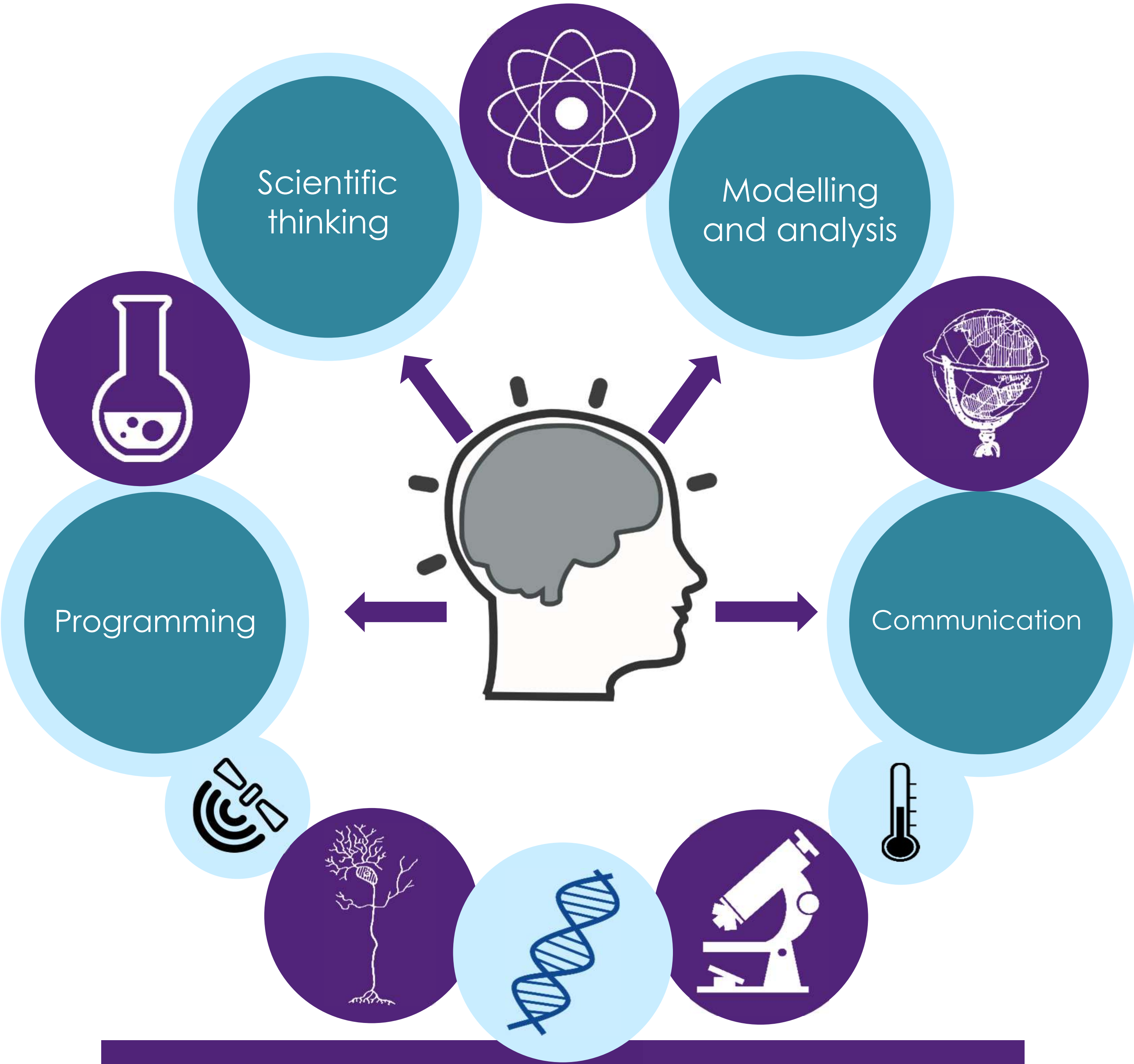
SCIE1000/1100
Theory and Practice in Science



Name: ______________________________

Contact details: ______________________________

Thirty-second edition, Semester 2, 2025

SCIE1000/1100

The core content falls into four key areas: scientific thinking, modelling, programming and communication. The content is taught in a highly ``interleaved'' manner rather than in separate blocks, but the figure below may help you to appreciate the course goals and content.

Thinking

- scientific units
- dimensional analysis
- mechanistic modelling
- phenomeno-logical modelling
- hypotheses
- history of scientific thinking
- inductive reasoning
- Popperian science
- quantitative reasoning

Modelling

- linear equations
- quadratics
- power functions
- sine functions
- exponentials
- logarithms
- average rates of change
- meaning of derivatives
- Newton's method
- areas under curves
- differential equations
- Euler's method
- systems of DEs
- Predator-prey models
- SIR models

Programming

- software design
- errors
- input and output
- variables
- calling functions
- conditionals
- loops
- arrays
- plotting
- writing functions

Communication

- units in communication
- communicating with graphs
- science in the media
- how to be precise, clear, and concise
- understand your purpose
- know your audience
- convey your key messages

Scientific Contexts

- blood alcohol
- Mars Climate Orbiter
- cancer
- thrombosis
- HIV screening
- atmospheric temperature
- Bicknell's thrush
- biodiversity
- sun protection
- seasons
- daytime hours
- migration of terns
- radioactive decay
- carbon dating
- pressure
- atmospheric CO_2
- pharmocokinetics
- heart disease
- apparent temperature
- metabolism of alcohol
- contraception
- exposure to nicotine
- glucose
- bioavailability of a drug
- bacteria growth
- oyster populations
- endangered turtles
- lynx and snowshoe hares
- disease models
- vaccination targets

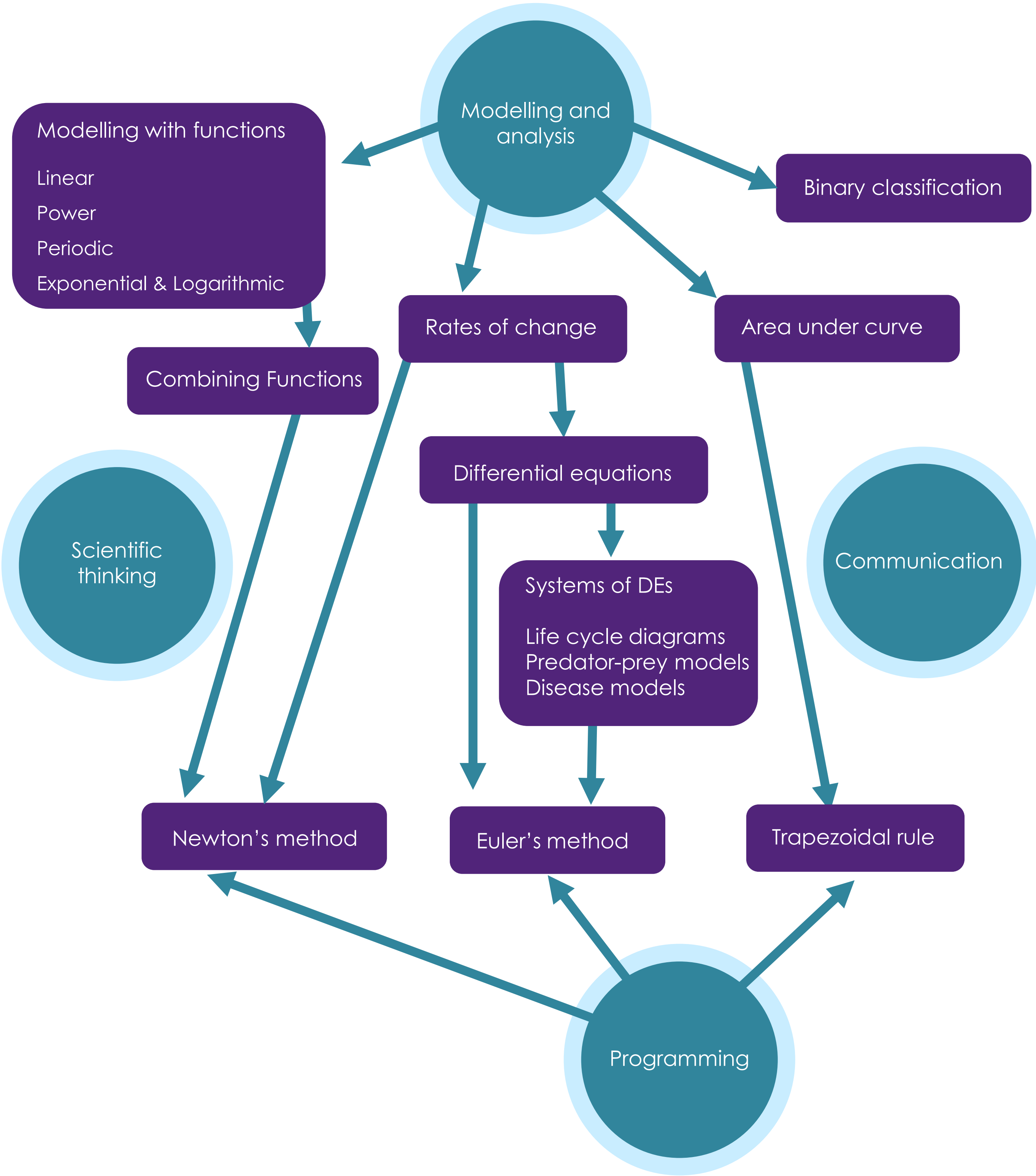

Modelling and analysis
Modelling with functions
Linear
Power
Periodic
Exponential & Logarithmic
Binary classification
Rates of change
Area under curve
Combining Functions
Differential equations
Scientific thinking
Communication
Systems of DEs
Life cycle diagrams
Predator-prey models
Disease models
Newton's method
Euler's method
Trapezoidal rule
Programming

Table of Contents

Chapter 1: Course introduction and overview

Lecture 1: A short discussion of nearly everything

Learning objectives

✓ Course staff and rationale

✓ Indigenous perspectives in science ✓ Introduce modelling, communication (units), philosophy and programming

✓ Develop critical evaluation skills

Scientific examples

✓ Blood flow

✓ Mars Climate Orbiter

Maths skills

✓ Developing a plausible mathematical model

1.1 Course staff

Welcome to the course! We have a dedicated interdisciplinary team to guide you through the learning materials:

Dr Mel Robertson-Dean
Mathematics
SCIE1000 Course Coordinator

A/Prof Tatsuya Amano
Environmental science

Dr Julian Steele
Physics

A/Prof Marcus Gallagher
Computer Science

A/Prof Steve Salisbury
Environmental science

Dr Peter Evans
Philosophy

Dr Guillermo Badia
Philosophy

Use of generative AI

Generative artificial intelligence (AI) has become a significant player in scientific endeavour in recent years. The ways in which it can be used are evolving at a phenomenal rate. However, it is important to understand that AI is one of many tools available to scientists in conducting their research. Generative AI will provide answers to almost any question posed, but critical thinking and evaluation are required in interpreting these answers.

You are welcome to make use of generative AI as a learning tool in this course. Please ensure that you are familiar with the usage policy for assessment items, as outlined in the course profile, and be sure to ask teaching staff if you have any questions on AI use.

Additional information on AI usage at UQ is provided here:

https://guides.library.uq.edu.au/tools-and-techniques/ai-student-hub

1.2 Science and Indigenous knowledges

Many of you will go on to become scientists in a range of disciplines, both in Australia and overseas. Contemporary scientists need to have a well-rounded skillset that not only includes an understanding of scientific methodologies and critical thinking, but also communication, leadership, citizenship and a sense of social and civic responsibility for a just and equitable society.

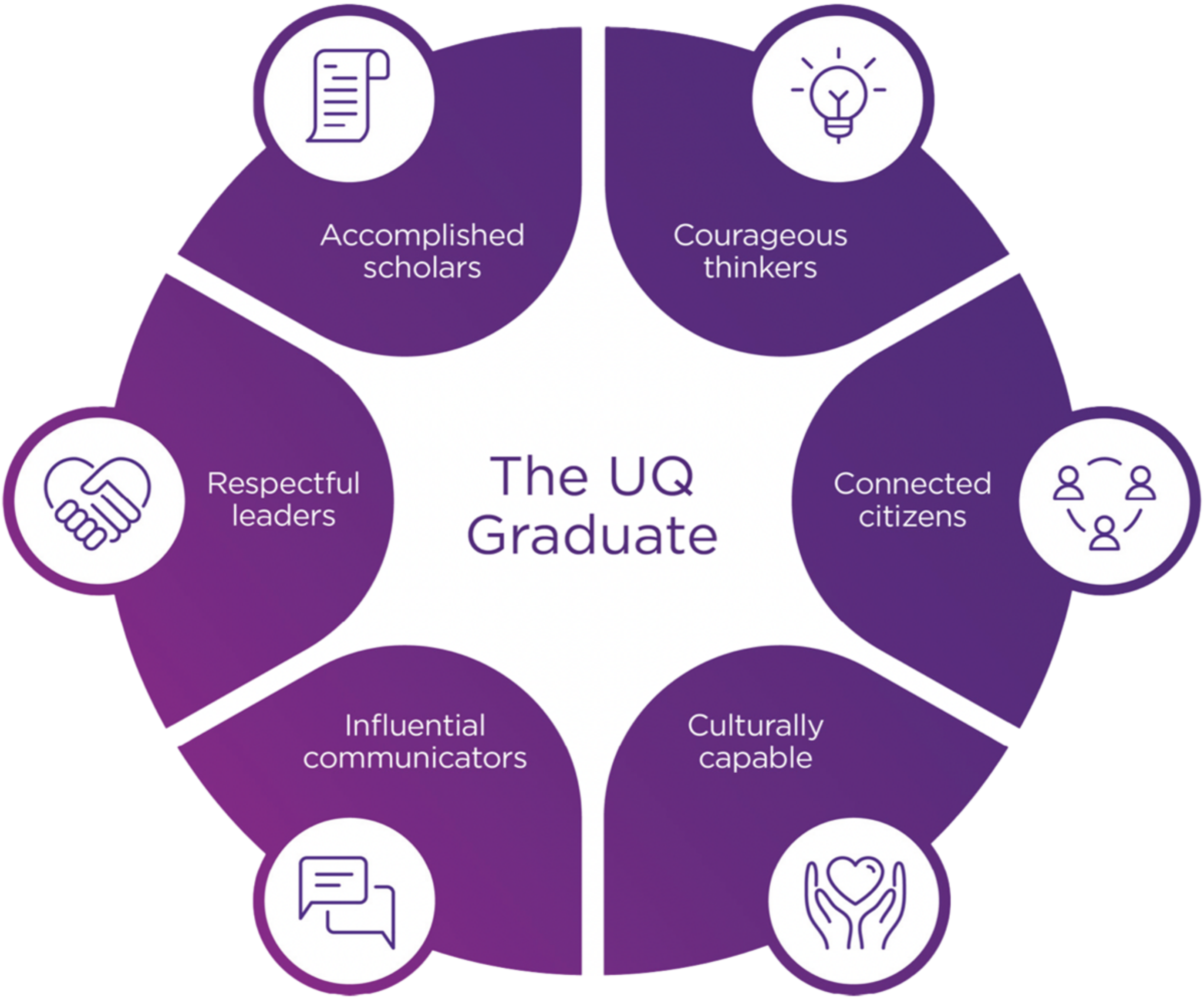


Figure 1.1: UQ Graduate Attributes. These high-level sets of skills, abilities, and attitudes encapsulate the essential and distinctive qualities of a UQ graduate. They also reflect the UQ Values and Mission.

One of UQ's Graduate Attributes addresses **cultural capability**:

> *"Graduates will have an understanding of, and respect for, Australian Aboriginal and Torres Strait Islander and global Indigenous peoples' values, cultures, and knowledge. They will have an appreciation of cultural and social diversity and work with a sense of social and civic responsibility towards a more just and equitable society."*

One aspect of advancing cultural capability in science is acknowledging that **there are multiple systems of knowledge that operate globally** (e.g., Indigenous knowledges and perspectives). Over the course of your degree, you may be exposed to 'other ways of knowing' as Indigenous knowledges are engaged with during teaching as part of UQ's Indigenising Curriculum strategy. This is part of a systematic effort by Indigenous and non-Indigenous experts to ensure equitable treatment and respect for Indigenous knowledges as valid ways of coming into understandings of the world (e.g., Universities Australia's Indigenous Strategy 2022–2025). Over millennia, Indigenous peoples globally have developed rich knowledge traditions grounded in observation, experience, and intergenerational knowledge transmission. These traditions contain detailed understandings of ecosystems, celestial bodies, human health and ways to live well, and more — many of which align with, complement, and often predate what is now categorised as 'modern science.'

While James Cook's first voyage to 'Terra Australis' was officially commissioned to observe the transit of Venus, he and the scientists aboard the Endeavour were also tasked with documenting and classifying human populations across the Pacific within a racial hierarchy to reinforce European superiority and justify imperial expansion. Since this expansion, science has been deeply entangled in the development of government policies that have profoundly impacted First Nations peoples. Discredited fields such as phrenology and eugenics were used to provide so-called scientific justifications for racial segregation policies in Australia, including policies which formed the basis for apartheid in South Africa. The legacies of these policies remain in living memory and continue to shape contemporary perceptions of science and its role in society.

The aim of this part of the lecture is to introduce you to the evolving role of what it means to be a contemporary scientist, particularly around the component of cultural capability with an emphasis on the importance of Indigenous cultures on these lands. This session will set the stage for you to engage with Indigenous perspectives in science throughout your programs, including in practical classes throughout this course.

What does it mean to be a contemporary scientist?

Question *1.2.1*

How does engaging with Indigenous knowledges in science make science more robust, ethical, and relevant?

Robust: Takes into account perspectives of others
ethical: not ignoring other people based soley on race
Relevant: ?

Case Study 1: **Palaeontology and indigenous knowledge**

Question *1.2.2*

Why have most aspects of Western palaeontology ignored Indigenous knowledges?

It uses a different knowledge system that doesn't put great emphasis on written preservation or mental attribution.

Photo 1.1: Indigenous knowledge of dinosaur tracks in the Kimberley region of NW Western Australia has completely transformed our understanding of Australia's dinosaur fauna from 130 million years ago. Here, Goolarabooloo Law Bosses Phil Roe (left) and Richard Hunter (right) show UQ palaeontologist A/Prof Steve Salisbury (centre) dinosaur tracks near Walmadany (James Price Point). As a result of this research partnership, Walmadany is now known to preserve the most diverse dinosaur track fauna in the world, and has been National Heritage listed. (Source: Damian Kelly.)

Question *1.2.3*

Stop and think: Write down some of your own reflections on ethical research collaborations with Indigenous communities.

Connecting theory with practice

Question *1.2.4*

Why do you think relationality matters in science?

1.3 Scientific endeavour

Science aims to **understand**, **explain**, **predict** and **influence** phenomena. Understanding science and thinking in a 'scientific manner', requires:

- *discipline knowledge and content* – the language, information, knowledge and skills specific to a discipline;
- *observation and data collection* – the processes and techniques used to collect useful data about particular phenomena;
- *scientific thinking and logic* – the conceptual process of performing systematic investigations, hypothesising, thinking critically and defensibly, and making valid deductions and inferences;
- *modelling and analysis* – the process of developing conceptual representations of phenomena, then using approximation, mathematics, statistics and computation in order to allow predictions to be made.
- *communication and collaboration* – the process of working with others, sharing information and resources;
- *curiosity, creativity and persistence* – the relatively intangible characteristics that include the ability to ask and answer 'interesting' questions, and solve difficult problems in novel ways;

In this course, we'll be developing your skills in scientific thinking and logic, in modelling and analysis, and in communication and collaboration. We will be building on your prior mathematical and scientific skills to examine studies that are real and important. Almost every example and case study covered in this course is based on a research paper or is a reasonably accurate model of a real situation. What could be more topical than rising CO_2 levels in the atmosphere? Or an understanding of pandemic modelling?

As compelling as the contexts are, memorising the specific details of any particular context is not your goal as a student. Instead, your focus will be on applying the underlying concepts to novel situations, understanding and practising how to think critically, developing and applying models, interpreting your results, and communicating with clarity and precision.

Gender and sex

In this course, and often in science more generally, one thing we do is to aim to keep models as simple as possible. When modelling medical phenomena, it can be useful to have a variable that considers the influence of a person's sex. First, it is important to recognise the distinction between a person's sex and a person's gender.

The World Health Organisation [90] says:

> *"Gender interacts with but is different from sex, which refers to the different biological and physiological characteristics of females, males and intersex persons, such as chromosomes, hormones and reproductive organs. Gender and sex are related to but different from gender identity. Gender identity refers to a person's deeply felt, internal and individual experience of gender, which may or may not correspond to the person's physiology or designated sex at birth."*

Often when we consider the influence of sex in scientific modelling, the relevant factors are related to *body fat percentage* and *hormones*. Under the hormonal effects of oestrogen, which is typically higher for females, comparatively larger amounts of fat tissue are deposited around the body; a factor that can influence a model. Thus, for the purpose of making simple models, we will sometimes use a binary choice for "sex"; for example, when modelling blood alcohol concentration. We hope that you recognise the limitations that come with making such an assumption in a model.

Question *1.3.1*

Let's start by considering what a scientist must do in conducting a scientific study. Suppose that your interest is to model and measure blood flow through the coronary artery. What must you consider, as a scientist, before even initiating the study?

1.4 Science and modelling

Models

Models are simplifications and approximations of reality, usually based on measured data, that allow us to understand phenomena, make predictions and evaluate possible impacts of interventions. All models must strike an appropriate balance between accuracy and complexity.

Models can be *physical* or *conceptual*. Examples of physical models include building scale models of bridges or dams and subjecting the model to tests. In this course, we will focus on conceptual models.

Some common ways of developing and presenting (conceptual) quantitative models are:

- words (descriptive text);
- values (for example, weight / height / age tables);
- pictures (such as graphs or flow diagrams);
- mathematics (using equations);
- computer programs.

Note that there is nothing "right" or "wrong" about each approach – each is suited to different uses and/or target audiences. Most models can be developed and presented in **all** of these ways.

Mechanistic Models vs Phenomenological Models

A **mechanistic model** is a model that is derived theoretically by examining the individual physical components of a system.

A **phenomenological model** is a model that is derived empirically by fitting a hypothesised relationship to the observed data.

Question *1.4.1*

Consider blood flowing through an artery. What key physical factors (of the artery and the blood) would be important in modelling the flow rate? Can you propose a mathematical model (equation) that describes the flow rate?

1.5 Science and philosophy

Image 1.1: *The School of Athens* (1510 – 1511), Raphael (1483 – 1520), Stanze di Raffaello, Apostolic Palace, Vatican. [85]

The School of Athens (left) depicts some famous scientists, mathematicians and philosophers, including Plato, Aristotle, Euclid, Socrates and Pythagoras. Science and knowledge play fundamental roles in human history, culture and society.

In order to understand phenomena in the world around us, we use a variety of mathematical and scientific techniques, real-world data, as well as "common-sense" in order to develop models. Whatever model we are considering, as scientists, we must compare its predictions to real-world data.

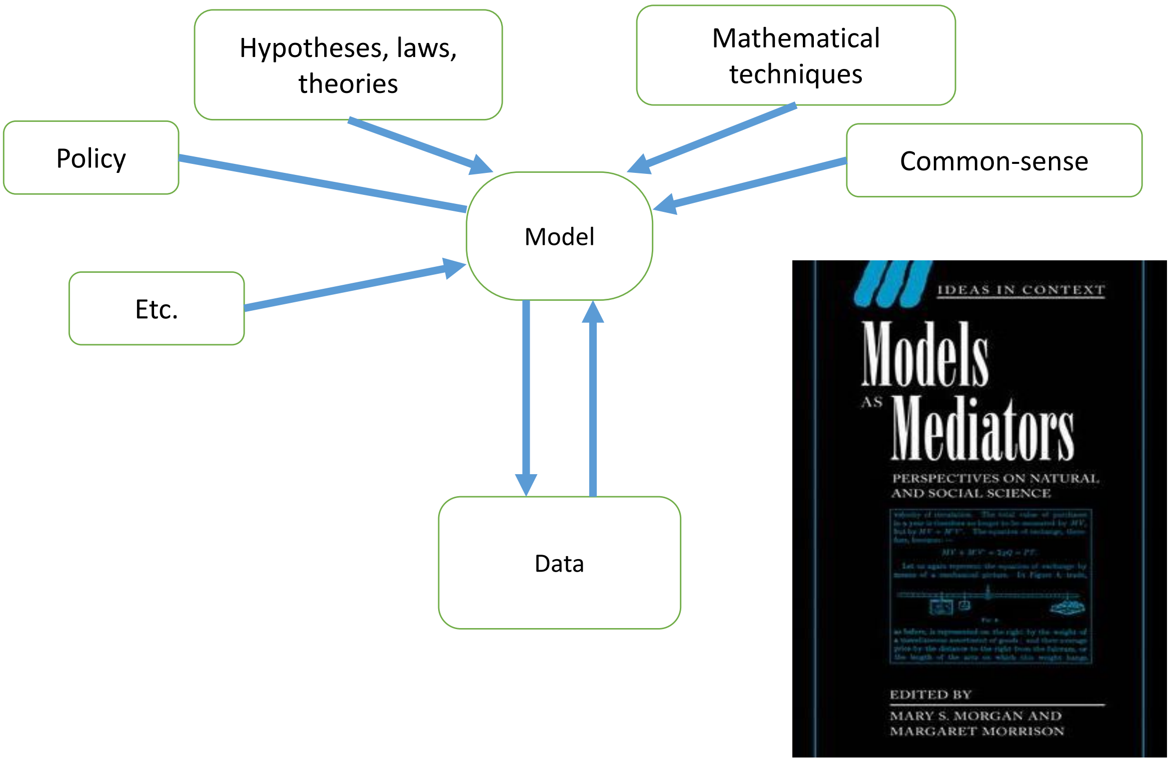


Figure 1.2: *Philosophy of Models: Models as Mediators* [41]

Question 1.5.1

The figure below shows climate model projections according to various modelling scenarios.

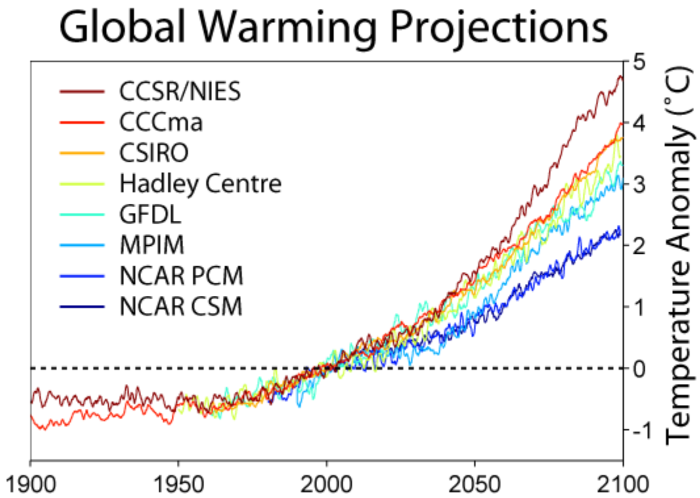


Figure 1.3: *Climate model predictions for global warming relative to global average temperatures in 2000, according to model scenarios considered by the IPCC Third Assessment Report* [77]

As we have just noted, a variety of considerations are required when developing a model. How is the fact that models are mediators illustrated in the example of the climate models?

1.6 Scientific units

A **unit of measurement** in science is an agreed upon quantity of something; any other quantity of the same kind can be described by giving the ratio between that quantity and the unit. The consequences of the poor use of units can be severe.

***Example** 1.6.1*

The Mars Climate Orbiter was launched in 1998 as part of a $USD330 million project, but in September 1999 it crashed into Mars. Here is an extract from the report into the accident [63]:

> *"During the 9-month journey from Earth to Mars, propulsion manoeuvres were periodically performed ... coupled with the fact that the angular momentum (impulse) data was in English, rather than metric, units, resulted in small errors being introduced in the trajectory estimate over the course of the 9-month journey. At the time of Mars insertion, the spacecraft trajectory was approximately 170 km lower than planned ... it was discovered that the small forces ΔVs reported by the spacecraft engineers for use in orbit determination solutions was low by a factor of 4.45 (1 pound force = 4.45 Newtons) because the impulse bit data contained in the AMD file was delivered in lb-sec instead of the specified and expected units of Newton-sec."*

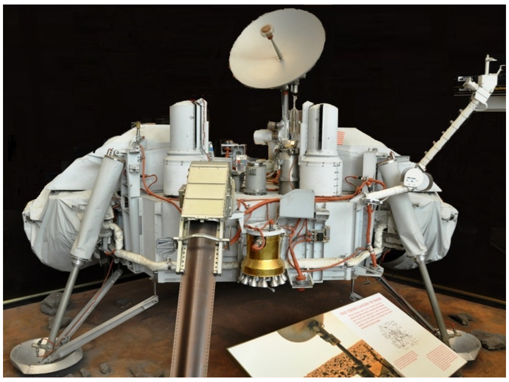


Photo 1.2: Mars Lander (proof test model) from the Viking program, launched 1975. (Source: PA.)

In this course, SI units will generally be used although we will encounter situations where other units have been introduced. Refer to Appendix D "The use of units in Science" for a review of scientific units.

About the use of units in this course

- We will regularly highlight the units of quantities that we calculate.
- We will use only scientific units (SI base units and combinations).
- A unit *must* always be included with the final values calculated (unless a value is unit-less). However, it is not necessary to include units in every intermediary step.
- We will generally work with two or three significant figures in our calculations (noting that uncertainties are not covered in this course).

Dimensional analysis is an important technique in science. It involves applying the following principle: an equation describing a physical situation can be true *only* if it is **dimensionally homogeneous**; that is, both sides of the equation have the same units. Dimensional analysis allows a quick check of whether a calculation is 'plausible'. If the dimensions do not match, then there **must** be an error.

Question 1.6.2

Use dimensional analysis to evaluate if our equation modelling blood flow rate is plausible according to the dimensions.

Chapter 2: Thinking and communicating

Lecture 2: Making sense in science

Learning objectives

✓ Understand the need for clear communication of scientific information

✓ Evaluate quantitative information presented by the media

✓ Interpret binary classification test tables

Scientific examples

✓ Breast and prostate cancer

✓ Birth control and thrombosis

✓ HIV screening

Maths skills

✓ Evaluate probabilities, percentages and ratios

Key literature

✓ *Helping Doctors and Patients Make Sense of Health Statistics* [23]

2.1 Motivation and Background

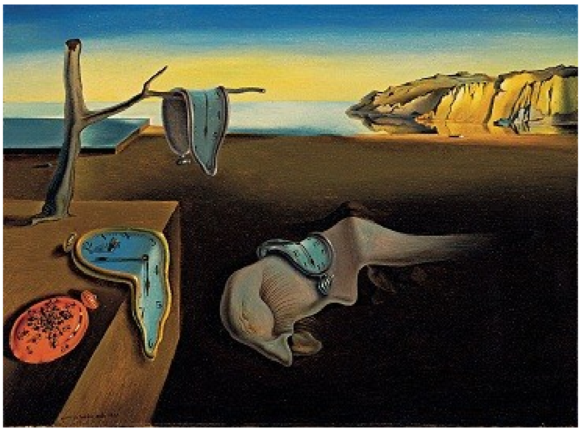


Image 2.1: *The Persistence of Memory* (1931), Salvador Dalí [9]

As we will see, a poorly worded health announcement in the UK led to widespread concern and fear about oral contraceptives. Research has shown that communicating risk using percentages is not necessarily understood by all in the targeted population group. Even doctors have been found to not properly understand some percentage-based risk calculations.

Here we consider the communication of scientific knowledge, particularly in the context of effectively communicating and interpreting quantitative data in a medical context. The goal is to motivate why clear scientific communication is so important. Appendix B "Communication in Science" provides detailed advice and examples, and we will work through these concepts throughout the course.

We are all producers and consumers of quantitative scientific information, in the form of scientific papers, assignments, media reports, the internet, and professional communications such as doctor/patient discussions.

As a *producer* of such information, we should aspire to be accurate, honest, logical, unambiguous, concise, precise, not excessively technical, and always mindful of the intended audience. Specifically, our aim when communicating in science is to be **precise**, **clear**, and **concise**. To ensure this, we will follow four principles:

- Being clear
- Knowing your audience
- Knowing your purpose
- Identifying key messages

These concepts are further explained Appendix B.

As a *consumer*, we should aspire to be thoughtful, reflective, sceptical, logical and analytical, while at the same time open-minded and accepting of evidence that may differ from our preconceptions or opinions. The media and internet provide a continual bombardment of facts, reports, summaries, interpretations and opinions, often covering sophisticated concepts but written and read by non-experts. In many cases there are errors (or deliberate falsities) in such communications. You should form the habit of critically evaluating information, data and (claimed) conclusions.

A useful approach when checking quantitative values in your own work, or the work of others, is *rough estimation*, which is the process of calculating approximate values. It involves building rough, conceptual models, and then evaluating them 'for sense'. Estimating 'gives an idea' whether a particular value is plausible. Often, we aim to find an approximate value within an *order of magnitude* of the correct value (that is, within a factor of 10 of the correct value).

2.2 Losing patients with mathematics?

Sometimes, particularly in a medical context, critically evaluating quantitative information is a matter of life and death. A paper from 2007 presents the following key findings [23]:

- Many people (doctors, patients, journalists and politicians) do not understand health statistics.
- Lack of understanding is due both to lack of knowledge, and intentional misrepresentation of information.

The following paragraph is a quote from the paper:

> "Statistical literacy is a necessary precondition for an educated citizenship in a technological democracy. Understanding risks and asking critical questions can also shape the emotional climate in a society so that hopes and anxieties are no longer as easily manipulated from outside . . ."

Question 2.2.1

In [23], researchers asked 450 American adults (aged 35-70; 320 had attended college; 62 had a postgraduate degree) for answers to the following questions:

> *"1. A person taking Drug A has a 1% chance of having an allergic reaction. If 1,000 people take Drug A, how many would you expect to have an allergic reaction?*
>
> *2. A person taking Drug B has a 1 in 1,000 chance of an allergic reaction. What percent of people taking Drug B will have an allergic reaction?*
>
> *3. Imagine that I flip a coin 1,000 times. What is your best guess about how many times the coin would come up heads in 1,000 flips?"*

Question *2.2.1 (continued)*

(a) What are the answers to the above three questions?

10 0.1% 500

(b) What are the ramifications of getting these answers incorrect for doctors, journalists and politicians?

Question *2.2.2*

In 1995, an emergency announcement in the UK warned that third-generation oral contraceptive pills doubled the risk of potentially life-threatening blood clots (thrombosis). The announcement led to widespread concern and fear, and many women ceased using the contraceptives. Reports estimate that in the following year there were an additional 13,000 abortions and 13,000 births, with 800 additional pregnancies in girls under 16 years of age. The announcement omitted the following relevant information:

- young women have an absolute risk of spontaneous thrombosis of 1 in 10,000.
- the absolute risk of thrombosis when taking second-generation oral

Question *2.2.2 (continued)*

contraceptive pills is about 1 in 7000.

- the relative risk of thrombosis increases by a factor of 4 to 8 during a Caesarean birth.
- the relative risk of thrombosis during and after pregnancy increases by a factor of around 4.
- the absolute risk of dying from thrombosis during or after an abortion is around 1.1 in 10,000.

(a) Define the terms "absolute risk" and "relative risk".

(b) The Australian Medical Association (AMA) website [2] states that:

"...in order to support and enhance the collaborative nature of the doctor-patient relationship, patients must be able to make informed choices regarding their health care. An informed choice is dependent on receiving reliable, balanced health information, free from the influence of commercial considerations, that is communicated in a manner easily understood by patients."

What key points about the third-generation contraceptive pill should have been conveyed in the public health announcement?

Case Study 2: **Cancer**

Cancer is the name for a large group of diseases affecting many different parts of the body. It arises from the uncontrolled, rapid growth of abnormal cells that interfere with the usual bodily functions.

Treatments of cancer include chemotherapy, radiation therapy and surgery. These can all have minor to major side effects, including fatigue, nausea, mouth ulcers, hair loss, cognitive problems, infection, anaemia, infertility, graft-versus-host disease, burns, cancer (!) or death.

Two commonly reported medical statistics are:

- the *5-year survival rate*, which is the percentage of people who are still alive five years after being diagnosed with a condition; and
- the *annual mortality rate*, which is the number of people dying from a given condition each year, often expressed as a rate per 100,000 people.

Question *2.2.3*

(a) The 5-year survival rate for prostate cancer in American men is 98%; for British men it is 71%.

(i) Assume that 1,000 British men and 1,000 American men receive a diagnosis of prostate cancer (at the same time). After 5 years how many men in each country are expected to have died?

(ii) Considering only the given data, which country do you think has the 'better' health system, and why?

Question *2.2.3 (continued)*

(b) The annual mortality rate for prostate cancer in American men is 26 deaths per 100,000; for British men it is 27 per 100,000. Considering only these data, which country has the 'better' health system, and why?

(c) The medical information given in Parts (a) and (b) is all correct. Explain how the (apparent) discrepancies could occur.

(d) Treatment for prostate cancer is invasive with many substantial side effects, including incontinence and impotence. Considering only prostate cancer, which country has the 'better' health system, and why?

2.3 Binary classification

A *binary classification test* aims to classify objects, people or things into one of two groups. Examples include many medical tests, such as determining whether or not an individual has (or is likely to have) cancer.

Binary classification test

The table below illustrates how we can represent the possible outcomes of such a test in tabular form.

Disease

	Yes	**No**
Test $+$		
Test $-$		

Most binary classification tests are imperfect: results can be *true positives*, *false positives*, *true negatives* or *false negatives*. Identify where these terms would each sit in the table above.

We can also represent this information in a *probability flowchart* which we can use to identify the number of true/false positives/negatives for a given test in a population. This is created as follows:

- This chart starts at the top with the total number, N, who are tested.
- This is then divided, on the next line, into those who have the disease (according to some 'gold standard' test) and those who don't have the disease.
- These are then each further subdivided on the last line into those that test positive according to the binary classification test (true positive, false positive) and those that test negative (true negative and false negative).

Accuracy, sensitivity, specificity and prevalence

The terms *accuracy*, *sensitivity*, *specificity* and *prevalence* are defined mathematically below. Explain what each term means and why it is important.

Disease

	Yes	**No**
Test $+$	A	B
Test $-$	C	D

$$N = A + B + C + D$$

$$\text{Accuracy} = \frac{A + D}{N}$$

$$\text{Sensitivity} = \frac{A}{A + C}$$

$$\text{Specificity} = \frac{D}{B + D}$$

$$\text{Prevalence} = \frac{A + C}{N}$$

Question 2.3.1

A paper [17] studied the effectiveness of combined mammography and ultrasound imaging to screen for breast cancer. A total of 203 patients returned "suspicious or malignant" test results, of whom 138 were later found to have cancer (via biopsy testing). A total of 2811 patients returned "normal or probably benign" test results, of whom 12 were later found to have cancer. Find the accuracy, sensitivity, specificity and prevalence of the combined procedures.

		Actual +	Actual −
test	+	TP	FP
test	−	FN	TN

		Actual +	Actual −
test	+	138	65
test	−	12	2799

$N = 3014$

$$\text{Accuracy} = \frac{138 + 2799}{3014} = 0.974 = 97.4\%$$

$$\text{Sensitivity} = \frac{138}{138+12} = 0.92 = 92\%$$

$$\text{Specificity} = \frac{2799}{2799+65} = 0.977 = 97.7\%$$

$$\text{Prevalence} = \frac{138+12}{3014} = 0.498 = 4.98\%$$

Question 2.3.2

Most binary classification tests are not 100% accurate. Clearly it is better to have more true positives and true negatives than false positives and false negatives. However, it is often the case that it is not possible to simultaneously optimise both the sensitivity and the specificity.

(a) Identify some negative impacts of false positive or false negative cancer test results.

(b) When might a higher false positive rate be tolerated? When might a higher false negative rate be tolerated?

(c) Are false positive results 'better' or 'worse' than false negative results?

Question 2.3.3

A paper [23] quotes an example in which 160 gynaecologists were asked:

"Assume you conduct breast cancer screening using mammography . . . You know the following information about the women in this region:

- *The probability that a woman has breast cancer is 1% (prevalence)*
- *If a woman has breast cancer, the probability that she tests positive is 90% (sensitivity)*
- *If a woman does not have breast cancer, the probability that she nevertheless tests positive is 9% (false-positive rate)*

A woman tests positive. She wants to know whether that means that she has breast cancer for sure, or what the chances are. What is the best answer?

A. The probability that she has breast cancer is about 81%.
B. Out of 10 women who test positive, about 9 have breast cancer.
C. Out of 10 women who test positive, about 1 has breast cancer.
D. The probability that she has breast cancer is about 1%."

(a) Without doing detailed calculations, what is **your instinctive** answer?

90% chance

		Actual	
		+	−
test	+	A	B
	−	C	D

$N = A + B + C + D$

$A + C = 0.01$

$\frac{A}{A+C} = 0.9$

$A = 0.9A + 0.9C$

$0.1A = 0.9C$

$A = 9C$

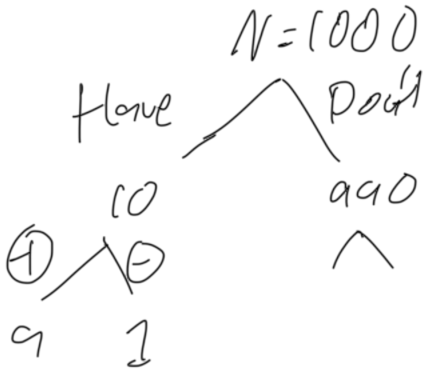
$C = 0.001$

$A = 0.01 - 0.001$

$A = 0.009$

Question 2.3.3 *(continued)*

(b) Investigate the answer to the question using a *probability flowchart* and a group of 1000 'typical' women. Convert your flowchart answers to a binary classification table.



(c) Based on your calculations in Part (b), what is the correct answer pertaining to risk of the condition having tested positive?

(d) Only 21% of gynaecologists in the study gave the correct answer for Part (a)! What are the implications for you and/or people with female hormones?

End of Case Study 2: Cancer.

Question *2.3.4*

In the 1980s, blood screening in Florida found that 22 people who had donated blood tested positive for HIV. Once notified of the test results, seven of these donors died by suicide. (At that time, HIV was not well known, and people were not regularly tested. Screening donors for the disease commenced after the discovery that transmission of HIV occurred through contact with infected blood.)

The HIV test has a very high *sensitivity* [percentage of infected individuals who correctly test positive] of about 99.9% and *specificity* [percentage of non-infected individuals who correctly test negative] of about 99.99%.

The *prevalence*, or rate of infection, for heterosexual men with low-risk behaviour, is around 1 in 10,000.

What is the (approximate) probability that someone who tests positive for HIV is infected?

Question 2.3.5

To investigate the quality of HIV counselling for heterosexual men with low-risk behaviour, an undercover client visited 20 public health centres in Germany, undergoing 20 HIV tests [23].

The client was explicit about belonging to a low risk group, as do the majority of people who take HIV tests. In the mandatory pre-test counselling session, the client asked:

'*Could I possibly test positive if I do not have the virus? And if so, how often does this happen?*'

The answers from the medical practitioners were:

No, certainly not	False positives never happen
Absolutely impossible	With absolute certainty, no
With absolute certainty, no	With absolute certainty, no
No, absolutely not	Definitely not ... extremely rare
Never	Absolutely not ... 99.7% specificity
Absolutely impossible	Absolutely not ... 99.9% specificity
Absolutely impossible	More than 99% specificity
With absolute certainty, no	More than 99.9% specificity
The test is absolutely certain	99.9% specificity
No, only in France, not here	Don't worry, trust me

(a) How would **you** answer the question?

Question 2.3.5 *(continued)*

(b) Recall that the Australian Medical Association (AMA) website [2] states:

"...in order to support and enhance the collaborative nature of the doctor-patient relationship, patients must be able to make informed choices regarding their health care. An informed choice is dependent on receiving reliable, balanced health information, free from the influence of commercial considerations, that is communicated in a manner easily understood by patients."

Two key aspects of communication are being clear and knowing your audience. Do you think any of the answers from the German doctors demonstrate these principles?

