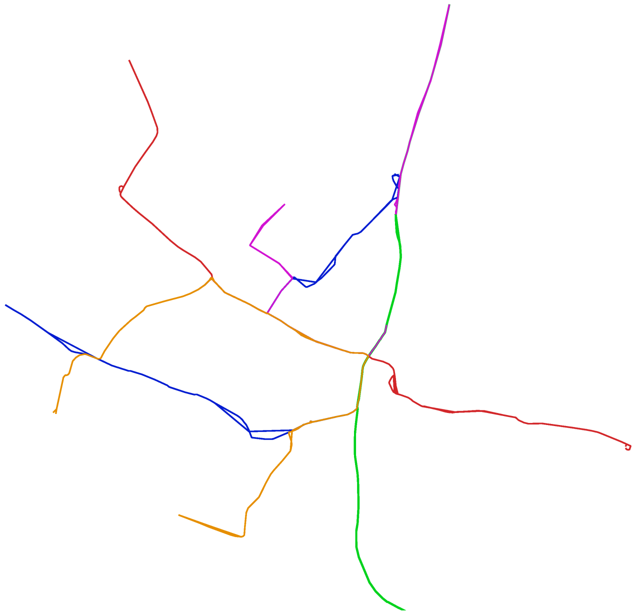


Example: $\mathcal{L} = \{\ \}, L((u, v)) = \{\ \}$

Line graph construction - Input data



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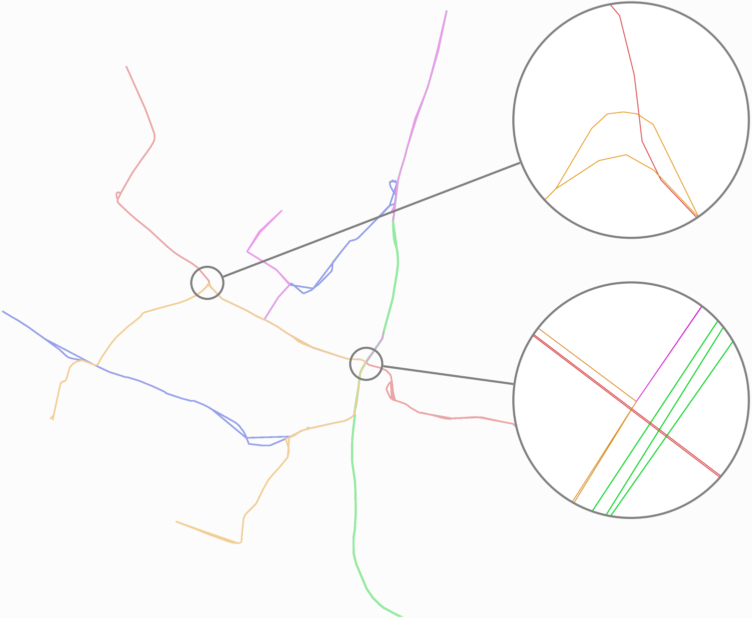

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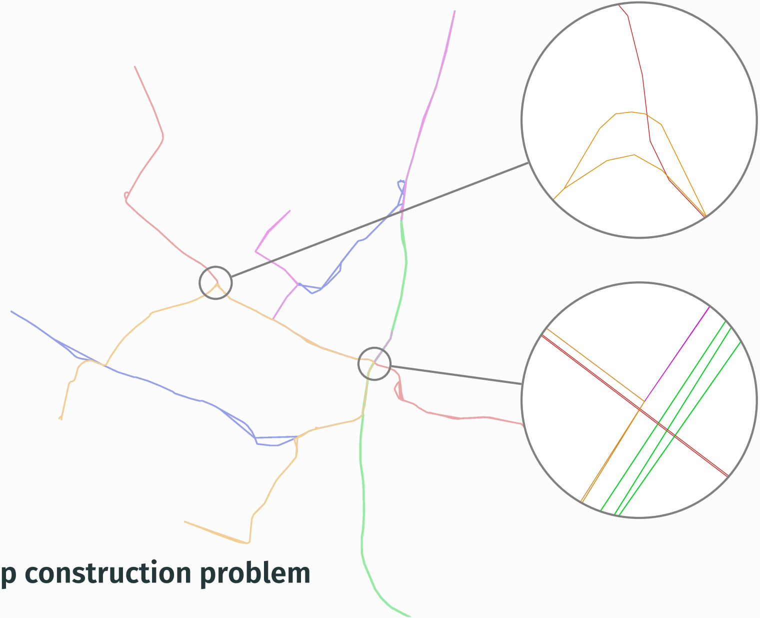
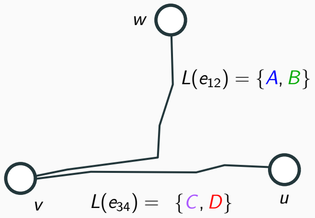


