BWSI-UAV Homework 1

Self-Assessment

Linear algebra

1

Let $a, b, c \in \mathbb{R}$ and S be the following symmetric matrix:

$$S = \begin{pmatrix} a & b \\ b & c \end{pmatrix}.$$

Note that since S is symmetric, all of its eigenvalues are real, and it has a complete basis of eigenvectors. In this question, you will compute a symbolic symmetric eigendecomposition of S.

1. What is S's characteristic polynomial $p_S(x)$?
2. What are S's eigenvalues?
3. What are the eigenvectors corresponding to each eigenvalue you computed in part (b)? (For the purposes of this question, you need not normalize these eigenvectors.)

2

A 3D object O has feature points at the following locations, expressed in the object's body-centric coordinate frame:

$$\mathbf{oP}_1 = \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}, \quad \mathbf{oP}_2 = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}, \quad \mathbf{oP}_3 = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}, \quad \mathbf{oP}_4 = \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix}.$$

Using a stereo camera, a robot observes this object, and measures the locations of these feature points as:

$$\mathbf{sP}_1 = \begin{pmatrix} -1.3840 \\ 4.5620 \\ -0.1280 \end{pmatrix}, \quad \mathbf{sP}_2 = \begin{pmatrix} -0.9608 \\ 1.3110 \\ -1.6280 \end{pmatrix}, \quad \mathbf{sP}_3 = \begin{pmatrix} 1.3250 \\ -2.3890 \\ 1.7020 \end{pmatrix}, \quad \mathbf{sP}_4 = \begin{pmatrix} -1.3140 \\ 0.2501 \\ -0.7620 \end{pmatrix}.$$

Differential equations

Given a nonlinear ordinary differential equation:

$$\dot{x} = f(x),$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ is a smooth map, it is often useful to study the behavior of the system (3) in a small neighborhood of a fixed point $x^* \in \mathbb{R}^n$ using its linearization about x^*:

$$\dot{h} = Ah,$$

where $A \triangleq \frac{\partial f}{\partial x}(x^*) \in \mathbb{R}^{n \times n}$ is the Jacobian of f evaluated at the fixed point x^*.

Suppose for a given problem that $n = 2$, and A is given by:

$$A = \begin{pmatrix} 3 & -1 \\ 3 & -2 \end{pmatrix}.$$

1. What are the eigenvalues of A? What are the corresponding eigenvectors?
2. What kind of stationary point is the origin? (Source, sink, saddle point?)
3. Using your answers to parts (a) and (b), sketch the phase portrait of the linearized system (4) in a neighborhood of the origin.

Problem 1 (AR Tags)

Please complete the two challenges at the end of the introduction to AR tag repository. To get started, clone the repository using the following command:

```
git clone https://github.com/BWSI-UAV/intro_to_ar_tags.git
```

Problem 2 (Camera Calibration)

Using the code description and photos in the class repository. Please find the camera matrix and calibration error. Make sure to save the matrix to a npz file(so you can load it later for mosaicing) and also print it to terminal.

Problem 3 (Photo Mosaic)

Figure 1: Example Final Mosaic

Using the calibration file from problem 2 and the images in the problem folder. Please generate panoramas of from the images in each folder and answer the following questions.

1. From which set of photos were the best features extracted?
2. How does the overlap percentage affect the quality of the final mosaic?
3. How does the final mosaic of the brick data set compare to the others? What do you attribute the differences to?

Problem 4 (Iterative Closest Point)

NOTHING FOR YOU TO SOLVE

Read problem 2 of the HW2 document found in the folder. After taking some time to digest it, take a look at my python implementation of the solution. We will discuss the problem setup and optimization later on in the course.