Chapter 3: Modelling - power functions

Lecture 3: **The powers of science**

Learning objectives

✓ Develop linear, quadratic and power models of real-world phenomenon

✓ Critically analyse what information is being provided by a model

Scientific examples

✓ Atmospheric temperature

✓ Climate change and Bicknell's thrush

✓ Biodiversity and species richness

✓ UV light and SPF

Maths skills

✓ Find a power function to fit given data

Key literature

✓ *Potential effects of climate change on birds of the northeast* [54]

3.1 Motivation and Background

Image 3.1: *Starry Night* (1508 – 1512), Van Gogh [87]

Mathematical modelling often involves developing a function (or equation) that can be compared with some experimental measurement. The function may be developed based on theory (mechanistic) or designed to fit the data (phenomenological). It is quite common to "linearise" the function and then apply statistical analyses to determine constants in the function.

In this chapter, and in following chapters, we will examine the use of a variety of functions in modelling scientific phenomena, largely drawn from existing scientific literature. We will start with "power" functions - functions where the independent variable is raised to some power. You will likely have previously encountered these functions but you might like to review Section C.1 in Appendix C which covers the pre-requisite mathematical tools that we will use. Use SOMSE, available through the course website, for further support.

3.2 Temperature

In this section we will look at the application of the linear function (power 1) as applied to the temperature of the atmosphere.

Case Study 3: **Higher than a kite**

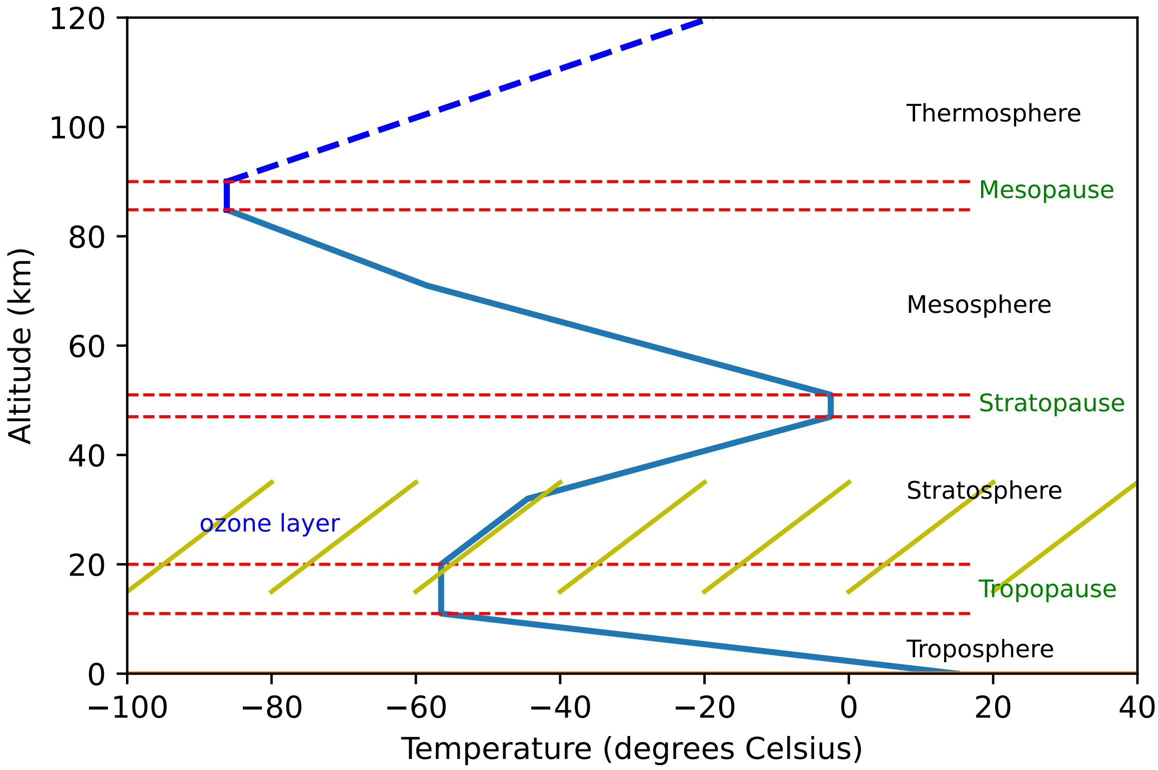
Scientists divide Earth's atmosphere into five primary regions: *troposphere*, *stratosphere*, *mesosphere*, *thermosphere* and *exosphere*.

The *International Standard Atmosphere* (ISA) [31] is a model which further divides the atmosphere from the surface of Earth to the base of the thermosphere into eight layers. (Layer 0 is closest to the surface.) The ISA models various properties of each layer, including temperature, pressure and density. Layers in the ISA are defined as atmospheric regions in which *temperature is a linear function of altitude.*

Layer	Name	Height at base (km)	Lapse rate (°C/km)	Temperature at base (°C)
0	Troposphere	0.0	−6.5	+15.0
1	Tropopause	11.0	+0	−56.5
2	Stratosphere	20.0	+1.0	−56.5
3	Stratosphere	32.0	+2.8	−44.5
4	Stratopause	47.0	+0	−2.5
5	Mesosphere	51.0	−2.8	−2.5
6	Mesosphere	71.0	−2.0	−58.5
7	Mesopause	84.9	NA	−86.2

Table 3.1: Some properties of the layers within the International Standard Atmosphere.

Table 3.1 and Figure 3.1 show various properties of the ISA temperature at different altitudes. In the model, the *lapse rate* is the rate at which temperature changes as altitude increases.



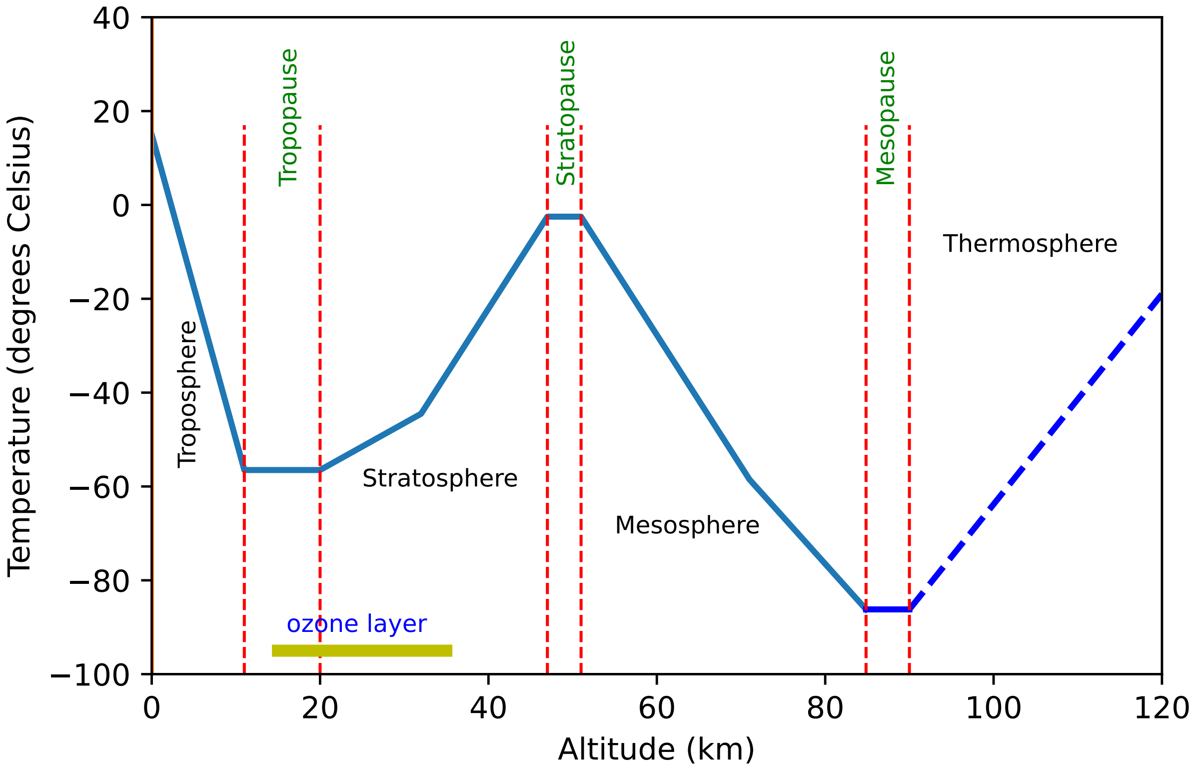


Figure 3.1: The relationships between temperature and altitude modelled by the ISA. The top graph shows altitude versus temperature, and the bottom graph shows temperature versus altitude. (The ISA does not model the thermosphere; temperature data displayed for the thermosphere are from other measurements.)

Question 3.2.1

(a) On an international flight, the following altitudes and external temperatures were reported on the in-flight information screen. The data are graphed in Figure 3.2.

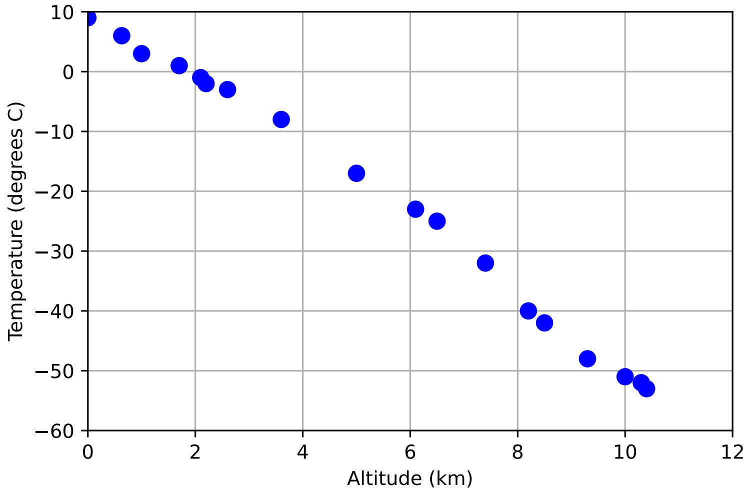


Figure 3.2: Measured external temperatures as a function of altitude.

Are these measurements consistent with the ISA model?

(b) How would the model for the temperature of ISA Layer 3 as a function of altitude differ from that of Layer 1?

End of Case Study 3: Higher than a kite.

3.3 Bend it!

Case Study 4: **Climate change and Bicknell's thrush**

Image 3.2: Bicknell's thrush, *Catharus bicknelli* [67].

Photo 3.1: Adirondack mountains, USA. (Source: PA.)

Example *3.3.1*

A paper [54] developed models for bird distributions using data from various altitudes, temperatures and locations in the north-eastern USA. The authors then used their models to predict the likely impact of rising temperatures on these distributions. Part of their study focused on Bicknell's thrush.

- The paper [54] built on earlier work that developed qualitative models Bicknell's thrush distributions with temperature.
- The authors combined these models to create a model for thrush distribution with respect to mean July temperatures across the breadth of their habitat.
- For the habitat in consideration, the study region was divided into cells, each 30 m by 30 m square.
- The study found that thrush habitats with July temperatures outside the range of 9.3 °C to 15.6 °C contained insignificant numbers of thrush.

Let T be a temperature within the range 9.3 °C – 15.6 °C. The proportion $p(T)$ of cells containing thrush is closely modelled by the quadratic function:

$$p(T) = -0.0747T^2 + 1.8693T - 10.918.$$

Question 3.3.2

The graph of the function $p(T)$ is shown in Figure 3.3.

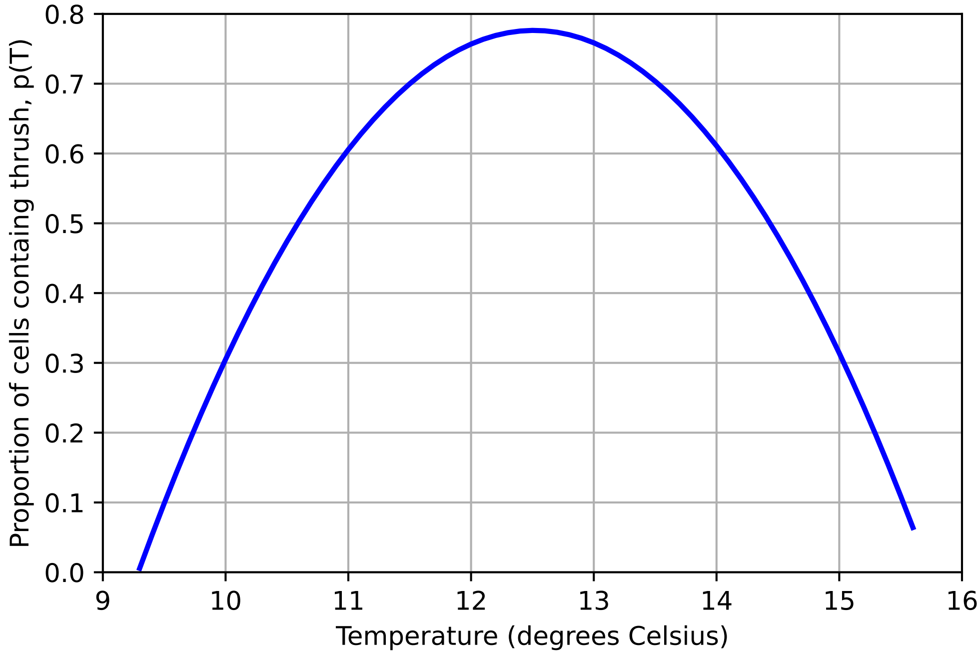


Figure 3.3: Distribution of Bicknell's thrush according to temperature.

(a) What is the probability that a thrush will be found in a sample area in which $T = 11$ °C?

(b) From the graph, at what (approximate) value of T is the thrush distribution most dense, and what is the (approximate) value of $p(T)$?

(c) There is no value of T for which $p(T) = 1$. Explain what this means and give reasons why it would happen.

***Question** 3.3.2 (continued)*

(d) Average temperature rises in the region over the next century are predicted to range from 2.8 °C under a low greenhouse gas emission scenario, to 5.9 °C under a high emission scenario.

(i) How do you think the graph in Figure 3.3 would change if the average temperature rose by a value within this range? Explain your answer.

(ii) Assuming a substantial rise in average July temperatures, what factors may be a concern to resident Bicknell's thrush?

(iii) What kind of survival strategies might the thrush utilise in response to temperature change?

End of Case Study 4: Climate change and Bicknell's thrush.

3.4 (Super) powers

Recall that linear and quadratic functions are examples of the more general group of *power functions*. Functions with different powers have graphs with different shapes, and hence can model different phenomena.

Case Study 5: **Species-area curves and biodiversity**

Photo 3.2: Counting species in the field. (Source: DM.)

Previously we discussed the abundance and distribution of a *single* species, *Bicknell's thrush*. Ecologists often study the *overall number of species* found in a region (sometimes called the *biodiversity* or *species richness*).

> **Species-area curves**
>
> In ecology, a *species-area curve* is a graph showing the number of distinct species observed, as a function of the size of the area surveyed.

Photo 3.3: Scribbly gum (*Eucalyptus racemosa*). Right: Scaly-breasted Lorikeet (*Trichoglossus chlorolepidotus*). (Source: PA.)

Rather than performing a full count for an entire region, data from a smaller area can be extrapolated to estimate the regional species richness.

Example 3.4.1

Consider a four hectare property in eastern Brisbane. We wish to estimate the number of distinct, naturally occurring, native plant species (individuals greater than 2 m in height), that occur on this land. Suppose that 30 cells (or *quadrats*), each 10 m by 10 m, are selected at random and for each cell we record the occurrence of new species not seen in previous cells. Table 3.2 shows information on the previously unseen species, including the cumulative total count of species observed so far.

Cell(s)	New species observed	Count
1	*Eucalyptus racemosa, Acacia fimbriata, Banksia integrifolia*	3
2	*Eucalyptus tereticornis, Alphitonia excelsa*	5
3	*Acacia disparrima*	6
4	*Acacia leiocalyx, Lophostemon suaveolens*	8
5	–	8
6	*Glochidion sumatranum*	9
7	–	9
8	–	9
9	*Eucalyptus crebra*	10
10	–	10
11 – 15	*Banksia robur, Melaleuca quinquinerva*	12
16 – 20	–	12
21 – 30	*Allocasuarina littoralis,Angophora leiocarpa*	14

Table 3.2: Information on additional observed species.

Figure 3.4 is a species-area curve summarising the data in Table 3.2.

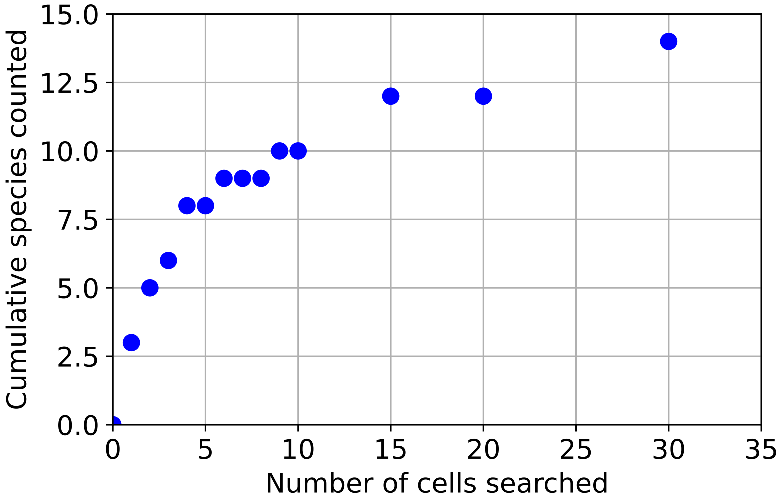


Figure 3.4: The number of distinct tree species recorded on the property.

The graph has a shape that is typical of many species-area curves: the number of distinct species initially rises rapidly as the area increases, but then rises less rapidly as the area becomes larger.

Equations for species-area curves

Species-area curves can be mathematically modelled using power functions, with power p between 0 and 1 (typically, p is between 0.2 and 0.5).

Their general form is $S(a) = Ma^p$, where S is the number of species occurring as a function of the area a, and M and p are constants depending on the geographical location, resource availability and similar factors.

Question *3.4.2*

With respect to a species-area curve:

(a) Discuss why species-area curves exhibit the general shape of this type of power function.

Question *3.4.2 (continued)*

(b) How do the values of M and p impact on the shape of the graph?

(c) What physical factors could affect the shape of the curve?

Example 3.4.3

Figure 3.5 shows the graph of $S(a) = 5a^{0.3}$ and the species data from Table 3.2, where a is the number of 10 m by 10 m cells (hence related to area).

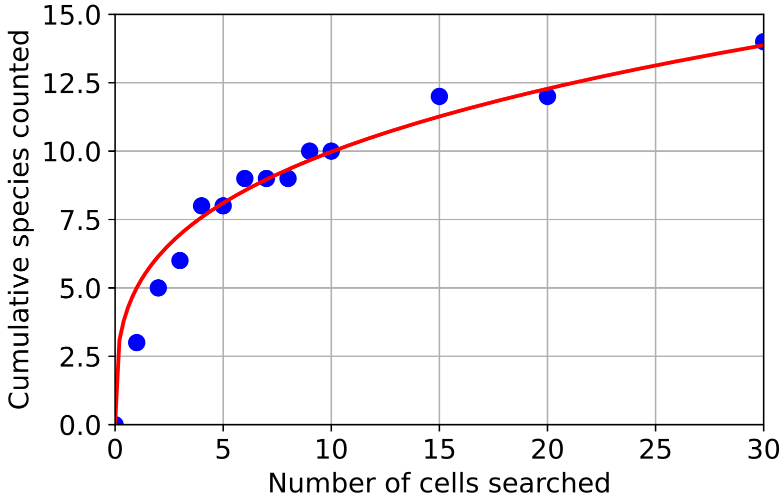


Figure 3.5: Modelling the species data from the 4 hectare property.

Question 3.4.4

Assume that this question refers to native, naturally occurring plants more than 2 m high, growing on land ecologically similar to the four hectares of land in the previous example (that is, the model shown in Figure 3.5 is appropriate).

(a) Estimate the species richness (total number of species) on the four hectare (40,000 m^2) property.

Question *3.4.4 (continued)*

(b) A typical conservation goal is to establish parks that preserve 10% of the representative land area. What fraction of species richness would be represented within such a park in the area near the four hectare property?

(c) Many people believe that the figure in Part (b) is too low. If the goal is to retain 75% of species, what proportion of land should be preserved?

Case Study 6: **Ban the tan, man**

Earlier, we saw that temperature in the lowest atmospheric layer (Troposphere) decreases as altitude increases, but temperature in the next layer (Stratosphere) increases from altitude 20 km to 50 km. The rise in temperature is due to interactions between the ozone layer and ultraviolet (UV) light.

UV light is electromagnetic radiation with wavelengths shorter than visible light, and can be divided into: UVA (wavelength 315 – 400 nm); UVB (wavelength 280 – 315 nm); and UVC (wavelength 100 – 280 nm).

The following sequence of chemical reactions occurs in the ozone layer:

$$\begin{aligned}
O_2\ (g) &\xrightarrow{\text{UVC light}} 2O\ (g)\\
O_2\ (g) + O\ (g) &\longrightarrow O_3\ (g)\\
O_3\ (g) &\xrightarrow{\text{UVB light}} O_2\ (g) + O\ (g)\\
O_3\ (g) + O\ (g) &\longrightarrow 2O_2\ (g)\\
2O\ (g) &\longrightarrow O_2\ (g)
\end{aligned}$$

The second and third reactions describe the *ozone-oxygen* cycle. The net result of these reactions is that harmful UV radiation is converted to heat, and ozone is conserved (no net loss of ozone).

The chemical reactions which occur in the ozone layer are extremely important to life on Earth:

- UVC light is very damaging to life, but is completely absorbed.
- Most UVB light is also absorbed; only around 1 part in 350 million reaches the surface of Earth. Exposure to this light causes sunburn, eye cataracts, visible ageing, genetic mutations in cells and skin cancer.
- Almost all UVA light reaches the surface of Earth.

The effectiveness of sunscreens at preventing UVB light from reaching the skin is measured by their *Sun Protection Factor*, SPF. When a product with

SPF n is correctly applied to the skin, it blocks a fraction of $(n-1)/n$ of the usual amount of UVB light.

Question 3.4.5

Write a function for the proportion of UVB light that is **not** blocked by sunscreen with SPF n, and draw a rough sketch of the graph of the function.

What proportion of UVB light is not blocked by sunscreen with SPF 30? SPF 50? SPF 100?

Should people buy sunscreen with SPF 100? Why or why not?

Chapter 4: **Modelling - periodic functions**

Lecture 4: **Give us a wave**

Learning objectives

✓ Interpret sine function models of real-world phenomenon

Scientific examples

✓ Seasons on Earth

✓ Daylight hours

✓ Tracking migration of terns

Maths skills

✓ Understand sine functions and their transformations

Key literature

✓ *Tracking of Arctic terns Sterna paradisaea reveals longest animal migration* [14]

4.1 Motivation and Background

Image 4.1: *The Great Wave off Kanagawa* (1829–1833), Katsushika Hokusai. Print at the Metropolitan Museum of Art [86]

Many phenomena in Science and nature *repeat* or *cycle*. These include: many aspects of weather and climate; ocean waves and tides; physiological processes, such as breathing and hormone levels; sound waves; and the voltages and currents in alternating current electricity.

In this chapter, we will discuss how to model cyclic or periodic phenomenon using a sine function. You should have encountered sine functions in previous study of mathematics. Review Section C.2 in Appendix C for the pre-requisite mathematical tools we will use in this chapter. Use SOMSE, available through the course website, for further support.

Some examples of naturally occurring cycles are shown the four graphs in Figure 4.1. Each graph shows climate-related data for Brisbane over a period of one year. If the graphs were extended over subsequent years, then an approximate cycling pattern would be observed. We can model such variations using a periodic or sine function.

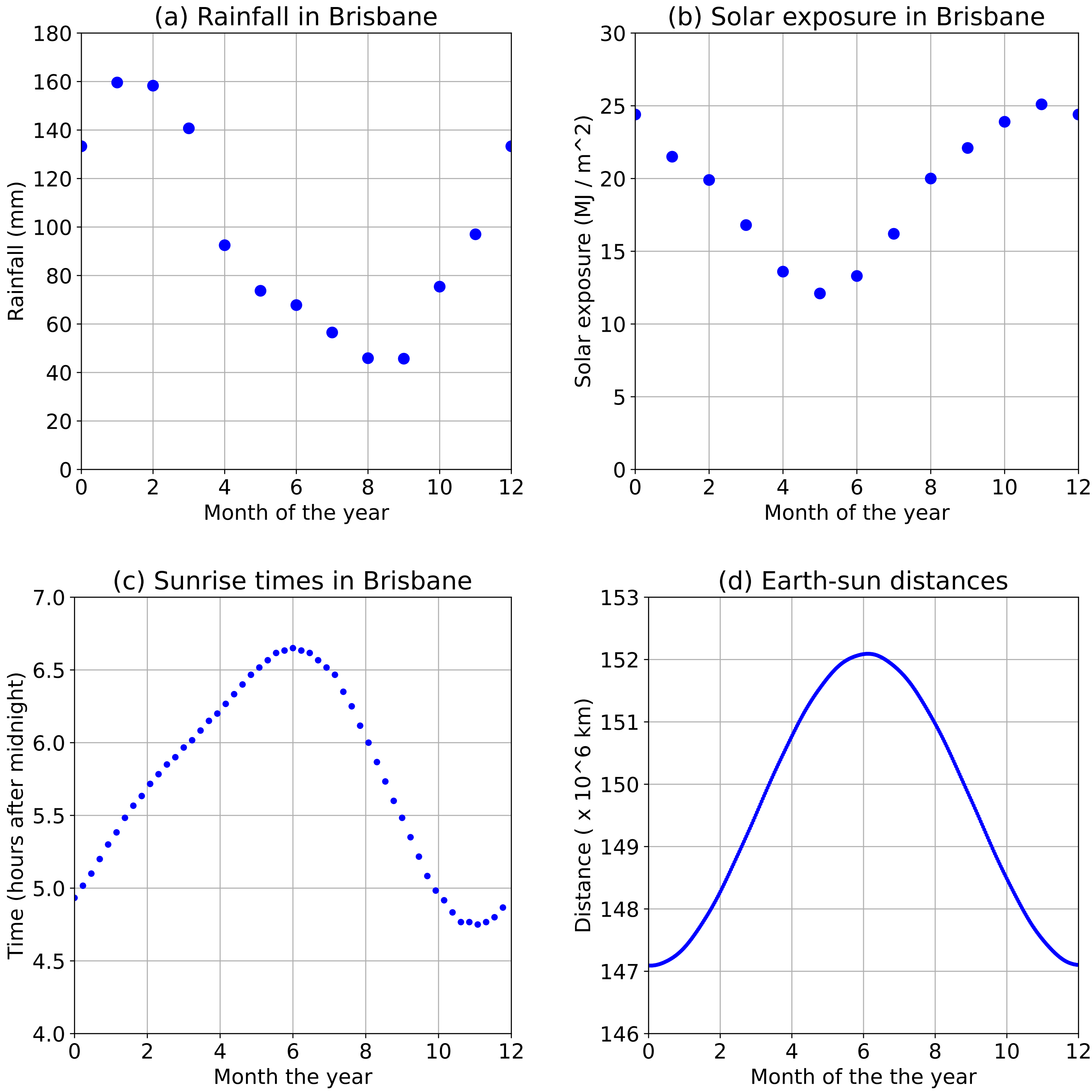
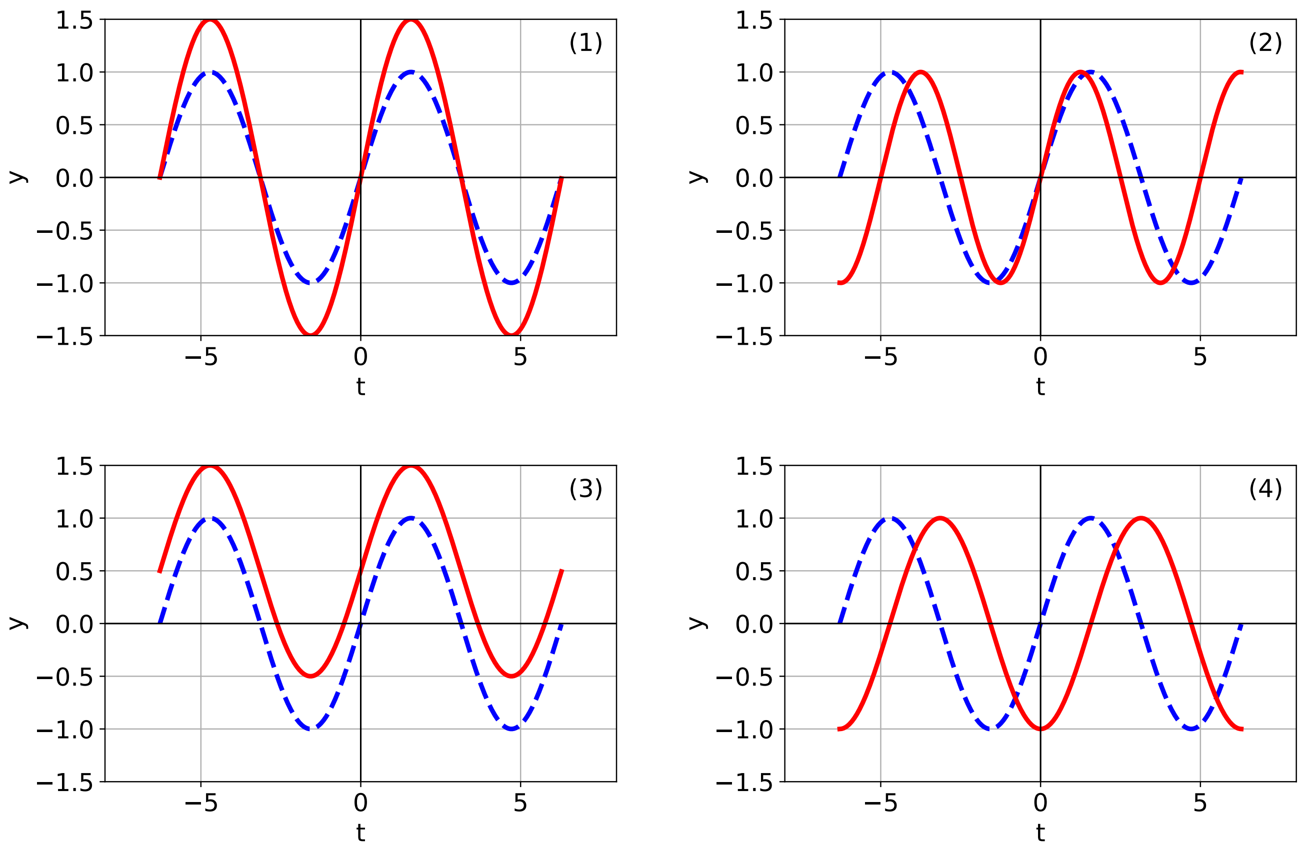


Figure 4.1: Four climate-related graphs. (a) Average monthly rainfall in Brisbane. (b) Average monthly solar exposure in Brisbane. (c) Weekly sunrise times in Brisbane. (d) Daily distances between the centres of Earth and the sun.

Question 4.1.1

As a check of your understanding, consider the following four transformed sine functions. Graphs are given below of each function. In each graph, the blue (dashed) curve shows a plot of $y = \sin t$ for two full cycles (from $t = -2\pi$ to $t = 2\pi$). The red curve shows a plot of one of the functions listed below. See if you can match each equation to its corresponding graph.

(a) $y = \sin(t) + 0.5$	**(b)** $y = 1.5\sin(t)$
(c) $y = \sin(t - \frac{\pi}{2})$	**(d)** $y = \sin(\frac{2\pi}{5}t)$



If you use a calculator to do some checking, *make sure that your calculator is set to radians NOT degrees.*

4.2 Seasons and cycles

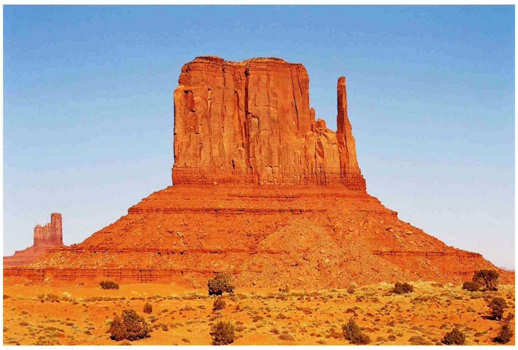
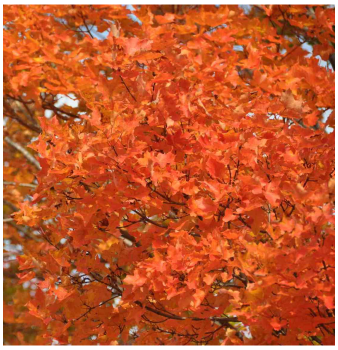





Photo 4.1: Spring – Lotus flower, *Nelumbo nucifera* (Tokyo, Japan); Summer – Monument Valley (Utah, USA); Autumn – sugar maple, *Acer saccharum* (Vermont, USA); Winter – Pine tree (Canyonlands, Utah, USA). (Source: PA.)

Photo 4.2: Migrating Canada Geese, *Branta canadensis*, New York State, USA. (Source: PA.)

Many animals undertake *migration*, during which they move from one area to another, and then return. This often happens on an annual basis, according to seasons or weather patterns. Migratory behaviour occurs in all major animal groups (birds, reptiles, mammals, amphibians, fish, insects and crustaceans); see [12].

Examples of migration include: wildebeest and zebra on the Serengeti plains in Africa; geese "flying south for winter" in the northern hemisphere; humpback whales travelling north along the Queensland coast during winter; and sea turtles returning to beaches to lay eggs.

Question *4.2.1*

What causes seasons?

The amount of sunlight available at a location on Earth on a given day can be modelled using *daytime*, defined as the time between sunrise and sunset. (This is independent of clouds or weather events.)

The **summer solstice** and **winter solstice** are the days with the longest and shortest daytimes (respectively). The **vernal equinox** and **autumnal equinox**, are the days in spring and autumn (respectively) with daytimes of exactly 12 hours.

On some occasions in the polar regions there is no sunrise or sunset for a period greater than one day. For simplicity, in such cases we say that the daytime is 24 hours (always light) or 0 hours (always dark).

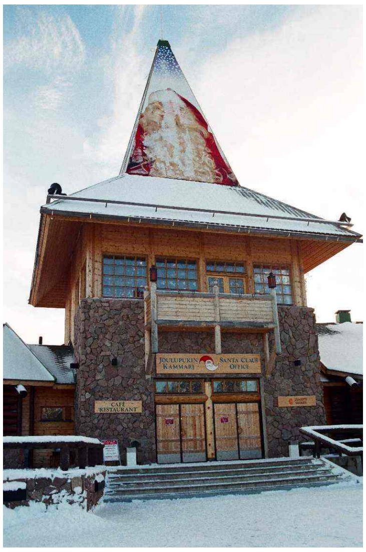


Photo 4.3: Seasonal destinations. the official home of Santa (Santa Claus Village, Finland). (Source: PA.)

Question 4.2.2

Discuss the daytime lengths in midsummer and midwinter in each of:

(a) Brisbane;

midsummer:

midwinter:

(b) Singapore (which is very close to the equator); and

midsummer:

midwinter:

(c) Santa Claus village, Rovaniemi, Finland (north of the Arctic Circle).

midsummer:

midwinter:

Question 4.2.3

On some international flights, in-flight maps show areas of night and day superimposed on the surface of Earth; see Figure 4.4 for two examples.

Photo 4.4: In-flight maps. (Source: PA.)

(a) Describe everything you can about the date and time of day in Brisbane when the first photograph was taken. Justify your answer.

(b) Describe everything you can about the date and time of day in Brisbane when the second photograph was taken. Justify your answer.

Case Study 7: **Modelling daytimes**

The general equation of a sine function that we will use is

$$y(t) = A \sin\left(\frac{2\pi}{P}(t - S)\right) + E,$$

where A is the amplitude, P is the period, S is the (horizontal) shift and E is the equilibrium (average) value.

Figure 4.2 shows the daytimes in Brisbane at weekly intervals throughout a calendar year. The graph of daytime lengths in every year will be very similar; clearly, the graph resembles a sine wave!

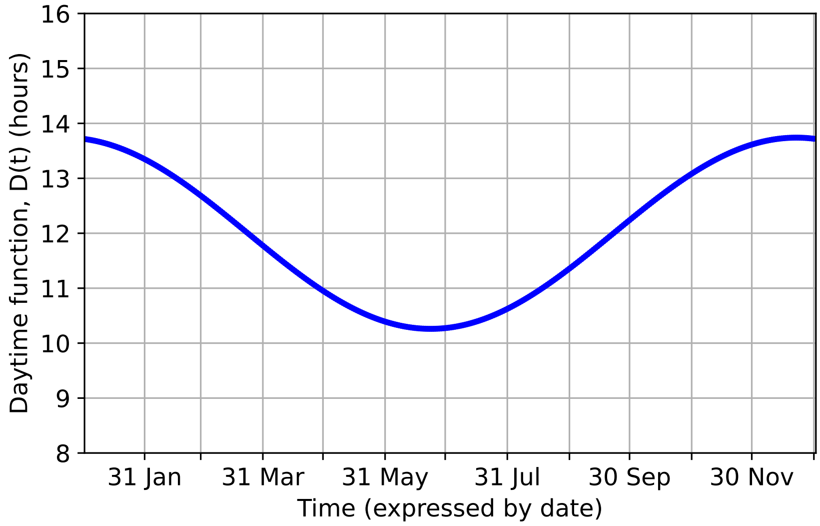


Figure 4.2: Daytimes in Brisbane during the year.

Question 4.2.4

Use the graph in Figure 4.2 to identify when the solstices occur in Brisbane (and how long are the daytimes), and when the equinoxes occur.

Every location on Earth has a *latitude*, which is a measure of its distance from the equator. On any given day, **every location with the same latitude has the same daytime length.**

At each location on Earth, the daytimes repeat in a yearly pattern. Therefore, they can be modelled using sine, as a function of the day of the year. (In reality, daytimes will vary slightly from these functions as days are discrete time steps whereas the Sun and Earth move continuously.)

Question 4.2.5

If t is the day number in the year (starting from $t = 0$ on January 1st), then the length of the daytime in hours at any point in the southern hemisphere is given by the function

$$D(t) = 12 + K \times \sin\left(\frac{2\pi}{365}(t - 264)\right)$$

where K is a constant determined by the latitude of the point. Near the equator $K \approx 0$ hours, and its value increases for more southerly locations. $D(t)$ for Brisbane is plotted in Figure 4.3, where $K \approx 1.74$ hours.

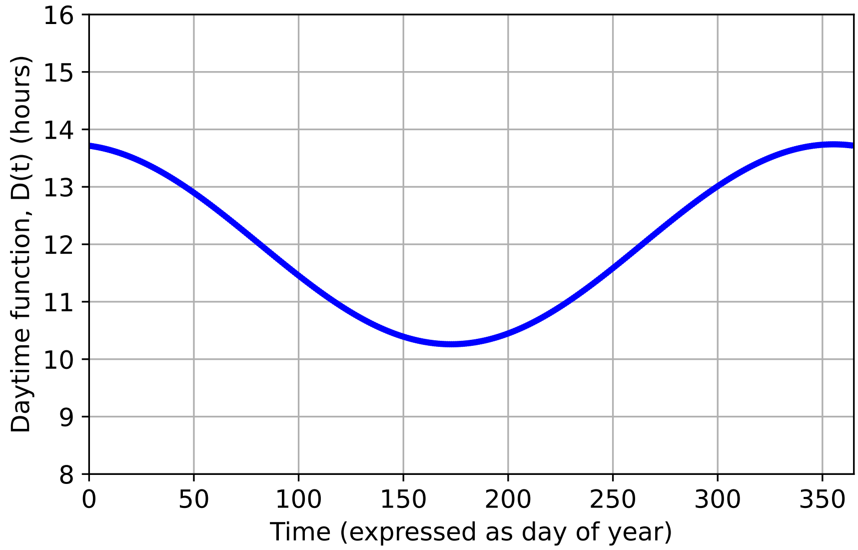


Figure 4.3: The daytime function for Brisbane.

Fill in the missing entries in the following table, describing the physical and mathematical significance of each term in $D(t)$ for Brisbane, and indicate these on the graph.

Question 4.2.5 *(continued)*

Value	**Mathematical meaning**	**Physical meaning**
12 hours	Equilibrium of the sine wave	Average number of daylight hours in a day
1.74 hours		
365 days	Period of the sine wave	Number of days in a year (follows a yearly cycle)
264 days		

Question 4.2.6

Briefly explain how to *mathematically* find when the solstices occur in Brisbane, using the function $D(t) = 12 + 1.74 \sin\left(\frac{2\pi}{365}(t - 264)\right)$.

End of Case Study 7: Modelling daytimes.

Question 4.2.7

Recall that, if t is the day number in the year (starting from $t = 0$ on January 1st) then the length of the daytime in hours at any point in the southern hemisphere is given by the function

$$D(t) = 12 + K \times \sin\left(\frac{2\pi}{365}(t - 264)\right)$$

where K is a constant determined by the latitude of the point.
In Brisbane, $K \approx 1.74$ hours, whereas $K \approx 1$ hour for Townsville, and $K \approx 3.3$ hours for Hobart. The graph for Brisbane is shown in Figure 4.4.

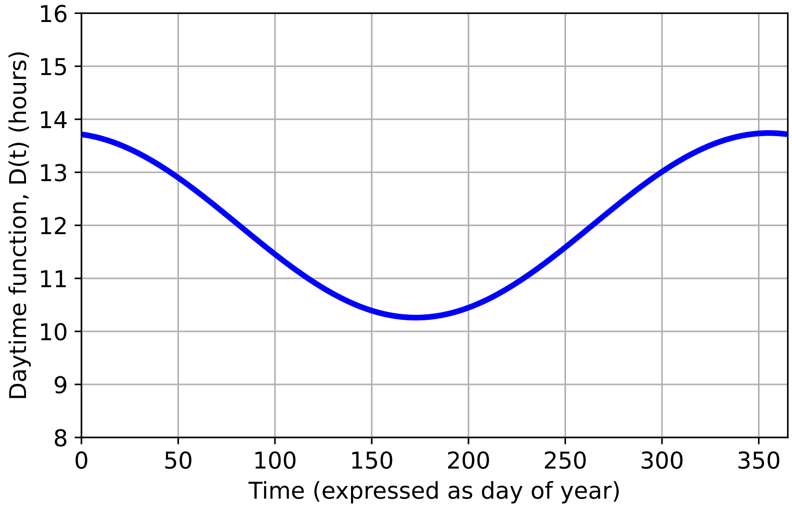


Figure 4.4: Daytimes in Brisbane over the year.

(a) Roughly sketch the graphs of $D(t)$ for Townsville and Hobart on the above graph.

(b) By how much is the daytime on the **summer** solstice in Hobart **longer** than in Townsville? By how much is the daytime on the **winter** solstice in Hobart **shorter** than in Townsville?

(c) Which town receives the most total daylight over the entire year?

Case Study 8: **To every thing, there is a season, tern, tern, tern**

The Arctic tern, *Sterna paradisaea*, is a seabird that migrates annually from its breeding grounds in the Arctic to the Antarctic and back.

Image 4.2: Arctic tern in flight [65]

Individual terns have been tracked travelling a distance of 400–700 km per day, and 80000 km in a year; this is the longest (known) migration of any animal. Figure 4.5 shows tracked migration routes of 11 Arctic terns (see [14]).