⇒ **Map construction problem**

Line graph construction - Non-station nodes



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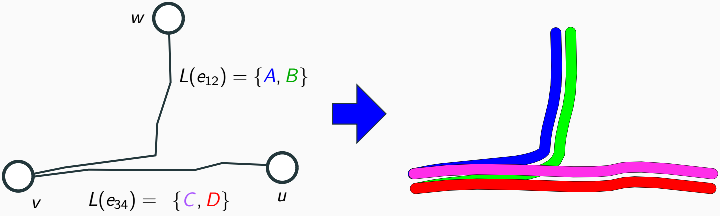

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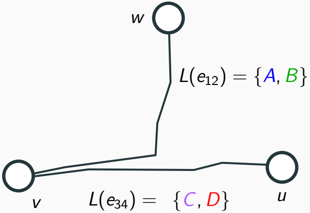

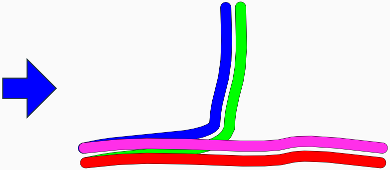

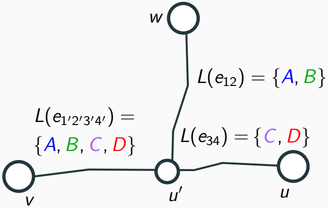

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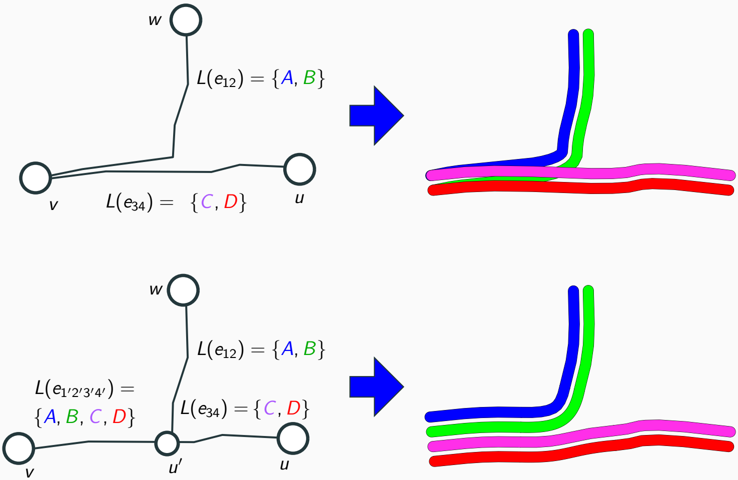
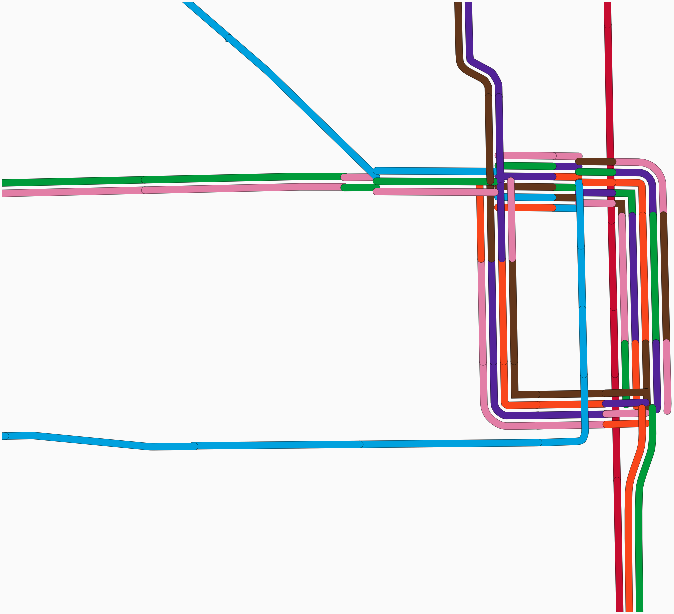


Results so far (1)



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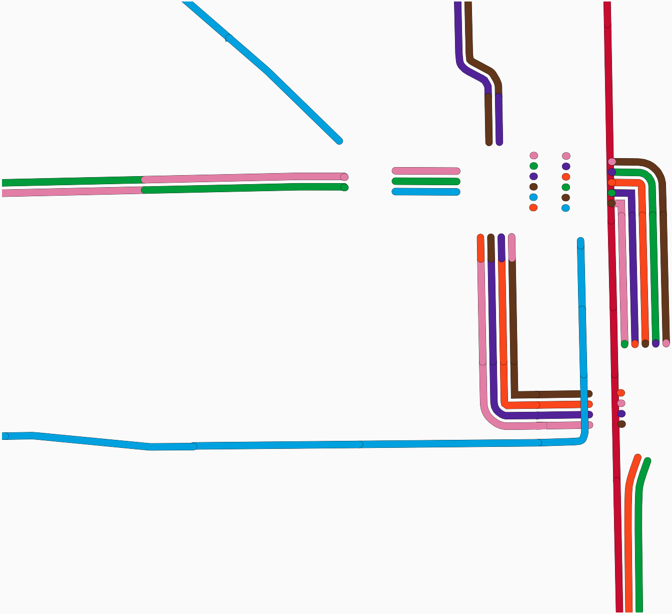

