STAT1301 Advanced Analysis of Scientific Data
Semester 2, 2025
Assignment 2

The due date/time is given on Blackboard. Please submit your assignment on Blackboard.

1. In a game of dice, two fair dice are rolled and the payout X is the product of the two dice outcomes.

    (a) What is the probability mass function (pmf) of X? [3 marks]

    (b) What is the expected value of X? [3 marks]

    (c) What is the standard deviation of the payout? [4 marks]

2. A laboratory is studying the growth of algae in a controlled environment. The growth of algae is measured by the amount of biomass produced (in grams), which can be modelled by a random variable X with probability density function (pdf)

$$f_X(x) = c\left(x^2 - 60x + 800\right) \quad \text{for } 0 < x < 20,$$

where c is a constant.

    (a) Find the value of c. [2 marks]

    (b) Find the cumulative distribution function (cdf) of X. [3 marks]

    (c) Find the expected value of (X). [2 marks]

    (d) The laboratory equipment can only detect biomass that exceeds 2 grams (the minimum detectable amount). What is the probability that the biomass exceeds 10 grams, given that it is detectable? [3 marks]

3. Each day, a quality control officer inspects a random sample of 25 products from the production line of a factory. The probability of a product passing inspection (being defect-free) is 0.25. If the product passes, the factory saves \$3 in repair costs. If the product fails, the factory incurs an additional \$1 cost for re-inspection after repair. Let X be the number of products that passes inspection on a given day, and Y be the net savings for the factory on that day.

$$\sigma$$

    (a) State the distribution of X, including all its parameters. [2 marks]

    (b) What is the minimum sample size needed so that the probability of finding at least one defect-free product exceeds 99%? [3 marks]

    (c) Calculate the expected value and variance of the factory's net savings. [3 marks]

    (d) What is the probability that the factory will save at least \$27 on a given day? [2 marks]

4. A storeroom in a warehouse maintains strict temperature control to ensure that sensitive materials are stored at optimal conditions. The temperature of the storeroom follows a normal distribution with mean μ and standard deviation σ degrees Celsius (°C). The storeroom has a temperature threshold of 8°C to avoid damaging the materials.

You may use statistical tables to answer this question, then use R to verify your results.

(a) Suppose the storeroom temperature is adjusted so that $\mu = 7.5$°C and $\sigma = 0.3$°C. What is the probability that the temperature of the storeroom will be between 7.2°C and 8°C?

[3 marks]

(b) Assume that $\sigma = 0.3$°C. What should μ be set to so that the storeroom temperature exceeds 8°C only 1% of the time? [3 marks]

(c) What is the largest standard deviation σ that will keep the temperature within 1°C of the mean with 95% probability? [4 marks]