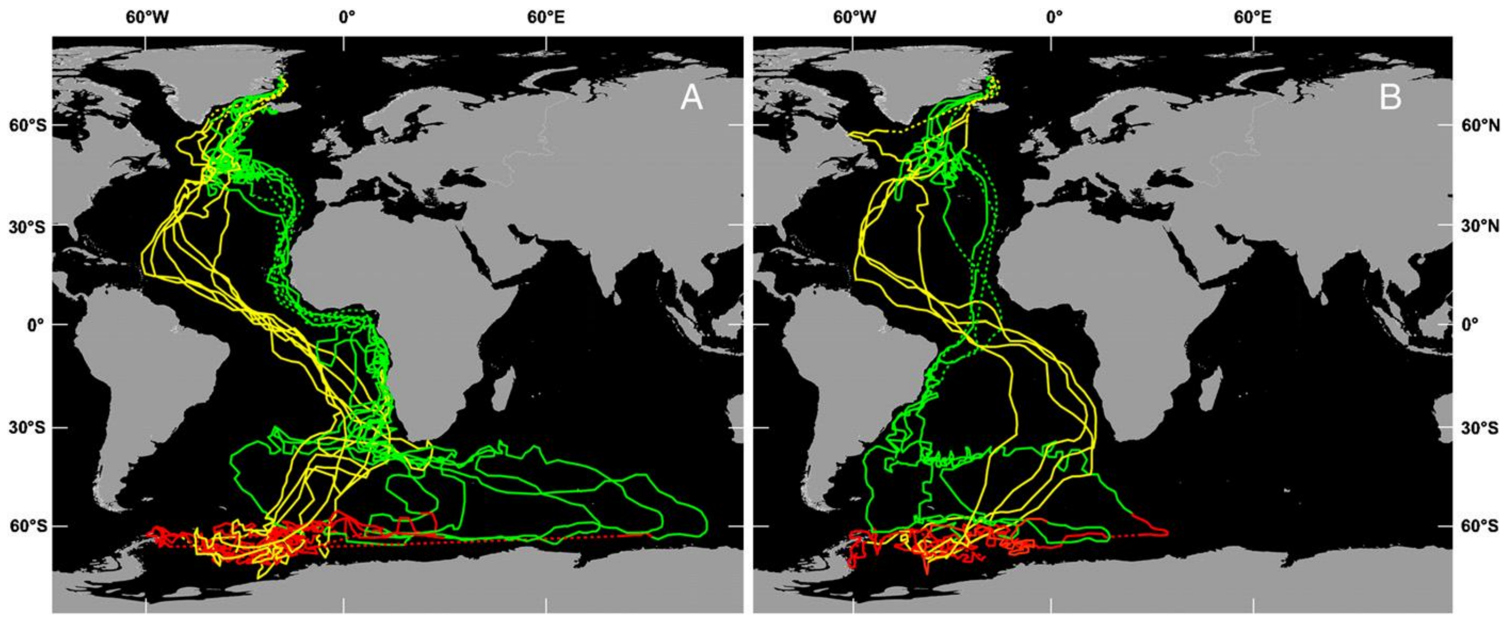


Figure 4.5: Interpolated geolocation tracks of 11 Arctic terns tracked from breeding colonies in Greenland ($n = 10$ birds) and Iceland ($n = 1$ bird). Green = autumn (post breeding) migration (August/November), red = winter range (December/March), and yellow = spring (return) migration (April/May). Two southbound migration routes were adopted in the South Atlantic, either (A) West African coast (n = 7 birds) or (B) Brazilian coast. (This text is an extract from [14].)

Question 4.2.8

Researchers in [14] attached miniature light loggers to individual birds in Iceland and in Greenland, and retrieved the data one year later. The light loggers have an internal clock and they record when they are exposed to light. For this question, assume the birds are in the southern hemisphere.

(a) Describe how researchers could have used this data to determine the latitude of a tern on any given day.

(b) Describe how the researchers could have used the data from the light loggers to determine the longitude of a tern on any given day.

The paper [14] states that:

> *"Locations were unavailable at periods of the year when birds were at very high latitudes and experiencing 24 h daylight. In addition, only longitudes were available around equinoxes, when day length is similar throughout the world. Overall, after omitting periods with light level interference and periods around equinoxes, the filtered data sets contained between 166 and 242 days of locations for each individual."*

Question *4.2.9*

Why did the researchers in [14] need to comment on high latitudes and the equinoxes?

End of Case Study 8: To every thing, there is a season, tern, tern, tern.

Chapter 5: Modelling - exponential functions

Lecture 5: All about that base

Learning objectives

✓ Interpret exponential function models of real-world phenomena

✓ Understand the form of log-plots

Scientific examples

✓ Radioactive decay

✓ Carbon dating

✓ Newton's Law of heating and cooling

✓ Atmospheric pressure

Maths skills

✓ Understand exponential functions and logarithms

✓ Doubling time and half-life

✓ Interpret log-lin and log-log plots

Key literature

✓ *AMS 14C age determination of tissue...* [4]

5.1 Motivation and Background

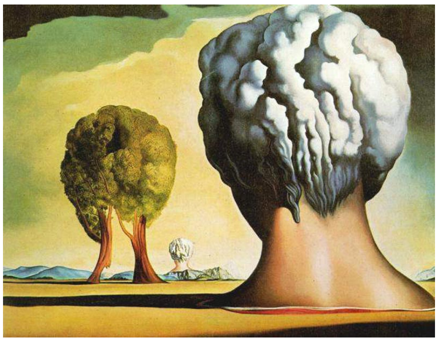


Image 5.1: *The Three Sphinxes of Bikini* (1947), Salvador Dalí (1904 – 1989), Morohashi Museum of Modern Art [10].

Exponential functions are useful for modelling many natural phenomena such as growing populations or radioactive decay of isotopes, as well as many "un-natural" phenomena such as compound interest.

Logarithms are closely related to exponential functions, and you will have used them in previous mathematical study to solve exponential equations.

In this chapter we will review some of the properties of these important functions and discuss some of the scientific contexts in which they naturally arise. You should have encountered exponential and logarithmic functions in previous study of mathematics. Review Section C.3 in Appendix C for the pre-requisite mathematical tools we will use in this chapter. Use SOMSE, available through the course website, for further support. A few key points relevant to the class are given below.

Science primarily studies phenomena that change. Often, the rate of change at any time is proportional to the amount that is currently there. This is typical of many populations. For example, each year the size of the global human population is increasing by around 1.5% of its current size.

Any phenomenon that has a rate of change proportional to the current amount follows an *exponential* function. (We will see why later.) An exponential function is of the form $y(t) = Ca^{kt}$, where a is the *base* of the exponent. C and k are constants. In many scientific contexts, Euler's number ($e \approx 2.718\ldots$) is used as the base, giving $y(t) = Ce^{kt}$.

Doubling time/Half-life

The **doubling time** for an exponentially growing quantity is the time it takes to increase to twice its current size.

The **halving time** or **half-life** for an exponentially decreasing quantity is the time it takes to decrease to half its current size.

Logarithms (or *logs*) are very closely related to exponential functions. Logarithms are the *inverse* of exponentiation (in much the same way that division is the inverse of multiplication).

Logarithms and exponentials

The relationship between exponentials and logarithms is:

- If $y = 10^x$ then $x = \log_{10} y$ (and vice-versa).
- If $y = e^x$ then $x = \ln y$ (and vice-versa).

5.2 Exponentials in action

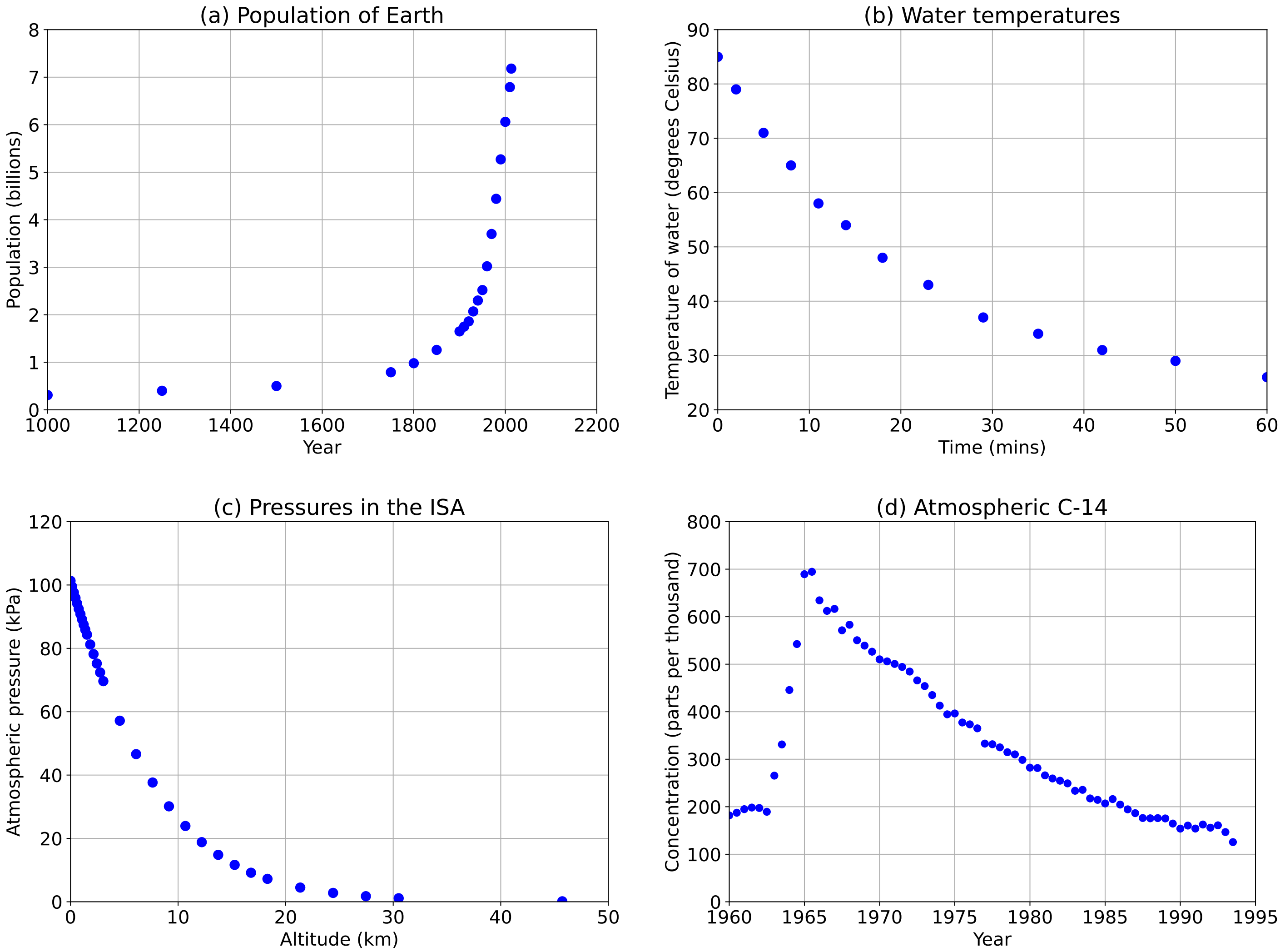


Figure 5.1: Four (possibly) exponential phenomena. (a) Population of Earth over 1000 years. (b) Measured water temperature in a simple experiment. (c) Atmospheric pressures in the international standard atmosphere. (d) Concentration of atmospheric radioactive Carbon-14.

Question 5.2.1

What do you think could have caused the observed changes of the atmospheric radioactive Carbon-14?

Case Study 9: **Radioactive decay**

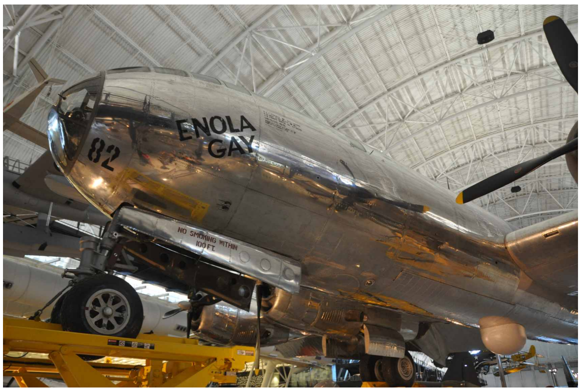


Image 5.2: The B-29 Superfortress bomber "Enola Gay", National Air and Space Museum, Virginia, USA. (Source: PA.)

Isotopes of an element behave the same way chemically but have different numbers of neutrons in the nucleus of the atom. One standard way of denoting isotopes is to write the name or chemical symbol of the element, hyphenated with its atomic mass. For example, deuterium (an isotope of hydrogen and the main ingredient in "Heavy water") is written as Hydrogen-2 or H-2.

Not all atoms remain the same over time; some undergo *radioactive decay*, which involves rearrangement of the nucleus of the atom, sometimes changing it into a different element.

Radioactive isotopes have useful applications in a range of sciences and industries, including chemistry, biology, medicine, physics and engineering. Therefore, it is important to understand how to model their decay.

Radioactive decay is spontaneous, so there is no way of knowing *when* a *specific* individual atom is going to undergo decay. However, it *is* known that in any given time period a certain *proportion* of the total quantity in a sample will have decayed. Thus, radioactive material undergoes continuous decay at a rate **proportional** to the **quantity** of material, so the decay is an exponential process.

Example 5.2.2

Half-lives vary greatly between radioactive elements. For example:

- Polonium-212 has a half-life of about 3×10^{-7} s.
- Uranium-238 has a half-life of about 4.5×10^{9} years.
- Carbon-14 has a half-life of about 5730 years.

Decay constant

The general form of the equation for exponential functions that we will use is $y(t) = Ce^{kt}$. For a radioactive element, the constant k is called the *decay constant* and reflects the rate of decay of the element. The decay constant is a property of the chemical isotope. The half-life can be calculated from the value of k, and vice-versa.

Example 5.2.3

Carbon-14 (C-14, also known as *radiocarbon*) is used to determine the age of organic-based artefacts (up to around 60,000 years).

Cosmic rays striking nitrogen in the upper atmosphere produce C-14. It then reacts chemically with oxygen to form radioactive carbon dioxide which permeates living creatures in a fixed proportion, either directly (by absorption from the atmosphere), or indirectly (via food chains).

When an organism dies, it ceases to accumulate C-14, and the remaining amount undergoes net decay over time. *Carbon dating* is the process of measuring the residual level of C-14 in organic artefacts, and thus deducing their age.

Question 5.2.4

The half-life of C-14 is 5730 years.

(a) Find the decay constant of C-14.

Question *5.2.4 (continued)*

(b) Consider the following extract from the paper [4].

In September 1991, a mummified corpse was found in a small rock depression on the Hauslabjoch, in the Ötztaler Alps, South Tyrol, Italy. Although the associated artifacts, including a copper axe, suggested a Late Stone Age origin, it was imperative to determine an absolute age for the Ice Man himself.

Researchers discovered that 57.6% of the expected original quantity of C-14 was present in a bone sample obtained from the mummified corpse. Deduce the approximate age of the corpse.

The corpse is now known as Ötzi (the Iceman), and is an extremely important archaeological find, being the oldest wet mummy in the world. Additionally, the clothing and equipment he carried is unique – no other organic material from the Copper Age has survived.

End of Case Study 9: Radioactive decay.

Case Study 10: **Hot stuff, cold stuff**

Moving an object with one temperature to a location with a different (but constant) temperature leads to a gradual change in the temperature of the object to match that of the new location. Energy (also called heat) is transferred to/from the object from/to the surrounds through processes such as conduction, convection and radiation. The rate at which this energy is transferred depends on the *temperature difference* between the object and its surrounds. Hence the temperature of the object as a function of time can be described by an *exponential function*.

Question 5.2.5

In an experiment, the temperature of hot water in a cooler, constant temperature container was recorded at various times over one hour; see Figure 5.2. The room (and container) temperature was measured to be 25 °C. Determine an equation for the water temperature at any time in minutes.

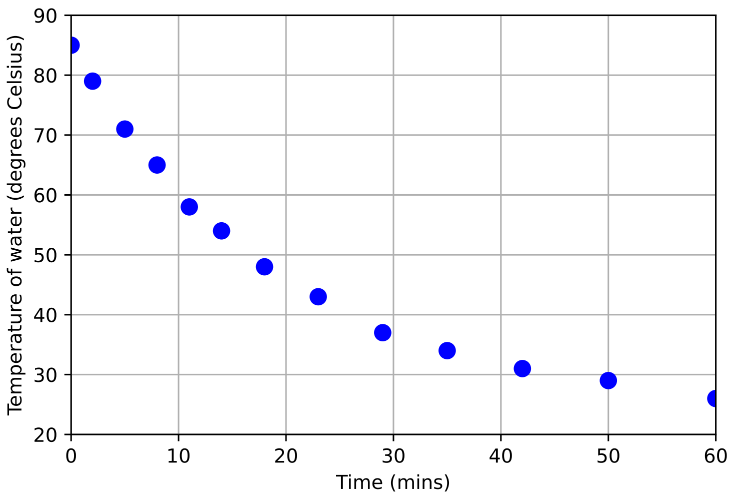


Figure 5.2: A graph of the measured temperatures.

Question *5.2.5 (continued)*

We can develop a computer program to model the temperature.

Program specifications: Write a program that plots the measured water temperatures and the function that models these temperatures.

Program 5.1: Temperatures

```
# Program to plot measured and modelled water temperatures.
import numpy as np
import matplotlib.pyplot as plt

# Measured temperatures (minutes, degrees C)
times = np.array([0,2,5,8,11,14,18,23,29,35,42,50,60])
temperature_data = np.array([85,79,71,65,58,54,48,43,37,34,31,29,26])

# Model
temperature_model = 60 * np.exp(-0.05 * times) + 25

# Draw graph
plt.plot(times, temperature_data, "bo", markersize=8, label="Data")
plt.plot(times, temperature_model, "r-", linewidth=3, label="Model")
plt.xlabel("Time (mins)")
plt.ylabel("Temperature of water (degrees Celsius)")
plt.xlim(0,60)
plt.ylim(20,90)
plt.grid(True)
plt.legend()
plt.show()
```

The output is shown in Figure 5.3.

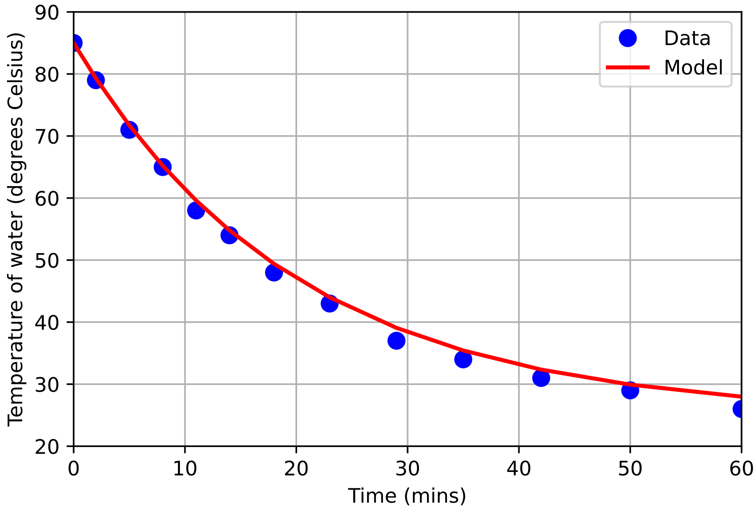


Figure 5.3: Modelled and measured water temperatures.

Question 5.2.6

Do you think the model shown in Figure 5.3 is a good fit to the given data? If you were to use this model, justify your choice. If you were to modify the model, what change or changes would you suggest and why?

End of Case Study 10: Hot stuff, cold stuff.

5.3 Logarithms in action

Logarithms provide a convenient mechanism for converting exponential data into a form that can make data analysis easier.

***Question** 5.3.1*

Assume some data are modelled by the exponential function $y(x) = Ce^{kx}$. Demonstrate how a logarithmic transformation of the data values results in a linear model. Interpret the y-intercept and gradient of the linear model. (Hint: if x and y are positive then $\ln(xy) = \ln x + \ln y$.)

Question 5.3.2

Earlier we saw that the *International Standard Atmosphere* (ISA) [31] models various atmospheric properties, including temperature, pressure and density. Figure 5.4 shows atmospheric pressures in kilopascals (kPa) at various altitudes in the ISA in a linear-linear and log-linear form.

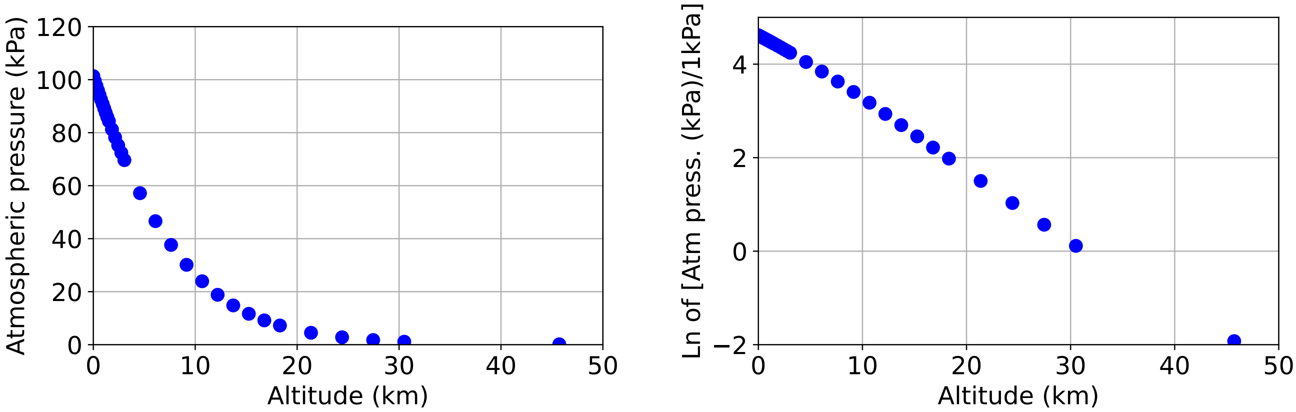


Figure 5.4: ISA pressure (linear and transformed data).

(a) Use Figure 5.4 and Question 5.3.1 to find an exponential model of pressure in the ISA.

Question *5.3.2 (continued)*

(b) When a jetliner is in flight, the pressure in the cabin is artificially raised to a higher level than the pressure outside. The *cabin altitude* is the altitude at which atmospheric pressure matches the pressure inside the cabin.

Modern planes typically cruise at an altitude of 12,000 m, but maintain a cabin altitude of about 2,000 m. Determine the pressure inside and outside the cabin when cruising. Note that, on the ground, atmospheric pressure is around 100 kPa.

Photo 5.1: Bang? (Source: PA.)

We can also use logarithms to help analyse power model data.

Question *5.3.3*

Assume we now have some data modelled by a power function of the form $y(x) = ax^p$. Demonstrate how a logarithmic transformation of this data can also result in a linear model. Again, interpret the y-intercept and gradient of the linear model. (Reminders: if x and y are positive then $\ln(xy) = \ln x + \ln y$ and $\ln x^p = p \ln x$)

Question 5.3.4

We can apply this approach to developing an equation for the species-area curve that we introduced when considering power functions. Figure 5.5 shows our earlier species-area data in linear and log-log form.

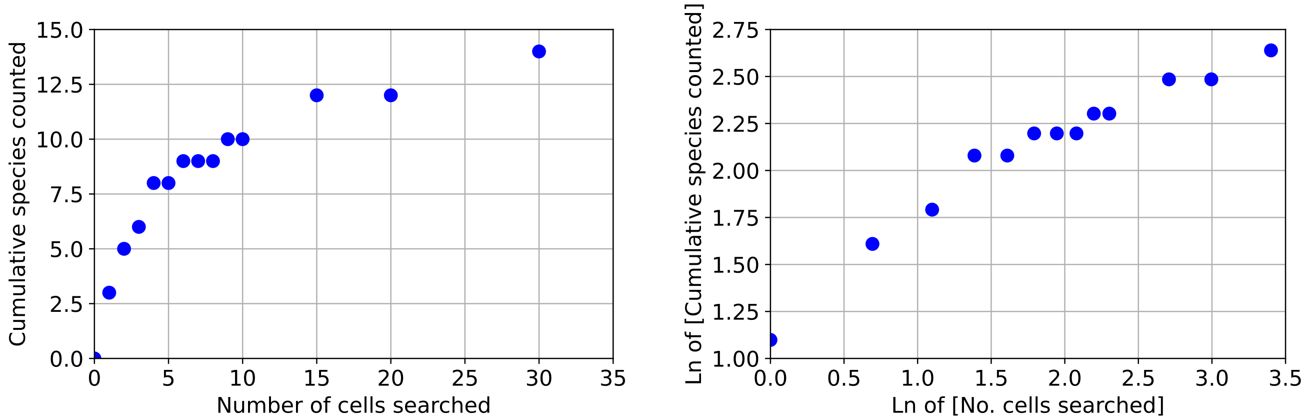


Figure 5.5: Species-area data (linear and transformed data).

Use Figure 5.5 and Question 5.3.3 to find a power law model fitting the data.

In summary

- A log-linear plot is useful in examining data that may be modelled by an *exponential function*.
- A log-log plot is useful in examining data that may be modelled with a *power function*.

Chapter 6: **Scientific reasoning - Part 1**

Lecture 6: **Philosophy - Part 1**

Learning objectives

✓ Clarify the module question, 'How is science rational?'

✓ Realise that some important questions are not able to be answered by experiment (e.g. how is science rational?)

✓ Develop clarity and precision in language (e.g. being clear about what we mean by the terms 'theory', 'law', 'proof' or 'model')

✓ Consider how models are created and used in science

✓ Examine how induction works ✓ Recognise the proper role subjectivity has in science

✓ Consider challenges to the idea that induction is a rational process, including that of justifying induction; formulate a version of the principle of induction that addresses some of these challenges

✓ Understand the problem of induction

✓ Recognise what special challenges models pose for the inductivist view of science

Image 6.1: Émilie Du Châtelet (1706-1749), mathematician, physicist and philosopher of the French Englightenment, portrait by Latour [73]

6.1 Introduction: science and the assumption of rationality

News Headlines:

"No Rational Person Can Deny Human-Induced Global Warming"

What does this mean? What, if anything, is scientific "rationality"?

Many people believe that science is rational and that this is because there is such a thing as *the scientific method* by means of which we are able to make reliable claims about the natural world. Results arrived at by means of the scientific method have a special status – *scientific knowledge is reliable knowledge*, unlike claims made on the basis of common sense. This is not to say that common sense never leads to truth or science always does, but rather that our grounds for accepting claims made in the name of science are stronger than those for accepting claims made on the basis of common sense – this by virtue of the method used for arriving at the claims made.

To illustrate the point, consider the conflict between the common sense view that the sun revolves around the earth (the earth feels stationary, the sun seems to move across the sky, etc.) and the scientific view that, in fact, the earth revolves around the sun. This conflict is typically resolved in favour of the scientific view. In general, where science and common sense conflict, common sense gives way.

So consider two public speakers A and B. A stands and proclaims that human-induced global warming is not occurring, B claims it is. When asked, A admits that he claims no global warming due to human activities is occurring because common sense suggests that any fluctuation is more likely to be part of a natural cycle of climate change. B, on the other hand, admits that she claims human-induced global warming is occurring because scientific evidence all points to the fact. Now, *regardless of who is in fact right, who do we have more reason to believe?* B would commonly be said to be the more credible of the two by virtue of the means employed for arriving at the claim – B's method is more reliable, more rational, than A's.

The general view underlying this resolution of the conflict seems to be, again, that scientific claims to knowledge have some kind of merit not shared by common sense claims. But if merit attaches to scientific knowledge then why? What makes science rational? The common answer, again, is that science employs a method which is rational and is the means by which scientific knowledge is arrived at. This is what we will be discussing in these workshops.

6.2 Getting Philosophical

Of course, at this point, in discussing the general nature of science, we are engaged in an activity other than science itself. No amount of scientific experimentation will tell us whether or how science is rational. Just think about it for a minute. Asking questions about what science is and how it works is not something that we can do in a lab. White coats, bunsen burners and experiments won't help. When we question our beliefs *about science* we step out of science itself to a more abstract level of discussion. We are engaged in *philosophical* debate and argument about the nature of science.

The following workshops are directed at introducing you to some of the philosophical issues that arise in attempting to explain the apparent rationality of science and scientific method.

6.3 Some Preliminaries

6.3.1 What is Science?

What is science? As we shall see, it is not a body of *facts*. Despite common views to the contrary, science is not in the business to putting forward proven facts. Scientific laws and theories are continually being overthrown in the face of problems, or anomalies, that the laws or theories fail to adequately account for – they are always *provisional* to some degree, as are the specific scientific claims that depend on them. Newton's Laws, for example, were never scientific facts. They were conjectures that were eventually overthrown by "better" laws, those of relativity theory.

Science is a way or method of thinking: thinking **critically** about **the empirical world** using **evidence** to try to justify **hypotheses**, **laws** and **theories** (collections of laws about some set of phenomena), put forward as **conjectures** that are subject to further critical testing against ever-increasing bodies of evidence.

Key Point: Science is not a body of facts. It is a way of thinking critically about the empirical world using evidence to make general conjectures.

Just think about the hypothesis of human-induced global warming. We appeal to evidence about humans increasing carbon dioxide levels in the atmosphere and their effect on the heat-retaining capacity of the atmosphere (the so-called "green house effect"), and go on to conjecture that *we* are thus the cause of a warmer climate. This conjecture is further tested by increasing bodies of evidence about past climate variation and its possible causes. And so it goes.

This conjecture is, of course, a hypothesis in the applied field of climate science. As applied science, it depends on a large number of even more fundamental physical and chemical laws, as well as associated mathematical principles that enable modelling of exponential growth of gas concentrations and summative effects of gas concentrations as a result of chemical reactions, etc. And these currently accepted laws and mathematical principles themselves are taken as (currently) justified. So how do we justify these? What justifies *these* fundamental laws and principles as acceptable?

For obvious reasons, we shall limit our discussion to *scientific* theory and set aside the many (interesting) issues surrounding the acceptability of mathematics used in science. The philosophy of mathematics is yet another area of philosophical enquiry with a heritage stretching back to the ancient Greek philosopher Pythagoras and beyond. The notion of mathematical truth, in particular, has been the subject of considerable study – most notably in the late 19th and early 20th centuries with key players like Bertrand Russell, David Hilbert and Kurt Gödel. Our focus, though, is squarely on scientific hypotheses, laws and theories.

6.3.2 Hypotheses, Laws, Theories and Models

Science is made up of many hypotheses and laws, and groups of them that work together to make up scientific theories – evolutionary theory, electromagnetic theory, and so on. Let's just stop for a minute to get clear on our terms here. As potential scientists you will come across the terms 'hypothesis', 'law' and 'theory' a lot, and the way they are used in science is sometimes different to how they get used "on the street", so some clarification may help here.

— **A Hypothesis:** a (scientifically testable) claim used to predict or explain some particular phenomenon or event.

  E.g. hypothesising that, since a ball began to move, it was acted on by some force. The hypothesis is that the ball was acted on by some force.

— **A Law:** a (scientifically testable) claim describing *a general regularity* in nature used to predict or explain some particular phenomenon or event.

  E.g. All bodies remain at rest unless acted upon by some force.

— **A Theory:** a set of interconnected laws and principles working together to form a *model* (typically involving significant idealisation) used to explain the general regularities themselves.

  E.g. Einstein's general theory of relativity (including $E = mc^2$, etc.) is a model that explains gravitational laws. Darwin's theory of evolution (including principles of natural selection, etc.) is a model that explains biological diversity.

Thus theories explain how the world works in general and why it works as it does by providing a model of the system in question. Hypotheses and laws are simply used to tell us why some particular thing happened but might themselves stand in need of explanation.

NB: Later in these workshops we'll pin down, more exactly, what we mean by "scientifically testable". For now, the above distinctions should be sufficiently clear.

Some more examples:

1. "Gravity is caused by undetectable particles exerting forces" may appear to be a hypothesis but is not (it is not testable – as we shall see later when discussing testability).

2. "The postie is sick" is a hypothesis (that one might offer to explain the lack of mail), but is not a law (it is not general).

3. "$F = ma$" is a law (describing a general regularity: when a force acts on an object it is caused to accelerate by an amount which when multiplied by the mass of the object, equals the force applied), but is not a theory. The relationship between the three quantities remains unexplained.

4. "Einstein's theory" explains general features of the observable world expressed by gravitational laws, etc.

Common Misuses and Abuses

Note that these key terms are not always used this way outside of science.

"Theory": People sometimes mean a mere *guess* or speculation lacking any support. (For example, "That is just a theory".) The former US President Ronald Reagan was famously reported as saying "Evolution is a theory, a scientific theory only…". He meant it was just a guess, arguing that it was no more reasonable to believe than creationism (the view that the world was created by God rather than evolved).

"Law": People sometimes mean a general regularity in nature that has been *proven*. (For example, "But that is a law. It cannot be wrong.")

Scientists generally use the terms 'theory' and 'law' in a way that is neutral concerning whether they have no support, some support, or very strong support. More generally, the terms 'hypothesis', 'law' and 'theory' do not indicate a difference in how well established or proven a scientific claim is; they indicate the kind of claims in question – a specific claim that is general (law) or not, or an explanatory set of claims (theory). When we want to indicate that we are speaking of the *currently supported or accepted* view, we often speak of *the* theory of evolution, or the law of gravity. (Here we indicate that one from among many candidates is accepted.)

Don't forget about models!

— **Models:** idealised, i.e., simplified or distorted, representations of portions of the world. Models can be theory driven, that is, arrived at by the application of theory. Models can also be data driven, that is arrived at on the basis of data. Often, models are theory and data driven.

E.g. The Keeling Curve is a data model of the accumulation of Carbon Dioxide in the Earth's atmosphere.

It is partly because models are idealised that they are so useful. Data would be unmanageable if we did not simplify/distort it, e.g., by modelling it with a simple function. Theories would often be hard, if not impossible, to test or apply if they were not simplified in the process of their application. The idealised nature of models, therefore, enables them to connect our theories, hypotheses and laws – our 'ideas' – with data – the world – and thus enables testing our ideas. Models are, as the philosophers of science Mary Morgan and Margaret Morrison neatly put it, mediators.


Image 6.2: Margaret Morrison [32] and Mary S. Morgan [81]

6.3.3 The Task Ahead

Scientists typically believe that their way of thinking about the world involves some method, "the scientific method", and this rational method gives a way of justifying hypotheses, laws and theories (the results of scientific activity) as scientific knowledge. So what is this "method" and how does it produce scientific knowledge?

6.4 Science and Inductive Reasoning

A popular view of scientific method is that science begins with particular observations of natural phenomena. From observation one logically arrives at general principles – scientific laws – by an inference known as *induction*.[1]

For example, imagine Boyle (1627-1691) studying the behaviour of a gas at constant temperature. He observes the following numerical measures of its volume at different pressures, in appropriate units:

Pressure	Volume
1	12
2	6
3	4
4	3

From an examination of these few measures he infers the general law that the product of the pressure and volume is constant (given constant temperature) – Boyle's Law.

Justification of scientific laws is thus by way of a special kind of generalising argument – using "inductive reasoning" to infer the law-like structure of the universe. Our belief in the laws of science is therefore rational since based on logical argument from evidence…or so the story goes according to the inductivist.

In the same way, Avicenna (the 10-11th century Arabic philosopher, scientist, physician and mathematician) gave the following example of inductive inference, referring to the purgative effects of scammony[2] (not to be tried at home!):

It is observed that ingestion of scammony is followed by
the discharge of red bile
This observation is repeated under circumstances in which
other possible causes of the discharge of red bile have been excluded
Hence, all scammony according to its nature withdraws red bile.

We seem to use this kind of inference all the time in daily life. Why expect that the fire will burn me? Because it has numerous times in the past. Why think that food will nourish me? Because it has numerous times in the past.

The main principle underlying this use of "inductive inference" is the idea that *what occurs frequently does not do so by chance.*

[1]"Induction" here is not to be confused with the mathematical inference known as "mathematical induction".

[2]A twining plant having a stout taproot, *Convolvulus scammonia*, found in Syria, Asia Minor, Greece, etc. The dried milky juice was used as a medicine from ancient times.

In this way, from observations of lunar eclipses and other phenomena concerning light one inductively infers general principles or laws – e.g. that light travels in straight lines; that opaque bodies cast shadows; and that certain configurations of opaque and luminous bodies place one opaque body in the shadow of another. (See the left side of Figure 6.1.)

These general principles or laws themselves can then serve as assumptions, along with some other facts concerning particular conditions etc., enabling one to *deduce* statements about phenomena. Continuing with the example above: the laws concerning opaque bodies and light, along with the fact that the earth and the moon are opaque bodies and the fact that on such-and-such a date the earth will pass across the line between the luminous sun and moon, can be used to deduce that there will be a lunar eclipse on that date. In this way we can make *predictions* about events not yet observed. (See the right side of Figure 6.1.)

Alternatively, by showing how an observation can be deduced from the laws one can progress from an already observed fact that the lunar surface has darkened to an understanding of *why* this took place. In this way, deducing the observation from laws about the nature of light and opaque bodies, along with particular facts, we can provide an *explanation* of events already observed.

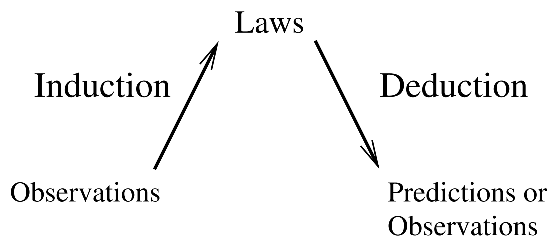


Figure 6.1: Induction and deduction.

On this account, "induction" is the crucial process used to arrive at laws. It allows us to infer some general law-like property or relation from a number of particular, observed cases or events. Our experience of some novel scientific phenomenon (for example, the effect of some drug on the human nervous system) is usually very undiscriminating; we do not initially see the general principles at work – we begin with a confused mass. However, over time, with sufficiently many repeated occurrences of the phenomenon, we are able to infer the general principles underlying the phenomenon – we reason our way to a universal law-like feature of the universe. By examining many cases we can inductively infer a formal pattern.

Key Point: Inductivist models of scientific thinking claim scientific laws are justified by *induction* from observations of a particular phenomenon.

6.5 The Renaissance: experimentation and mathematics

Notice the use made of an actively pursued set of experimental results in the second premise of Avicenna's inductive inference. Later, in the 16th and early 17th century, the Renaissance development of instruments like the telescope, microscope and accurate clocks meant that much more sophisticated experiments could be undertaken (e.g. in the microscopic realm and the astronomical realm). And more extensive use was made of increasing amounts of experimental evidence that was becoming available. The emphasis on active experimentation to acquire new and relevant data for theorising became increasingly significant.

Another significant Renaissance development was the increasing use of mathematics and mathematical modelling in scientific theorising.[3] In the Renaissance the idea of a "clockwork universe" developed – a conception of the universe as like a giant mechanical clock (typically with God the great Watchmaker who set it all in motion) governed by laws amenable to precise mathematical modelling, and there was a shift from qualitative analysis to quantitative analysis and measurement. Scientists began to theorise about motion with idealised mathematical models invoking frictionless planes, perfectly spherical bodies, etc. This reached its greatest expression in Newton's Laws of Motion, whose mathematical simplicity and wide applicability confirmed a view of the cosmos as essentially mathematical in nature.

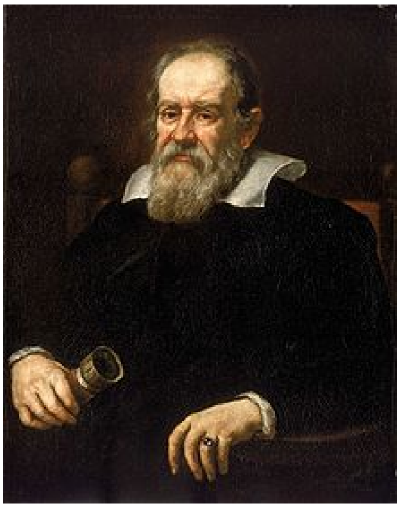


Image 6.3: Galileo, looking a touch haggard [76]

A century before this great Newtonian triumph, Galileo had put the point clearly:

> *Philosophy [i.e. science] is written in that vast book which stands forever open before our eyes, I mean the universe; but it cannot be read until we have learnt*

[3]It seems hard to imagine science without sophisticated mathematics involved but, remember, the calculus of infinitesimals – developed by Gottfried Leibnitz and Isaac Newton, and necessary for the modelling of motion, acceleration, population growth, etc. – and probability theory – developed by Blaise Pascal – were not developed until the 17th century.

> *the language and become familiar with the characters in which it is written. It is written in the mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.*[4]

The book of nature is written in the language of mathematics, and that is why you need to get some mathematical skills under your belt before getting anywhere in science. If you don't like the maths, blame the Renaissance![5]

Key Points: In the Renaissance, technological innovation led to more *active experimentation* and an emphasis on the pursuit of observational evidence. Renaissance science also emphasised the *mathematical* structure of the scientific universe.

QUESTION: Could we do science purely *qualitatively* – i.e. using notions like *strong*, *weak*, *hot*, *cold*, *near*, *far*, *fast*, *slow*, etc. – instead of the Renaissance push to quantitative measurement? If not, why not?

Notes:

[4] *Il Saggiatore (The Assayer).* Quoted in A.C. Crombie's *Grosseteste and Experimental Science*, Oxford (1953) p. 285.

[5] For some examples of this increasingly mathematical approach to science see: H. Kearney, *Science and Change 1500-1700*, Weidenfeld & Nicolson (1971), pp. 66-7; A.C. Crombie, *Science, Optics and Music in Medieval and Early Modern Thought*, Hambledon Press (1990), p. 325.

6.6 A Common View of Science

Consistent with the account of scientific method described earlier, a popular view of science is described by Alan Chalmers as follows:

> Scientific knowledge is proven knowledge. Scientific theories are derived in some rigorous way from the facts of experience acquired by observation and experiment. Science is based on what we can see and hear and touch, etc. Personal opinion or preferences and speculative imaginings have no place in science. Science is objective. Scientific knowledge is reliable knowledge because it is objectively proven knowledge.[6]

The notion of "proof" referred to here will, for the inductivist, be proof by inductive inference. On this "inductivist" interpretation of the popular view then, scientific reasoning is inductive and scientific method yields reliable knowledge through the application of inductive reasoning from the "facts of experience".

There are problems here though:

(i) The idea that scientific knowledge is *proven* knowledge can mislead us to think of such knowledge as certain. It is not.

(ii) The idea that science is objective and that "personal opinion or preferences and speculative imaginings have no place in science" is misguided.

(iii) The reliability of induction as a form of proof is difficult to justify.

Let us turn firstly to the problems concerning (ii).

6.6.1 Discovery versus Justification

With a little thought it seems obvious that there is a distinction to be made, first clearly drawn by John Herschel in the 1830s, between the means by which scientific theories are *discovered* – the context of discovery – and the means by which they are to be *justified* – the context of justification. How we discover a theory – i.e. the means by which we come to have the theory in our minds – is one thing, whereas establishing a theory as rationally acceptable – i.e. justifying it – is another thing.

The common view described above seems to wrongly characterise science by ruling as illegitimate that highly imaginative and creative aspect of science whereby practitioners "cook up" theories for consideration and testing. Who ever came up with the idea that there could be "dark matter" or "dark energy" and what in heaven's name were they on

[6] A. Chalmers, *What Is This Thing Called Science?*, (1976) p. 1.

when they thought up the idea? Who cares? What is relevant is simply whether or not such an idea (indeed, whether any scientific law, hypothesis or theory) can be justified. For, irrespective of how someone came up with the idea, what matters from the point of view of understanding the universe is whether or not such an idea can be justified.

With this in mind, we do not need to give any account of the process whereby we *discover* scientific theories. We do not need to claim that the process is in any way rational or reliable, let alone suggest that it proceeds by way of induction. An account of scientific method as essentially inductive then need only be committed to the view that scientific laws are *justified* by induction. Personal opinion and preferences, and speculative imaginings play a key role in science. Science is, in this sense, clearly subjective. Imaginative theorising by individual subjects puts hypotheses on the table for consideration which might otherwise never have been considered (these hypotheses are not "objectively given" to us), and some of the greatest honours in science go to those who have used their subjective imagination in ways that are ingenious, and which have produced theories which have subsequently come to be seen as justified. But how are they justified? This, according to those advocating an inductive scientific method, is by way of inductive inference. And so to problem (i).

Key Point: The discovery of scientific hypotheses – a process entirely separate from justification – is a *creative* process that infuses science with subjectivity.

6.6.2 Inductive Inference and Fallibilism

What *exactly* is the logic of inductive inference? Consistent with Avicenna's example, inductive inference, according to one of its most famous advocates, the philosopher J.S. Mill (1806-1873), consists in inferring from a finite number of observed instances of a phenomenon, that it occurs in *all* instances of a certain class that resemble the observed instances in certain ways.[7] For example, from the fact that ingestion of scammony is observed on a number of occasions to produce red bile we infer that its ingestion always produces red bile. Similarly, from the fact that Nicole, Bianca, etc. are all mortal we infer, by induction, that all humans are mortal. Let's look at this in more detail.