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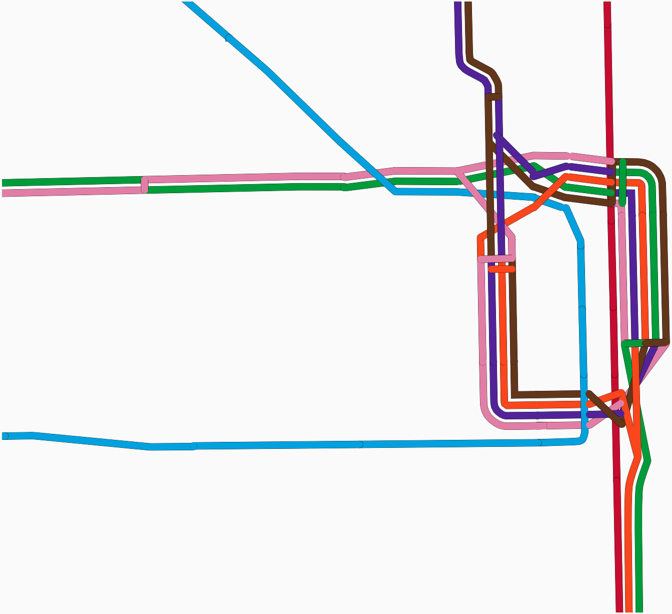
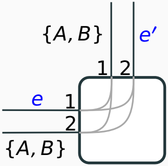


Line-ordering optimization - Baseline ILP



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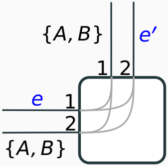

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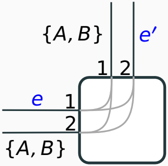

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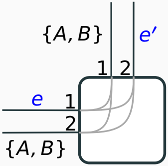

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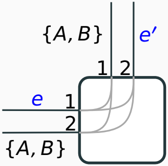

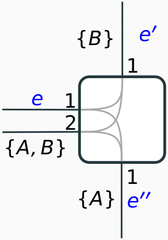

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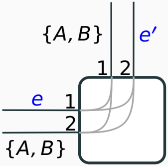

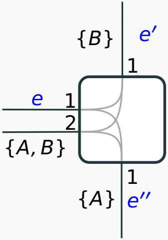

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$\Rightarrow \mathcal{O}(|E|M^2)$ variables, $\mathcal{O}(|E|M^6)$ constraints

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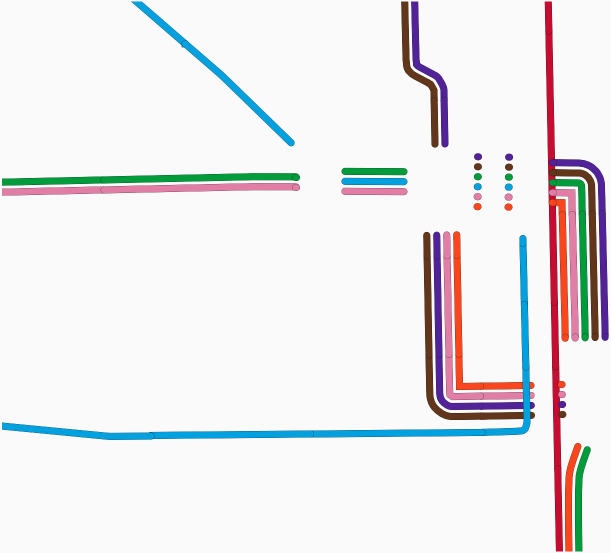

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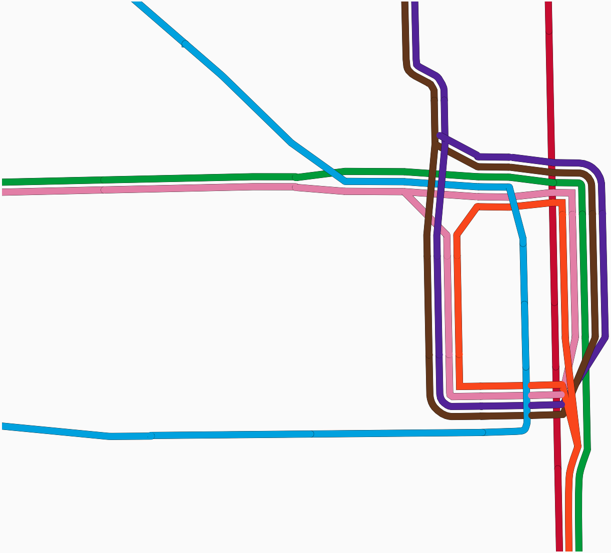

