Supplementary Material for "A Probabilistic Approach for Predicting Vessel Motion"

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A. Units and range of measurements

TABLE I: Measurements

Measurement	Range	Unit
ϕ	$[-90°, 90°]$	decimal/DMS
λ	$[-180°, 180°]$	decimal/DMS
ψ	$[0°, 359.9°]$	degree
U	—	knot
χ	$[0°, 359.9°]$	degree

B. Parameters in Equations (8)—(11)

TABLE II: Parameters

Parameter	Value	Parameter	Value
a	6378137	d_1	$10^{-1}/\sqrt{60}$
e	0.08181919	d_2	$10^{-3}/\sqrt{60}$
h	0	K_p	1
T	107.3	K_d	20
K	0.185	Δ	500

C. Prediction metrics for each measurement

TABLE III: Comparative Experiment Results

Measurement	Proposed method			EKF		
	GE	GER	EAPI	GE	GER	EAPI
2	322.65	24.97	130.26	370.54	28.68	162.16
3	101.26	4.53	40.79	144.70	6.47	95.68
4	665.07	18.60	41.65	2278.62	63.73	183.51
Average	**362.99**	**16.03**	**70.90**	931.28	32.96	147.11

D. Approximation algorithm for Equations (4)—(7)

Algorithm 1 Gaussian Assumed Density Approximation for Sequential Bayesian Inference

Input: The prior distribution $p(\mathbf{x}_N|\mathbf{y}_{1:(N-1)}) = \mathcal{N}(\mathbf{m}_N^-, \mathbf{P}_N^-)$.

Procedure:

1) Calculate the mean and covariance of the posterior distribution $p(\mathbf{z}_N|\mathbf{y}_{1:N}) = \mathcal{N}(\mathbf{m}_N^*, \mathbf{P}_N^*)$:

$$\begin{aligned}
\mathbf{Q}_N &= \mathbf{H}_\mathbf{x}(\mathbf{m}_N^-)\mathbf{P}_N^-\mathbf{H}_\mathbf{x}^T(\mathbf{m}_N^-) + \mathbf{R}_N,\\
\mathbf{K}_N &= \mathbf{P}_N^-\mathbf{H}_\mathbf{x}^T(\mathbf{m}_N^-)\mathbf{Q}_N^{-1},\\
\mathbf{m}_N &= \mathbf{m}_N^- + \mathbf{K}_N(\mathbf{y}_N - \mathbf{h}(\mathbf{m}_N^-)),\\
\mathbf{P}_N &= \mathbf{P}_N^- - \mathbf{K}_N\mathbf{Q}_N\mathbf{K}_N^T,\\
\mathbf{m}_N^* &= \begin{bmatrix}\mathbf{m}_N\\ \mathbf{m}_s\end{bmatrix}, \mathbf{P}_N^* = \begin{bmatrix}\mathbf{P}_N & \mathbf{0}\\ \mathbf{0} & \mathbf{P}_s\end{bmatrix}.
\end{aligned} \tag{1}$$

2) Integrate the following equations up to time t_f:

$$\begin{aligned}
\frac{d\mathbf{m}}{dt} &= \mathbf{g}(\mathbf{m}),\\
\frac{d\mathbf{P}}{dt} &= \mathbf{P}\mathbf{G}_\mathbf{z}^T(\mathbf{m}) + \mathbf{G}_\mathbf{z}(\mathbf{m})\mathbf{P} + \mathbf{L}(\mathbf{m})\mathbf{L}^T(\mathbf{m}).
\end{aligned} \tag{2}$$

where $\mathbf{G}_\mathbf{z}(\mathbf{z})$ and $\mathbf{H}_\mathbf{x}(\mathbf{x})$ represent the Jacobian matrices of $\mathbf{g}(\mathbf{z})$ and $\mathbf{h}(\mathbf{x})$ with respect to $\mathbf{z}$ and $\mathbf{x}$, respectively. The initial conditions for (2) are $\mathbf{m}_N^*$ and $\mathbf{P}_N^*$, respectively. The corresponding solutions are given by:

$$\mathbf{m}_f^* = \begin{bmatrix}\mathbf{m}_f^-\\ \mathbf{m}_s\end{bmatrix}, \mathbf{P}_f^* = \begin{bmatrix}\mathbf{P}_f^- & *\\ * & \mathbf{P}_s\end{bmatrix}. \tag{3}$$

Output: The prior distribution $p(\mathbf{x}_f|\mathbf{y}_{1:N}) = \mathcal{N}(\mathbf{m}_f^-, \mathbf{P}_f^-)$.