Observational (singular) statements

The simple inductivist account claims that science starts with observation. The scientist, with normal, unimpaired senses records what she sees without prejudice. Impartial reports as to how the world operates are justified by the use of the senses. Statements reporting these particular facts – often referred to as *observational statements* – serve as the basis for the derivation of scientific laws. Examples of some simple observation statements are:

At midnight on Jan. 1 1975, Mars appeared at position x in the sky.

Mrs Smith struck her husband.

[7]J.S. Mill, *A System Of Logic*, Vol. I, p. 354.

> The water boiled at 100 degrees Celsius at sea level (at place p and time t).

Observing what is the case will establish such statements as true or false at a particular place and time. Such statements are *singular statements*; they describe a particular event or state of affairs at a particular place and time.

Universal (general) statements

Scientific statements however describe general patterns in nature. For example:

> Planets move in ellipses around their sun. (Astronomy)
>
> Animals in general have an inherent need for some kind of aggressive outlet. (Psychology)
>
> Water always boils at 100 degrees Celsius at sea level. (Physics)

These statements refer to *all* events of a particular kind at any place and time; they are not about particular events or states of affairs but suitably general. The laws and theories of science involve general statements of this kind; they are *universal statements*.

The problem then for those who think that science starts with particular observation statements is to explain how one can justifiably arrive at universal statements from particular ones. For example: just because water boiled when heated to 100°C at sea level by person x_1 at place p_1 and time t_1, and by person x_2 at place p_2 and time t_2, and by person x_3 at place p_3 and time t_3, and ... and by person x_n at place p_n and time t_n, how can we thereby justifiably infer that the result holds in general, for all future times, places and persons? How can the general be justified on the basis of the particular? How can a set of observations about how the universe is now justify claims as to how the universe is in general?

Inductive reasoning

The inductivist reply is that under certain conditions we can generalise. So long as the following conditions are met, we may legitimately generalise to an appropriate universal statement:

1. The number of observation statements forming the basis of a generalisation is large.
2. The observations are repeated under a wide variety of conditions.
3. No accepted observation statement conflicts with the derived universal law.

Condition (1) helps rule out anomalous cases (e.g. a defective measuring instrument) and stops one jumping to conclusions prematurely. Condition (2) implies that it is not enough to increase our base of singular statements by simply repeating tests on the same subject under the same conditions. To rule out the possibility of the observed phenomenon being due to some hidden factor we ought to test for the phenomenon under as varied conditions as possible. For example, the claim ‘All liquids contract when frozen’ would seem a justified generalisation if water was not considered; testing liquids under a wide variety of conditions will include testing the liquid water which is unusual in that it expands when frozen. Now obviously if water is observed to expand when frozen then the universal law ‘All liquids contract when frozen’ is not justified. Hence condition (3) is necessary.

This kind of reasoning – from a finite list of singular statements to a universal statement, from some to all – is called *inductive reasoning* and the process of reasoning thus is called *induction.*[8] The simple Inductivist position can be summed up by saying: science is based on inductive inference.

> *Principle of Inductive Inference:*
> If a large number of As have been observed in the past, under a wide variety of conditions, to possess the property B without exception we can infer that all As have the property B.

Scientific knowledge is built up from and justified by a secure base of particular observation statements by induction.

Science then, on this account, is justified by its use of induction in inferring, from particular observations, general laws and theories; *scientific statements can be inductively justified by experience.* Scientific statements, based on observational and experimental evidence (i.e. the facts) are contrasted with statements of other kinds – those based on pure logic or mathematics, authority, tradition, prejudice, or any other foundation. Scientific statements are derived in a rigorous and objective manner from objective facts. Science is a body of such knowledge and scientific progress then is the piecemeal addition of laws and theories to that body of knowledge; the accumulation of facts, and new laws and theories arrived at via induction form the ever-growing observational base. This cumulative conception of scientific knowledge is sometimes called ‘the bucket theory’.

Fallibilism

So the notion of proof that the inductivist relies on is inductive “proof”. But it is important to realise that inductive proof falls short of certainty. Call it “proof” if you want, but it would be wrong to think inductive inference is anything like mathematical proof. When we prove Pythagoras’s Theorem, we justify it as true, and because of the nature of mathematical proof we then take it to be established with *certainty.* There is no “probably” about it. This

[8]NB: there are other forms of inductive reasoning, like inferring from the fact that water has always boiled at 100^0C at sea level in the past that it will do so when I next boil it. The problems we go on to discuss apply equally to these other forms but for simplicity we shall concentrate on the simple form presented here.

is not the case with inductive proof. Inductive inference cannot establish laws as certain. At best, it makes them *highly likely*.

For example, if water has been observed, on numerous occasions, to boil at 100°C at sea level without exception, at best that only makes it *likely* or *probable* that all water boils at 100°C at sea level. It is always left open to further contradictory evidence (evidence that we must recognise may be "out there", for all we know). We must recognise that our scientific investigations yield results that are clearly *fallible*. No-one in their right mind these days would claim that a currently "proven" scientific law is established as certain and so beyond revision. Since we cannot assume we have all the relevant data, scientific laws are always to be considered open to revision. In fact, we are constantly revising our scientific understanding of the world on the basis of new evidence and this involves admitting that we didn't have things "quite right" previously – a polite way of saying we were, in fact, *wrong* in what we previously thought!

If you think about it, everything we claimed to know about the scientific structure of the world in the past has been shown to be wrong. Scientists were once confident, for example, that Newton's laws of motion were absolutely certain, but new scientific evidence found in the early twentieth century led to developments in physics that resulted in their rejection in favour of more general relativity theory. Of course, you may want to say that Newton's laws weren't "wrong", they were just too general – they are a correct account of motion at low speeds. But that is to admit that, as general laws of motion (what they were put forward as describing), they *were* wrong. They are not true. More recently, scientists claim to have new evidence that suggests that they cannot account for some 80 percent of matter in the universe! They now speak of "dark matter" and "dark energy" – the stuff they can't yet detect but suppose is there. What revisions of our scientific laws will this lead to? We'll see. The point is, we can never discount the possibility of new evidence forcing revisions of our scientific understanding.

Science is an activity of constant testing of what we think we know – our *fallible* claims to knowledge – and constant searching for new information that might further confirm or refute what we think we know.

Bearing this in mind, the corresponding Principle of Inductive Inference should reflect this. It now says (and perhaps this what many had in mind all along when it comes to induction):

> *The Weakened Principle of Inductive Inference:*
> If a large number of As have been observed in the past, under a wide variety of conditions, to possess the property B without exception we can infer that all As probably have the property B.

The common-sense account of science mentioned earlier that sees science as providing objectively *proven* knowledge is mistaken if 'proof' is read as 'proved with certainty'. Inductive proof can, at best, establish laws as reasonable-given-the-evidence.

Accepting this fallibilist shift then, we may ask why we ought to accept the weakened principle? Can such a principle be justified? And so to problem (iii) identified earlier.

Key Points: Simple induction proceeds from the *singular* to the *general*. Legitimate inductive inference is subject to certain *conditions* (1 - 3 above). Unlike mathematical proof, scientific proof is *fallible* – induction only justifies scientific laws as *probable*.

6.6.3 The Problem of Induction

As we have seen, any conclusion reached by inductive reasoning is always fallible. This raises an interesting worry about scientific knowledge: if inductive reasoning is always fallible, how are we ever to *justify our belief* in conclusions that we reach by inductive reasoning? The problem of trying to answer this question is known as the problem of induction.

Consider this argument for the claim that the sun will rise tomorrow: the sun has risen every day up to now; therefore, the sun will continue to rise in the future. This is clearly an inductive inference, as we infer a general regularity from a number of particular observations. But how do we go about justifying this inference?

To begin with, we could argue that this inference is known without looking at the world, purely on the basis of logic or reasoning. An example of an inference that is known on the basis of logic might be "If no As are Bs and all Cs are Bs then no As are Cs". If you take a moment to think about this inference, you should be able to recognise that it is true – you can do this just on the basis of logical considerations. Justifying this inference does not require anything but a nice peaceful room and pure thinking alone. So too with "$2 + 2 = 4$". You don't need to know anything about how the world actually happens to be, no particular facts, to justify it. So can we justify our inductive inference that the sun will continue to rise just on the basis of logic and thus without looking at the world?

The short answer is: No! The conclusion of the inference implies that some regularity will *always* be the case. But no matter how many observations we make of particular cases, we can never observe enough cases to ensure that the regularity will always be the case – the number of particular observations seems to pale into insignificance next to the potentially infinite number of possible situations we are making claims about by way of the regularity. Our evidential support for some law or theory always appears insignificant compared with the full strength of our general claim. This is, of course, just the fallibility of science as we've discussed above.

Consider an analogy. Suppose you are out on a lake in heavy fog. Everywhere you can see in your vicinity is water. This is how things are in your bit of the world at your particular time and you are utterly ignorant of everything else. You have only ever lived in a boat on water, with the rest of the world obscured by fog. How confident could you be in inferring, by induction, that the universe in general, throughout space (i.e. everywhere) and time (i.e. always), is water? Not very. How things seem here and now is not a very reliable guide to how they are everywhere and always. Yet, the (weakened) inductivist is essentially in this position, it seems. From a finite number of observations over (at best) a few hundred years, in our little local part of the cosmos, how confident can we be that we can know with a high probability how things are in the cosmos in general – as when we claim to have confidence in a scientific law? Not very. Our justification is very weak, as it involves judging the nature of the cosmos in its potential infinity from evidence of a small, finite part.

(A lot of effort has gone into trying to develop a notion of 'probability' and an inductive logic that will enable us to logically calculate or estimate the degree of support theories attain given the body of evidence in their favour but that story continues and we leave it to those interested to read further... [9])

It should not be so surprising that this effort to justify our inference failed. This is because scientific claims simply are not purely a matter of logic. That is, we cannot simply sit in a dark cupboard and think to decide whether some scientific claim is true; on the contrary, we need to open the door and have a look at the world outside to make that assessment. Consider a scientific hypothesis like, "SCIE1000 students are the smartest students on campus". Thinking and pure reason alone, without any experience of how the world happens to be, cannot justify this claim. Sitting in a dark room with no idea of the outside world, one cannot prove it. We can only justify the claim given some knowledge of how the world is, after we have investigated the particular facts "on the ground", as it were.

So perhaps we can justify our inductive inference that the sun will continue to rise by looking at the world. That is, if we can find out enough about how the world is then perhaps we can discover that this general principle is, as a matter of fact, true. We might say something like the following. Whether the sun continues to rise is dependent upon whether the Earth continues to rotate about its axis (as well as around its orbit) after we lose sight of the sun at sunset tonight. We have confidence that the Earth will continue to rotate about its axis given the direct observation that the Earth has continued to spin at a more-or-less constant angular velocity throughout the duration of our lives (and the indirect collective "observation" amongst humans that it has done so for many millennia).

But we have a problem with potential justification of our inductive inference: it is based on an assumption that what we have observed to be the case in the past will continue to be the case in the future. That is, we are assuming that the world around us is uniform, in that the way the future will be will resemble the way the past has been. We can put this assumption explicitly into our argument: the sun has risen every day up to now; nature

[9] See: W. Salmon, *The Foundations of Scientific Inference*, (1966).

operates uniformly; therefore, the sun will continue to rise in the future.

It certainly seems reasonable to assume that nature will behave as it has always done; the regularities observed in nature do not seem likely to be disrupted. But how do we justify *this* claim? It may be that our universe is really quite irregular but we find ourselves in a special pocket in which things are regular just at the moment. Once we move out of this pocket, we would have very little justification for believing the assumption that the universe were regular, that nature is uniform. If we could indeed justify this assumption then we would have good reason to justify many of the inductive inferences made by science. So how *do* we justify this assumption?

The problem here is that it looks like the only justification for this assumption that we could possibly have is itself an inductive inference. That is, it seems that the only justification we can provide for the assumption that nature is uniform is by appealing to our experience: based on our observations of the world around us, the world is uniform; things continue to behave in the way that we observe them to behave; in the past, nature has operated uniformly. But to make this claim is to make a generalisation from particular observations of the uniformity of nature to a generalisation about uniformity. It is itself an inductive claim! Thus, using the uniformity of nature as a justification for the inductive inferences made by science is circular, as we would be justifying all our inductive inferences inductively.

We might think that the weakened principle of induction could help us escape the problem of induction. But this is not the case. Saying that we are only inferring what is *probably* true might sound more modest and cautious, but it doesn't actually help us justify the inference. The issue is this: how do we know that the fact something has happened many times in the past makes it *likely* to keep happening in the future? The appeal to probability only makes sense if we can show that the observed regularity gives us some rational grounds for expecting it to continue—yet that is precisely what is in question. So if we ask why we are entitled to regard some inference as *probably* correct, we are left in the same position as before: either we appeal to a past success rate of similar inferences (which is itself an inductive generalisation), or we simply assert that such inferences are reasonable, both of which are circular. Thus weakening the principle of induction does not escape the core problem: the issue remains that any justification for the inference, even in this weaker form, still relies on the very thing we are trying to justify—inductive reasoning itself.

This is a problem for science. Inductive reasoning relies on the assumption that nature is uniform and the only way to justify that assumption is by using inductive reasoning; the inductive inferences of science are thus unjustified. So, we have a problem justifying the inductivist's account of what science is.

Key Points: Induction seems impossible to justify without using inductive inference.

So there is a problem trying to justify the principle of induction. This is known as "The

Problem of Induction" or "Hume's Problem" since it was the Scottish philosopher David Hume who brought the problem into sharp relief in his *Treatise On Human Nature* (1739).[10] The inductivist who claims scientific knowledge based on observation seems unable to defend their use of the principle of induction.

6.6.4 A Quick Puzzle about Models and Induction

Induction, as we have described it, involves drawing general conclusions from observations. But models, recall, simplify or distort reality. They are not accurate representations of what we observe in reality. Our data is, for example, never quite a straight line even when we model it as such. How then can our models be inductively inferred from what we observe? Does inductivism apply to models?

A natural response here is that our models are not inferred from data. Rather, models are part of an attempt to interpret data. We start from assumptions about data and use these to develop models, including models of data. This idea is developed in our discussion of the hypothetico-deductive account of science.

Image 6.4: David Hume [82]

Notes:

[10] You might like to look at Alan Musgrave's very readable account of the problem of induction and some responses in his *Common Sense, Science and Scepticism*, Cambridge University Press (1993), esp. Ch. 8 & 9.

Chapter 7: **Scientific reasoning - Part 2**

Lecture 7: **Philosophy - Part 2**

Learning objectives

✓ Understand the Popperian response to the problem of induction

✓ Understand the Popperian (falsificationist) hypothetico-deductive account of science

✓ Understand how falsifiability might be thought to be the hallmark of good science

✓ Recognise what special challenges models pose for falsificationism ✓ Recognise problems with the (falsificationist) hypothetico-deductive model of science

✓ Understand the Kuhnian criticism of falsificationism

✓ Understand the Kuhnian evolutionary account of science, including the notions of a paradigm and a paradigm shift

✓ Become familiar with the philosophy essay assignment

7.1 Popperian Science

7.1.1 The Hypothetico-Deductivist Account of Science – Falsificationism

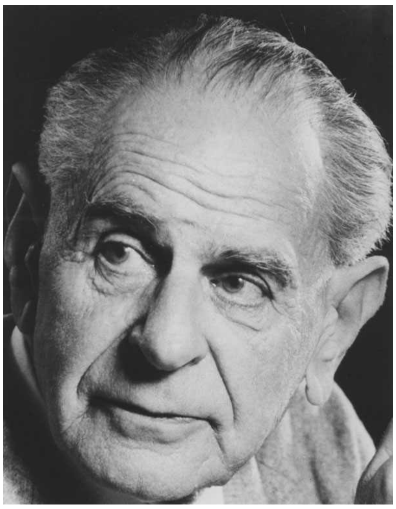


Image 7.1: Karl Popper [78]

The philosopher of science Karl Popper certainly thought that induction (and the presumption of the uniformity of nature) was unjustifiable. He argued that we shouldn't pin our hopes on induction as an account of how science works because we'd then be relying on an unacceptable principle of induction. Popper claimed that science didn't, in fact, need induction at all. We can explain how science works and why it is rational without any need to rely on anything as suspect as induction. His account of science and scientific method has been widely accepted by the science community and has become known as "the hypothetico-deductive method", sometimes also called "falsificationism". And, properly understood, it contrasts in a number of respects with the "common-sense view" mentioned earlier.

According to Popper, science proceeds by conjectures (hypotheses) and refutations (deductive inferences that show the hypothesis to be false) – hypothesis and deduction (hence the name *hypothetico-deductivism*. Rather than trying to *prove* universal laws that constitute scientific knowledge by induction from observational evidence, as the inductivist would have us believe, science is in the business of proposing bold conjectures as laws describing what we see around us and then subjecting these conjectures to stringent tests to see if the law can withstand attempts to *falsify* it (hence the alternative name *falsificationism*). If it can withstand the tests then, though we are not in a position to claim it as true (when are we ever in such a position? – this is the force of the problem of induction) we may claim it as the best "law" currently available and so rational to believe, at least for the time being.

The popular notion that science is a body of established fact is entirely mistaken; conjectured scientific laws are, at any given time, those which have not yet been shown to be false and, on balance, are the best we currently can conceive of to account for the nature of the world around us.

7.1.2 Inductive Proof vs. Falsification

Popper's proposed solution to the problem of induction derives its force from a logical difference between the (supposedly inductive) proof of a universal claim and the (clearly deductive) falsification or disproof of a universal. Consider the claim 'All metal expands when heated'. No number of instances of metals expanding when heated can be sufficient to prove the claim true yet a single instance of a metal not expanding when heated will be enough to prove the claim false – i.e. "falsify" it. The claim says that all As are B, yet if we can find a single A that is not B then the universal claim will be shown to be false. Unlike inductive proof, this is uncontroversial.

The falsity of a universal statement can be conclusively inferred from certain singular (particular) statements. Thus there *is* a clear and uncontroversial logical relation between singular and universal statements: singular statements, though they cannot inductively prove universal ones, can falsify them. It is this justifiable logical relation that Popper relies on to explain how it is that observation and scientific data relates to scientific laws and theories.

Laws are not justified by being proved by the data. However, laws that are not disproved by the data are preferred over those that are disproved by the data.

Falsifiability

Let us look more closely at just what we mean when we describe some statement as falsifiable. The following claims are all falsifiable in the Popperian sense:

1. 'It never rains on Wednesdays.'
2. 'All metals expand when heated.'
3. 'Heavy objects fall straight downwards if not impeded.'
4. 'When a ray of light is reflected from a plane mirror, the angle of incidence is equal to the angle of reflection.'

Claim (1) can be falsified if the world is observed to be such that it rains on some Wednesday. Claim (2) can be falsified if at some particular time a metal is observed not to expand under heating. Claim (3) can be falsified if a heavy object, which was not impeded in any way, was observed not to fall straight downwards. Claim (4) would be falsified if some ray of light at some time were observed to be reflected from a plane mirror in such a way that the angle of incidence was different from the angle of reflection.

The definition of "falsifiability" then is the following: *a hypothesis is falsifiable if there exists a logically possible observation statement or set of observation statements that are inconsistent with it – i.e., which if established as true would falsify the hypothesis.*

Scientific laws then (given the requirement that they be falsifiable) are testable in spite of their being unprovable; they can be tested by systematic attempts to falsify them.

Examples of claims that are not falsifiable are:

Logical Truths — e.g. 'Either it is raining or it is not raining.'

Definitional Truths — e.g. 'All bachelors are unmarried males.'

Mathematical Truths — e.g. '$2 + 2 = 4$'

Certain Modal Truths — e.g. 'Luck is possible in sporting situations.'

This latter example is the stock-in-trade of many fortune-tellers and newspaper astrologists. Such claims can never be shown to be false because they are not capable of being falsified.

As further examples of seemingly unfalsifiable claims, consider the following:

1. 'The cosmos doubled in size overnight.'
2. 'God created the earth 6,000 years ago complete with fossil record.' (To test our faith perhaps – in this way, it is argued, we can consistently argue for creationism.)

3. 'The world came into existence only five minutes ago, complete with a "history".'

And what about:

4. 'Survival of the fittest'?

Falsifiability as the Demarcation of Scientific Statements

The falsificationist's view is that scientific hypotheses must provide information as to how the world is, and so therefore how it is not. In other words, scientific hypotheses must have some information content in the sense of ruling out certain possibilities. In fact, scientific statements, in general, whether hypotheses, laws or simply observation statements, must have information content in this sense. They must, in this sense, be "testable". Claims that are true or false regardless of how the world is tell us nothing about the world itself, are not falsifiable (i.e. not testable), and thus are not scientific (though they may appear to be scientific). Because scientific statements (including scientific hypotheses) make definite claims about the world they have informative content and so must be falsifiable.

This simple fact is used to test whether statements count as "scientific" or not. If I tell you that you may be lucky in sport today I might appear to be making a prediction about your future. In this sense it may appear that I am making a scientific claim about the future. Or, consider the claim that the electron may curve anti-clockwise in the cloud chamber and not clockwise. These claims are not falsifiable; they are not testable. (They only say that something *might* happen, and the mere *possibility* is not falsified by its *actually* not happening.) They rule nothing out. (Anything is *possible*!)

Key Point: In order to be scientific, a hypothesis must be *falsifiable*. We cannot inductively prove such statements but we can deductively *disprove* them.

7.1.3 How Scientific Knowledge Advances

Scientific hypotheses then are by their very nature falsifiable. And the scientific method proceeds by putting forward such hypotheses, however arrived at. Then subjecting them to stringent testing to see if they are, in fact, false. Observations are compared to the consequences predicted by the hypothesis. If the observations conflict with the prediction then the (falsifiable) hypothesis is actually *falsified*. If not, if they pass stringent testing against our observations, then the hypothesis can be provisionally accepted (and if it is suitably *general* then it will be a provisionally accepted *law*) until and unless it is later falsified. If a law is falsified then we can establish this deductively, and we then go on to look for new bold hypotheses that will explain all that the old rejected law will explain and moreover which will explain the observation that led to the rejection of the old law.

There is no room for induction in this picture. Deriving predictions needs only deductive logic, and so too inferring that a hypothesis is false given our observations. Nothing on this picture "proves" hypotheses; they are simply useful conjectures that have been put forward and not yet been shown false.

The Hypothetico-Deductive Account

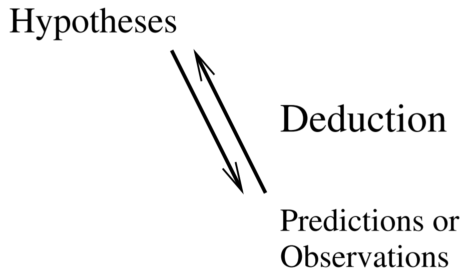


Figure 7.1: The Hypothetico-deductive account.

According to the hypothetico-deductive account then all our knowledge is, of its nature, provisional and will always be. At no stage are we in a position to be able to "prove" what we know to be true – it is always possible that it will turn out to be false. As such, the hypothetico-deductive account *rejects* the possibility of any confirmation of scientific claims. It is a simple fact of our intellectual history that nearly everything we have ever claimed at any time to know has later turned out to be false. A good example is Newton's Laws; they must have seemed so secure for two centuries until the Relativistic-turn early last century.

It is a mistake to try to prove a proposed law or theory to be true; to do so is to look for more assurance than will ever be available. What we can do is justify our preference for one candidate law (i.e. hypothesis) or theory (i.e. a collection of hypotheses) over another. (For example, the other was falsified, or was not scientific, or though scientific and not yet falsified was less informative so less useful than its preferred rival.) As theories are falsified we look for new bold conjectures that will explain all that the old rejected theory will explain and moreover which will explain the observation that led to the rejection of the old theory. *We learn from our mistakes; science progresses by trial and error.*

The popular notion that science is a body of established fact is entirely mistaken. Nothing in science is permanently established, it is changing all the time and not through the addition of new facts as the inductivist would have us believe. If we are rational then we will base our decisions on "the best available knowledge at the time", which is exactly what science provides us with.

Key Point: Popper's hypothetico-deductive account, falsificationism, describes science as a process of proposing hypotheses then deducing consequences from them for testing. Hypotheses are accepted as provisional unless they are falsified by observation.

7.1.4 Hypothetico-Deductivism and the Problem of Induction

So, it is claimed, at no point does induction play a role in assessing the status of scientific knowledge; the problem of induction does not arise. To be sure, we may as a matter of psychological fact invoke inductive methods to think up a conjectured law or hypothesis just as we might have hit upon it in a moment of sublime inspiration, blind drunkenness, or a dream but the psychological means whereby the law or hypothesis was arrived at tells us nothing about its status (as acceptable or unacceptable). We do not have to face the problem of justifying *how we came up with* our conjecture, whether it was by inductive inference or our dream-inspired method; the way we think up some law or hypothesis is not something requiring justification. What *does* require justification is why we might persist with such a claim and take it as something we can work with rather than abandoning it – this is done by seeing if it can pass those tests applied to it, not by any use of the problematic principle of induction.

Key Point: Since hypothetico-deductivism does not rely on induction to justify scientific laws, there is no problem of induction.

7.1.5 A Quick Puzzle About Models and Falsificationism

Models, recall, simplify or distort reality. This means that we know, from the outset, that our models are not entirely accurate; they are, strictly speaking, false. But if they are false, it makes no sense to try and falsify them/to try to test their truth. What, then, are we doing when we are testing our models? Does Popper's falsificationism apply to models?

A natural, but not unproblematic, response here is that in testing our models we are testing whether they are good enough for our purposes rather than trying to falsify them.

Notes:

7.1.6 More Problems

"Great" you say. Leaving aside the issue of how to test models, problem solved. We can now describe scientific method as essentially hypothetico-deductive, thus establish our belief in scientific laws and theories as rational, and go on to explain how it might be that (returning to where we began) no rational person could deny human-induced global warming. Using (now) rationally justified scientific laws we can show how the hypothesis of human-induced global warming best explains the data we are confronted with, and so by inference to best explanation, rationally infer that such a phenomenon exists. All the evidence, along with rationally justified laws, points to the hypothesis and so we are rationally justified in accepting it.

But – and there is nearly always a 'but' in philosophical debate, as there often is in scientific debate – a hypothetico-deductive account does raise some interesting questions. Firstly, some have doubted that the hypothetico-deductive method can actually avoid the use of inductive justification of scientific laws. Recall the claim in §7.7.2: "Laws are not justified by being proved by the data. However, laws that are not disproved by the data are preferred over those that are disproved by the data." How are we to decide which hypothesis to prefer from among the infinitely many that have not been disproved? And why should we trust hypotheses that have not been proved? Popper might try to respond that induction would perhaps be required if we were trying to decide which hypotheses to believe, but all we are doing is preferring available non-falsified hypotheses over available falsified ones.

Secondly, Popper presents cases where *one* theory is being tested against our experimental data, but hypotheses are tested in groups. When we "test" a theory, we are assuming a lot of other theories in the background. So if we find anomalous results should we reject the theory being "tested" or one of the other auxiliary hypotheses operating in the background?

Lastly, even when anomalies *are* detected, we frequently do not go on to reject the theory thought to be at issue. Often we retain theories that have been successful and proved themselves powerful and look for ways to reconcile ourselves to the anomaly.

Popper's response to these last two challenges involved suggesting that it is falsifiability that helps us to decide which hypotheses and theories to abandon. We make the choices that maximise the falsifiability of our system of hypotheses and theories. For example, Popper says that it actually is okay to protect a theory from falsification on the basis of another theory that provides new, falsifiable predictions and thus that increases the falsifiability of our overall system.

7.2 Kuhn's View of Science and its Challenges

7.2.1 Scientific Paradigms and Revolutions

The observation that scientists sometimes protect theories from falsification led in the latter half of the 20th century to accounts of science, including elaborations of falsificationism, that emphasise the social and political aspects of scientific progress. A number of philosophers of science have contributed to this view of science, including Thomas Kuhn in his study of science and scientific method – *The Structure of Scientific Revolutions* – and Helen Longino in her study of the sociology of science – *Science as Social Knowledge*. Since Kuhn's account is a particularly famous work in this direction, we will focus on his picture of science.

In short, Kuhn recognised by looking at the history of scientific practice that, far from a willingness to discard scientific theories or hypotheses as Popper suggests, most scientists tend to take a much more conservative attitude towards their theories. Scientific progress in Kuhn's picture happens in a series of distinct stages.

Pre-science

According to Kuhn, an emerging scientific field of research starts out with disparate and disorganised activity – think of the discipline of alchemy in the early 18th century. This activity gains structure when some new ideas enable a breakthrough in solving some problem of the field. If the proposed solution is impressive enough, and promises to be a successful way to tackle the problem, other researchers will be naturally drawn towards using these new tools to solve other problems of the field. In this way, a consensus can emerge amongst a group of researchers concerning the foundations of the field: a solution to a problem will come wrapped up with a particular theory of the problem, define significant experimental procedures, employ a certain type of scientific language, and even a set of assumptions about the way the world is.

Paradigms

A set of shared theoretical beliefs, values, language, instruments, techniques, and assumptions about the world to which a community of scientists are committed is called by Kuhn a *paradigm*. We can think of a paradigm as both a particular way of viewing the world and a particular way of doing science. A paradigm provides an organised and coordinated structure within which scientists can work.

For example, 18th century French scientist Antione Lavoisier made a concerted effort to combine all the disparate experimental results of alchemy into the framework of a single theory with a new set scientific language – today we call this framework chemistry. He pushed for the mathematisation of chemistry, he invented instruments and experimental techniques, and he standardised the conceptual framework for understanding chemical processes. Lavoisier established the paradigm.

A researcher who wishes to join a scientific community working within a particular paradigm, according to Kuhn, gains knowledge of the paradigm through a specific scientific education. Students of the paradigm will learn the techniques, language, and conceptual framework so that they can participate in paradigm-specific scientific work.

Normal science

Kuhn called this sort of scientific work *normal science.* Scientists undertaking normal science are trying to explain as many of their observations as possible within the framework of the paradigm. When new observations are made, scientists adapt the paradigm to integrate into the paradigm these new facts about the world. This process improves the match between the paradigm and the world. This process is called by Kuhn *puzzle-solving.* Normal science progresses by the accumulation of solutions to such puzzles.

Inevitably, though, scientists will experience difficulties accommodating all new observations. A novel observation that cannot be integrated into the paradigm is an *anomalous* observation. When faced with an anomalous observation, scientists are inclined to ignore or explain away such facts if at all possible. This picture of science is in stark contrast to Popper's picture, where scientists take the falsification of a theory to follow logically from an anomalous observation.

Crisis

It is only through the accumulation of a series of particularly troublesome anomalies that scientists begin to lose confidence in their ability to undertake normal science within a paradigm. Scientists simply cannot continue doing their research under this cloud of doubt about the paradigm. When this happens, according to Kuhn, the community of scientists enters into a *crisis.* Upon entering a crisis, scientists begin exploring new foundations for the field, new ways of doing science, or new fundamental assumptions. In short, scientists begin looking for a new paradigm.

The new paradigm must account for all the phenomena that the old paradigm accounted for, and also the anomalous observations that the old paradigm could not account for. Once a framework emerges that solves the old and new problems, scientists are naturally drawn to the framework as a problem solving tool.

Revolution

When a critical mass of scientists adopt the new framework, this framework emerges as the new paradigm in the scientific field. According to Kuhn, this renewal of the paradigms is called, a *scientific revolution.*

Significantly for Kuhn's picture, the process by which a scientific revolution occurs does not follow a strict set of rules. Thus, the decision to opt for a revision of a paradigm, or which choice of revision is best, is not one that is rationally compelled. For this reason, the revolutionary phase of science is particularly open to competition among differing ideas and rational disagreement about the relative merit of each idea.

Each paradigm or way of doing science comes with its own methods of investigation, ways of explaining phenomena, important applications and characteristic types of argument. So, each paradigm comes with its own standards of what good science amounts to. Another way of putting this is by saying that paradigms are *incommensurable*: they have no common measure or standard that would allow their rational comparison. But then it seems that rationally choosing between theories from different paradigms will be a challenge. Proponents of each paradigm will have to use their own standards of good science in criticising their rivals' theories and thus to simply assume that the rival theories are problematic. In such a situation, according to Kuhn, there is no fully objective or rational, way of deciding which theory is correct.

Kuhn claims that these revolutionary cycles drive scientific progress. The problem solving capability of the new paradigm will exceed the problem solving capability of the old paradigm. However, this improvement of problem solving capability does not represent an improved proximity to the 'truth'. Rather, Kuhn favours an evolutionary view of scientific progress whereby science improves by allowing its theories to evolve in response to puzzles, and progress is measured by its success in solving those puzzles.

Image 7.2: Helen Longino [74]

7.2.2 Responding to Kuhn

There are, however, possible responses to Kuhn. First, if it really is true that we cannot rationally decide between rival paradigms, then it seems the rational thing to do is not to choose between them. In such a situation, we have to accept that we do not know which paradigm is correct. Further, scientists can continue to develop rival paradigms alongside each other. Indeed, we can encourage individual scientist to work in multiple paradigms.

Second, choosing between paradigms need not amount to begging the question by using the standards of one paradigm to judge the correctness of rival paradigms. There is always the possibility that those working within a paradigm will come to realise, using the standards of their own paradigm, that their paradigm is failing and needs to be abandoned.

Finally, there may at least be some standards for judging which paradigms are preferable that are, or should be, shared by different paradigms and that allow us objectively to decide between paradigms. This was the strategy preferred by Imre Lakatos in his 1970 text *Falsification and the Methodology of Scientific Research Programmes*. Lakatos agreed with Kuhn that when a paradigm confronts anomalies, it is sometimes rational to protect the paradigm from falsification. However, Lakatos also thought that we can compare the empirical successes and failures of rival paradigms over time and use such a comparison rationally to decide between the paradigms. If one paradigm has been making interesting new predictions over time and, further, these predictions have been successful while the new predictions of a rival paradigm invariably fail and give rise to anomalies, the paradigm making the new, successful predictions can rationally be preferred over its rival. In this way, Lakatos develops something like Popper's views, discussed in 7.7.6, about when it is okay to protect theories from falsification.

7.2.3 Beyond Kuhn

Kuhn and Lakatos were by no means the only ones to develop an evolutionary account of science. Grace and Theodore de Laguna, for example, already develop an account that is in some ways more sophisticated than Kuhn's in their 1910 book *Dogmatism and Evolution: Essays in Modern Philosophy*, The MacMillan Company.

There is also the later work of the radical Paul Feyerabend (see his entertaining 1975 essay "How to Defend Society Against Science", for example – `http://www.galilean-library.org/manuscript.php?postid=43842`) who argued that there is no such thing as *the* scientific method. He claims that close study of the history of science shows that there is *no* set of rules, *no method*, that describes "scientific progress". More recently, there is Deobarah Mayo's updated version of falsificationism, according to which some severe tests *can* reliably select true hypotheses (see her *Error and the Growth of Experimental Knowledge*, Chicago University Press (1996)).

We won't pursue this further. But there is a lot more to be said here. Those interested might look at Alan Chalmers' very readable book *What Is This Thing Called Science?*, available from the Library – much of the notes so far follow this text. (There are a number of editions but most after edition 1 are very good.) A very good introduction to some of the historical material is J. Losee's *A Historical Introduction to the Philosophy of Science*, 3rd ed. (1993), and D. Oldroyd's *The Arch of Knowledge*, University of New South Wales Press (1986). See also P. Godfrey-Smith's *Theory and Reality*, University of Chicago Press

(2003). Further material is also mentioned in the notes and footnotes.

Notes:

7.3 Exercise

Create your own glossary by writing down definitions of the following terms:

(a) Induction

(b) Singular statement

(c) Universal statement

(d) Hypothesis

(e) Fallible

(f) Weak induction

(g) Hypothetico-deductive method

(h) Falsifiable

(i) Falsified

(j) Paradigm

(k) Scientific revolution

(l) Incommensurability

Chapter 8: Modelling - combined functions

Lecture 8: Merging models

Learning objectives

✓ Analyse more complex models of real-world phenomenon

Scientific examples

✓ Atmospheric carbon dioxide

✓ Pharmacokinetics

✓ Heart disease

✓ Apparent temperature

Maths skills

✓ Understand when and how to combine functions

✓ Understand the form of surge functions and their graphs

Key literature

✓ *Exchanges of atmospheric CO_2 and $^{13}CO_2$ with the terrestrial biosphere ...* [34]

8.1 Motivation and Background

Image 8.1: Right panel of *The Garden of Earthly Delights* (1503 – 1504), Hieronymus Bosch (c. 1450 – 1516), Museo del Prado, Madrid. [75]

In the earlier chapters we looked at various functions that could be used to model scientific phenomena. This included power functions, cyclic functions and exponential functions. However, such functions alone will often not be sufficient to generate a useful model.

In this chapter we will explore some more complex functions, such as those with several variables and those which are combinations of functions we have previously seen.

8.2 The Keeling curve

Case Study 11: **Atmospheric CO_2**

The broad scientific consensus is that:

- Earth is undergoing a period of significant climate change;
- global temperatures are likely to rise over coming years;
- the warming is related to increasing concentrations of carbon dioxide (CO_2) in the atmosphere; and
- the increase in atmospheric CO_2 concentration is a result of human activity.

A famous, long-running study has monitored atmospheric CO_2 concentrations at the Mauna Loa observatory in Hawaii since 1958. When these data are plotted, the graph is called the *Keeling curve*, named after the initiator of the study, David Keeling.

Gases in the lower atmosphere mix fairly well, so scientists consider the Keeling curve data to be representative of the atmospheric CO_2 concentration world-wide. The Scripps Institution of Oceanography (which runs the study) describes the Keeling curve as "*... almost certainly the best-known icon illustrating the impact of humanity on the planet as a whole ...*"

Data from ice-core samples show that CO_2 levels remained relatively constant at 280 ppm for thousands of years, but the level started increasing in the 19th century.

When SCIE1000 was first offered in 2008, the level of atmospheric CO_2 was about 380 ppm. By July 2016, concentrations were consistently above 400 parts per million (ppm).

Figure 8.1 is a plot of the Keeling curve data, taken from [34, 58].

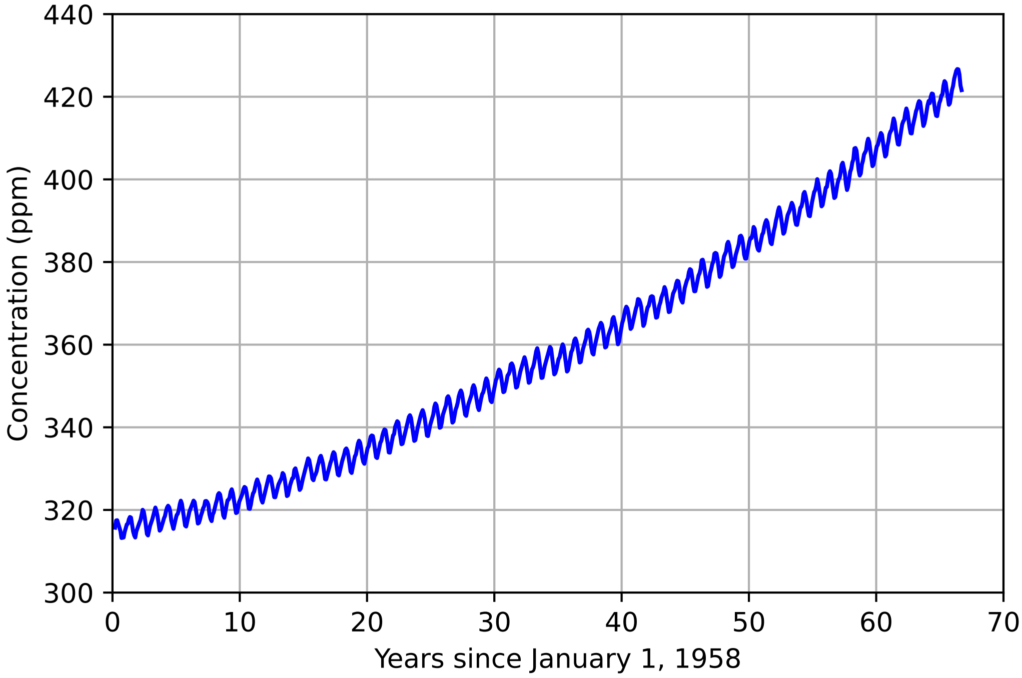


Figure 8.1: The Keeling curve.

Question *8.2.1*

(a) Describe the main features of the Keeling curve graph.

(b) What physical factor(s) could cause these features?

We will now develop our own mathematical model of the Keeling curve. We can use an increasing function such as a power function ($p > 1$) together with a sine function to model the measured concentrations.

Question *8.2.2*

Give a rough sketch of the shape of the graph in each case:

(a) a power function added to a sine function;

(b) a power function multiplied by a sine function.

Which type of model is more appropriate to model the Keeling curve?

Question 8.2.3

Consider the following three models of the Keeling curve.

- Model Q+S: $y(t) = 0.014t^2 + 0.7t + 315 + 3.5\sin\left(2\pi(t - 0.15)\right).$
- Model P+S: $y(t) = 1/3t^{1.37} + 315 + 3.5\sin\left(2\pi(t - 0.15)\right).$
- Model E+S: $y(t) = 280 + 35e^{0.022t} + 3.5\sin\left(2\pi(t - 0.15)\right).$

Figure 8.2 plots graphs of the Keeling curve and all three models.

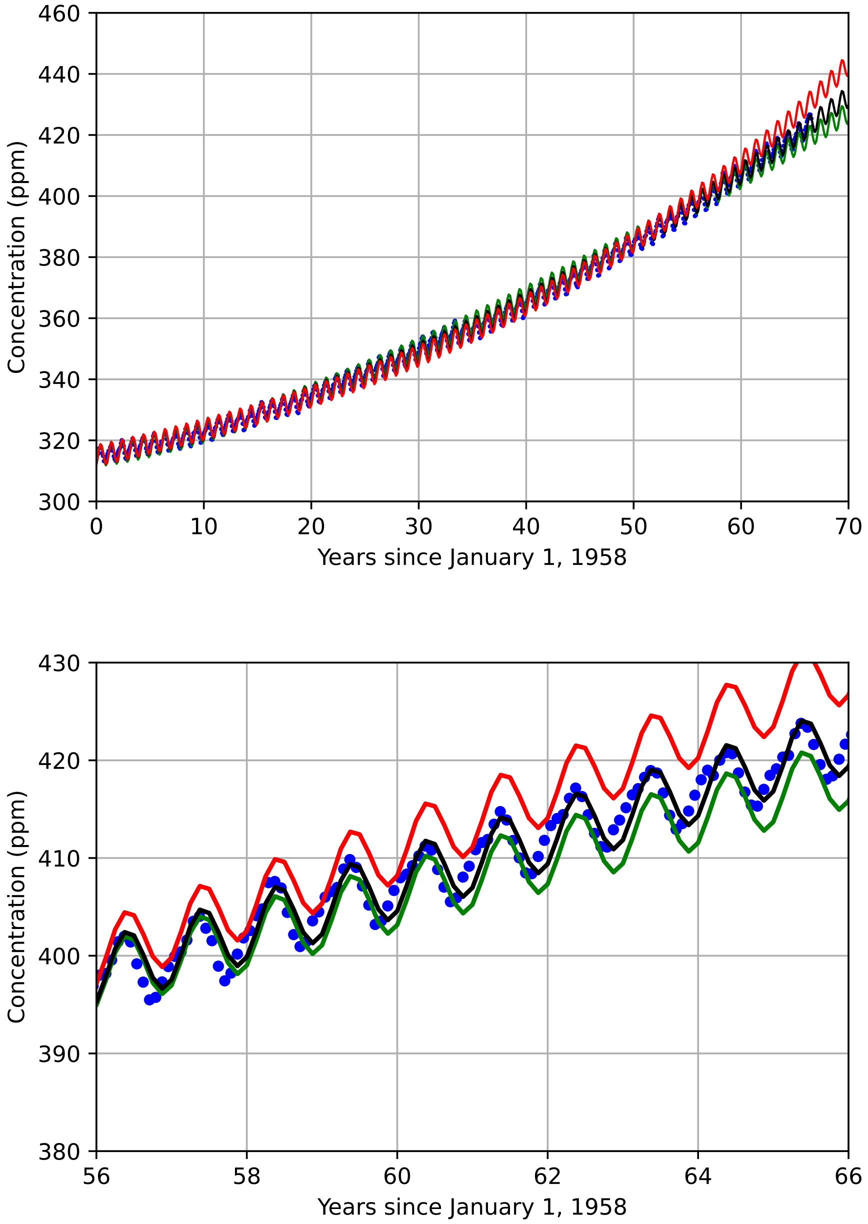


Figure 8.2: The Keeling curve and the three models for all years (top) and recent years (bottom).