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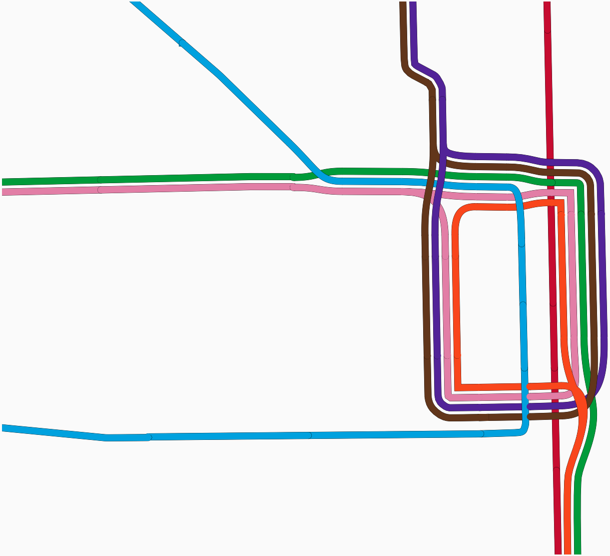
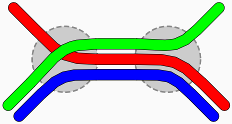
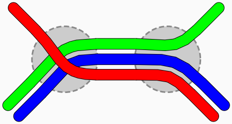


Line-ordering optimization - Line separations



vs.



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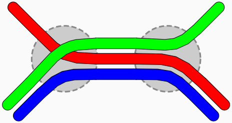

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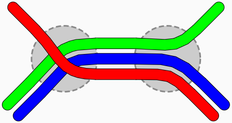

2 crossings

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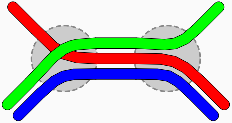

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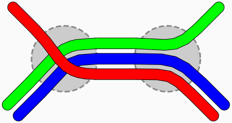

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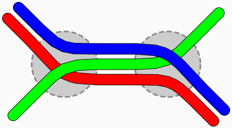

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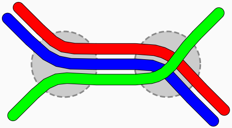

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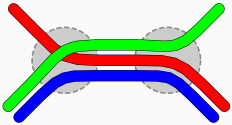

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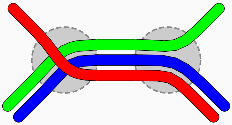

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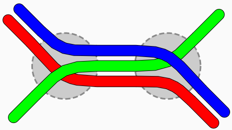

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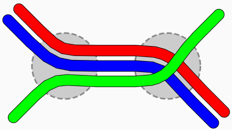

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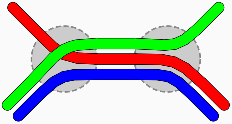

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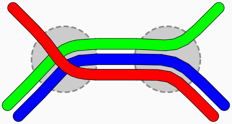
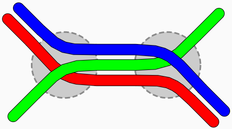
1 separation

vs.



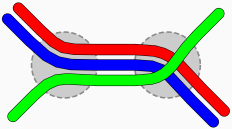
2 crossings



2 crossings

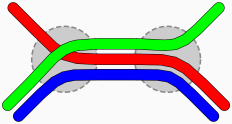
1 separation

vs.



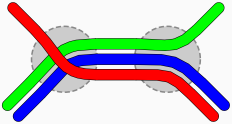
2 crossings

Line-ordering optimization - Line separations

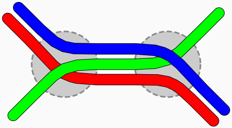


1 crossing
1 separation

vs.

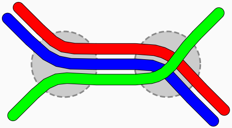


2 crossings
0 separations



2 crossings
1 separation

vs.



2 crossings
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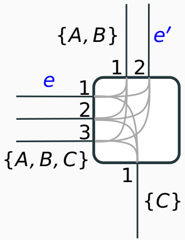
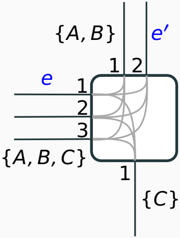
Line-ordering optimization - Line separations (ctd.)



- **Idea:** If two lines A, B continue from e to e', set a binary separation variable $x_{ee'A\|B} = 1$ if they are next to each other in e, but no in e'

Line-ordering optimization - Line separations (ctd.)



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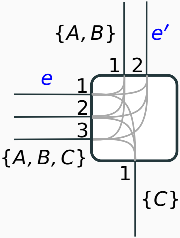

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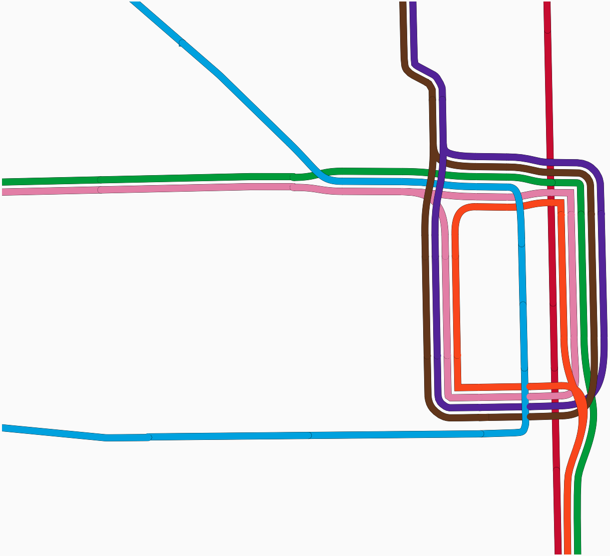
$\Rightarrow$ Still $\mathcal{O}(|E|M^2)$ variables, $\mathcal{O}(|E|M^2)$ constraints

