

Ques 1. Eigenvalues

[ANIRODH GARG
2281005]

$$\text{As } \begin{bmatrix} 0 & 5 & 0 & 4 \\ 5 & 0 & 4 & 0 \\ 0 & 3 & 0 & 2 \\ 3 & 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

For matrices A (with eigenvalues $\omega_i: 1 \leq i \leq j$) and B (with eigenvalues $\omega_k: 1 \leq k \leq l$) we have the property that the eigenvalues of $C = A \otimes B$ are $\omega_i \omega_k: 1 \leq i \leq j, 1 \leq k \leq l$.

Thus the eigenvalues are: $\lambda = \frac{7 \pm \sqrt{57}}{2}$

$$\lambda = \frac{-7 \pm \sqrt{57}}{2}$$

$$\lambda = \frac{-7 - \sqrt{57}}{2}$$

$$\lambda = \frac{7 - \sqrt{57}}{2}$$

b) Like this

$$\text{But } B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$$

As the constituent matrices are same as the last part, the eigenvalues remain the same!

$$\lambda = \frac{7 \pm \sqrt{57}}{2}$$

\therefore same eigenvalues as before

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c) Similarly,
$$\begin{bmatrix} 25 & 20 & 20 & 16 \\ 15 & 10 & 12 & 8 \\ 15 & 12 & 10 & 8 \\ 9 & 6 & 6 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix} \otimes \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$$

$$\lambda = \frac{53 + 7\sqrt{57}}{2}, \frac{53 - 7\sqrt{57}}{2}, -2$$

-2 has a multiplicity of 2.