Question *8.2.3 (continued)*

(a) Which of the three models of the Keeling curve is correct? Why?

(b) Figure 8.3 extrapolates the models to the year 2058 (100 years after the Keeling study commenced). Which curve corresponds to each model?

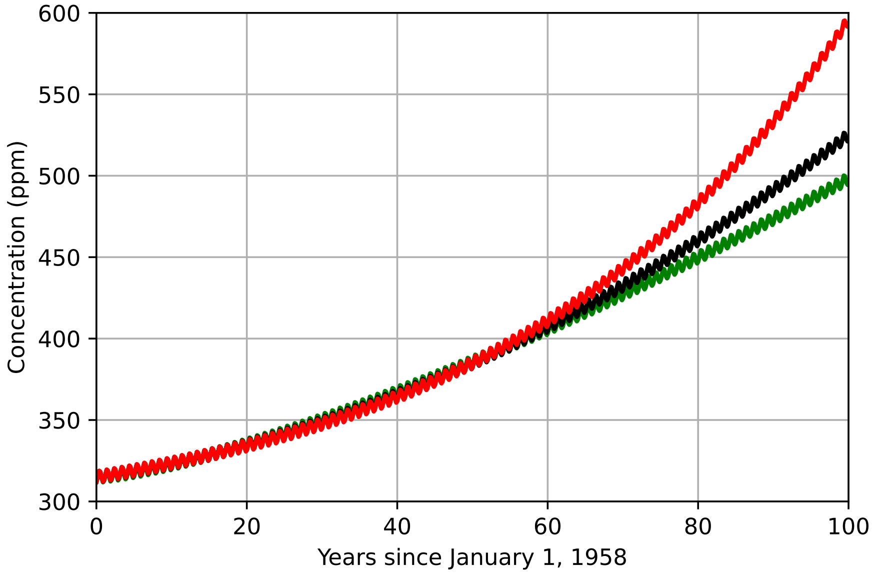


Figure 8.3: The three models of the Keeling curve, extrapolated to the year 2058.

End of Case Study 11: Atmospheric CO_2.

8.3 Drugs in the blood and surge functions

Mathematics and functions are particularly important when modelling the *change* in drug concentrations over time, as they help to predict the *impact* of the drug and the *timing* of subsequent interventions. In class we will examine such models – some key concepts are given below.

Some drug-related terminology

Broadly speaking, a *drug* is any externally derived chemical substance introduced into an organism that affects the function of that organism. Drugs may enhance physical or mental well-being, and include both medicinal and so-called recreational drugs.

Pharmacology studies the properties of drugs and their effects on living organisms.

Pharmacokinetics studies what happens to drugs inside the body, particularly the extent and rates of **absorption**, **distribution**, **metabolism** and **excretion**.

Drug concentrations

After the administration of a drug, key determinants of its impact on the body are the drug **concentration** in the bloodstream, which is commonly measured as mass per volume (such as mg/L), and the **time** over which that concentration occurs. Concentrations can be measured at various times after drug administration and plotted on a *drug concentration curve*.

Case Study 12: **Zoloft and depression**

Depression is one of the most common mental health problems. Unlike many health problems, depression (and other mental illnesses) can occur more frequently in young adults than in older adults. There are multiple treatments available for depression, including a variety of therapy-based treatments, and pharmacological interventions.

Zoloft (and a number of generically branded equivalents) is the drug *sertraline hydrochloride*, which is an antidepressant of the SSRI class (Selective Serotonin Reuptake Inhibitor).

The Consumer Medicine Information fact sheet explains that SSRIs "...are thought to work by blocking the uptake of a chemical called serotonin into nerve cells in the brain. Serotonin and other chemicals called amines are involved in controlling mood".

Zoloft is the most commonly prescribed antidepressant in Australia, and one of the most prescribed drugs overall on the Australian Pharmaceutical Benefits Scheme. Zoloft is taken orally as a pill. The usual dosage ranges from 25 mg per day to 200 mg per day. Zoloft has a number of comparatively mild side effects (including insomnia, loss of appetite, and some sexual impairment), and is generally believed to be both effective and well tolerated.

Question *8.3.1*

Figure 8.4 shows the average blood sertraline concentrations for 11 young women involved in a study [55] (with straight lines interpolating data points). Participants received daily oral doses of sertraline over 30 days (to achieve 'steady state' concentrations), then a final dose was administered and blood concentrations monitored. Such drug concentration curves (for sertraline or other drugs) allow pharmacologists to observe, measure and analyse various factors.

Question *8.3.1 (continued)*

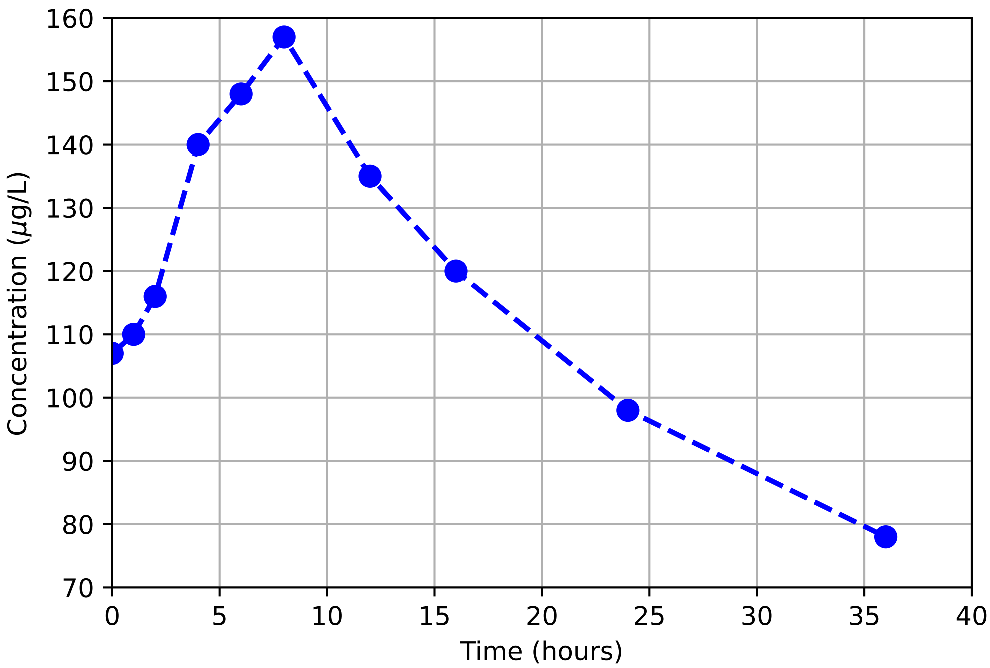


Figure 8.4: Blood sertraline concentrations in young women.

Use the figure to determine each of the following (also mark on the graph where appropriate):

(a) the peak drug concentration C_{max};

(b) the time t_{max} at which C_{max} occurs;

(c) the *half-life* $t_{1/2}$ of the drug, which is the time taken for the concentration to fall to half of its previous value;

(d) the times at which the maximum rates of drug absorption/removal occur;

(e) the "total exposure" of the body to the drug.

Example 8.3.2

Compare some of the features/observations in Example 8.3.1 with the following taken from the sertraline fact sheet at *www.pbs.gov.au*.)

"**Pharmacokinetics:** In humans, following oral once-daily dosing over the range of 50 to 200 mg for 14 days, mean peak plasma concentrations (C_{max}) of sertraline occurred between 4.5 to 8.4 hours post dosing. The average terminal elimination half-life of plasma sertraline is about 26 hours. Based on this pharmacokinetic parameter, steady-state sertraline plasma levels should be achieved after approximately one week of once-daily dosing. Linear dose-proportional pharmacokinetics were demonstrated in a single dose study in which the C_{max} and area under the plasma concentration time curve (AUC) of sertraline were proportional to dose over a range of 50 to 200 mg.

Dosage: Adults (18 years and older) The usual therapeutic dose for depression is 50 mg/day. ...patients not responding to a 50 mg/day dose may benefit from dose increases up to a maximum of 200 mg/day. Given the 24 hour elimination half-life of sertraline, dose changes should not occur at intervals of less than 1 week. The onset of therapeutic effect may be seen within 7 days ....

Use in Children and Adolescents aged less than 18 years: Sertraline should not be used in children and adolescents below the age of 18 years for the treatment of major depressive disorder. The efficacy and safety of sertraline has not been satisfactorily established for the treatment of major depressive disorder in this age group.

Overdosage: On the evidence available, sertraline has a wide margin of safety in overdose. Overdoses of sertraline alone of up to 13.5 g have been reported. Deaths have been reported involving overdoses of sertraline, primarily in combination with other drugs ...."

The general shape of the blood sertraline concentration curve shown in Figure 8.4 is typical of many drug concentration curves. The corresponding functions are sometimes called *surge* functions.

Surge functions

In a **surge** function, the value initially rises rapidly before falling off exponentially over time. A general equation for a surge function is

$$f(t) = at^p e^{-bt}$$

where the values of a, p and b depend on the phenomenon $(0 < p < 1)$. Figure 8.5 shows the general shape of a surge function.

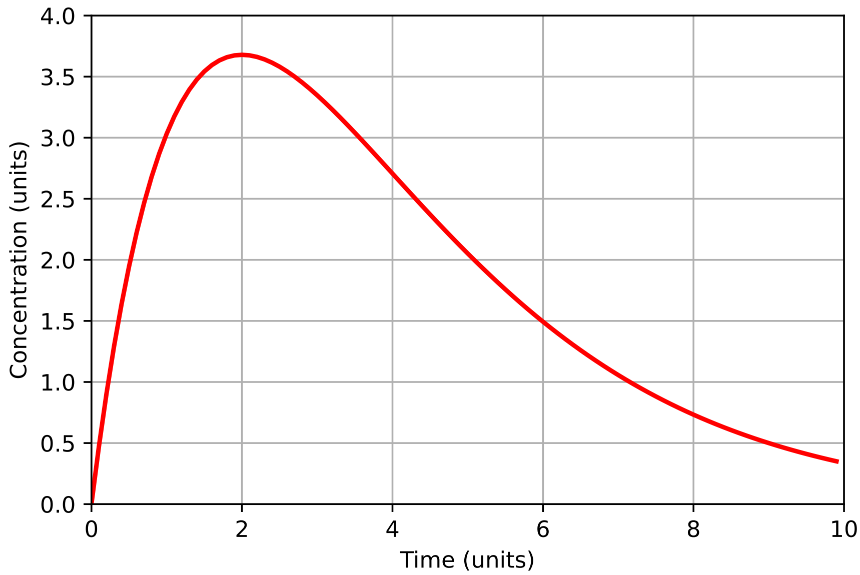


Figure 8.5: General shape of a surge function.

A surge function, as defined above, will reach a maximum when $t = \frac{p}{b}$.

Question 8.3.3

(a) Give some physical reasons why blood concentration curves typically take the form of surge functions.

Question 8.3.3 *(continued)*

(b) Explain mathematically why functions of the form $f(t) = at^p e^{-bt}$ have a 'surge function shape'.

(b) Soon we will study some examples of surge functions, including blood concentrations of:

- paracetamol: $C_1(t) = 14t^{0.6}e^{-0.5t}$ μg/mL.
- a long-lasting contraceptive: $C_2(t) = 0.87t^{0.15}e^{-0.0008t}$ ng/mL.

Without drawing them, briefly discuss how the graphs of C_1 and C_2 would appear, including their similarities and differences. Time is measured in hours for both. Ignore the differences in concentration units.

End of Case Study 12: Zoloft and depression.

8.4 Predicting heart disease

Case Study 13: **To the heart of the matter**

Diseases of the circulatory system (including heart disease and stroke) are the leading cause of death in many western societies. Individuals, doctors and public health bodies all have an obvious interest in predicting the risk of suffering cardiovascular disease. In medicine and population health, risks are often specified as a probability of an identified event occurring in a given time period.

Question *8.4.1*

Which factors or data are crucial when developing a model for estimating the likelihood that a person will suffer from coronary heart disease (CHD) in the next 10 years? Does each factor increase or decrease the risk?

***Question** 8.4.1 (continued)*

What is your "gut feeling" of the likelihood that your lecturer will suffer from CHD in the next 10 years?

Until comparatively recently, little was known about the general causes of heart disease and stroke, although the rates of cardiovascular disease (CVD) in many societies had been rising for some time.

In 1948, a study into heart disease commenced in Framingham, Massachusetts, which has become one of the best-known longitudinal health studies. [1].

The Framingham study (which continues today) has monitored the cardiovascular health of participants, identified a range of risk factors for CHD and included these factors in a mathematical risk model.

One of the resources produced from the Framingham Study is a CHD Risk Prediction score sheet, used to predict the likelihood that a person will suffer CHD in the next ten years. The sheet is a tabular representation of a more complex mathematical formula combining a range of functions.

[1] All information from the Framingham study has been reproduced with permission from the National Heart, Lung, and Blood Institute as a part of the National Institutes of Health and the U.S. Department of Health and Human Services.

Step 1: Age

Age (Years)	Points Female	Points Male
30-34	-9	-1
35-39	-4	0
40-44	0	1
45-49	3	2
50-54	6	3
55-59	7	4
60-64	8	5
65-69	8	6
70-74	8	7

Step 2: LDL cholesterol

LDL (mmol/L)	Points Female	Points Male
≤ 2.59	-2	-3
2.60-3.36	0	0
3.37-4.14	0	0
4.15-4.91	2	1
≥ 4.92	2	2

Key	
Colour	Risk
green	very low
white	low
yellow	moderate
rose	high
red	very high

Step 3: HDL cholesterol

HDL (mmol/L)	Points Female	Points Male
≤ 0.9	5	2
0.91-1.16	2	1
1.17-1.29	1	0
1.30-1.55	0	0
≥ 1.56	-2	-1

Step 4: Blood pressure (F: Female, M: Male)

Systolic (mm Hg)	Diastolic (mm Hg) < 80 Points	80-84 Points	85-89 Points	90-99 Points	≥ 100 Points
≤ 120	F: -3 M: 0				
120-129		0			
130-139			F: 0 M: 1		
140-159				2	
≥ 160					3

Note: When systolic and diastolic pressures provide different estimates for point scores, use the higher number.

Step 5: Diabetes

Diabetes	Points Female	Points Male
No	0	0
Yes	4	2

Step 6: Smoker

Smoker	Points Female	Points Male
No	0	0
Yes	2	2

Step 7: Sum points from Steps 1-6

Category	Points
Age	
LDL	
HDL	
Blood pressure	
Diabetes	
Smoker	
Point total	

Step 8: Determine risk from point total

Point total	10 Year CHD risk Female	Male
≤ -3	1%	1%
-2	1%	2%
-1	2%	2%
0	2%	3%
1	2%	4%
2	3%	4%
3	3%	6%
4	4%	7%
5	5%	9%
6	6%	11%
7	7%	14%
8	8%	18%
9	9%	22%
10	11%	27%
11	13%	33%
12	15%	40%
13	17%	47%
14	20%	≥ 56%
15	24%	≥ 56%
16	27%	≥ 56%
≥ 17	≥ 32%	≥ 56%

Step 9: Compare to others of the same age

Age (Years)	Average 10 Yr risk	Low 10 Yr risk
30-34	F: <1% M: 3%	F: <1% M: 2%
35-39	F: 1% M: 5%	F: <1% M: 3%
40-44	F: 2% M: 7%	F: 2% M: 4%
45-49	F: 5% M: 11%	F: 3% M: 4%
50-54	F: 8% M: 14%	F: 5% M: 6%
55-59	F: 12% M: 16%	F: 7% M: 7%
60-64	F: 12% M: 21%	F: 8% M: 9%
65-69	F: 13% M: 25%	F: 8% M: 11%
70-74	F: 14% M: 30%	F: 8% M: 14%

Note: low risk was calculated for an individual of the same age, with normal blood pressure, LDL 2.60-3.36 mmol/L, HDL 1.45 mmol/L, non-smoker and no diabetes.

Figure 8.6: Framingham CHD risk assessment sheet for males and females

Question 8.4.2

Use the Framingham CHD risk assessment sheet in Figure 8.6 to estimate the probability that your lecturer will suffer CHD within 10 years. Compare this with your answers to Question 8.4.1.

Question 8.4.3

Consider the different risk factors stipulated in the risk assessment sheet. What are some of the limitations the model might face in accurately assessing the risk that some individual will suffer CHD? Can you think of any other potential risk factors that are not included in the assessment?

Question 8.4.4

Do these limitations mean that we cannot use this model to make any kind of risk assessment? I.e. is the Framingham CHD risk assessment simply wrong?

End of Case Study 13: To the heart of the matter.

8.5 Apparent temperature

Apparent temperature relates to how a person "perceives" the actual temperature. This feeling is impacted by the rate at which energy is lost from the body to the surrounds.

Case Study 14: Apparent temperature for Aussies

Most weather apps now include a feature that explains what the "apparent temperature" is, or what the temperature "feels like".

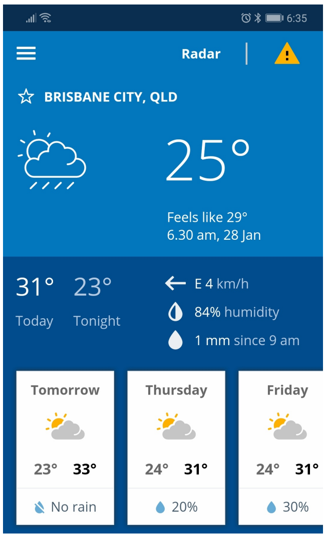


Photo 8.1: Snapshot of The Bureau of Meteorology mobile weather app. (Source: SD.)

Question *8.5.1*

Derive a plausible equation that models apparent temperature. (Hint: start by deciding which factors are important, whether they increase or decrease the apparent temperature.)

Example *8.5.2*

The model which is used by the Australian Bureau of Meteorology is based on the following function developed in [61]. Let T be the ambient air temperature in °C, H denote the relative humidity (%), and v be the wind speed in m/s. The perceived apparent temperature AT in °C can be modelled by:

$$\mathrm{AT} = T + 2.015\left(\frac{H}{100}\right)\exp\left(\frac{17.27T}{237.7+T}\right) - 0.7v - 4.00,$$

where we note that $\exp(x)$ is another way to write e^x.

Question *8.5.3*

A simplified version of the model for apparent temperature in Australia is given by

$$\mathrm{AT} = T + 2\left(\frac{\mathrm{H}}{100}\right)e^{0.06T} - 0.7v - 4.$$

(a) Is this consistent with the trends we saw in Question 8.5.1?

Question *8.5.3 (continued)*

(b) Suppose that on one of the days during the 2009 heat wave in Mildura, at noon there was a relative humidity of 60%, wind speed of 3 m/s, and the apparent temperature AT was 4°C higher than the ambient air temperature T. What was the ambient air temperature T at that time?

End of Case Study 14: Apparent temperature for Aussies.

Chapter 9: Rates of change

Lecture 9: Average and instant change

Learning objectives

✓ Understand the meaning and applications of average rates of change

✓ Understand the meaning and use of derivatives

Scientific examples

✓ Metabolism of alcohol

✓ Depo-Provera for contraception

Maths skills

✓ Calculate average rates of change

✓ Understand how to apply Newton's method

✓ Using derivatives to find maxima or minima

Key literature

✓ *Pharmacokinetics of ethanol after oral administration in the fasting state* [88]

9.1 Motivation and Background

Image 9.1: *Skull with a burning cigarette* (c1885), Vincent van Gogh (1853 – 1890), Van Gogh Museum, Amsterdam. [83]

We are often interested in studying phenomena whose values change over time. The primary mathematical tool that considers rates of change is the *derivative*. You should have encountered derivatives in previous study of mathematics. See Section C.4 in Appendix C for the prerequisite mathematical tools we will use in this chapter. In this course we do not focus on how to *find* derivatives, but we instead focus on how to *use* and *interpret* them. We will also use *Newton's method*, which makes use of the derivative, to estimate a solutions to equations that cannot be solved analytically. You can use SOMSE for further support on derivatives and Newton's method.

The concept of one quantity changing as another quantity changes is crucial to understanding and modelling many processes in science, engineering, social sciences and economics. We will cover two methods for analysing rates of change: *average* rates of change and *instantaneous* rates of change.

The *average rate of change* measures the average rate at which a particular parameter changes over a finite time period using values at the start and end of the time period.

Example 9.1.1

The concentration of atmospheric CO_2 has risen by about 90 ppm over the last 60 years. Hence the average rate of change over this time is:

$$\frac{90 \text{ ppm}}{60 \text{ years}} = 1.5 \text{ ppm year}^{-1}.$$

Rather than using the average rate of change between two points, it is often more useful to consider the *instantaneous* rate of change at a point. The mathematical term for an instantaneous rate of change is *derivative*.

As part of this workshop, we will examine Blood Alcohol Concentration (BAC). We will use the definition of a 'standard drink' as one containing 10 g of alcohol. Usually, the measure of BAC is the percentage of total blood volume that is alcohol (or equivalently, grams of alcohol per litre of blood). In Australia the legal blood alcohol content for driving is 0.05%, or 0.5 g/L.

Unlike many other drugs, the rate of alcohol metabolism is roughly constant (called a *zero-order* reaction in Chemistry) as the metabolising capacity of enzymes within the liver are saturated. The exact rate of metabolism varies between individuals, influenced by factors such as age, mass (weight) and sex. A graph of BAC from the time drinking commenced will show a rapid initial rise during the absorption phase, prior to a decline in concentration during the elimination phase. The rate of alcohol metabolism is approximately constant for much of the period during which alcohol is being removed from the body. Hence, a graph of BAC from the time of peak concentration shows a linear decline until metabolism is almost complete.

9.2 Sobering rates of change

Case Study 15: **Whisky (Blood Alcohol Concentration)**

Question 9.2.1

Figure 9.1 shows some BAC measurements (see [88]). Let $B(t)$ represent the straight line modelling the BAC from $t = 1$ hour to $t = 6$ hours.

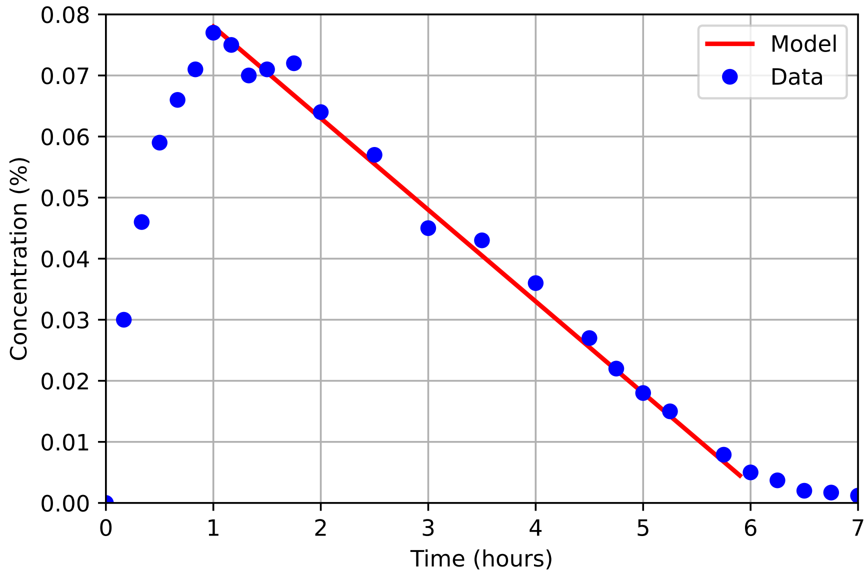


Figure 9.1: Measured blood alcohol concentrations.

Fitting a linear model to match the line drawn above, we find

$$B(t) = -0.015t + 0.093,$$

where B is in % and t is in hours.

(a) Find $B'(t)$ (include units).

(b) Interpret, in words, what $B'(t)$ physically represents.

***Question** 9.2.1 (continued)*

(c) Figure 9.2 shows measured BACs after researchers administered four different controlled doses of alcohol to study participants (see [88]). Discuss the value of B' for each of the graphs. What are the implications?

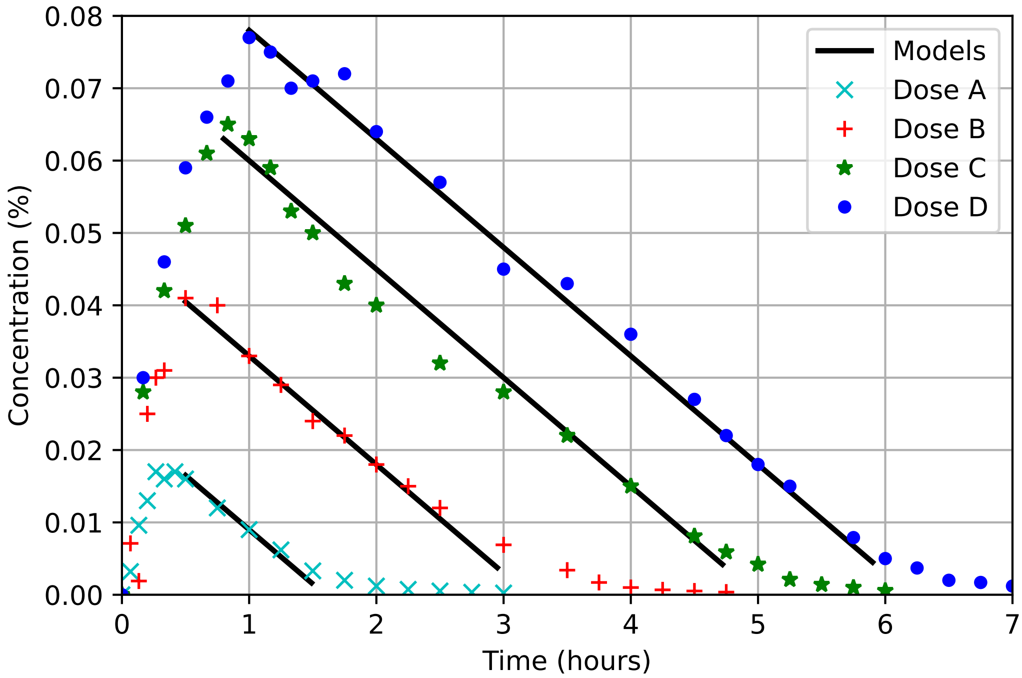


Figure 9.2: Measured BACs after administration of four different controlled doses of alcohol.

End of Case Study 15: Whisky (Blood Alcohol Concentration).

Case Study 16: **Drink deriving**

In practice (particularly in legal cases), models of BAC use the *Widmark formula*, developed in 1932. The equation is:

$$B = \frac{A}{rM} \times 100\% - Vt$$

where B is the BAC at time t since commencing drinking, A is the amount of alcohol consumed in grams, V is the rate at which the body eliminates alcohol measured in % per time period, M is the body mass in grams and the *Widmark factor* r estimates the proportion of body mass that is water.

The precise value of r depends on factors such as sex, age and percentage body fat. Reasonable estimates are $r \approx 0.7$ for adult males and $r \approx 0.6$ for adult females. The typical value for V is 0.015 % hr^{-1}.

Question 9.2.2

(a) What is the physical meaning of the term rM in the Widmark formula?

(b) The Widmark formula is: $B = \dfrac{A}{rM} \times 100\% - Vt$. Sketch rough graphs of B against time for a "typical" male and a "typical" female who each consume the same amount of alcohol (that is, assume A is constant).

Question 9.2.2 *(continued)*

(c) Use the Widmark formula to justify Australian government guidelines that, to remain below the legal driving BAC limit (0.05%), within the first hour "men should drink at most two drinks and women at most one". Note that 1 standard drink contains 10 g of alcohol.

Question 9.2.3

In the Widmark formula, the absorption term assumes that the body absorbs alcohol **immediately** after consumption. The following variation, from [51], takes into account absorption time.

$$B = \frac{A}{rM} \times \left(1 - e^{-kt}\right) \times 100\% - Vt$$

where k is the rate at which the body absorbs alcohol.

Question *9.2.3 (continued)*

(a) Expand this variation of the Widmark formula and compare it with the "standard" Widmark formula (which is $B = \dfrac{A}{rM} \times 100\% - Vt$.) Sketch rough graphs of each.

(b) Recall that $B = \dfrac{A}{rM} \times \left(1 - e^{-kt}\right) \times 100\% - Vt$.

If t is measured in hours, what are the units of k?

(c) What factors could influence the value of k?

Question 9.2.3 *(continued)*

(d) Let t_{max} be the time at which BAC reaches its maximum value B_{max}. For larger values of k, will t_{max} be larger, smaller or unchanged? Why? Will B_{max} be larger, smaller or unchanged? Why?

Question 9.2.3 *(continued)*

(f) When consuming alcohol with food in the stomach, $k \approx 2.3/\text{hr}$, but when the stomach contains no food, $k \approx 6/\text{hr}$. When a "typical" man of mass 80 kg consumes 4 standard drinks with food in his stomach, we have

$$\begin{aligned} B(t) &\approx 0.0714(1 - e^{-2.3t}) - 0.015t \\ B'(t) &\approx 0.164e^{-2.3t} - 0.015. \end{aligned}$$

Find t_{max} and B_{max} for this man.

(g) If the same man consumes the same amount of alcohol on an empty stomach, we have $t_{max} \approx 0.56$ hours and $B_{max} \approx 0.0605$ %. Compare this with your answer to Part (f), and relate this to Part (d).

We can now write a program to calculate the blood alcohol concentration of a person after consuming a specified amount of alcohol.

Program specifications: Write a program that plots BAC curves up to five hours after alcohol consumption on both a full and empty stomach, for men or women of varying masses and for various levels of alcohol consumption.

Program 9.1: BACs and food consumption

```
# Program to compare BACs for men and women of varying masses
# and levels of alcohol consumption, on full and empty stomachs.
import numpy as np
import matplotlib.pyplot as plt

alcohol_mass = float(input("Mass of pure alcohol consumed (in g)? "))
mass_person = float(input("Person's mass (in kg)? "))
sex_person = float(input("Type 1 if male, 2 if female: "))
if sex_person == 1:
    mult = 100 * alcohol_mass / (0.7 * mass_person * 1000)
else:
    mult = 100 * alcohol_mass / (0.6 * mass_person * 1000)
times = np.arange(0, 5.1, 0.01)

# Calculate the BAC at each time.
bac_full = mult * (1 - np.exp(-times * 2.3)) - 0.015 * times
bac_empt = mult * (1 - np.exp(-times *   6)) - 0.015 * times

# BAC cannot be negative.
for i in range(0, np.size(times)):
    bac_full[i] = max(bac_full[i], 0)
    bac_empt[i] = max(bac_empt[i], 0)

# Plot graph.
plt.plot(times,bac_full,"k--",linewidth=2, label="Full stomach")
plt.plot(times,bac_empt,"b-",linewidth=2, label="Empty stomach")
plt.grid(True)
plt.xlabel("Time (hours)")
plt.ylabel("BAC (%)")
plt.xlim(0,5)
plt.ylim(0,0.07)
plt.legend()
plt.show()
```

Figure 9.3 shows the output from running the above program for an 80 kg male consuming four standard drinks (40 g alcohol):

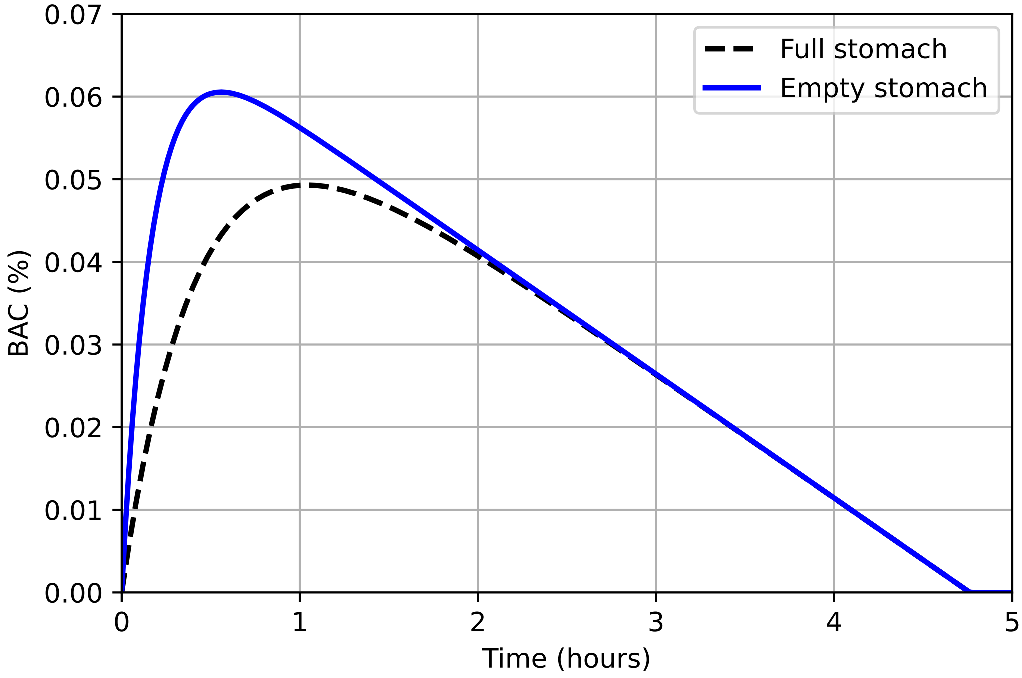


Figure 9.3: Predicted BACs when consuming alcohol on a full stomach compared to an empty stomach.

Question *9.2.4*

Briefly compare the graph in Figure 9.3 with your answers to Question 9.2.3. What are some of the physical implications of the graph?

End of Case Study 16: Drink deriving.

9.3 Derivatives and Newton's method

Case Study 17: Contraceptive calculations

Each of the many different methods of contraception has advantages and disadvantages. Table 9.1 compares the effectiveness of various methods of contraception, based on data given in [13].

Photo 9.1: Various types of contraceptive including: oral contraceptive, condoms, injected contraceptives and traditional herbal methods. (Source: PA.)

Method	Typical use	Ideal use	1-year
Chance	85	85	
Spermicides	26	6	40
Periodic Abstinence	25	1 – 9	63
Cap			
Parous Women	40	26	42
Nulliparous Women	20	9	56
Sponge			
Parous Women	40	20	42
Nulliparous Women	20	9	56
Diaphragm	20	6	56
Withdrawal	19	4	
Condom	14	3	61
Oral pill	5	0.1	71
IUD	0.1 – 2.0	0.1 – 1.5	80
Depo-Provera IM 150 mg	0.3	0.3	70
Female Sterilisation	0.5	0.5	100
Male Sterilisation	0.15	0.10	100

Table 9.1: The expected percentage of females who will experience an unintended pregnancy when using various methods of contraception for a year, through either typical use or ideal (very careful) use. Also shown is the average percentage of females continuing to use that method of contraception after one year.

Depo-subQ Provera 104 is a long-term female contraceptive administered as an injection every $12-13$ weeks. The active ingredient in a standard 0.65 mL dose is 104 mg of the artificial female hormone medroxyprogesterone acetate (MPA), which is similar to progesterone. It is 99.7% effective, which is very high when compared to many other forms of contraception.

Several commonly quoted benefits of MPA are convenience and reliability. However, studies have identified some side effects, including breakthrough bleeding, reduced libido, weight gain and potentially, reduced bone density.

Example *9.3.1*

Table 9.2 shows some pharmacokinetic parameters of MPA after a single subcutaneous injection of Depo-SubQ Provera 104 in healthy women. The data are based on results in [13], with a sample size of $n = 42$ women.

	C_{max} (ng/mL)	t_{max} (day)	C_{91} (ng/mL)	AUC_{0-91} (ng day/mL)	$AUC_{0-\infty}$ (ng day/mL)	$t_{1/2}$ (day)
Mean	1.56	8.8	0.402	66.98	92.84	43
Min	0.53	2.0	0.133	20.63	31.36	16
Max	3.08	80.0	0.733	139.79	162.29	114

Table 9.2: Pharmacokinetic parameters of MPA.

In Table 9.2:

- C_{max} = peak blood concentration;
- t_{max} = the time at which C_{max} occurs;
- C_{91} = blood concentration at 91 days;
- AUC_{0-91} = the area under the concentration-time curve over 91 days;
- $AUC_{0-\infty}$ = the area under the concentration-time curve over an indefinite time period; and
- $t_{1/2}$ = half-life removal of MPA from the body.

Question 9.3.2

The blood concentration of an injected long-lasting female contraceptive (medroxyprogesterone acetate or MPA) in ng/mL can be modelled by the function $C(t) = 1.4t^{0.15}e^{-0.02t}$ where t is in days. The graph of $C(t)$ is shown in Figure 9.4.

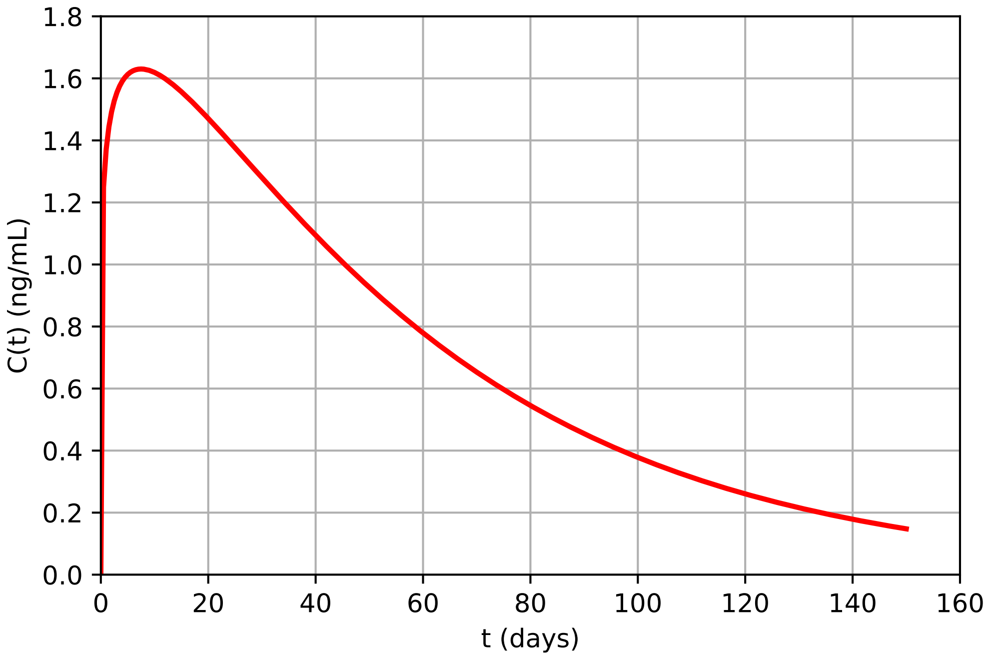


Figure 9.4: Modelled blood concentration after an injection of MPA.

(a) The minimum blood concentration for reliable contraception is 0.3 ng/mL. Estimate the time at which reliable contraception ceases.

(b) Write an equation that should be solved to accurately find this time.

(c) Do you think you could solve the equation in Part (b)?

Some equations are difficult or impossible to solve **exactly**. An alternative is to find an **approximate** solution, using *solution-finding* algorithms, which involve repeatedly applying similar mathematical steps or *iterations*.

Usually, a *numerical error* is associated with the approximate solutions. These errors can often be reduced by performing more iterations.

Newton's method is an iterative solution-finding algorithm which uses an *initial estimate* of a solution and a derivative to find a solution. Newton's method does not always *converge* to a solution, but will usually converge if the initial estimate is 'good enough'.

Note that Newton's method only solves equations of the form $f(x) = 0$. Before applying Newton's method, the equation may need rearranging. For example, to use Newton's method to solve the equation in Part (b) of Question 9.3.2, we instead solve $C(t) - 0.3 = 0$.

Newton's method - informal description

To solve $f(x) = 0$:

1. Choose an initial estimate of the solution.
2. Calculate a new estimate using the old estimate and the derivative. (The new estimate is hopefully better than the old.)
3. Stop if the new estimate is sufficiently accurate or if too many steps have been taken. Otherwise, return to Step 2.

Newton's method is based on equations of straight lines! Let the initial estimate of an unknown solution of $f(x)$ be x_0. Newton's method calculates the next estimate x_1 by extending a line from the point $(x_0, f(x_0))$ to the x-axis, with the slope of the line equal to the value of the derivative f' at the point x_0; see Figure 9.5.

Question 9.3.3

Annotate the following figure to describe how Newton's method repeatedly uses a guess and the tangent line to find the next guess.

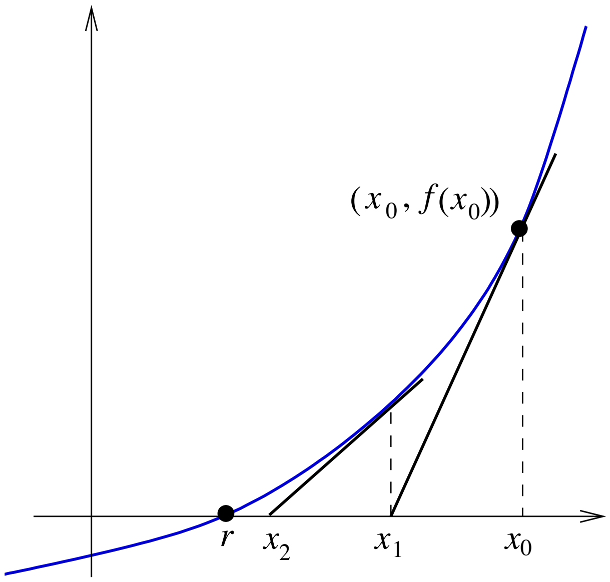


Figure 9.5: Two steps of Newton's method.

Newton's method - formal description

To solve $f(x) = 0$:

1. Let x_0 be an initial estimate of a solution of f that is 'sufficiently close' to an actual solution of f. At the ith iteration ($i = 0, 1, 2, \ldots$), x_i is the current approximation of the actual solution.

2. Calculate the next estimate: $$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

3. (a) If the value of x_{i+1} is sufficiently accurate then stop; x_{i+1} is the estimated solution.

(b) If x_{i+1} is not sufficiently accurate after a certain number of steps then stop, because the method is probably not converging to a solution. Choose a 'better' value for x_0 and start again.

(c) Otherwise, return to Step 2.

Example 9.3.4

Use Newton's method to estimate $x = \sqrt{12}$.

Newton's method is used to solve equations of the form $f(x) = 0$. We want to solve $x = \sqrt{12}$, which does not have the right form. However, we know that $x = \sqrt{12}$ will be a solution to $x^2 - 12 = 0$, and this equation has the form for Newton's method. Hence, let $f(x) = x^2 - 12$; we aim to solve $f(x) = 0$.

To apply Newton's method, we first need to find the derivative and then choose an initial estimate of the solution:

- Because $f(x) = x^2 - 12$, we have $f'(x) = 2x$.
- We know that $\sqrt{12}$ is between 3 and 4, so we will use $x_0 = 3$ as the initial estimate of the solution. (We could choose other estimates but $x_0 = 3$ is likely to be "close" to the solution.)

Now we have everything we need to use Newton's method. Recall that the equation for finding the next estimate of the solution is:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.$$

Performing three steps of Newton's method gives the results shown in Table 9.3, with the last column showing the sequence of approximations to the solution.

i	x_i	$f(x_i) = x_i^2 - 12$	$f'(x_i) = 2x_i$	x_{i+1}
0	3	-3	6	3.5
1	3.5	0.25	7	3.4642857
2	3.4642857	0.001275	6.92857	3.4641016

Table 9.3: Using three iterations of Newton's method to find $\sqrt{12}$.

After three steps, the estimate of $\sqrt{12}$ is $x_3 = 3.4641016$ which is actually accurate to seven decimal places.

We will now return to finding the time at which reliable contraception ceases after an injection of MPA. To do this, we will use derivatives and Newton's method to solve the equation thus determine the timing of a follow-up injection.