Results so far (3)



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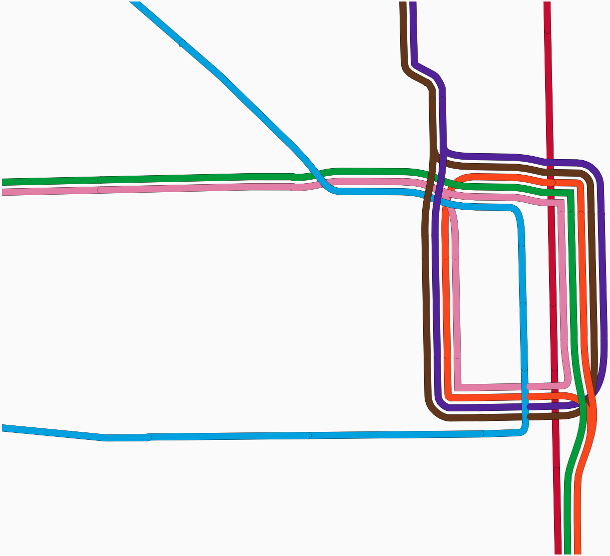
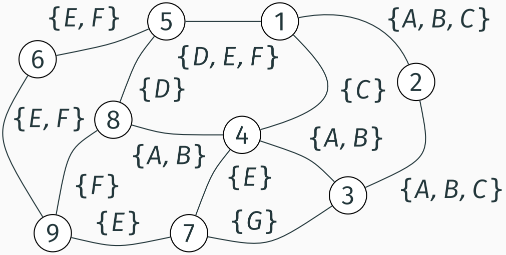


Line-ordering optimization - Core optimization graph



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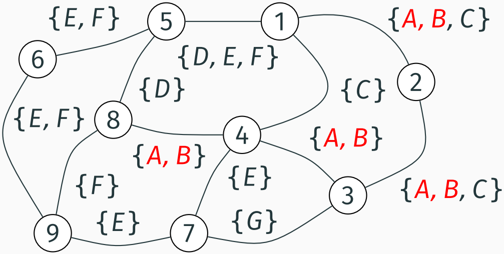

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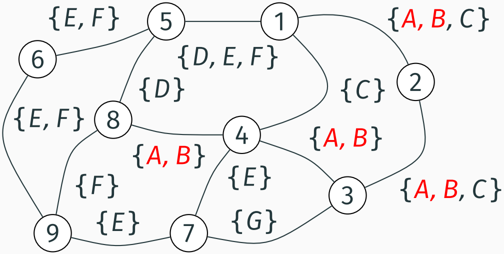
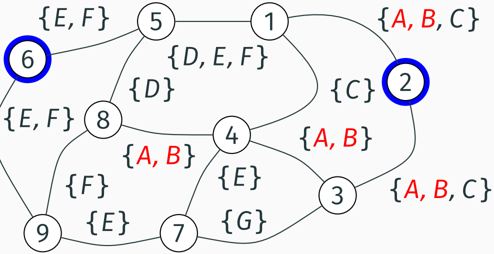


- Combine lines A, B that always occur together into a single new line ($A, B \rightarrow X$)

Line-ordering optimization - Core optimization graph



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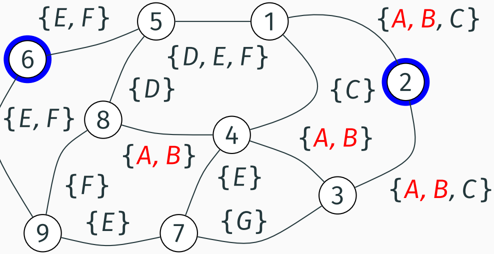
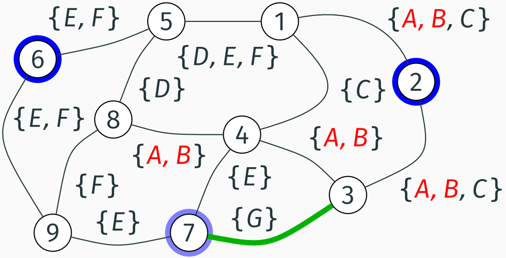


- Combine lines *A*, *B* that always occur together into a single new line ($A, B \longrightarrow X$)
- Delete nodes with degree 2 if adjacent edges have the same lines and merge these edges

Line-ordering optimization - Core optimization graph



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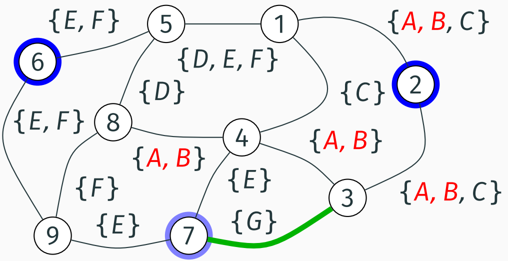

