

Ans 3:

$$\begin{aligned} \|Au\|^2 &= \langle Au, Au \rangle = u^T A^T A u \\ &= \langle u, A^T A u \rangle \\ &= \langle u, u \rangle \\ &= \|u\|^2 \end{aligned}$$

$$\begin{aligned} \therefore \|Au\|^2 - \|u\|^2 &= \langle u, A^T A u \rangle - \langle u, u \rangle \\ &= \langle u, (A^T A - I)u \rangle = 0 \end{aligned}$$

for all $u \in V$
 $\Rightarrow A^T A = I$, similarly can be shown that $AA^T = I$

$$\begin{aligned} \|Au\|^2 &= \langle Au, Au \rangle = \langle u, A^T A u \rangle \\ &= \langle u, u \rangle = \|u\|^2 \\ &\quad [A^T A = I] \end{aligned}$$

$$\|Au\| = \|u\| //$$