Reliable contraception ceases when $C(t) = 0.3$ ng/mL, so $C(t) - 0.3 = 0$ and hence the function $f(t)$ for Newton's method is:

$$f(t) = 1.4t^{0.15}e^{-0.02t} - 0.3.$$

The derivative is:

$$f'(t) = 1.4e^{-0.02t}\left(0.15t^{-0.85} - 0.02t^{0.15}\right).$$

We can use $t_0 = 50$ days as the initial estimate for the solution. (Remember that you usually have a choice of many different initial values.)

Question *9.3.5*

Use one step of Newton's method to find a "better" approximate solution to $f(t) = 0$ for this example.

A Python program can be written to perform further steps.

Program specifications: Write a Python program to apply Newton's method to find the follow-up time for a contraceptive injection.

Program 9.2: Using Newton's method for contraception

```python
# Use Newton's method to find the follow-up time for a
# contraceptive injection.
import numpy as np

# Define the function and its derivative.
def func(t):
    return 1.4 * pow(t,0.15) * np.exp(-0.02*t) - 0.3

def func_dash(t):
    val1 = 1.4 * np.exp(-0.02*t)
    val2 = 0.15 * pow(t,-0.85) - 0.02*pow(t,0.15)
    return val1 * val2

# Input initial estimate.
val = float(input("What is the initial estimate? "))

# Loop through steps of Newton's method.
i = 0
while abs(func(val)) > 0.0001:
    val = val - func(val) / func_dash(val)
    i = i + 1
    print("Step ", i, ":", round(val,3))

# Ouput
print("Estimated time is:", round(val,3), "days")
```

Output for Program 9.2

```
What is the initial estimate? 50
Step  1 : 89.769
Step  2 : 108.467
Step  3 : 112.302
Step  4 : 112.44
Estimated time is: 112.44 days
```

As seen, it gave: $t_4 \approx 112.440$ days after four steps. Thus, the blood concentration of MPA decreased to 0.3 ng/mL at around 112 days, or about 16 weeks (the manufacturer recommends 12-13 weeks with some safety margin).

End of Case Study 17: Contraceptive calculations.

Chapter 10: **Area under the curve**

Lecture 10: **You, me and AUC**

Learning objectives

✓ Interpret the meaning of the area under a curve in various scientific contexts

Scientific examples

✓ Exposure to alcohol, nicotine and glucose

✓ Bioavailability of a drug

Maths skills

✓ Estimate the area under a curve

✓ Apply the trapezoid rule

Key literature

✓ *Pharmacokinetics of paracetamol (Acetaminophen) after intravenous and oral administration* [53]

10.1 Motivation and Background

Image 10.1: *The Drunks* (1629), Diego Velazquez (1599 – 1660), Museo del Prado, Madrid [80]

We have noted that in pharmacology, the area under a drug concentration curve has an important physical meaning. Specifically, a key determinant of the impact of a drug once it has entered the bloodstream is the total exposure of the body to the drug, which is the area under the curve (AUC). When a function is known, it may be possible to calculate the area under the curve using definite integrals. A brief reminder of this is included in Section C.5 of Appendix C.

In this chapter we will study areas under curves (AUCs) and their application to various scientific contexts. While we will not cover integration techniques, we will discuss several methods of finding or estimating AUCs, mainly focussing on the *trapezoid rule*. More importantly, you will need to know how to use and interpret the results. Use SOMSE, available through the course website, for further support and practice.

10.2 Areas under curves

Given a graph, the *area under the curve* or *AUC* of that graph is the area bounded by that curve, the x-axis and two points on the x-axis. The AUC often has a useful physical meaning, which depends on what is being graphed.

***Question** 10.2.1*

What is meant by the AUC in each of the following.

(a) A graph of velocity versus time.

(b) A graph of electricity consumption in a household versus time.

(c) A graph of chlorine concentration in water versus time.

Case Study 18: **Dying for a drink**

Photo 10.1: Left: mellow and yellow. Right: better red than dead. (Source: DM.)

Question 10.2.2

Figure 10.1 shows a graph with a line fitted to some measured blood alcohol concentrations (see [88]).

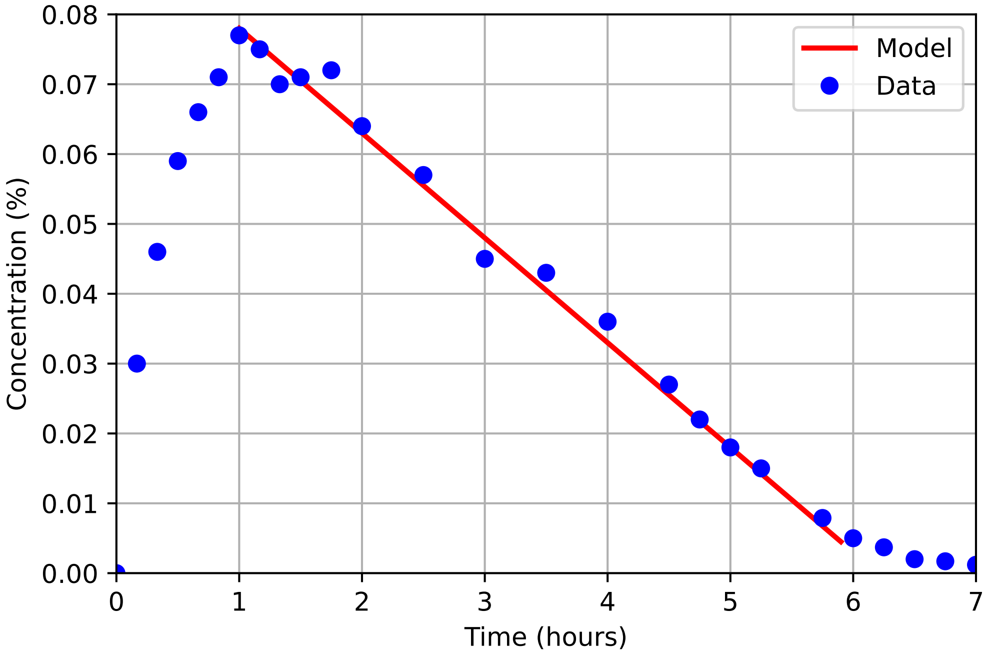


Figure 10.1: A graph of measured BACs.

(a) What are the units of the AUC in the graph?

Question 10.2.2 *(continued)*

(b) What does the AUC represent and why is it significant?

In addition to the immediate risks associated with alcohol consumption (such as accidents), the risk of many negative long-term health effects is increased by both the frequency and volume of consumption. Thus, long-term health risks are affected by the *exposure* to alcohol, (that is, the area under the alcohol concentration curve).

The "standard" *Widmark formula* is:

$$B = \frac{A}{rM} \times 100\% - Vt.$$

The formula, does not account for absorption, but it can be used to estimate the area under the alcohol concentration curve. Since each standard drink contains 10 grams of alcohol, and alcohol is removed from the blood at a rate of $0.015\%/h$, we have

$$B = \frac{10n}{rM} \times 100\% - 0.015t,$$

where B is measured in %, n is the number of standard drinks, M is the person's mass in grams, and t is measured in hours.

Recall that r is the proportion of the person's mass that is water. Typically, $r \approx 0.7$ for males (on average) and $r \approx 0.6$ for females (on average).

Question 10.2.3

The Widmark formula is used to estimate blood alcohol content (BAC); see Question 9.2.2. For a 'typical' 80 kg man drinking n standard drinks, his estimated % BAC at time t in hours since commencing drinking is

$$B = \frac{10n}{560} - 0.015t.$$

(a) Define the *total exposure to alcohol* E as the AUC of B from $t = 0$ until the BAC reaches 0 again. Find an expression for E for this man. [Hint: You will need to find an expression for the time at which his BAC returns to 0.]

***Question** 10.2.3 (continued)*

(b) Assume that long-term damage to internal organs from consumption of alcohol is proportional to the total exposure to alcohol E (which is simplistic, but not unreasonable). Discuss the impact on E of "one extra drink for the road".

Question *10.2.3 (continued)*

(c) A particular 'typical' man with mass 80 kg consumes two standard drinks every day. A second 'typical' man with the same mass consumes 14 standard drinks once a week, but does not drink at any other time. Estimate the weekly value of E for each man. What are some of the physical ramifications of your answer in relation to binge drinking?

(d) For a 'typical' woman of mass 60 kg, $E = 0.0257n^2$. Find the ratio of the values of E for the 'typical' woman and 'typical' man. What does this mean?

Program specifications: Write a Python program that uses the Widmark formula to graph the total exposure to alcohol for a person who consumes from zero to 14 standard drinks.

Program 10.1: Wilful exposure (to alcohol)

```
# Calculate exposure to alcohol.
import numpy as np
import matplotlib.pyplot as plt

# Input
mass_person = float(input("Person's mass (in kg): "))
sex_person = float(input("Enter 1 if male, 2 if female: "))

# Estimate exposure for each number of drinks
if sex_person == 1:
    mass_water = 1000 * mass_person * 0.7
else:
    mass_water = 1000 * mass_person * 0.6
no_drinks = np.arange(0, 15)
peak_bac = 1000 * no_drinks / mass_water
time_bac_0 = peak_bac / 0.015
auc = time_bac_0 * peak_bac / 2.0

# Plot
plt.plot(no_drinks, auc, "bo", markersize=6)
plt.grid(True)
plt.xlabel("Number of drinks")
plt.ylabel("Total exposure (% hours)")
plt.xlim(0, 14)
plt.ylim(0, 3)
plt.show()
```

The program was run with inputs of 80 kg and 1 (male), see Figure 10.2.

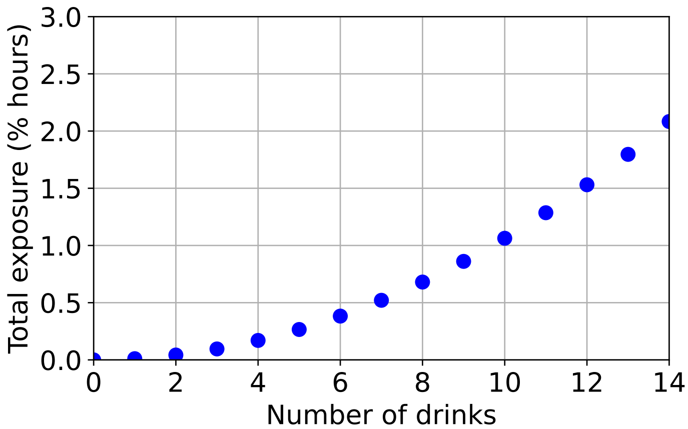


Figure 10.2: Program output showing total exposure to alcohol according to drinks consumed.

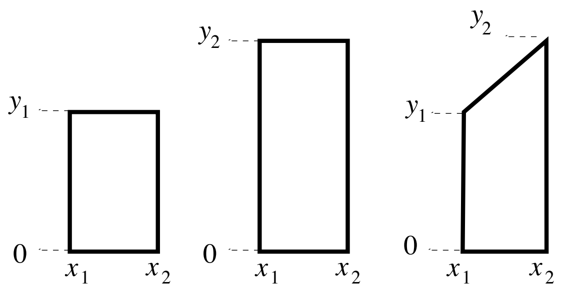
End of Case Study 18: Dying for a drink.

Often, AUCs are used in practical applications in which the only available information is a collection of measured data values, and the function $f(x)$ is **not** known. In such cases, AUCs are estimated, by summing the areas of geometric shapes of "narrow" width, such as rectangles (called *Riemann sums*), or *trapezoids* (called the *trapezoid rule*).

Areas of rectangles and trapezoids

We will fit a rectangle or a trapezoid to consecutive data points, and then sum these areas to find the total area. Consider two data points (x_1, y_1) and (x_2, y_2). We can fit rectangles using either the left height (y_1) or the right height (y_2), or we can fit a trapezoid.

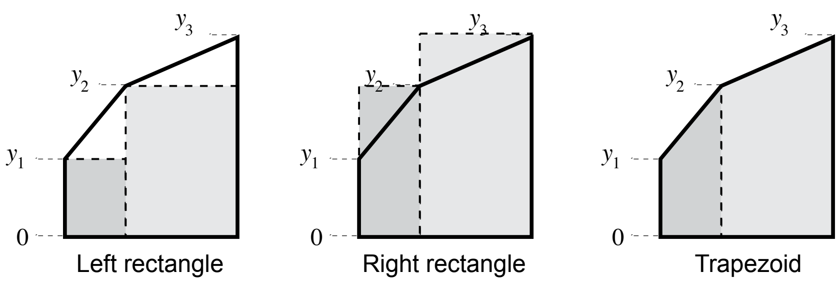
Ensure that you can derive the expressions for the areas of the rectangles and the trapezoid as given below.



$$A_{\text{left rect}} = (x_2 - x_1)y_1; \quad A_{\text{right rect}} = (x_2 - x_1)y_2$$

$$A_{\text{trapezoid}} = (x_2 - x_1)\frac{(y_1 + y_2)}{2}$$

The following figure demonstrates how these shapes are used over successive sets of data points when calculating total areas.



Case Study 19: **Smoking areas**

Question *10.2.4*

Let $N(t)$ be an unknown function representing the blood nicotine concentration of a person after smoking a cigarette. Figure 10.3 shows some concentrations, measured experimentally (see [3]). Data points are joined by straight lines.

t (min.)	0	6	20	35	65	95
Value (ng/mL)	5	15.4	11.3	9.8	8	7

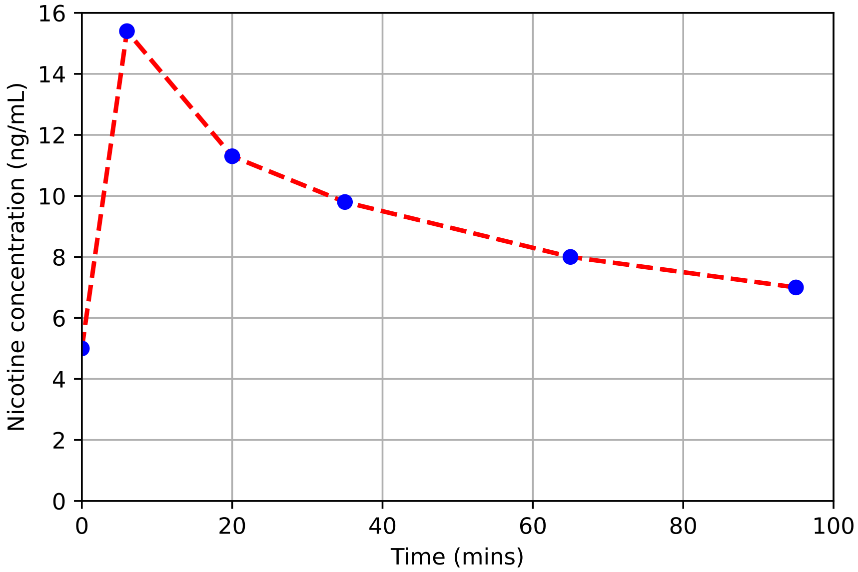


Figure 10.3: Measured blood nicotine concentrations after smoking.

The following calculations use areas of rectangles to estimate the AUC of the nicotine concentration curve from 0–95 minutes. Link the calculations to rectangles on the graph and then calculate for the remaining points.

$$
\begin{array}{rcrcl}
(6-0) & \times & 5 & = & 30 \\
(20-6) & \times & 15.4 & = & 215.6 \\
(35-20) & \times & 11.3 & = & 169.5
\end{array}
$$

Question 10.2.5

Repeat Question 10.2.4 but instead use the trapezoid rule.

t (min.)	0	6	20	35	65	95
Value (ng/mL)	5	15.4	11.3	9.8	8	7

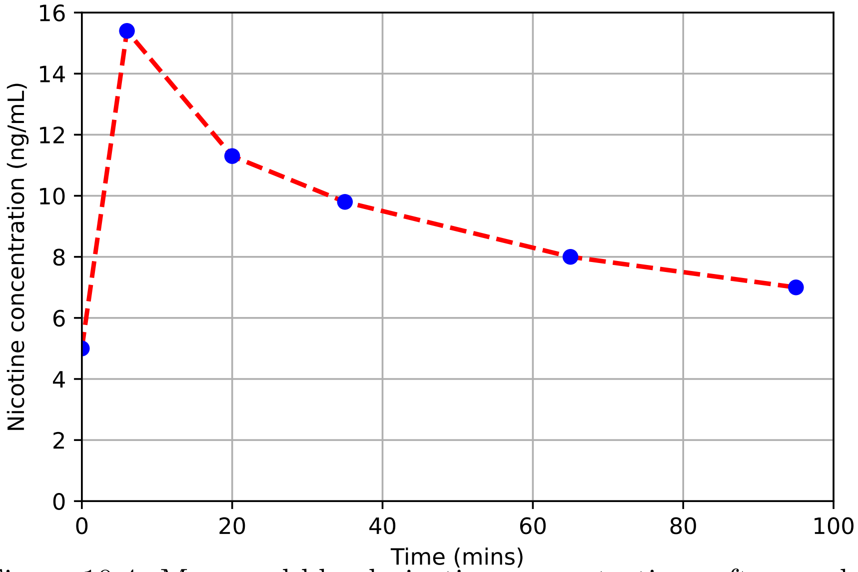


Figure 10.4: Measured blood nicotine concentrations after smoking.

$$\begin{aligned}
(6-0) &\times (5+15.4)/2 &= 61.2 \\
(20-6) &\times (15.4+11.3)/2 &= 186.9 \\
(35-20) &\times (11.3+9.8)/2 &= 158.25
\end{aligned}$$

We can perform the previous calculations more efficiently using a program.

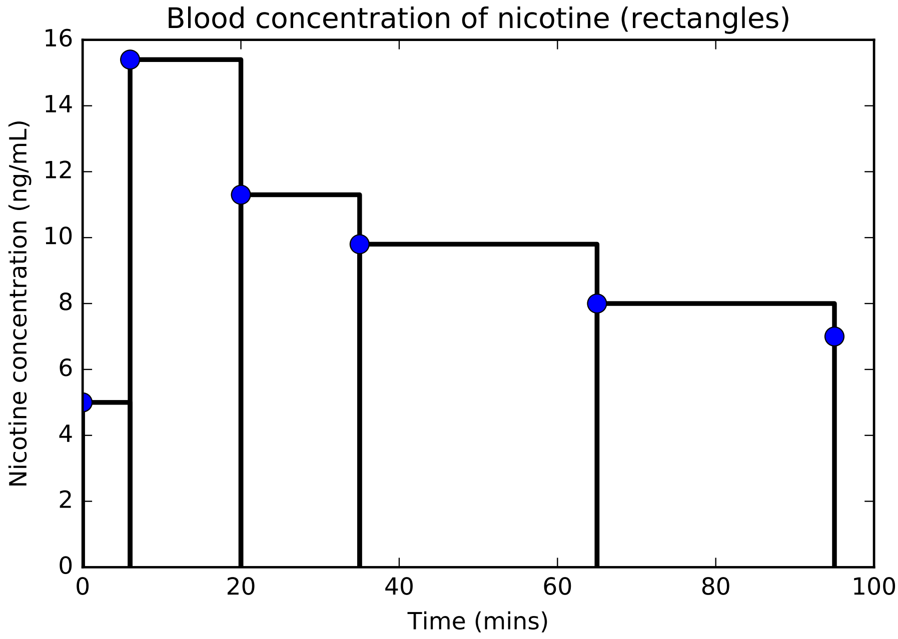
Program specifications: Write a Python program that estimates the AUC for $N(t)$ using rectangles or the trapezoid rule. The program must output the total AUC and draw a graph showing the shapes used in the sums.

Program 10.2: AUC for nicotine

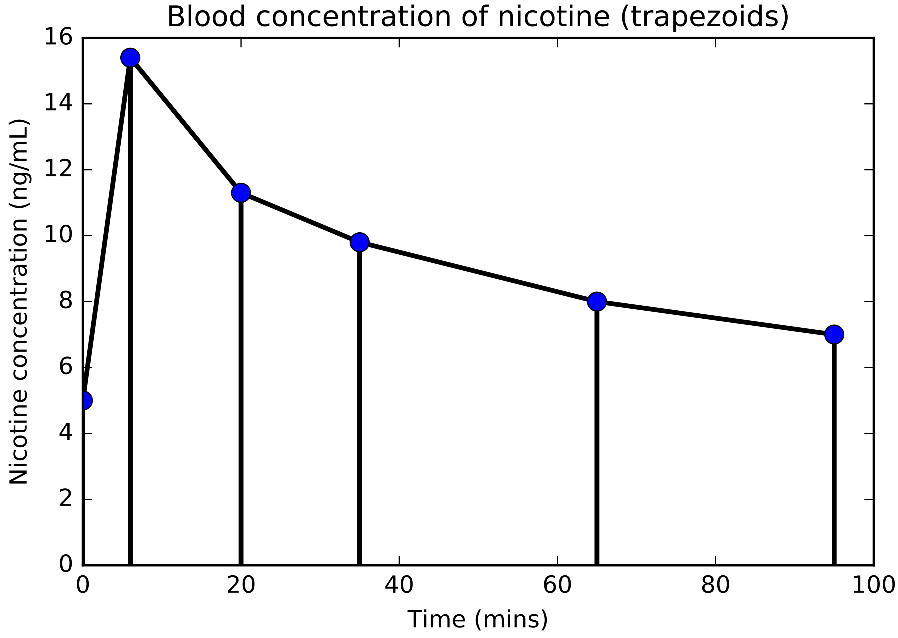
```python
# Use rectangles or trapezoids to estimate AUC for nicotine.
import numpy as np
import matplotlib.pyplot as plt

# Initialise variables
shape = float(input("Type 1 for rectangles, 2 for trapezoids: "))
times = np.array([0, 6, 20, 35, 65, 95])
concentrations = np.array([5, 15.4, 11.3, 9.8, 8, 7])
area = 0

# Sum the areas of each shape
for i in range(1, np.size(times)):
    width = times[i] - times[i-1]

    if shape == 1:
        height = concentrations[i-1]
        shape_t = np.array([times[i-1], times[i-1], times[i], times[i]])
        shape_c = np.array([0, height, height, 0])
    else:
        height = (concentrations[i-1] + concentrations[i])/2
        shape_t = np.array([times[i-1], times[i-1], times[i], times[i]])
        shape_c = np.array([0, concentrations[i-1], concentrations[i], 0])

    area = area + height * width

# Plot each shape
    plt.plot(shape_t, shape_c, "k-", linewidth=2)

# Finish plot and give the output.
print("The estimated AUC is", area, "ng min / mL")
plt.plot(times, concentrations, "bo", markersize=8)

plt.xlim(0,100)
plt.ylim(0,16)
plt.xlabel("Time (mins)")
plt.ylabel("Nicotine concentration (ng/mL)")

if shape == 1:
    plt.title("Blood concentration of nicotine (rectangles)")
else:
    plt.title("Blood concentration of nicotine (trapezoids)")

plt.show()
```

Here is the output from running the above program twice:

```
Type 1 for rectangles, 2 for trapezoids: 1
The estimated AUC is 949.1 ng min / mL
```




```
Type 1 for rectangles, 2 for trapezoids: 2
The estimated AUC is 898.35 ng min / mL
```



End of Case Study 19: Smoking areas.

10.3 All in the blood

Case Study 20: **Sweet P's**

The hormone *insulin* regulates conversion of glucose into usable energy in the body. *Diabetes mellitus* is a group of chronic diseases in which insufficient insulin is produced, or insensitivity to insulin develops. This leads to high levels of blood glucose.

There are three main types of diabetes: *type* 1 (insulin-dependent diabetes, typically present in early childhood, representing about 10–15% of all cases), *type* 2 (non-insulin-dependent diabetes, which accounts for 85–90% of all cases) and *gestational diabetes* (developed in 15–18% of pregnancies).

Typical signs of diabetes include:

- *polyuria* (excessive urination, often with a sweet taste)
- *polydipsia* (excessive thirst)
- *polyphagia* (excessive hunger).

Diabetes, particularly type 2, is becoming increasingly common in societies with a "western lifestyle". Once type 1 or type 2 diabetes becomes established, it is usually permanent. Around 1.7 million Australians have diabetes, although only about half of them are aware of it.

The Framingham heart study has shown that diabetes significantly reduces life expectancy (by 7.5 years for men aged over 50, and 8.2 years for women).

Untreated diabetes can cause blindness, kidney failure and cardiovascular disease including blockages in small arteries. Some patients require amputations after blocked peripheral circulation causes the death of soft tissue.

An *Oral Glucose Tolerance Test* (OGTT) is a common test for diabetes. Prior to taking the test, the patient fasts for around 12 hours. During the test, the patient is administered a measured oral dose of glucose, with blood samples taken immediately prior to ingestion of the glucose and at various intervals for 2 hours afterwards.

The graph in Figure 10.5 compares the measured blood glucose levels for a non-diabetic person with those from a hypothetical diabetic person.

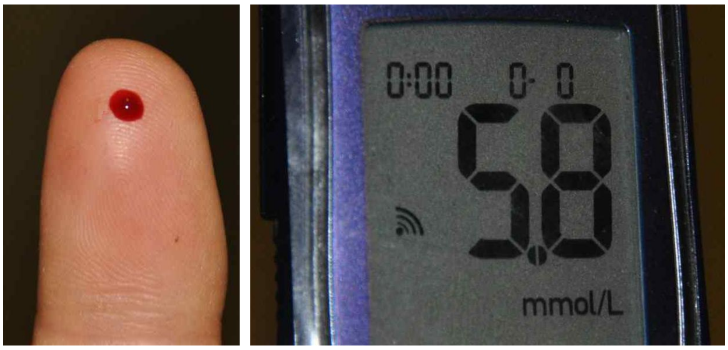


Photo 10.2: Left: bloody finger. Right: measured blood glucose concentration. (Source: PA.)

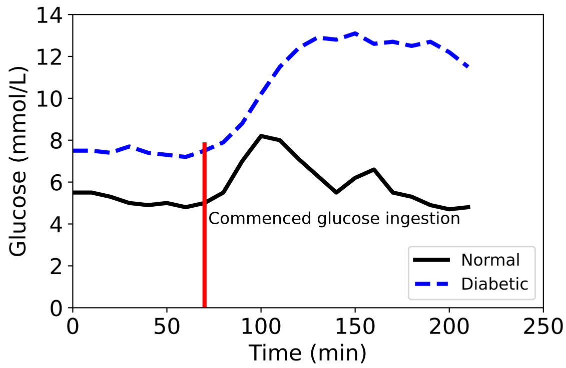


Figure 10.5: Normal blood glucose levels and levels indicative of diabetes.

Table 10.1 shows the World Health Organisation guidelines for blood glucose levels indicating various stages of health or disease.

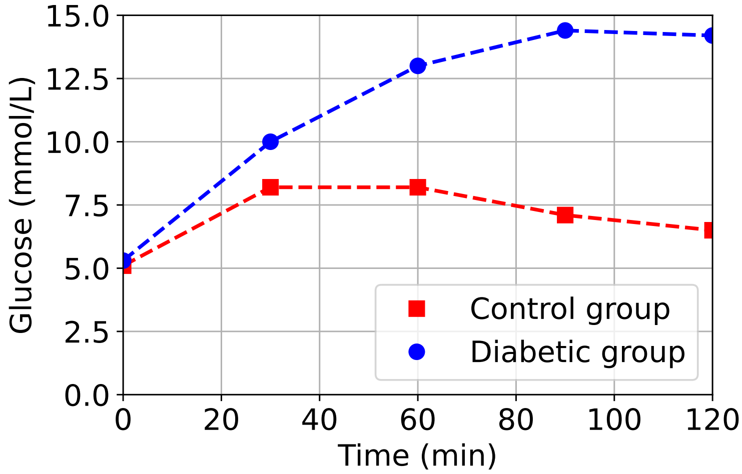
	Blood glucose level (mmol/L)			
time (hr)	Normal	IFG	IGT	DM
$t = 0$	< 6.1	$\geq 6.1, < 7.0$	< 7.0	≥ 7.0
$t = 2$	< 7.8	< 7.8	≥ 7.8	≥ 11.1

Table 10.1: World Health Organisation guidelines for blood glucose levels as indicators of: Impaired Fasting Glycaemia (IFG); Impaired Glucose Tolerance (IGT or *pre-diabetes*); and Diabetes Mellitus (DM).

It is possible that an individual blood glucose measurement might be within the normal range at some instant in time, but outside that range over a longer time period. It is often very useful to analyse AUCs as well.

Question 10.3.1

Figure 10.6 shows measured blood glucose levels for a diabetic and control groups, adapted from [27]. Data points are interpolated by straight lines.



time (mins)	diabetic (mmol/L)	control (mmol/L)
0	5.3	5.1
30	10	8.2
60	13	8.2
90	14.4	7.1
120	14.2	6.5

Figure 10.6: Mean blood glucose levels for diabetic and non-diabetic (control) groups.

(a) Roughly estimate the AUC for the control group.

(b) Use the trapezoid rule to calculate the AUC for the control group.

Question *10.3.1 (continued)*

(c) The paper [27] found that the "glucose AUC" for the control group is around 265 mmol/L/min. Comment on the units and compare the given AUC for the control group with your values in Parts (a) and (b).

(d) How does the concept of measuring the AUC for a blood glucose curve relate to the concept of *total exposure to alcohol* discussed earlier.

End of Case Study 20: Sweet P's.

Case Study 21: **Bioavailability of drugs**

Drugs can be administered via many routes, including:

- orally, such as the contraceptive pill;
- as a gas, such as nicotine from a cigarette;
- through the skin, such as a nicotine patch;
- nasally, such as "snorted" cocaine;
- intravenously, such as chemotherapy drugs for treating cancer;
- sublingually, such as nitroglycerin used to treat angina; and
- rectally, such as a paracetamol suppository.

Different routes of drug delivery are required depending on the drug type, duration and frequency of treatment, and the condition of the patient. Oral administration is common, but other routes may be more convenient for drugs that cause nausea or vomiting, or for patients who cannot swallow.

After administration of a drug, it typically needs to pass through a number of stages before it enters general circulation and has a chance to act. This can have a substantial impact on the proportion of the dose available to achieve the desired pharmacological impact. For example, the following *first pass effects* reduce the availability of orally-administered drugs:

- how readily and rapidly the drug dissolves in the digestive tract;
- whether the drug is damaged by acidic stomach contents;
- whether the drug is partially metabolised by bacteria in the gut;
- how much of the drug is absorbed across the intestinal wall;
- the digestive health of the individual (for example, vomiting or diarrhoea may cause mechanical expulsion of the drug); and
- how much of the drug is metabolised in the liver prior to entering general circulation (because blood travels from the small intestine to the liver and then to the rest of the body).

After administering a drug by a given route, its relative **bioavailability** F is the fraction of the dose that enters general circulation compared to a dose administered via a more direct route, usually *intravenously* (IV).

Definition of bioavailability

If $R(t)$ is the blood concentration of a drug after giving a dose by some route and $I(t)$ is the concentration after an IV dose of **the same size**, then the bioavailability F of the drug administered via this route is

$$F = \frac{\displaystyle\int_0^\infty R(t)\,dt}{\displaystyle\int_0^\infty I(t)\,dt}.$$

Question 10.3.2

Explain the physical meaning of the expression for F. What range of values is F likely to take on?

Question 10.3.3

In [53] and [40], on separate occasions, test subjects were each administered 1000 mg doses of paracetamol. In [53], doses were intravenous and in oral tablet form. In [40], an aqueous dose was administered as a rectal suppository. Figure 10.7 plots the blood concentrations of paracetamol obtained for these subjects over the following six hours.

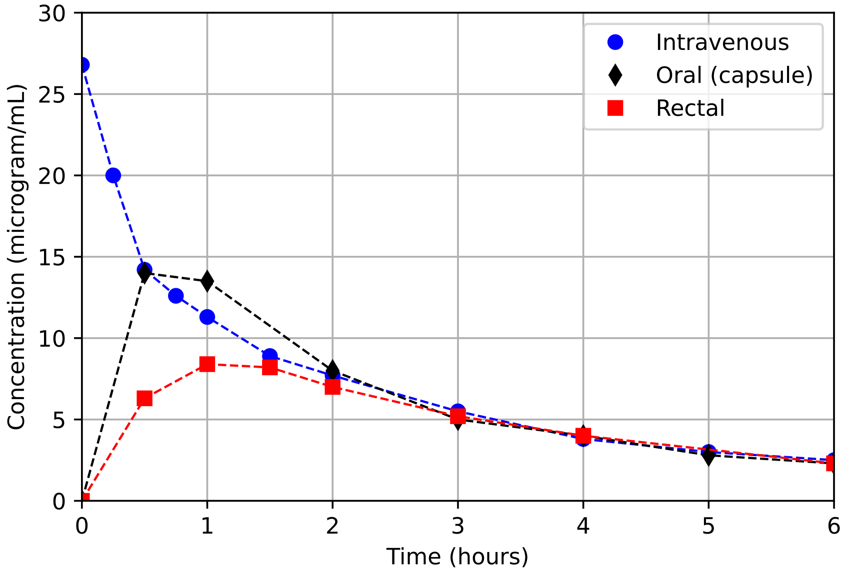


Figure 10.7: Blood concentration curves for paracetamol administered in various ways.

(a) Discuss the shapes of the three curves in Figure 10.7.

(b) **Estimate** the bioavailabilities of the oral and rectal doses.

***Question** 10.3.3 (continued)*

(c) For the IV dose (see [53]), the blood paracetamol concentration in μg/mL at time t in hours after dosing is modelled using the equation

$$I(t) = 13.8e^{-2.55t} + 13e^{-0.28t}.$$

Figure 10.8 plots the measured values and $I(t)$. Using integration, the AUC for this curve is 51.84 μg hr/mL. In [53], the AUC for the oral tablet dose is around 44 μg hr/mL. Calculate the bioavailability of the oral tablet dose.

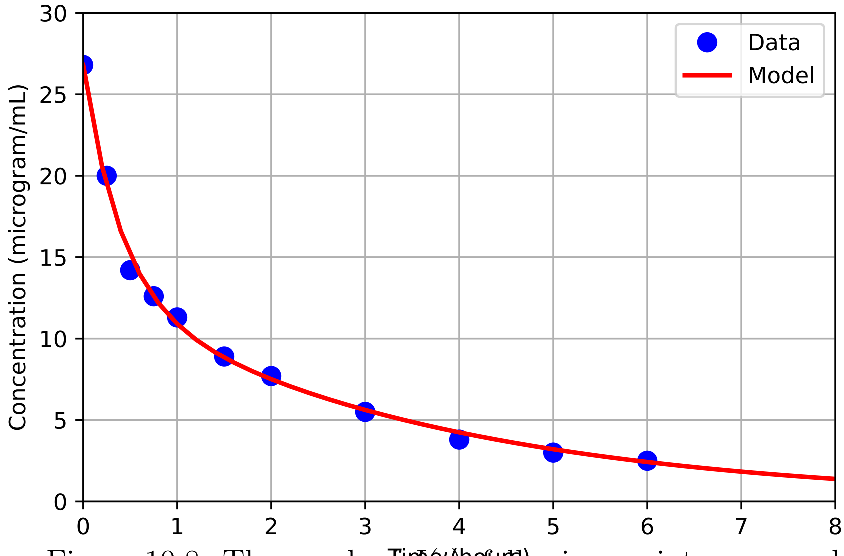


Figure 10.8: The graph of $I(t)$, following an intravenous dose of 1000 mg of paracetamol.

(d) In [40], the AUC given for the rectal dose is around 2290 μg min/mL. Calculate the dose bioavailability.

(e) Compare your answers to Parts (c) and (d) with those to Part (b).

End of Case Study 21: Bioavailability of drugs.

Chapter 11: Differential equations and populations

Lecture 11: What's a DE?

Learning objectives

✓ Understand what a differential equation (DE) is and how they arise naturally in scientific contexts

✓ Analyse the form of population growth under limited resource constraints

✓ Understand the meaning and use of carrying capacity

Scientific examples

✓ Bacteria and oyster population growth

Maths skills

✓ Know the exponential and logistic DEs and their solutions

✓ Understand how and why Euler's method works

Key literature

✓ *When is it optimal to delay harvesting? ...* [33]

11.1 Motivation and Background

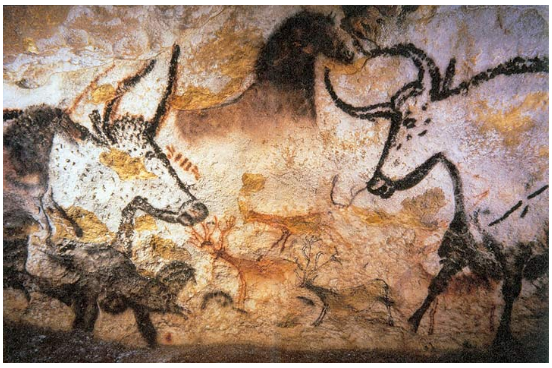


Image 11.1: *Lascaux Cave*, Depiction of aurochs, horses and deer, UNESCO World Heritage Site, Part of prehistoric sites and decorated caves of the Vésère Valley [79]

Differential equations are very useful, "natural" tools for modelling a huge range of phenomena in science and other areas. For example, if you know that a population is changing in size at a rate proportional to its current value, then you can write and solve a simple DE to represent what is happening. Many natural phenomena display this kind of relationship, so knowing how to write and solve the DE allows the phenomena to be studied.

Here we investigate changing phenomena in the context of life cycles and populations, and explore how to model these phenomena. The mathematics that underpins this section is the *differential equation* or DE. We will begin by introducing differential equations (DEs), and then study the exponential and logistic DEs, and use them to model changes in the sizes of simple populations. We will also learn how to use *Euler's method* to solve DEs numerically. Use SOMSE for further support on DEs and Euler's method.

Typically, developing a mathematical model of a phenomenon involves deriving one or more equations that predict the *value* of the phenomenon. Sometimes, this is difficult or impossible.

Instead, it may be possible to write an equation for how the value is *changing*, then use mathematical techniques to deduce information about the *value*.

An equation for the rate at which the value of a phenomenon is changing is called a *differential equation*. To understand differential equations, it is essential to be clear about the distinction between the *value* of a phenomenon, and the *rate at which that value is changing*.

Differential equations

If y is an unknown function of t, then a **differential equation** is an equation that involves a combination of t, y and/or the derivatives of y.

In all of the examples we will study, the DE will be of the form $y' = \ldots$. (That is, the DE will be an equation for the rate of change of y.)

A particular function y is called a **solution** to a DE if the DE is true when y and its derivative(s) are substituted into the DE.

Thus, a solution to a DE is **another function** which, when **substituted** into the DE, makes the DE **true**.

Just as with any mathematical model, there are two steps to modelling with DEs:

- *writing* the equations;
- and then *solving* them.

We will discuss approaches for both steps in this workshop.

11.2 Writing differential equations

DEs are often very "natural" ways of representing phenomena. That is, it often makes "more sense" to write an equation for a rate of change of some value than to write an equation for the value.

Question *11.2.1*

Write a DE to model each of the following.

(a) The straight line distance $D(t)$ travelled by a car increases by 10 m/s.

(b) A population of *E. coli* $N(t)$ has a growth constant of 1% per minute. Assume this growth rate continues.

(c) According to the Australian Bureau of Statistics, during 2013, the Australian population $P(t)$ changed due to: a birth rate of 1.34%; a death rate of 0.64%; an inward migration of 504000 people; an outward emigration of 268000 people. Assume these changes continue indefinitely.

(d) The *von Bertalanffy growth model* states that the rate of increase of the length $L(t)$ of a shark of age t years is proportional to the difference between a fixed maximum length M and its current length $L(t)$. The constant of proportionality is an intrinsic positive growth rate r.

Once a DE has been written for the *rate of change* of a phenomenon, that DE can (sometimes) be *solved* to give the *value* of the phenomenon. This (usually) requires an additional piece of measured information, such as the value of the phenomenon at some time, often $t = 0$.

Example *11.2.2*

Refer to Question 11.2.1. The following additional information applies in each case.

(a) Initially the car has travelled 0 metres.

(b) The number of live bacterial cells on a contaminated slice of ham is initially 580.

(c) The human population of Australia at the start of 2013 was 22.9 million.

(d) In a study on sharks [25], it is found that the length of a typical shark at birth is 110 cm. They also noted that the maximum length of a female *sand tiger* or *grey nurse* shark is $M = 296$ cm; and that the intrinsic growth rate is $r = 0.11$ yr^{-1}.

We can solve some DEs *analytically*, using integration and algebra (but obtaining solutions will not be covered in this course). For others we need to apply numerical approaches to obtain an *approximate* solution. We will explore DEs with known solutions and also introduce an approach to solve DEs numerically.

11.3 The exponential DE

Earlier we studied exponential growth and decay. On Page 71, we state

Any phenomenon that has a rate of change proportional to the current amount follows an exponential function.

This occurs precisely because such phenomena satisfy simple DEs whose solutions are exponential functions.

DE for exponential growth and decay

Any quantity $N(t)$ whose rate of change at any time is proportional to the value of N, with rate of change equal to a constant r per time period, follows the DE

$$N' = rN.$$

The solution to this DE is the exponential function

$$N(t) = N_0 e^{rt},$$

where N_0 is the value of N at $t = 0$.

Many populations can be modelled effectively using exponential functions, for some periods of time. Let's revisit one that we briefly discussed above.

Case Study 22: **Poo**

Escherichia coli (usually shortened to *E. coli*) are bacteria commonly found in the lower intestine of warm-blooded animals, including humans. Most strains of *E. coli* are harmless in the digestive system, or even beneficial to the host.

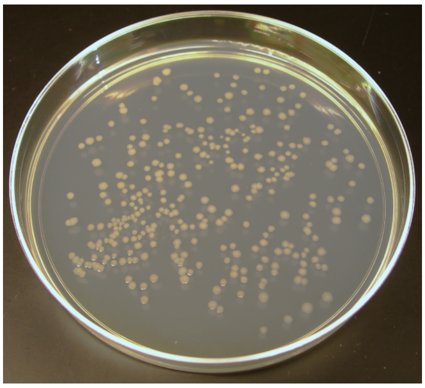


Image 11.2: E. coli colonies on a plate [71]

However, some strains produce toxins, and can cause food poisoning, gastrointestinal infections and urinary tract infections. One such strain is O104:H4, which caused outbreaks of illness in Europe in 2011. Around 50 people died and more than 4000 became ill. Contamination was traced to a farm that grew organic sprouted vegetables.

Because *E. coli* can survive outside the body for some time, tests for *E. coli* are often used to identify faecal contamination in environmental samples or foods during hygiene checks. Under simplifying assumptions (such as relatively unlimited resources), the rate of increase of a population of *E. coli* at any time is proportional to the population size at that time. Hence the population follows an exponential function, and it makes sense to discuss the *doubling time* of the population.

Under favourable conditions, the doubling time for a population of *E. coli* may be an hour, or even shorter. The rapid growth rate is one reason why good hygiene standards are important in food preparation.

When studying populations of bacteria, microbiologists commonly count *colony-forming units* (CFU), which is the number of *live* bacterial cells (direct counts of individuals include both dead and living cells).

Question 11.3.1

A study [60] investigates *E. coli* contamination of pre-cooked meat. Researchers contaminated some ham with an *E. coli* CFU count of 10^7 and then sliced the ham. The same blade was then used to slice clean ham. The number of CFUs were then counted on each of 100 slices of the second ham, showing that Slice 1 contained around 580 and Slice 100 contained 9.

(a) Assuming a growth rate of *E. coli*, $r = 1\%$ per minute, write down the DE for the slice 1, and its solution.

Question *11.3.1 (continued)*

(b) How many CFUs will be on Slice 1 after 24 hours, assuming it is stored under ideal growing conditions for *E. coli*?

(c) The mass of 1 CFU is approximately 1×10^{-12} g and the mass of the Earth is approximately 6×10^{24} kg. According to the exponential model, how long would it take for the mass of the *E.coli* on Slice 1 to reach the mass of the Earth?

End of Case Study 22: Poo.

11.4 Euler's method

We noted that some DEs have analytical solutions but, for many more complex DEs, finding an exact solution is often not possible with such an approach. Instead, we can attempt to find approximate solutions using numerical algorithms. One numerical algorithm for finding an approximate solution to a DE is called *Euler's method*.