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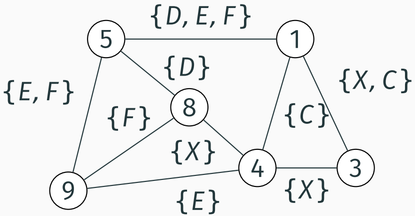

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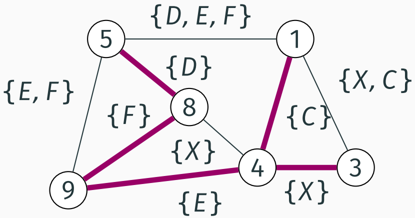

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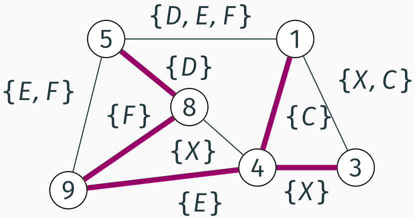

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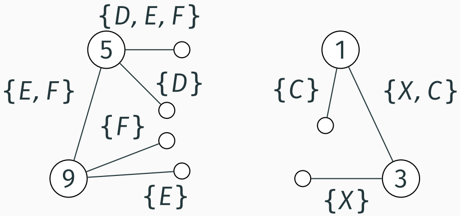

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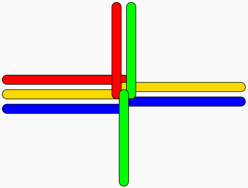
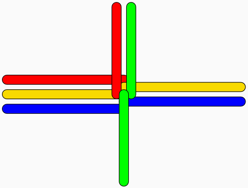
Rendering



1. Render parallel lines

Rendering



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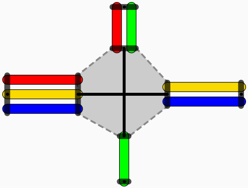
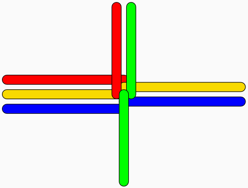


2. Free node space

Rendering



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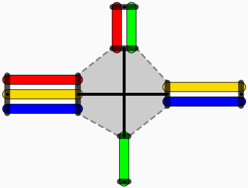

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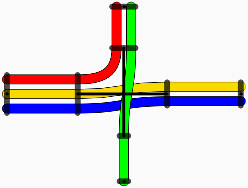
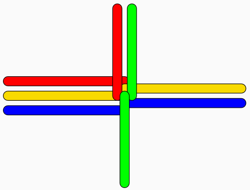


3. Render inner node connections

Rendering



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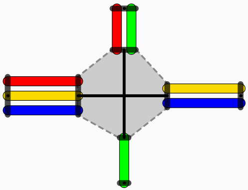

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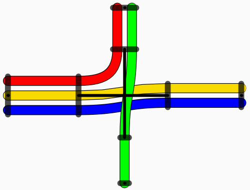

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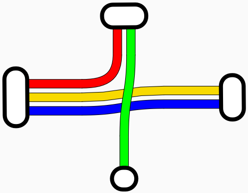
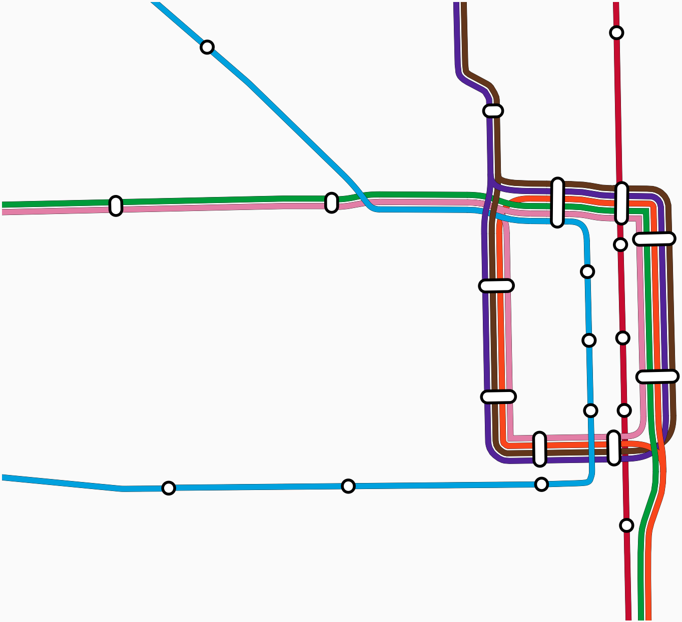
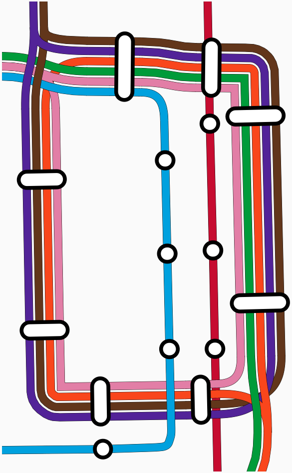


4. Render stations

Results so far (4)



Results so far (4)



