$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \quad (20)$$

$$\hat{u}_i = y_i - \hat{y}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i \quad (21)$$

sum of squared residuals $\triangleq SSR = \sum_{i=1}^{n} \hat{u}_i =$

$$= \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2 \quad (22)$$

$$\frac{\partial SSR}{\partial \hat{\beta}_0} = \frac{\partial}{\partial \hat{\beta}_0} \left[\sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2 \right] \xrightarrow[\text{out}]{\text{move } \Sigma}$$

$$= \sum_{i=1}^{n} \left[\frac{\partial}{\partial \hat{\beta}_0} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2 \right]$$

$$= \sum_{i=1}^{n} \left[2 \cdot (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) \cdot (-1) \right]$$

$$= -2 \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0 \quad (14)$$

$$\frac{\partial SSR}{\partial \hat{\beta}_1} = \sum_{i=1}^{n} \left[\frac{\partial}{\partial \hat{\beta}_1} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2 \right]$$

$$= \sum_{i=1}^{n} \left[2 \cdot (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) \cdot (-x_i) \right]$$

$$= -2 \sum_{i=1}^{n} x_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0 \quad (15)$$

$$\text{(14)} \Rightarrow 0 = \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \hat{\beta}_0 - \sum_{i=1}^{n} \hat{\beta}_1 x_i = \sum_{i=1}^{n} y_i - n\hat{\beta}_0 - \hat{\beta}_1 \sum_{i=1}^{n} x_i$$

$$\Rightarrow \hat{\beta}_0 = \frac{\sum_{i=1}^{n} y_i - \hat{\beta}_1 \sum_{i=1}^{n} x_i}{n} = \bar{y} - \hat{\beta}_1 \bar{x} \qquad \text{(17)}$$

$$\text{(15)} \Rightarrow 0 = \sum_{i=1}^{n} \left(x_i y_i - \hat{\beta}_0 x_i - \hat{\beta}_1 x_i^2\right) = \sum_{i=1}^{n} x_i y_i - \hat{\beta}_0 \sum_{i=1}^{n} x_i - \hat{\beta}_1 \sum_{i=1}^{n} x_i^2$$

$$\Rightarrow \hat{\beta}_1 \sum_{i=1}^{n} x_i^2 \overset{(17)}{\underset{\hat{\beta}_0}{=}} \sum_{i=1}^{n} x_i y_i - \left(\frac{\sum_{i=1}^{n} y_i}{n} - \frac{\hat{\beta}_1 \sum_{i=1}^{n} x_i}{n} \right) \sum_{i=1}^{n} x_i$$

$$= \sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} y_i \cdot \sum_{i=1}^{n} x_i}{n} + \hat{\beta}_1 \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}$$

$$\Rightarrow \hat{\beta}_1 \left[\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n} \right] = \sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i}{n}$$

$$\Rightarrow \hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}$$

$$= \frac{n \cdot \sum_{i=1}^{n} x_i \sum y_i - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} = r \frac{\sigma_y}{\sigma_x}$$

$$= \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2 \cdot (y_i - \bar{y})^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \qquad \text{(19)} = \frac{Cov(x, y)}{Var(x)}$$