Euler's method is a useful approach, that you have possibly used before without knowing, based on a simple observation: if the value of some quantity is changing at a certain amount per time period, then it is possible to estimate the future value as follows:

(future value) = (current value) +
(estimated change per time period) × (number of time periods)

Example *11.4.1*

The human population on Earth in 2014 was 7.295 billion, and was expected to grow at about 80 million per year. We can calculate the population in 2017 as

$$\begin{aligned} P_{2017} &= P_{2014} + (80 \times 10^6 \text{ per year}) \times (3 \text{ years}) \\ &= 7.295 \times 10^9 + 0.240 \times 10^9 \\ &\approx 7.535 \times 10^9 \end{aligned}$$

As can be seen in this example, we do not know the functional form of the population, but we do have a function that describes the *growth*. Here, that function is a constant that is equal to 80 million.

Euler's method proceeds by approximating the unknown function as a series of short **straight lines**, starting from the initial point, each with:

- **width** equal to a chosen step size h;
- **slope** equal to the slope calculated using the DE; and
- **height** equal to the width multiplied by the slope.

The following is a more formal description of Euler's method.

Euler's method

Given an unknown quantity y, a DE of the form $y' = \ldots$, and a value of y (say y_0) at a given time (say t_0):

1. Choose a small *step size* h, and start at the initial point (t_0, y_0).
2. Use the DE to find the slope y_i' at the current time, by substituting the current values of t_i and y_i into the DE.
3. Advance the current point to the end point of a short straight line, by setting $t_{i+1} = t_i + h$ and $y_{i+1} = y_i + h \times y_i'$.

   The new point (t_{i+1}, y_{i+1}) is the next approximate function value.
4. If t has reached the desired end-point then stop, else return to Step 2.

Question *11.4.2*

Draw a diagram illustrating Euler's method.

Example *11.4.3*

Consider the *E coli* example where there was a growth rate of 1% per minute, with an initial CFU population of 580. The population was modelled by the DE

$$N' = 0.01N \qquad \text{where} \qquad N_0 = 580.$$

Use Euler's method to estimate the population after 60 minutes, using a step size of $h = 30$ minutes.

Example 11.4.3 *(continued)*

This population can be modelled using a computer program.

Program specifications: Develop a Python program that uses Euler's method to calculate *E coli* populations and compare with the exact solution.

Program 11.1: *E. coli*

```python
# Uses Euler's method to model the E. coli population.

import numpy as np
import matplotlib.pyplot as plt

# Initialise variables.
initialpop = float(input("Initial CFU population: "))
maxtime = float(input("Enter the maximum time (in minutes): "))
stepsize = float(input("Enter step size (in minutes): "))
steps = int(maxtime/stepsize + 1)

# Create arrays - time in minutes, CFU population is unitless
time = np.zeros(steps)
time[0] = 0
ecolipop = np.zeros(steps)
ecolipop[0] = initialpop

# Step through Euler's method for specified time
for i in range(1,steps):
    d_ecolipop = 0.01 * ecolipop[i-1]
    time[i] = time[i-1] + stepsize
    ecolipop[i] = ecolipop[i-1] + stepsize * d_ecolipop

# Calculate exact solution
timeexact = np.arange(0, maxtime+0.1, 0.1)
ecolipopexact = initialpop * np.exp(0.01 * timeexact)

# Output the graph.
plt.plot(time, ecolipop, "bx", label="Euler (approx)")
plt.plot(timeexact, ecolipopexact, "r", label="Exact")
plt.xlabel("Time (minutes)")
plt.ylabel("Number of E. coli CFUs")
plt.xlim([0, maxtime])
plt.grid(True)
plt.legend()
plt.show()
```

Example *11.4.3 (continued)*

Figure 11.1 shows the output from running the program with the input values used in the previous example.

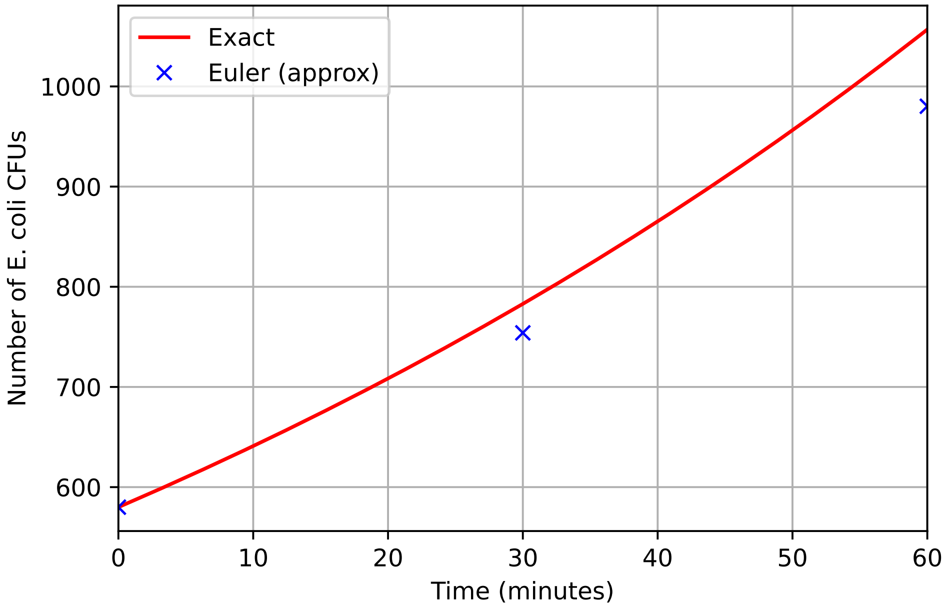


Figure 11.1: *E. coli* population modelled using Euler's method.

How will our answer differ if we repeat the problem in the previous example using a smaller step size?

There are some important things to know about Euler's method:

- It gives an **approximate** solution, not an exact one. There will be numerical inaccuracies in the answer, particularly over a large range of t values.
- The choice of step size is very important: smaller values will give a more accurate answer, but take longer to calculate.
- Despite these limitations, the method can give very good approximate solutions to quite difficult problems.

11.5 Limited scope for growth

Exponential growth models are *unconstrained*, so the growth continues indefinitely with a constant proportional rate of increase, say r. This can be accurate over limited time periods, but in reality populations cannot continue to show unconstrained growth forever. Many *constrained* growth models assume that there is a maximum population size.

Carrying capacity of an ecosystem

The **carrying capacity** K of an ecosystem is the maximum population size of a particular organism that is supported by resources within the ecosystem. Resources may include food, water, shelter and sunlight. The carrying capacity for a particular organism often changes over time; for simplicity, we will assume it remains constant.

A population size below the carrying capacity will typically increase towards the carrying capacity, whereas a population size above the carrying capacity (which may occur when, for example, a lake is overstocked with fish) will typically decrease to the carrying capacity.

Logistic DE

We can model constrained growth using the logistic DE where the function $N(t)$ changes at a rate proportional to its value (with unconstrained growth rate r), **and also** in reverse proportion to how close the value is to a carrying capacity K.

$$N' = r\left(1 - \frac{N}{K}\right)N.$$

We can see that N' is a quadratic function in N. For small N (relative to the carrying capacity), N' increases as N gets larger. At $N = K/2$, N' is a maximum. For increasing values of N above $K/2$, N' then decreases.

Question *11.5.1*

What does the logistic DE predict for N' in the following extreme cases:

(a) N is approximately equal to K?

(b) N is much less than K?

Question *11.5.2*

Based on the information provided, make a rough plot of N against time t assuming a starting value for N that is small compared to the carrying capacity K.

11.6 Oysters in Chesapeake Bay

Case Study 23: **Overfishing annoys an oyster**

Chesapeake Bay is a large estuary on the east coast of the United States, near the states of Virginia and Maryland. The bay has a surface area of more than 11000 km^2, with a shoreline length of more than 18000 km.

In the past, the bay supported a diverse range of flora and fauna, including an abundant shellfish population, most notably oysters. However, it has experienced serious environmental degradation due to over-use, overfishing, and polluted runoff from agriculture, urban areas and industry.

Photo 11.1: Oysters. (Source: MG.)

Substantial *marine dead zones*, which are areas of water so low in oxygen that they are unable to support life, are often found within the bay.

Harvesting oysters is a long-term commercial industry in Chesapeake Bay, however, the size of the population (and hence the harvest) has drastically reduced, due to over harvesting and environmental damage.

In the 1980s there was a significant decline in oyster harvesting.

Considerable research, money and education are being devoted to developing and implementing a sustainable, comprehensive management strategy.

Figure 11.2 shows the historical annual official harvest data for eastern oysters in the Maryland part of the bay.

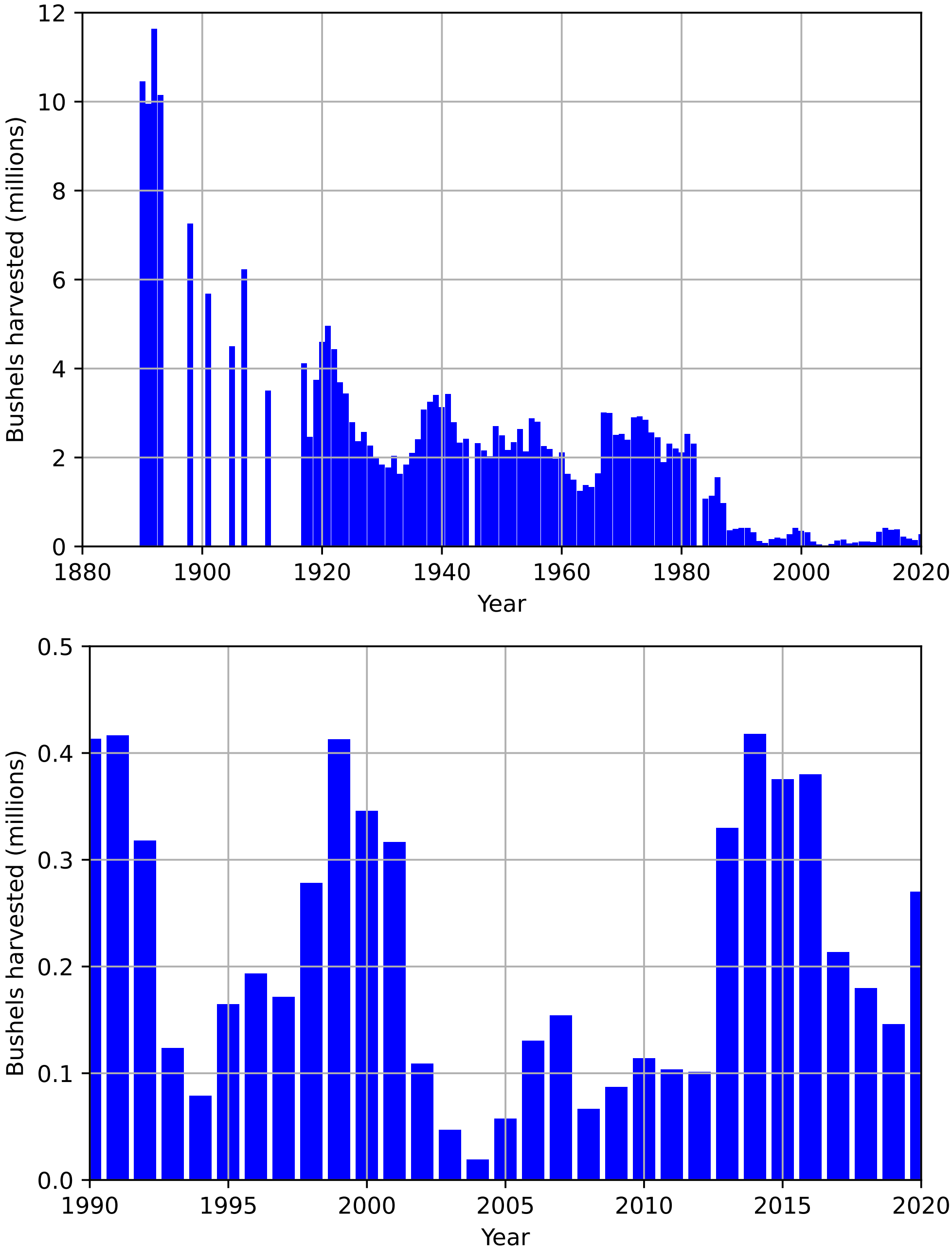


Figure 11.2: Annual harvest of Chesapeake Bay eastern oysters. Top: long term. Bottom: recent. [28]

The paper [33] reports a study of the population and value of market-sized oysters in the Maryland part of the Chesapeake Bay. Using data over the period 1994 – 2007, researchers developed a model which separately accounted for natural growth, oyster mortality due to disease, ecological value of the oysters, harvesting revenue and other parameters. Using a simplified approach (combining natural growth and losses due to disease), we can estimate an effective unconstrained growth rate of market-sized oysters to be around $r = 0.095$ per year for the same time period. The estimated carrying capacity of the Maryland part of the bay is around 5×10^9 market-sized oysters.

Question 11.6.1

(a) Write a logistic DE for the population $N(t)$ of market-sized oysters, assuming no harvesting.

(b) In 2001, the population of market-sized oysters in Chesapeake Bay was 639×10^6. Find the annual increase in the population size at that time.

(c) The 2001 harvest was 120×10^6 oysters. Is this sustainable? Why?

Question *11.6.1 (continued)*

(d) Find the largest number of oysters that could be harvested *from the 2001 population* each year without reducing the total population.

(e) If there was no oyster harvesting conducted for a few years, how would your answer to Part (d) change? Why?

(f) In resource management, especially fisheries management, the *Maximum Sustainable Yield* (MSY) of a population is defined to be the *largest possible harvest size that could be maintained indefinitely*. Explain how the MSY relates to population growth rates. What is the **population size** at which the MSY can be sustainably harvested?

Question 11.6.1 *(continued)*

(g) Find the MSY of oysters in Chesapeake Bay. Noting that there are around 350 oysters per bushel [33], comment on the harvest rates in Figure 11.2.

(h) A logistic growth model of a population with unconstrained growth rate $r = 0.095$ per year, current population $N_0 = 639 \times 10^6$, carrying capacity $K = 5 \times 10^9$ and no harvesting predicts the following population:

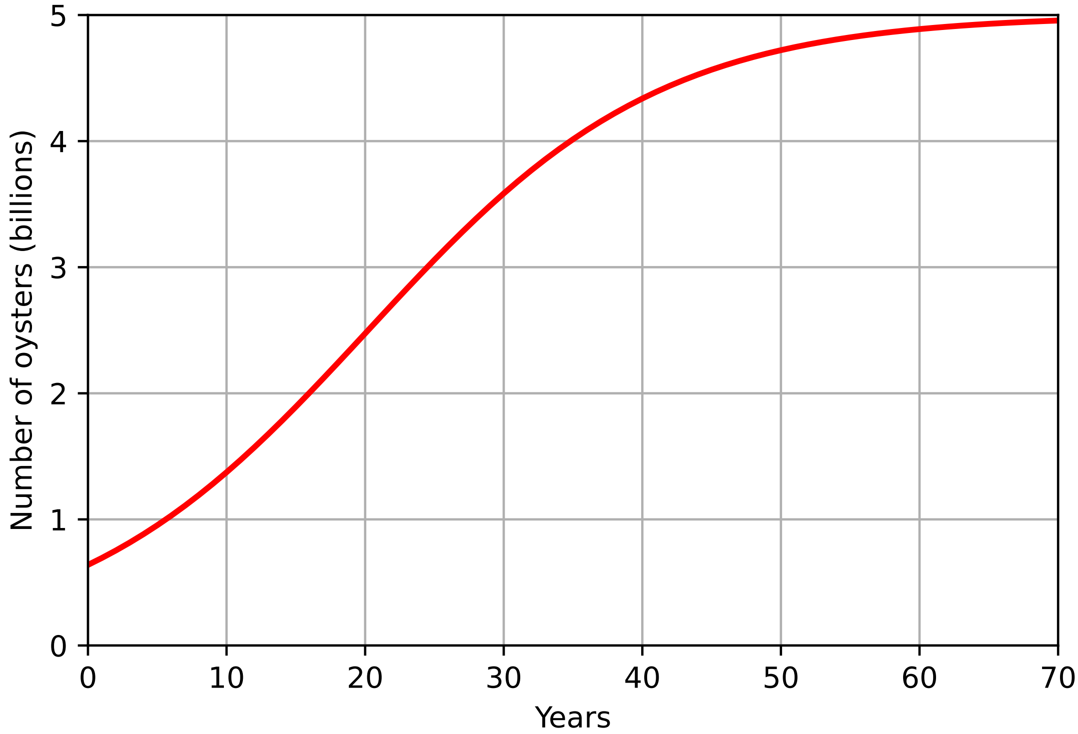


Figure 11.3: Logistic growth of the Chesapeake Bay oyster population with no harvesting.

***Question* 11.6.1** *(continued)*

Make some brief recommendations to assist the government with long-term oyster stock management in Chesapeake Bay.

Photo 11.2: Over-exploited? (Source: MG.)

***Question** 11.6.1 (continued)*

(i) The 2008 paper [33] considers the economic value ("net present value") of the oyster industry, which also considers expected net returns of future harvests. They explain that unrestricted harvesting is unsustainable. They recommend that decreasing harvesting rates will increase the net present value, and shutting down the fishery for a number of years will allow stock to recover and significantly increase the net present value. They noted:

- As a result of habitat degradation, they may have overestimated the carrying capacity, so to test the sensitivity of the results to this parameter, they re-ran the models with a reduced carrying capacity and achieved a near identical optimal harvest rate as the original model.
- To test the sensitivity of their results to the intrinsic growth rate, they tested different growth rates. For example, even when cutting the growth rate in half, the net present value for the optimisation model's recommendation was still six times greater than the net present value for the unrestricted harvesting policy.

Given what you know from the philosophy of science component of the course about models being strictly false, how does the authors' clarification help give confidence in their recommendations?

End of Case Study 23: Overfishing annoys an oyster.

Chapter 12: **Systems of DEs**

Lecture 12: **Stages of life**

Learning objectives

✓ Draw and interpret life cycle diagrams

✓ Understand how to get a system of DE's from a life cycle diagram

✓ Solve and interpret systems of DE's

Scientific examples

✓ Endangered turtles

✓ Canadian lynx and snowshoe hare

✓ Disease models and vaccination targets

Maths skills

✓ Understand and be able to use Euler's method for systems of DE's

Key literature

✓ *A Stage-Based Population Model for Loggerhead Sea Turtles ...* [7]

12.1 Motivation and Background

Image 12.1: *The wild hunt: Åsgårdsreien* (1872), Peter Nicolai Arbo (1831 – 1892), Nasjonalgalleriet, Oslo. [66]

Here we introduce some simple systems of DEs that allow us to model organisms with multiple life stages, and situations in which multiple populations interact, such as predator/prey relationships. We will use systems of DEs to model the spread of infectious diseases and investigate the potential impact of pandemics. We approximate solutions to systems of DEs using Euler's method. You can use SOMSE for further support on systems of DEs and Euler's method.

The DE models we have studied so far have all modelled a single, distinct phenomenon. Often, multiple factors interact, requiring more sophisticated models. For example:

- in predator-prey relationships, changes in population sizes of *two* species are interrelated;
- in species with multiple distinct life stages, changes in the population sizes within each stage depend on the numbers in other stages; and
- the rates at which epidemics spread through populations are influenced by the number of infected individuals **and also** by the number of susceptible individuals.

Typically, models for these more complex situations use a *system* of DEs (that is, more than one DE). Just as with single DEs, analytical solutions exist for some systems of DEs, but other systems require approximate solutions. Euler's method can be used to solve a system of DEs approximately, by applying a single iteration to each equation in turn, and then repeating.

Photo 12.1: Predators: Siberian tigers, *Panthera tigris altaica.* (Source: PA.)

We previously modelled populations using exponential and logistic DEs. In each case we assumed that populations were *homogeneous*; that is, every individual in the population had an identical impact on population growth.

Many organisms have different life stages, each with substantial differences in typical survival rates and reproduction rates. For example, in many species, small juveniles have a low survival rate and do not reproduce, whereas mature individuals have a high survival rate and typically do reproduce. Hence, simple models based on single DEs are inaccurate for more advanced organisms, particularly those with long life spans. In such cases, systems of DEs give rise to better models.

In one type of model, populations are classified into groups based on their *life stages*, such as *juvenile* or *breeding adult*. Rather than applying a constant growth rate to every individual in the population, a system of DEs includes:

- the *distribution* of the population amongst the distinct groups;
- differing rates of *reproduction* and *death* within groups; and
- the *transitions* of individuals between groups.

Life-cycle diagrams are useful aids to writing the equations in a system of DEs. These diagrams show the rates of *transition* between stages.

Life-cycle diagram

Life-cycle diagrams represent all possible transitions between stages in the life-cycle of an organism. Each stage is represented as a circle in the diagram, with a directed solid arrow joining Stage A to Stage B whenever it is possible for an individual to transition from Stage A to Stage B. Each arrow has an associated number, which is the *rate* of transition. We also use a dashed arrow to represent reproduction rates to distinguish it from transitions between stages (an adult does not *become* the offspring).

In order to draw the life-cycle diagram for an organism, we need to know the number of stages, all possible transitions to and from each stage, including reproduction, transitions due to the passage of time, and deaths, as well as the number or probability associated with each possible transition. Once we have drawn a life-cycle diagram, we can use it to write a system of DEs for the number of individuals in each stage. We will show how this is done in this class.

12.2 Going through a difficult stage

Case Study 24: **Total turtle turmoil**

The loggerhead sea turtle (*Caretta caretta*) is a large marine turtle, reaching a length of around 1 m and a mass of more than 100 kg. Individuals often live for more than 50 years. The species is distributed throughout temperate, subtropical and tropical regions, and nests in a number of countries, including Australia. The species is listed as threatened, largely due to human activity, so is likely to become endangered within the foreseeable future.

Image 12.2: Loggerhead sea turtle [69]

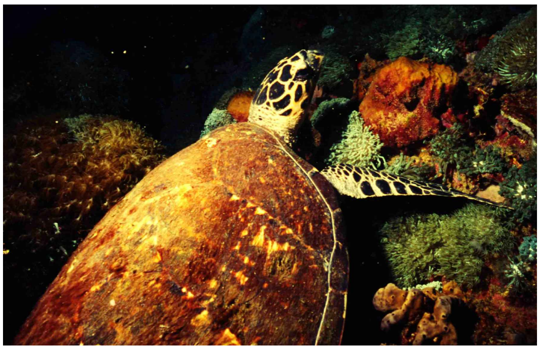


Photo 12.2: Sea turtle species. (Source: DM.)

Researchers in [5] and [7] found that these turtles move through seven distinct stages during their life cycle, and developed a population model based on these stages. We will study a simplified version of their model, with the seven stages collapsed into three for ease of calculation. Table 12.1 shows the life stages used for the simplified model, along with the estimated proportion of the total turtle population, and the global number of individuals, in each stage.

Stage	Description	Age (years)	Proportion	Global population
H	hatchlings	< 1	0.20651	1,445,570
Y	youth	$1 - 23$	0.79097	5,536,790
A	breeding adult	$24 - 54$	0.00252	17,640

Table 12.1: Loggerhead sea turtles classified into three life stages.

Each year, turtles transition with the following (rounded) probabilities:

- Hatchlings become youths with probability $p = 0.675$ or die with probability $p = 0.325$.
- Youths become breeding adults with probability $p = 0.000434$ and youths die with probability $p = 0.230$.
- Breeding adults produce new hatchlings (77.4 per adult), and die with probability $p = 0.191$.

The estimated global population across all life stages was 7 million.

Question 12.2.1

(a) Draw a life-cycle diagram for the three life stages of this turtle.

(b) Write a system of DEs for the turtle population.

Question *12.2.1 (continued)*

(c) Write down the general form of the equations that you would use to determine H, Y and A after one step of Euler's method.

(d) Use the relevant equation with a step size of 1 year to estimate the number of hatchlings after one year.

This population can be modelled using a computer program.

Program specifications: Develop a Python program that uses Euler's method with a step size of 1 year to model the turtle population over 30 years.

Program 12.1: Turtles

```python
# Uses Euler's method to model the turtle population.
import numpy as np
import matplotlib.pyplot as plt

# Initialise variables.
maxt = 30
Hpops = np.zeros(int(maxt+1))
Ypops = np.zeros(int(maxt+1))
Apops = np.zeros(int(maxt+1))
Hpops[0] = 1445570
Ypops[0] = 5536790
Apops[0] = 17640
stepsize = 1

times = np.arange(0, maxt+stepsize, stepsize)

# Step through Euler's method for 30 years
for i in range(1,np.size(times)):
    dH = -Hpops[i-1] + 77.4 * Apops[i-1]
    dY = 0.675 * Hpops[i-1] - 0.230434 * Ypops[i-1]
    dA = 0.000434 * Ypops[i-1] - 0.191 * Apops[i-1]
    Hpops[i] = Hpops[i-1] + stepsize * dH
    Ypops[i] = Ypops[i-1] + stepsize * dY
    Apops[i] = Apops[i-1] + stepsize * dA

# Output the graph.
plt.plot(times, Hpops/1E6, "bx", markersize=8,label="Hatchlings")
plt.plot(times, Ypops/1E6, "r+", markersize=8,label="Youths")
plt.plot(times, Apops/1E6, "gs", markersize=6,label="Adults")
plt.plot(times, (Hpops+Ypops+Apops)/1E6, "ko", markersize=6,label="Total")
plt.xlabel("Time (years)")
plt.ylabel("Number of turtles (millions)")
plt.grid(True)
plt.legend()
plt.show()
```

Figure 12.1 shows the output from running the program.

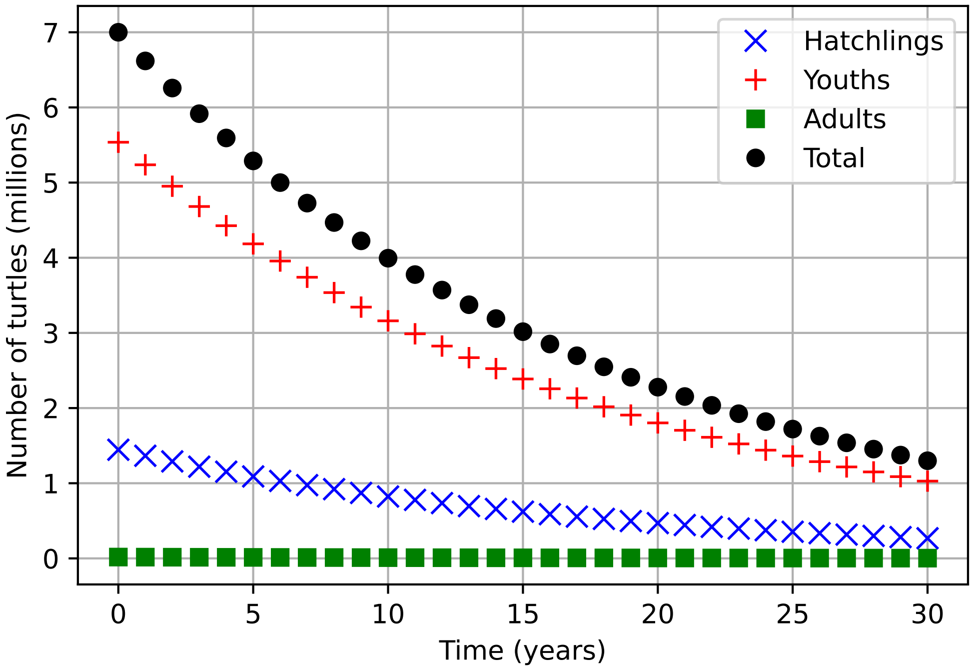


Figure 12.1: Turtle population modelled using Euler's method.

Question *12.2.2*

Researchers and authorities have proposed various conservation strategies for the sea turtle; see [8]. Briefly discuss some possible strategies, and explain how the population model would change to reflect them.

End of Case Study 24: Total turtle turmoil.

12.3 Eat or be eaten

In addition to modelling individual organisms with multiple life stages, systems of DEs can model interactions between multiple species. For example, the classical *predator/prey* problem in ecology considers what happens to the populations of two species when one preys on the other. In laboratory situations there is control over these interactions. In nature, inter-species interactions are highly complex.

Here we will introduce a realistic predator/prey model. In general, such models are based on the assumptions that:

- the prey has no other predators, and the predator no other prey;
- there is no significant change to the environment or species' genetics;
- the prey species is not resource limited and so grows rapidly; prey growth is regulated solely by predator consumption.
- predator population growth depends on the availability of prey; the predator declines to extinction in the absence of prey.
- there is no limit on the amount of prey the predators are able to consume.

The best-known predator/prey model is the **Lotka-Volterra** model.

Lotka-Volterra model

Let $P(t)$ and $Q(t)$ be the population sizes of a predator and prey species respectively, at any time t. The following system of DEs forms the *Lotka-Volterra model*:

$$\begin{aligned} Q' &= \quad aQ - bPQ \\ P' &= -cP + dPQ \end{aligned}$$

where a, b, c and d are positive constants whose values depend on various characteristics of the species and their physical interactions.

***Question** 12.3.1*

Explain the meaning of each term in the Lotka-Volterra equations.

Case Study 25: **Snowshoe hares and Canadian lynx**

Image 12.3: Canadian lynx [68] and a snowshoe hare [72]

The Canadian lynx, *Lynx canadensis*, is a member of the feline family distributed predominantly in Canada and Alaska. Lynx are carnivorous, with individuals weighing 8 to 15 kg, and living for up to 15 years.

The primary food source (up to 95%) of the Canadian lynx is the snowshoe hare, *Lepus americanus*. The hare has large hind feet (for moving on snow) and turns white in winter.

People have hunted these lynx and hares for their fur for many years. Harvest records dating from the 1730s allow long-term population estimates.

Figure 12.2 (from [36]) graphs these data over 90 years, and shows a series of reasonably regular fluctuations in the sizes of both populations.

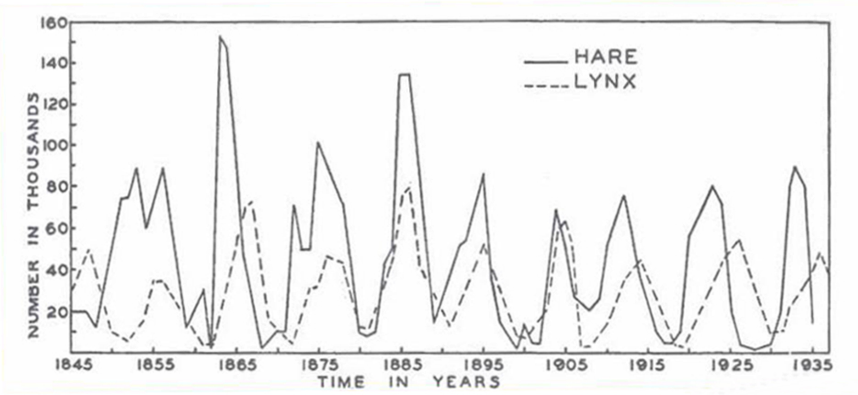


Figure 12.2: Numbers of Canadian lynx and snowshoe hares. (Source: [36].)

Question 12.3.2

Let $L(t)$ and $H(t)$ be the populations of lynx (predators) and hares (prey) respectively, **in thousands**. The Lotka-Volterra equations are:

$$H' = aH - bHL \qquad\qquad L' = -cL + dHL$$

(a) If either population suddenly became extinct, what does the model predict will happen to the other population? Simplify the differential equations and then sketch rough graphs of the populations.

(b) What would you expect to happen in reality?

Example 12.3.3

Table 12.2 and Figure 12.3 show data from the Canadian Government and the Hudson's Bay Company, estimating the populations of hare and lynx in part of their range from 1900 to 1920. (All populations are in thousands.)

Year	Hares	Lynx	Year	Hares	Lynx	Year	Hares	Lynx	Year	Hares	Lynx
1900	30	4	1905	20.6	41.7	1910	27.1	7.4	1915	19.5	51.1
1901	47.2	6.1	1906	18.1	19	1911	40.3	8	1916	11.2	29.7
1902	70.2	9.8	1907	21.4	13	1912	57	12.3	1917	7.6	15.8
1903	77.4	35.2	1908	22	8.3	1913	76.6	19.5	1918	14.6	9.7
1904	36.3	59.4	1909	25.4	9.1	1914	52.3	45.7	1919	16.2	10.1
									1920	24.7	8.6

Table 12.2: Populations of lynx and hares (in thousands).

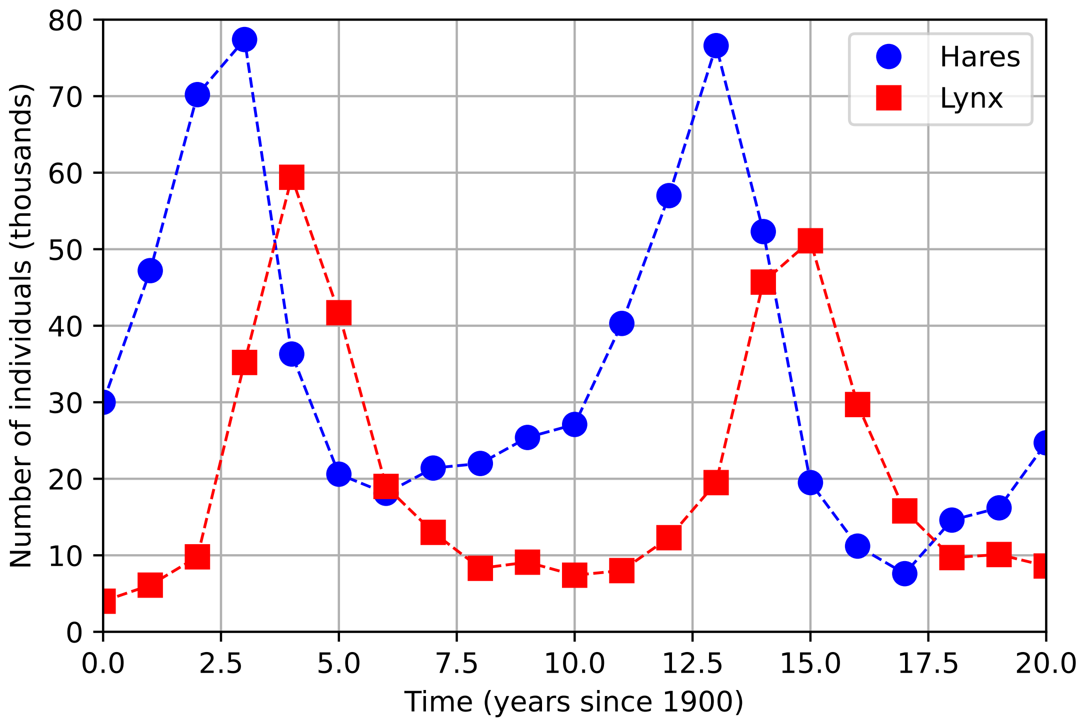


Figure 12.3: Graph of the populations of lynx and hares (in thousands).

Experimentation and analysis show that for this time period, reasonable values for the constants a, b, c and d in the Lotka-Volterra equations for the lynx-hare example are: $a = 0.484$, $b = 0.028$, $c = 1$ and $d = 0.032$ (each per year).

Now we can use Euler's method to model the population sizes.

Program specifications: Develop a Python program that uses Euler's method with a step size of 0.01 years to model the populations of lynx and hares.

Program 12.2: Lotka-Volterra model of hares and lynx.

```python
# Uses Euler's method and Lotka-Volterra equations to model
# populations of lynx and hare from 1900 to 1920.
import numpy as np
import matplotlib.pyplot as plt

# Initialise constants and variables for Euler's method
stepsize = 0.01
times = np.arange(0, 20+stepsize, stepsize)
a = 0.484
b = 0.028
c = 1
d = 0.032

# Create arrays and set initial conditions
hares = np.zeros(int(np.size(times)))
lynx= np.zeros(int(np.size(times)))
hares[0] = 30.0
lynx[0] = 4.0

# Step through Euler's method
for i in range(0,np.size(times)-1):
    dh =  a * hares[i] - b * hares[i] * lynx[i]
    dl = -c * lynx[i] + d * hares[i] * lynx[i]
    hares[i+1] = hares[i] + stepsize * dh
    lynx[i+1] = lynx[i] + stepsize * dl

# Output graphs
plt.plot(times, hares, "b-", linewidth=2, label="Hares")
plt.plot(times, lynx, "r-", linewidth=2, label="Lynx")
plt.xlabel("Time (years since 1900)")
plt.ylabel("Number of individuals (thousands)")
plt.xlim(0,20)
plt.ylim(0,70)
plt.grid(True)
plt.legend()
plt.show()
```

Example 12.3.4

Below is the output of the above program.

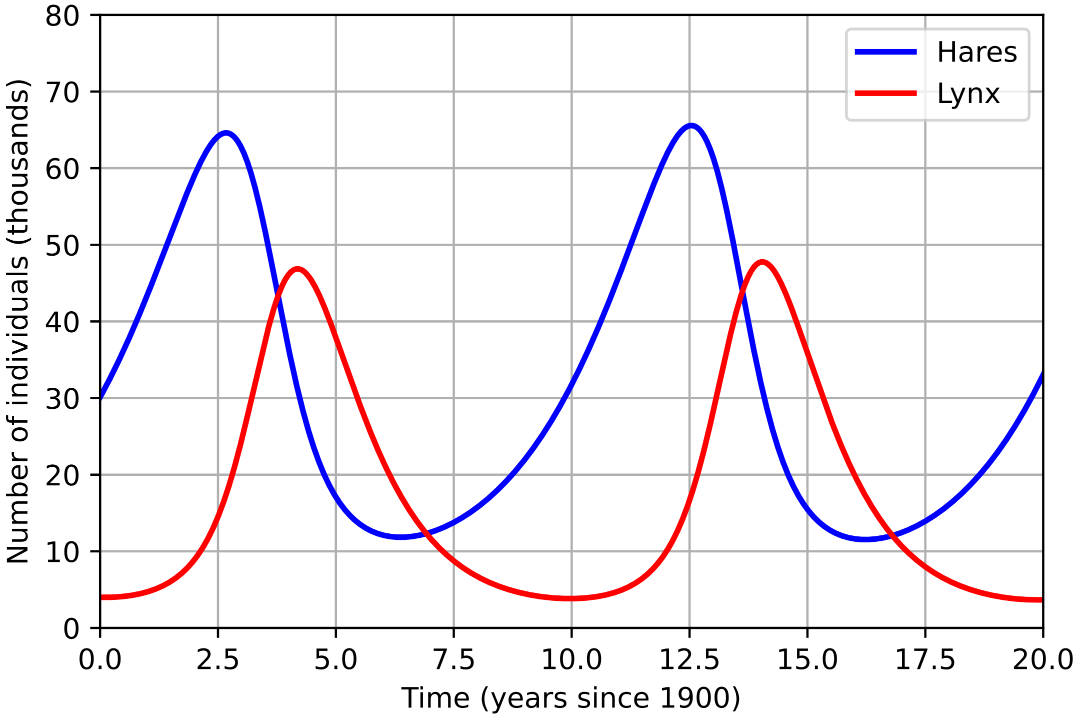
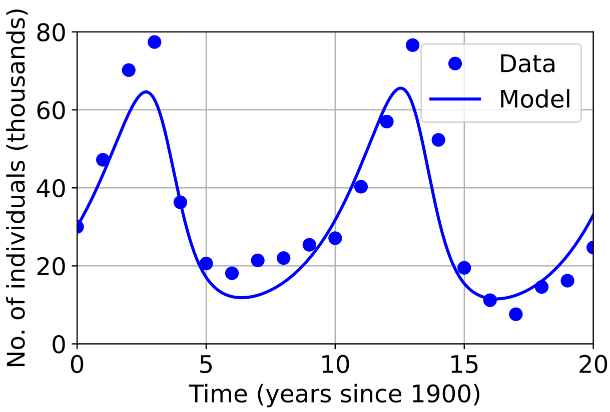


Figure 12.4: Modelled hare and lynx populations.

At time $t = 0$ years (corresponding to year 1900), data show that there were 30 (thousand) hares and 4 (thousand) lynx in the monitored region.

Figure 12.5 compares the modelled population sizes over 20 years with the real (measured) data for each population.



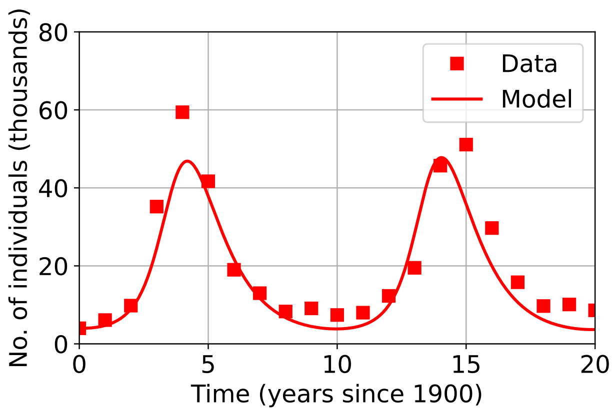


Figure 12.5: Real and modelled populations for hare (left) and lynx (right).

Question 12.3.5

(a) Comment on the results in Example 12.3.4.

(b) Critically evaluate the following possible media statement:

> *A survey has shown that the populations of lynx and snowshoe hares are both in decline. We need to act promptly or else one or both species will become extinct.*

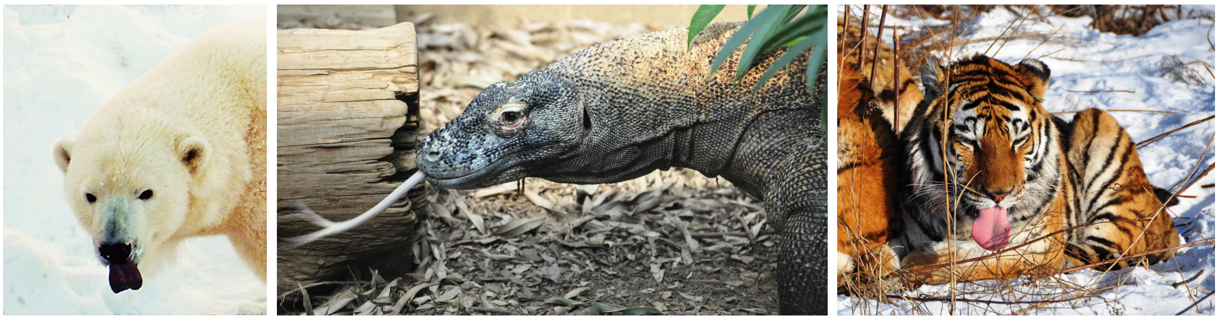


Photo 12.3: Three top predators. Left: polar bear, *Ursus maritimus*. Centre: Komodo dragon, *Varanus komodoensis*. Right: Siberian tiger, *Panthera tigris altaica*. (Source: PA.)

End of Case Study 25: Snowshoe hares and Canadian lynx.

12.4 Epidemics and SIR models

In this section, we will use systems of DEs to model the large-scale spread of communicable disease through a population over time.

Epidemic

A large-scale occurrence of disease in a human population is called an **epidemic** if new cases of the disease arise at a rate that "substantially exceeds what is expected" in a given time period. Localised occurrences are called **outbreaks**, and global occurrences are often called **pandemics**.

Modelling diseases is important to understanding how they spread, and how their impact may be mitigated through approaches such as quarantine, vaccination and public health campaigns.
Modelling disease spread often begins with estimates of the number of secondary infections that typically arise from an individual with the disease, and the rate at which individuals recover from the disease.

Basic reproduction number and infectious period

The **infectious period** of a disease is the average length of time during which an infective individual can infect a susceptible individual. Many diseases are infectious before symptoms become apparent.

The **basic reproduction number** R_0 of a disease is the average number of secondary infections caused by a single infective individual in a completely susceptible population, in the absence of any preventive interventions.

The value of R_0 is determined by factors including how infectious the disease is, how it is spread and the duration of the infectious period.

Photo 12.4: Images commemorating the bubonic plague in Eyam, the "Plague Village", UK. Left: stained glass 'Plague Window'. Centre: first page of the list of names of villagers who died from plague in 1665–6. Right: tombstone. (Source: PA.)

Table 12.3 gives information for some well-known communicable diseases.

Disease	Transmission method	R_0	Infectious period
Rubella	Airborne droplet	≈ 5	2 weeks
Measles	Airborne droplet	$12 - 18$	10 days
Whooping cough	Airborne droplet	$12 - 17$	3 weeks
Mumps	Airborne droplet	$4 - 7$	14 days
Swine flu	Airborne droplet	$1.4 - 1.6$	6 days
Seasonal influenza	Airborne droplet	$2 - 3$	6 days
COVID-19	Airborne droplet	$3 - 10$	$5 - 10$ days
Polio	Faecal/oral	$5 - 7$	$6 - 20$ days
HIV/AIDS	Sexual contact	$2 - 5$	unlimited
Syphilis	Sexual contact	≈ 1.5	up to 2 years
Human papillomavirus	Sexual contact	$1 - 3$	very variable
Pneumonic plague	Airborne droplet	≈ 1.3	2 days (100% death rate)

Table 12.3: Transmission methods, infectious periods and values of R_0 for some communicable diseases.