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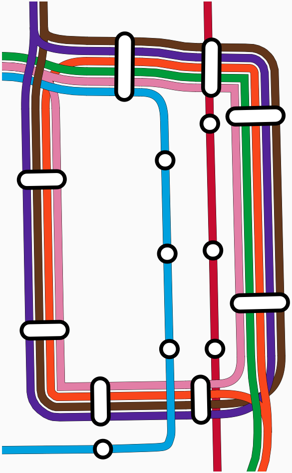

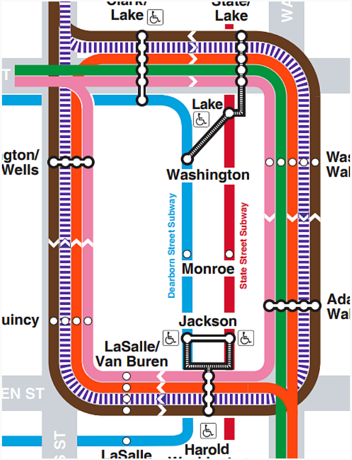

Evaluation - Line Ordering

T = number of (consecutive) line swaps necessary to transform offical map into our map

	Off. map		Our map		
	×	||	×	||	**T**
Freiburg	7	1	7	0	**2**
Dallas	3	1	3	0	**1**
Chicago	26	0	27	0	**1**
Stuttgart	65	5	64	2	**4**

Evaluation - ILP Solution times

ILP solution times for Chicago, on baseline graph

	rows × cols	GLPK	CBC	GU	×	||
Base	41k × 861	—	—	—	22	4-7
Impr.	1.4k × 982	**9s**	**1s**	**41ms**	22	4-7
+ Sep.	1.9k × 1.2k	**47m**	**19s**	**1.8s**	27	0

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ILP solution times for Chicago, on core graph

	rows × cols	GLPK	CBC	GU	×	||
Base	8.2k × 266	—	**47m**	**2m**	22	4-7
Impr.	394 × 285	**0.8s**	**0.1s**	**10ms**	22	4-7
+ Sep.	505 × 338	**23s**	**3.8s**	**0.3s**	27	0

Future work

- Additional rules for core graph reduction
  (work in progress)

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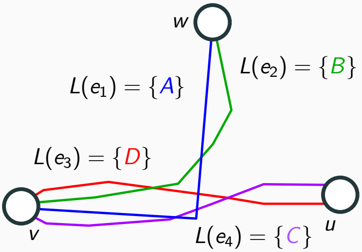
- Other sources for input line graph than schedule data (e.g. OSM, work in progress)
- Octilinearize line graph for (non-overlay) schematic metro maps (work in progress)

Thank you!

http://loom.informatik.uni-freiburg.de

Line graph construction - Shared segment collapsing



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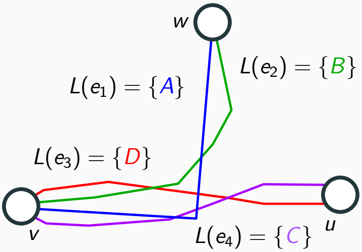

- Repeatedly collapse (segments of) two edges e and f within a distance $\hat{d}$

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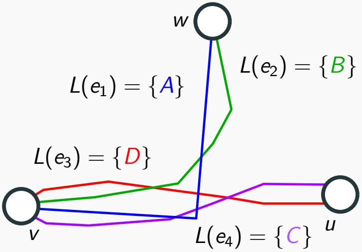

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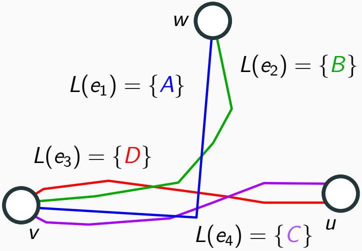

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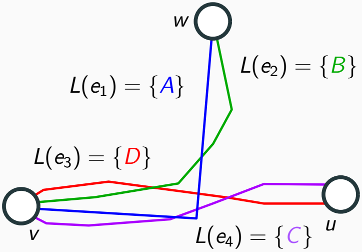

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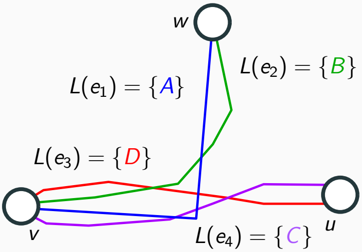

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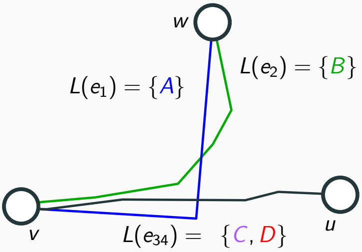

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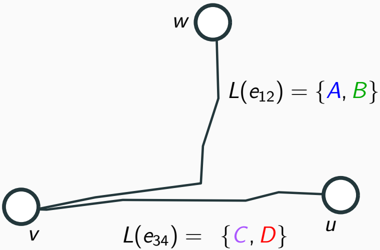

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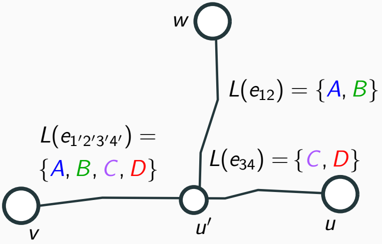

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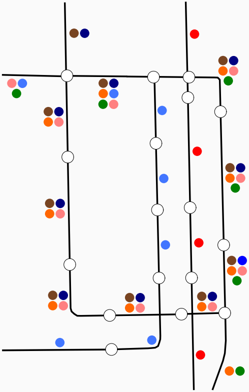
Line-ordering optimization - Exhaustive approach



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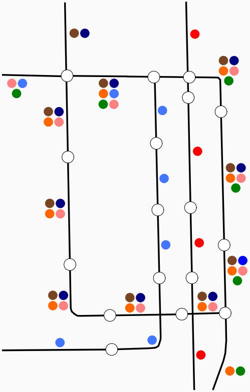

- 23 edges

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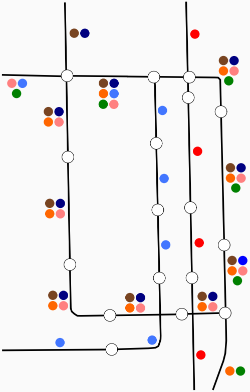

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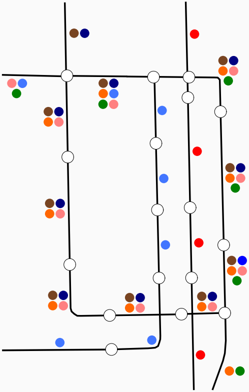

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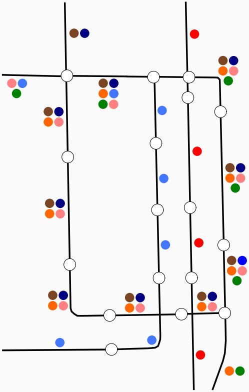

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$\Rightarrow$ **Naive exhaustive search infeasible**

Baseline ILP - Details

Each line must only be assigned one position:

$$\forall l \in L(e) : \sum_{p=1}^{|L(e)|} x_{elp} = 1.$$

Each position must only be assigned once:

$$\forall p \in \{1, \ldots, |L(e)|\} : \sum_{l \in L(e)} x_{elp} = 1.$$

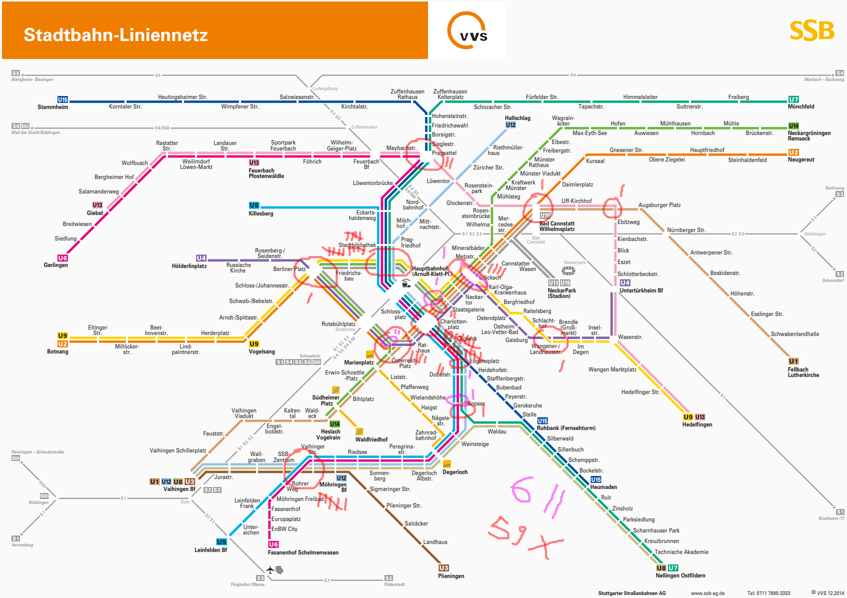
Constraints for ensuring that $x_{ee'AB} = 1$ if a crossing occurs:

$$x_{eA1} + x_{eB2} + x_{e'A2} + x_{e'B1} - x_{ee'AB} \leq 3$$

$$x_{eA2} + x_{eB1} + x_{e'A1} + x_{e'B2} - x_{ee'AB} \leq 3$$

...etc

Stuttgart map - annotated



Dataset dimensions

	t_{extr}	$\lvert\mathcal{S}\rvert$	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert\mathcal{L}\rvert$	M
Freiburg	0.7s	74	80	81	5	4
Dallas	3s	108	117	118	7	4
Chicago	13.5s	143	153	154	8	6
Stuttgart	7.7s	192	219	229	15	8
Turin	4.9s	339	398	435	14	5
New York	3.7s	456	517	548	26	9

Core graph dimensions

	$\|V\|$	$\|E\|$	$\|\mathcal{L}\|$	M
Freiburg	20	21	5	4
Dallas	24	24	7	4
Chicago	23	24	8	6
Stuttgart	50	58	15	8
Turin	91	124	14	5
New York	110	138	23	9