Infection rate and recovery rate

The **infection rate** a is the rate at which secondary infections arise from a single infective individual, and is defined to equal the basic reproduction number divided by the infectious period (IP). Thus, $a = \dfrac{R_0}{IP}$.

The **recovery rate** b is the rate at which an infective individual recovers, and is defined to equal 1 divided by the infectious period. Thus, $b = \dfrac{1}{IP}$.

SIR model of epidemics

The *SIR* epidemic model classifies a population into three distinct **compartments** or groups, and uses a system of DEs to predict the changes in the number of people in each group. At any time t:

(1) The *susceptible* compartment $S(t)$ is the group of people who are susceptible to the disease.

(2) The *infective* compartment $I(t)$ is the group of people who have the disease and can infect susceptible people.

(3) The *recovered* compartment $R(t)$ is the group of people who had the disease but are no longer infectious. We assume that recovered individuals are not able to be re-infected.

The only possible transitions in the simple SIR model are that: a susceptible person can become infective; and an infective person can become recovered. The model assumes that there are no births or deaths, and that the disease is spread by contact between susceptible and infective individuals in a sufficiently large population that mixes homogeneously.

In a (constant) population of N people, where $N = S(t) + I(t) + R(t)$, the DEs for the SIR model are:

$$\begin{aligned} S' &= -a \times \frac{S}{N} \times I \\ I' &= a \times \frac{S}{N} \times I - bI \\ R' &= b\, I \end{aligned}$$

where a is the infection rate and b is the recovery rate.

Question 12.4.1

Explain the meaning of each of the terms in the SIR model.

Case Study 26: **Rubella**

Rubella (or **German measles**) was (and in some countries, still is) a common disease, particularly in childhood. In most cases, symptoms are very mild, and may even pass unnoticed. However, if a woman is infected during the first 20 weeks of pregnancy, then spontaneous abortion can occur (in about 20% of cases), or the child may be born with congenital rubella syndrome (CRS), which is a range of incurable conditions including deafness, blindness and intellectual impairment. The risk of developing CRS in an unborn child is as high as 90% if the mother is infected during the first 10 weeks of pregnancy.

There was a rubella epidemic in the USA between 1962 and 1965. Data from [48] show that during 1964–65 there were 12.5 million rubella cases leading to 11,000 abortions (spontaneous and surgical) and 20,000 infants born with CRS (12,000 deaf, 3,580 blind, 1,800 with intellectual impairment) During that epidemic, 1% of all children born in New York were affected.

Example *12.4.2*

Consider a city population of 1.5 million people (the approximate population of New York in 1962). Assume that there is just 1 person infective with rubella, and that everyone else is susceptible. Determine the differential equations and the initial conditions to model a rubella outbreak.

Now we can develop a computer program to model the rubella epidemic.

Program 12.3: SIR model of rubella.

```
# This program uses Euler's method and the SIR equations to
# model the spread of rubella in a susceptible population
import numpy as np
import matplotlib.pyplot as plt

# Initialise variables for rubella; values of a and b are per week.
N = 1500000
a = 2.5
b = 0.5

# Initialise variables for Euler's method. The stepsize is 0.1 week.
weeks = 20
stepsize = 0.1
times = np.arange(0, weeks + stepsize, stepsize)
SA = np.zeros(int(np.size(times)))
IA = np.zeros(int(np.size(times)))
RA = np.zeros(int(np.size(times)))

# Set the initial number of people in each category.
IA[0] = 1
SA[0] = N - IA[0]
RA[0] = 0

# Step through Euler's method.
for i in range(0, np.size(times)-1):
    dS = -a * SA[i] * IA[i]/N
    dI =  a * SA[i] * IA[i]/N - b * IA[i]
    dR =  b * IA[i]
    SA[i+1] = SA[i] + stepsize * dS
    IA[i+1] = IA[i] + stepsize * dI
    RA[i+1] = RA[i] + stepsize * dR

# Output
plt.plot(times, SA/1E3, "b--", linewidth=3, label="Susceptible")
plt.plot(times, IA/1E3, "r-",  linewidth=3, label="Infective")
plt.plot(times, RA/1E3, "k-.", linewidth=3, label="Recovered")
plt.xlabel("Time (weeks)")
plt.ylabel("Number of people (thousands)")
plt.xlim(0,20)
plt.ylim(0,1600)
plt.legend(loc="center right")
plt.grid(True)
plt.show()
```

Example 12.4.3

Figure 12.6 shows the program output for a period of 20 weeks.

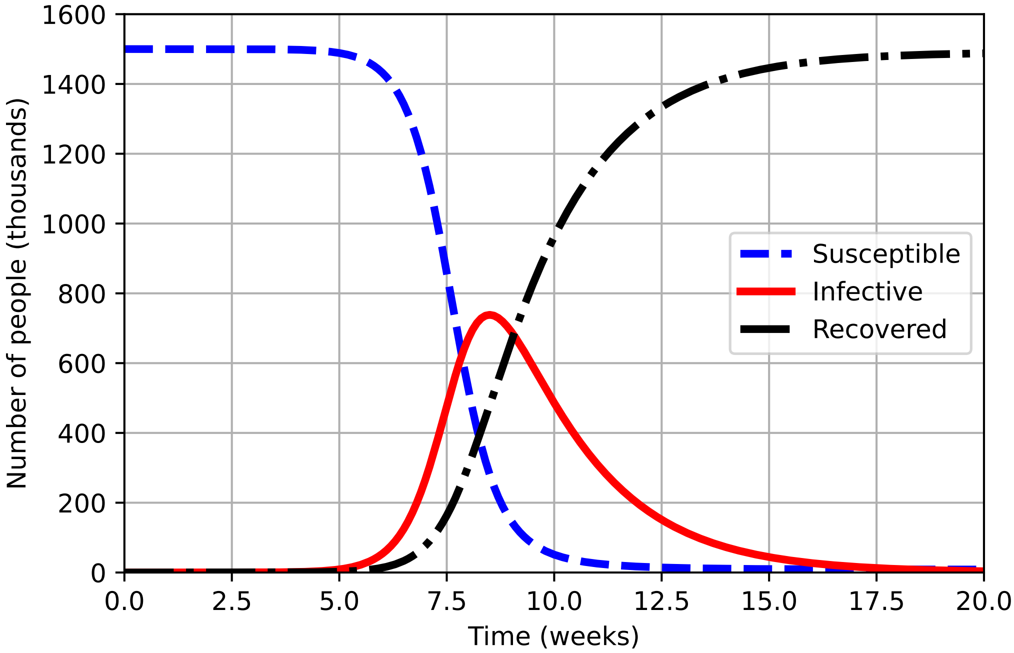


Figure 12.6: A rubella epidemic modelled using Euler's method, showing the numbers of people who are susceptible, infective and recovered.

What key aspects of the epidemic are predicted by the model (as shown in figure 12.6)?

End of Case Study 26: Rubella.

Case Study 27: **Vaccinations**

A rubella vaccine was introduced in 1969 and is routinely administered in many countries. In Queensland, the Department of Health recommends all children have combined MMR (measles, mumps and rubella) vaccines at the ages of 12 months and 4 years.

Vaccination campaigns have greatly reduced the incidence of rubella and the frequency of outbreaks. The Center for Disease Control and Prevention announced that rubella was eliminated from the USA in 2004. In January 2008, at least four babies in Sydney became infected with rubella. All were less than 12 months old, under the age for vaccination with the MMR vaccine.

Pre-emptive vaccination can help by reducing the number of susceptible people, therefore lowering the likelihood of the disease spreading.

Question *12.4.4*

An epidemic occurs if introducing infective people into a population causes an increase in the number of infectives in the population (that is, $I' > 0$). If a is the infection rate and b is the recovery rate then the DE for I' is:

$$I' = a \times \frac{S}{N} \times I - bI.$$

(a) Show that for an epidemic to occur, the proportion of susceptible people in the population must be more than $b/a = 1/R_0$ (in the SIR model).

Question *12.4.4 (continued)*

(b) Mass public vaccination aims to vaccinate a certain proportion of people. What level of coverage do authorities typically aim for? Why?

Question *12.4.5*

Explain why the target vaccination rate for measles is (at least) 95%. What is the figure for rubella?

Question 12.4.6

Earlier we modelled a rubella epidemic in a city with 1.5 million susceptible people and 1 infective person. Figure 12.7 shows the predicted numbers of infective people $I(t)$ under five scenarios, with *preventative* vaccination rates of 0%, 20%, 40%, 60% and 80%.

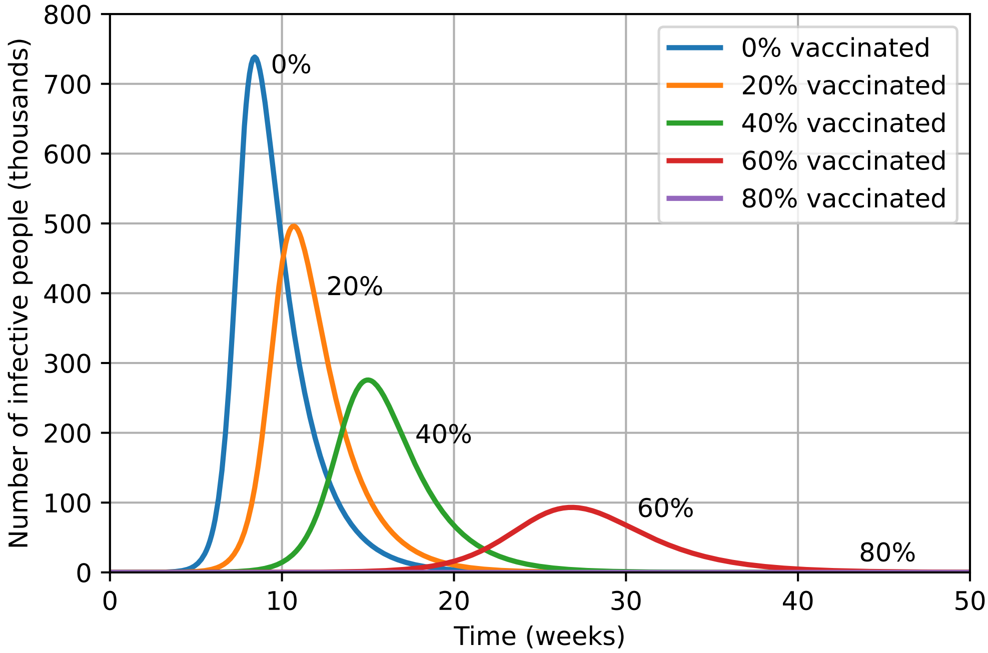


Figure 12.7: The effect of different vaccination rates on a rubella epidemic.

Interpret and explain the graphs. What are the benefits of increased vaccination rates?

End of Case Study 27: Vaccinations.

Appendix A: **Programming manual**

A.1 Getting started

A *computer program* is a list of commands (written into a text file) which instructs a computer to perform specific calculations and operations.

If you want to write a list of instructions for a person to follow, you must write in a language (comprising a vocabulary, rules of grammar, and so on) that the reader understands. Similarly, if you want to write a list of commands for a computer to follow, you must write in a language that the machine understands. An established vocabulary and grammar for commanding a computer is called a *programming language*.

There are many different programming languages, each suited to various uses. In SCIE1000 we will use the language *Python*.[1]

The standard vocabulary of Python includes *reserved words*, which are words with precise meanings in the language, and *literals* like numbers that are typed into the file. Commands are built out of reserved words, literals, and the following elements:

Variables allow data values to be stored and manipulated. Python allows variables that store single data values, and variables called *arrays* that store multiple data values.

Boolean expressions are logical statements that evaluate into true or false. In Python, boolean expressions are built using variables, numbers, the values `True` and `False` mathematical symbols such as "`>`", and logical words such as `and`, `or`.

Function calls direct the computer to complete a computation described in a pre-written piece of code called a *function*. Functions can be written

[1] Python was named after *Monty Python's Flying Circus*. We use Python because it is modern, freely available, fairly easy to learn, used in real science applications, and illustrates many important general computing concepts. Python users include Youtube, Google, Yahoo!, CERN and NASA.

by the programmer themselves, or located in *modules*, which are libraries of useful code.

Commands vary between computer languages, however the following command types are typical:

Comments are ignored by the computer, but make programs easier to understand for people reading them. In Python programs, lines commencing with `#` are comments.

Assignment commands set the value of a variable. In Python, the equals sign "`=`" is the assignment operator. A typical assignment command has the form `variable = calculation`.

Input commands allow data to be entered into the program from the keyboard or a file. In Python, the command `input` is used to obtain data from the keyboard.

Output commands display data on the screen or write it to a file. In Python, the command `print` displays text, while `plot` and `show` together display a graph.

Calculations permit the computer to perform a range of mathematical calculations. Python supports all standard calculations. The results of a calculation are often stored in a variable for later use.

Conditional execution allows the computer to execute certain commands if, and only if, a boolean expression is true. Python supports a number of conditional commands, including `if-elif-else`.

Loops repeat commands. Python has `for loops` which repeat something a pre-determined number of times, and `while loops` which repeat something while a condition is satisfied.

A.2 Basic use of Python

We write and run Python programs throughout semester. We start with a brief description of what a program is, and how to write, save and run programs. Then we cover some useful Python commands to use in your programs.

The usual approach to Python programming is to write a sequence of commands, save them to a file, and then run them. Saving programs in files means that:

- Programs can be run multiple times, on different computers.
- Programs can be debugged more easily. For example, once a section of a program is thoroughly tested, it does not need any further testing.
- Teams of people can design and write different parts of the program.
- Problems of much greater complexity can be solved.

A.2.1 Creating Python programs

The specific methods for creating, saving and running Python programs depend on the application in which you write and run your code. You are strongly encouraged to download Anaconda, a free and open-source distribution of the Python and R programming languages, for your own computer (instructions are available in the Learning Resources section of Blackboard).

The Anaconda distribution contains Jupyter Notebook and Spyder: two applications in which you can write and run Python code. They are designed for different purposes:

- Jupyter Notebooks makes it easy to produce interactive documents, where you can write and execute "snippets of code" in between pieces of written text. Your tutorial worksheets are prepared in Jupyter notebook.
- Spyder is designed for writing and running code only, and has enhanced features for debugging your code. You may prefer Spyder when writing a stand-alone program.

A.2.2 Comments in Python programs

The language Python is designed to be as "readable" as possible by English speakers. The vocabulary borrows from the English language, and the grammar rules are somewhat similar to English as well. Even so, the technical nature of the task means that it is sometimes difficult to read Python code and understand what it is doing.

One way to help the reader (who may be you, several weeks or months after you wrote the program!) quickly understand what your code is doing is to include *comments*. Any line starting with `#` in a Python program is a comment. The computer does not execute a comment, it just skips over it looking for the next command.

Just as annotations or margin notes can help the reader understand a dense and highly technical passage in a textbook, comments help the reader understand a piece of code. Always use comments where appropriate.

A.2.3 Importing modules

The vocabulary for Python provides a basic level of functionality. A more sophisticated vocabulary of instructions, including many mathematical functions and other useful operations, can be accessed by directing the computer to read through, or "import," one or more *modules* before it processes your program. We will use the the module called `numpy` for mathematical functions. Your programs should thus start with:

```
import numpy as np
```

This line instructs the computer to read through the module `numpy` and remember all of the new vocabulary it finds. We will introduce a second module when plotting.

Warning

Omitting "`import numpy as np`" when required will cause an error if you are using mathematical functions (such as `sin`) or arrays.

A.2.4 Printing to the screen

The *print* command outputs text and the results of calculations to the screen.

Printing text

To print text to the computer screen, use the `print` command in the following ways:

`print()`	prints a blank line.
`print("message")`	prints the message.
`print(expression)`	prints the **value** of the mathematical expression.

When printing multiple items (which may include a mixture of expressions and messages), items must be separated by commas. Note that the items will be *separated by a single space*.

Anything within quotation marks is printed *as it is*, whereas anything not inside quotation marks is evaluated as an expression and the *answer* is printed.

The following program demonstrates use of the `print` command. Note that line numbers are added for ease of reference and are not part of the program. If you wish to see how it works, type this program into a Python edit window, save it and then run it. Take care to type everything correctly.)

Program A.1: Printing things

```
1 # Print some messages.
2 print("This is a message")
3 print("This is first", "and this is second")
4
5 # Print the results of some calculations.
6 print(3+4)
7 print("3*4 =", 3*4)
```

Note that lines 1 and 5 of the program contain comments. Also, you can use blank lines (such as lines 4) to make your program more readable.

Output for Program A.1

```
This is a message
This is first and this is second
7
3*4 = 12
```

A.2.5 Numerical calculations

As we have demonstrated, Python can perform standard mathematical operations. The following table shows how to do this. (In each case, the letters a and b represent numbers.)

Mathematics	**Python**	**Mathematics**	**Python**
$a+b$	`a+b`	$a-b$	`a-b`
$a \times b$	`a*b`	$a \div b$	`a/b`
a^b	`a**b`	(...)	`(...)`

Exponentiation

You may have seen `a^b` used to represent a^b on your calculator. In Python `a^b` means something completely different so be careful to use `a**b`.

The following program demonstrates some mathematical operations.

Program A.2: Simple calculations

```python
# Simple calculations:
print(3+2, 3-2, 3*2, 3/2)
print("3 squared =", 3**2)

# Python correctly applies order of operations:
print(2+3*4, "and", (2+3)*4)
```

Output for Program A.2

```
5 1 6 1.5
3 squared = 9
14 and 20
```

Use spaces within expressions (almost) anywhere, to make the program easier to read and understand. When using spaces, remember that your main goal is *communication*; use your judgement about what works. The following example demonstrates one approach.

Program A.3: Spacing inside Python programs

```
# Adding one space between numbers and symbols is reasonable.
print(6 + 4)

# You normally do not use space between brackets and numbers.
print((2 + 3) * (6 - 4))

# Sometimes spaces are used to show order of operations.
print(2 + 3*4 + 5*6)
```

Note that in Python, the symbol `e` represents scientific notation when used while writing numbers. For example, 6.02×10^{23} is displayed as `6.02e+023`, and 3×10^{-4} as `3e-04`.

A.3 Variables, functions and input

A.3.1 Variables

A program can "remember" values by *assigning values* to *variables*. The programmer can create and use as many variables as needed.

Each variable has a name, chosen by the programmer, which is used to access it. When Python encounters a word that it does not recognise, it assumes that the word is the name of a new variable. To introduce a variable, you must use it on the left-hand side of an assignment command. For example, to assign a value to a new variable called `level`, use the command

$$\texttt{level} = \texttt{expression}$$

where `expression` is either a value (such as `3` or `-2.25`), or an equation (such as `2 + 4`). Python calculates the result from the expression on the right hand side of the equals sign, and *assigns* that value to the variable `level`.

After you have introduced a variable and assigned it a value, Python will recognise the name as referring to whatever value the variable has been assigned. You can use that variable name in subsequent calculations. You can use another assignment statement to assign a new value to the same variable. The following program gives some examples.

Program A.4: Variables

```
1 width = 20
2 height = 45
3 print("For a rectangle of size", width, "by", height)
4 print("the area is", width * height)
5 perimeter = 2 * width + 2 * height
6 print("and the perimeter is", perimeter)
7 
8 width = width + 5
9 height = height + 5
10 print("For a rectangle of size", width, "by", height)
11 print("the area is", width * height)
12 perimeter = 2 * width + 2 * height
13 print("and the perimeter is", perimeter)
```

In line 1, a new variable called `width` is introduced. Python knows this is a new variable because `width` is not a reserved word, and this is the first time it encounters it in the program. The variable is assigned the value 20. In lines 3, 4 and 6 the variable name `width` appears, in each case the value 20 is used in the place indicated. In line 8, `width` is assigned a new value, equal to the value it had (20) plus 5; so the new value is 25.

Output for Program A.4

```
For a rectangle of size 20 by 45
the area is 900
and the perimeter is 130
For a rectangle of size 25 by 50
the area is 1250
and the perimeter is 150
```

Choosing variable names

Always choose *meaningful* names for your variables, to make the program easier to understand. Examples of names are `x`, `height`, `NumFish` and `x7`. Do not use spaces or other “special” characters in variable names. Also, note that variable names are case sensitive, so `numfish` and `NumFish` are different.

A.3.2 Python functions

Just as a program can remember values by storing them as variables, it can also remember pieces of code in the form of functions. A function is a piece of code that acts on one or more values and produces some output. When you instruct the computer to run a function, you are said to be "calling" the function.

A function is not executed at the point in the program where it is defined, rather, it is executed at the place(s) in which it is called. To call a Python function that has already been defined, type the name of the function followed by parentheses enclosing the value, or list of values, to use.

Here is a list of some mathematical functions in Python and a description of the value they return:

`sqrt(value)`	square root of the value
`sin(value)`	sine of the value (the value is in radians)
`exp(value)`	e raised to the given power
`abs(value)`	the absolute value of *value*
`log(value)`	natural logarithm of value.
`log10(value)`	base-ten logarithm of value.
`round(value, digits)`	returns the value rounded to the specified number of digits after the decimal.

These mathematical functions (apart from `round`) come from the module `numpy` (which must be imported). They need to have '`np.`' written before them in your code as demonstrated in the following example.

Program A.5: Functions

```
import numpy as np

val = 9
print("The square root of 9 equals", np.sqrt(val))
print("e^1 = ", np.exp(1))
print("When rounded to two decimal places, e^1 = ", round(np.exp(1), 2))
print("When rounded to the nearest integer, e^1 = ", round(np.exp(1), 0))
print("log to base 10 of 1000 equals",np.log10(1000))

# Evaluate sin of 90 degrees. First, convert to radians.
angleDeg = 90
angleRad = angleDeg * np.pi/180
print(angleDeg,"degrees =", angleRad,"radians.")
print(np.sin(angleRad))
```

Output for Program A.5

```
The square root of 9 equals 3.0
e^1 =  2.71828182846
When rounded to two decimal places, e^1 =  2.72
When rounded to the nearest integer, e^1 =  3.0
log to base 10 of 1000 equals 3.0
90 degrees = 1.5707963267948966 radians.
1.0
```

A function must be defined before it can be called. Many useful functions are defined in the module `numpy`. Provided that your program begins with the line

```
import numpy as np
```

you can use any of the functions that have been defined there. These functions are efficient and extremely well-tested.

The constant `pi`, which (approximately) equals π is also included in `numpy`.

A.3.3 The input function

It is often useful or necessary to ask the user to enter some input from the keyboard. The `input(...)` function prints a message on the screen, waits for the user to enter data with the keyboard followed by the `Enter` key, and returns the data they entered so that it can be stored as a variable. For example, the command

```
user_age = input("Please enter your age: ")
```

will display the message "Please enter your age:", and cause the program execution to wait for the user to enter data with the keyboard. When the user finally hits the `Enter` key, whatever they typed will be stored in the variable `user_age`.

Users sometimes enter strange things. If the user responds to the prompt above by typing "`a\%6J@`", and the program goes on to use `user_age` as if it is a number, things might get weird. It is good programming practice to make sure that the data entered is of the type expected. In this course we will only ask the user to input numbers.

We can make sure that what the user enters is a number by calling the input function inside the function `float`[2]. The `float` function tries its best to interpret what the user entered as a decimal number; if it cannot, the program execution will cease with an error message.

Program A.6: Input

```python
# Input two values, then multiply and divide them.
a = float(input("Type a number: "))
b = float(input("Type another number: "))

prod = a * b
quot = a / b

print(a, "*", b, "=", prod)
print(a, "/", b, "=", quot)
```

Output for Program A.6

```
Tell me a number: 8
Tell me another number: 2
8.0 * 2.0 = 16.0
8.0 / 2.0 = 4.0
```

A.4 Software errors and bugs

All computer programmers make errors. An error in a program is called a *bug*. A key skill in programming is minimising the number of errors, and then identifying and fixing any that occur. There are many different types of errors, including incomplete problem descriptions, design faults in the software, unanticipated 'special cases', coding errors and logic errors.

In real life, the consequences arising from programming errors can be serious: they have caused plane crashes, rocket explosions, and failure of entire transport systems. In this course we will not give you control of aeroplanes, rockets or even small transport systems. The impact of any programming errors will be minor. You may feel a bit frustrated and need to ask for help, but no lasting damage will occur. Rather, you should learn from the process of finding and fixing your errors.

[2] Float is short for "floating point decimal", a phrase which refers to the format in which the computer stores the value in its memory.

A.4.1 Avoiding Errors

When writing programs, make sure that you:

- Understand the question **before** you start programming;
- Think about the best and most logical way to solve the problem;
- Consider planning your program on paper first;
- Put comments in your program so you know what you are trying to do;
- Test your programs on a range of data;
- Check some output carefully to make sure it is correct; and
- Pay attention to any error messages!
- Try writing a few lines at a time, then running your program to make sure those lines are doing what you think they are doing.

Fear not!

Do not be afraid of error messages! Never let the fear of error messages stop you from playing around with Python and trying different commands. Getting an error message **does not** mean that you will fail the course. If it helps you to figure out what you did wrong, then you have learned something! Equally important, do not ignore error messages. They give you useful advice about what is going wrong.

The following Python program illustrates a number of errors.

Program A.7: Multiple errors

```
# Input two values, then multiply and divide them.
a = float(input("Tell me a number: "))
b = float(input("Tell me another number: "))

prod = a * b
quot = a / bb

print(a, "*", b, "=", prod)
print(a, "/", b, "=", prod)
```

Output for Program A.7

```
Tell me a number: 12
Tell me another number: 4

Traceback (most recent call last):
  File "inputerror.py", line 6, in <module>
    quot = a / bb
NameError: name 'bb' is not defined
```

To help you to identify the error:

1. The **last line** of the error message indicates **what** error occurred. In this case, it says

   `NameError: name ‘bb’ is not defined`

2. The third last line (line 6) indicates **where** the error was detected. In this case, it says:

   `File "inputerror.py", line 6, in <module>`

   This gives the name of the file and the line number where the error occurred.

The information in the error message allows the error to be located and then identified. In line 6 of the program in the example above, the programmer has accidentally typed ‘`bb`’ instead of ‘`b`’. Because the variable `bb` does not already have a value, the program cannot divide `a` by `bb`.

If a program contains multiple errors, Python will display the message for the first one it encounters. After you find and fix the error, Python may give a different error message. This is usually a good sign, indicating that the first error is fixed and you can move on to the next one.

Here is the output from running the previous program with `bb` changed to `b`.

Output for Program A.7

```
Tell me a number: 12
Tell me another number: 4
12.0 * 4.0 = 48.0
12.0 / 4.0 = 48.0
```

Look carefully at the output — although there was no error message, there is still an error. The output says that `12.0 / 4.0 = 48.0`, which is incorrect. Hence you should *always check your output*, as it may be incorrect even if there is no error message. In this case, line 9 should print the variable `quot`.

Finally, if you enter `0` for the second input number, then an error message will arise, saying that you cannot divide by zero! This is a type of "special case" error, which only arises for certain values. To avoid such errors, you need to test your program on a range of "special cases".

For reference, three common error messages are:

Error	**Explanation and possible causes**
`SyntaxError`	The command is not understood by Python. Perhaps: • You used incorrect bracket types (e.g. `( )` instead of `[ ]`) • You have forgotten a bracket • Your indentation is incorrect (wrong number of spaces at the start of a line)
`NameError`	There is no variable with the given name. Perhaps: • You have forgotten to import a module (such as `numpy`). • You have mistyped the name of a variable. • You have forgotten to set a starting value for a variable.
`IndexError`	You have used an invalid index to an array or sequence.

A.5 Conditionals

A.5.1 Introduction to conditionals

Programs often require the computer to do different things depending on whether a condition is satisfied. For example, you might want to print various messages depending on a user's age. Python supports this by means of the *if-statement*, demonstrated below.

Program A.8: The basic `if`

```
age = float(input("What is your age? "))
if age >= 18:
    print("You can vote.")
print("Finished!")
```

Output for Program A.8

```
What is your age? 24
You can vote.
Finished!
```

Output for Program A.8

```
What is your age? 17
Finished!
```

Here are some things to note about the conditional command.

- In the first line, the word `if` and the colon `:` are essential.
- The text between `if` and `:` must be a boolean expression (a logical expression that is either true or false).
- After the first line, any lines that are indented by four spaces will run if, and only if, the condition is true.
- Lines of the program that occur after any indented lines are outside the scope of the conditional command and therefore will run whether the condition is true or false.

A.5.2 Boolean Expressions

Boolean expressions are logical statements which are either true or false. In Python they often take the form of a comparison between two variables, or a variable and a value. Python supports a number of operations for comparing variables and values:

Operation	**Mathematics**	**Python**
Greater than	$a > b$	`a > b`
Less than	$a < b$	`a < b`
Greater than or equal	$a \geq b$	`a >= b`
Less than or equal	$a \leq b$	`a <= b`
Equal to	$a = b$	`a == b`
Not equal to	$a \neq b$	`a != b`

Notice that the operator for checking whether two things are equal in Python is `==` and not just a single `=` sign.

Multiple conditions can be combined using the Python commands **and**, **or**, **not**, matching standard English usage of the words.

A.5.3 The `else` statement

In the previous example, the program prints a message if it is legal to vote, but gives no output if voting is not legal. When programming, we often have **two** possible situations – run some commands if a condition is true, and run other commands if the condition is false. This is done in the following way.

Program A.9: An `if-else`

```
# Is it legal to vote?
age = float(input("What is your age? "))
if age >= 18:
    print("You can vote.")
else:
    print("You cannot vote.")
print("Finished!")
```

Output for Program A.9

```
What is your age? 24
You can vote.
Finished!
```

Output for Program A.9

```
What is your age? 17
You cannot vote.
Finished!
```

If there are more than two conditions to check then the command `elif` is used; it means “else if”. Once again, we can use this to extend our example.

Program A.10: An `if-elif-else`

```
# Is it legal to vote?
age = float(input("What is your age? "))
if age > 18:
    print("You can vote.")
elif age == 18:
    print("You can vote for the first time.")
else:
    print("You cannot vote.")
print("Finished!")
```

Output for Program A.10

```
What is your age? 24
You can vote.
Finished!
```

Output for Program A.10

```
What is your age? 17
You cannot vote.
Finished!
```

Output for Program A.10

```
What is your age? 18
You can vote for the first time.
Finished!
```

A.6 Loops

A.6.1 Introduction to loops

Programs often require some commands to run multiple times. For example, to model the growth of a population over 50 years, rather than writing 50 identical sections of code it is more convenient to write a single section, and run it 50 times. The programming concept which allows lines of code to execute multiple times is called a *loop* and there are *two types*:

For loops are used when we know ahead of time how many repetitions we want. For instance, print `hello` ten times.

While loops are used when we need to repeat something *while* some condition is satisfied. For instance, rolling a dice until a six is rolled.

A.6.2 For Loops

We use *for loops* when we want to repeat something a *fixed number* of times. For instance, we can print the first five squares and cubes, starting from zero.

Program A.11: Simple For Loop

```python
for i in range(0, 5):
    print(i, i**2, i**3)
print("Finished!")
```

Output for Program A.11

```
0 0 0
1 1 1
2 4 8
3 9 27
4 16 64
Finished!
```

Notice the `print` statement was *repeated* five times and that the variable `i` was set first to zero, then one, then two, then three, and finally four — this is called *incrementing* `i`.

Further notice `i` was *not* set to five. The keyword `range` is left *inclusive* and right *exclusive*. That is to say

`range(i, j) = [i, i+1, i+2, ..., j-1]`

as in `range(4, 10) = [4, 5, 6, 7, 8, 9]`.

We may also increment by something other than one by writing:

`range(i, j, k) = [i, i+k, i+2*k, ..., i+n*k]`

where `n` is the greatest integer such that `i+n*k < j`. For instance

`range(-3, 8, 2) = [-3, -1, 1, 3, 5, 7]`.

Here are some things to note about *for loops*.

- The keyword `for` and the colon `:` are essential, and the text between `for` and `:` must be a `range`.
- Any lines subsequent to `for i in range(m, n):` that are indented by four spaces will repeat and have access to the incrementing variable `i`.
- Lines of the program that occur after the indented block will run once the range has been exhausted.

A.6.3 Nesting Loops

Multiple loops and conditionals can be *nested* within each other; indent by an extra four spaces each time.

Program A.12: Nested For Loop

```
for i in range(1, 3):
    for j in range(1, 4):
        print("i =", i, "j =", j)
```

Output for Program A.12

```
i = 1 j = 1
i = 1 j = 2
i = 1 j = 3
i = 2 j = 1
i = 2 j = 2
i = 2 j = 3
```

Notice that the *inner loop* (the one incrementing `j`) is repeated by the *outer loop* (the one incrementing `i`).

A.6.4 While Loops

While loops are used when we cannot determine a priori how many repetitions we need. For example, we do not know how many tries a user requires to enter their password — this is potentially infinite!

In this course we use while loops when prompting the user for input because we often want to ensure what is entered by the user is valid somehow. For example, the we can continually prompt the user until a given number is typed:

Program A.13: While Loop Prompt

```
command = int(input("Enter a number (0 to exit): "))
while command != 0:
    command = int(input("Enter a number (0 to exit): "))
print("You have quit.")
```

Output for Program A.13

```
Enter a number (0 to exit): 1
Enter a number (0 to exit): 2
Enter a number (0 to exit): 0
You have quit.
```

Here is some code demonstrating how to use a *while loop* to print out the first five squares and cubes. It is identical in behaviour to our previous for loop example, though requires some extra instructions.

Program A.14: While Loop

```
# Print squares and cubes of numbers from 0 to 4.
i = 0
while i<5:
    print(i, i*i, i*i*i)
    i = i + 1
print("Finished!")
```

Output for Program A.14

```
0 0 0
1 1 1
2 4 8
3 9 27
4 16 64
Finished!
```

Here are some things to note about *while loops*.

- In the first line, the word `while` and the colon `:` are essential, and the text between `while` and `:` must be a boolean expression.
- After the first line, any lines that are indented by four spaces will run while the condition is true.
- Lines of the program that occur after any indented lines will run once the condition is false.

Make sure you understand what is happening in the example loop above. A *loop control* variable, called `i`, is initially set to a value of 1. Each time the loop runs, the value of `i` is increased by 1, ensuring that the loop runs the required number of times, and stops when the condition `i<5` is false.

A.6.5 Infinite Loop

A *while* loop continues to run commands in the loop body until the condition is false. You must take care to choose a condition that will stop the loop at some stage. Consider the following loop:

Program A.15: Infinite loop

```
i = 1
while i < 5:
    print("forever...")
```

Notice that nothing within the body of the loop changes the value of `i`, so the condition `i < 5` is always true, and the loop will never terminate. This is called an *infinite loop*.

Stopping infinite loops

If you run a Python program and it seems to be taking a long time, it **may** contain an infinite loop. If you suspect that a running program contains an infinite loop, you can terminate it by pressing the "stop" button in the Juypter tool bar (if you are working in Spyder, press either the "stop" button on the top right of the console or `Ctrl+C`).

A.7 Arrays

A.7.1 Introduction to arrays

Thus far, we have only used Python to store individual data values in variables. A Python *array* is a different type of variable, one which allows *multiple* items of data to be stored in the **same** variable. We need to import the module `numpy` before we can use array commands. The following program creates and prints an array called `primes`.

Program A.16: Our first array

```python
import numpy as np

# Create an array containing the first 5 prime numbers.
primes = np.array([2, 3, 5, 7, 11])
print("Primes are:", primes)
print("Primes is an array with", np.size(primes), "elements")
```

Output for Program A.16

```
Primes are: [ 2  3  5  7 11]
Primes is an array with 5 elements
```

Here are some things to note about arrays.

- Arrays are variables, so must have meaningful names.
- Python uses square brackets [and] to distinguish arrays from other variables. These are shown in Line 4 of the above program, and in the output.
- The `array(...)` function provides one way to create an array. See below for a discussion of other ways.
- Line 5 uses the `print` command to display the entire contents of an array.
- The `size` function returns the number of elements in an array. The above array holds five values, so its `size` is 5.

A.7.2 Three functions for creating arrays

We shall discuss three different functions for creating an array:

The function `array(...)` was used in the program above. Use this function when your array is small and you know exactly which entries will go into the array at the time you are making it.

The function `zeros(...)` creates an array filled with zeros. It takes one value, an integer which tells it how many entries the array should have. To make sure that the value you pass to the function is understood to be an integer, you may need to enclose the value in the function `int(...)` as shown in the example below.

Sometimes, particularly when plotting graphs, you want to create an array filled with many equally spaced values. The Python function `arange` accomplishes this. This function is essentially the same as `range`, but special for arrays (`arange` as in array-range).

The function `arange` takes three values: the first value to be placed in the array; a value which indicates when to stop; and a value which indicates how far apart values in the array should be. For example, the Python function

```
X = np.arange(a,b,s)
```

creates an array `X` of values starting at `a`, increasing by an equally spaced step of `s` each time, and stopping at the **last value less than** b.

The following example programs demonstrates the use of each function.

Program A.17: Creating arrays

```
import numpy as np

# Create three arrays to demonstrate the functions
# array(...), zeros(...) and arange(...)

X = np.array([2, -1.2, 13, 4.578])
print("Array X is", X)

Y = np.zeros(int(5))
print("Array Y is", Y)

Z = np.arange(1, 2.5, 0.25)
print("Array Z is", Z)
```

Output for Program A.17

```
Array X is [  2.     -1.2    13.      4.578]
Array Y is [ 0.  0.  0.  0.  0.]
Array Z is [ 1.    1.25  1.5   1.75  2.    2.25]
```

A.7.3 Operations on arrays

Another way to create an array is by applying Python commands to already existing arrays. Most Python commands we have already seen also act element-by-element on entire arrays at once, producing new arrays as the result. Pay particular attention to Lines 6 and 7 of the following example program.

Program A.18: Creating new arrays from old

```
1 import numpy as np
2
3 # Create arrays of primes and powers of 10
4 primes = np.array([2,3,5,7,11])
5 pows = np.array([0.01, 0.1, 1, 10, 100, 1000])
6 primeSq = primes * primes
7 pows = np.log10(pows)
8 print("Squares:  ", primeSq)
9 print("log(pows):", pows)
```

Output for Program A.18

```
Squares:     [   4    9   25   49  121]
log(pows): [-2.  -1.   0.   1.   2.   3.]
```

A.7.4 Accessing individual array entries

In addition to dealing with an entire array, it is often useful to access individual entries in the array. The **index** of an entry refers to the *position* of that entry in the array (somewhat similar to the room numbers in the corridor of a building). To access individual entries, type the name of the array, immediately followed by the index surrounded by square brackets. For example, `A[i]` refers to the value at position `i` in the array `A`.

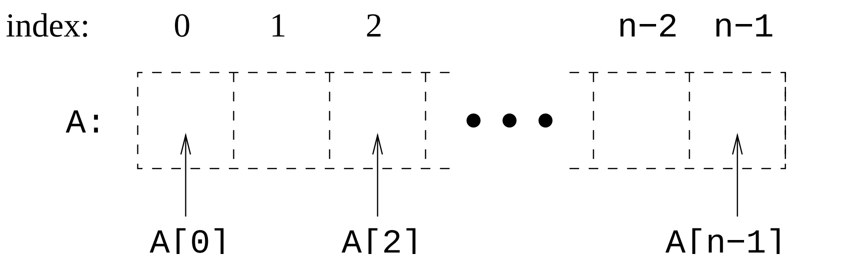


Figure A.1: The index of each entry in an array A with n entries

In Python, the **first** entry in an array has index `0`. This is important to remember! If `A` is an array of size n (so it contains n entries), then valid values of the index are from 0 to $n-1$ (inclusive). This is illustrated in Figure A.1.

The following program demonstrates how to access individual array entries.

Program A.19: Accessing individual array elements

```
import numpy as np

# Create an array of primes and access various entries
primes = np.array([2,3,5,7,11])
print(primes)
print(primes[0], primes[4])
primes[0] = 13
primes[1] = 2 * primes[1] + 1
print(primes)
```

Output for Program A.19

```
[ 2  3  5  7 11]
2 11
[13  7  5  7 11]
```

A.7.5 Arrays and loops

The following program uses the function `zeros(int(n))` to create an array with `n`-many cells set to zero[3] then uses a loop to place values in the array.

Program A.20: Arrays and loops

```
import numpy as np

# Create an empty array then put values in it
X = np.zeros(int(5))
for i in range(5):
    X[i] = i*i

print(X)
```

Output for Program A.20

```
[  0.   1.   4.   9.  16.]
```

Notice the behaviour of `range` is precisely what we need it to be — start from the zero index and exclude the fifth position.

A.8 Writing functions

A.8.1 Why write new functions?

Earlier we saw that Python includes mathematical functions such as `sqrt` and `sin`. Creating your own, new functions can be very useful. Once written, you can reuse them in multiple places or in different programs, without rewriting the commands each time.

Working with functions involves two related but distinct activities:

1. *creating*, or *defining*, the function. You *define* a function by giving it a name, by naming the values that will be passed to the function, and by specifying the Python commands that actually do what the function

[3] Using `int`, while not strictly necessary, ensures that Python treats the number `n` as an integer. There are various data types for numbers, which is a topic that goes beyond the scope of this course.

requires.

2. *using*, or *calling*, the function. You *call* the function by typing its name in one of your Python commands, followed by parentheses enclosing the values you are passing to the function.

A.8.2 Writing a new function

In an earlier example, before using trigonometric `sin` we needed to convert the angle from degrees to radians. Here is some Python code demonstrating this.

Program A.21: Degrees to radians, 1

```
angleDeg = 90
angleRad = angleDeg * np.pi/180
print(np.sin(angleRad))
```

Converting from degrees to radians is a common calculation, so it may be useful to create a new function called (say) `toRadians` to do the conversion. Once the new function is written, the Python code could become

Program A.22: Degrees to radians, 2

```
angleDeg = 90
angleRad = toRadians(angleDeg)
print(np.sin(angleRad))
```

The second line has changed – now the new function performs the conversion to radians. Of course, before using the new function, you need to write it. Here is a Python program showing the new function.

Program A.23: Converting to radians

```
def toRadians(deg):
    # This function converts degrees to radians.
    rad = deg * np.pi/180
    return (rad)
```

Pay careful attention to this example, as it demonstrates a number of important aspects of writing new functions. The first line and the last line are particularly important. Take the time to understand what is happening.

First line:

- The *name* of this function is `toRadians`. You should always choose **meaningful names** for your functions, to help you remember what the functions do.
- The word `def`, the brackets `(...)` and the colon `:` are essential!
- The word `deg` inside the brackets says that one value will be passed to the function and it will be referred to as `deg`. In this function (in degrees) to be converted to radians. The common terminology is that this value is *passed into the function*. The values passed to a function are sometimes called *arguments* to the function. You can choose any name for these arguments.

Last line:

- The last line 'returns' the value calculated by the function; in this case, it is the angle converted to radians.

Remaining lines:

- Except for the first line (or any comments), **every line** in the function **must** be indented; that is how Python can tell where the function ends. Here we will always use **four spaces** for indentation.
- These lines must perform the calculations required by the function, and create the value to be returned.

Here is an example showing how to use the new function.

Program A.24: Using a new function

```
import numpy as np

def toRadians(deg):
    # This function converts degrees to radians.
    rad = deg * np.pi/180
    return (rad)

angleDeg = float(input("Enter the angle in degrees: "))
angleRad = toRadians(angleDeg)
print(angleDeg,"degrees equals", angleRad, "radians.")
print("and sin() of this equals", np.sin(angleRad))
```

Output for Program A.24

```
Enter the angle in degrees: 90
90 degrees equals 1.57079632679 radians.
and sin() of this equals 1.0
```

A.8.3 Some notes on functions

Note that:

- Variables that are given values **inside** a function are **not accessible** outside the function, even if you use the same name in both places.
- You can call other functions from within a function.
- You can define multiple functions inside the same file. Remember that indenting shows where the body of each function starts and ends.
- The lines inside a function are **only** used when you call the function.
- A Python function is not restricted to only performing mathematical calculations. It can do anything that a program does.
- If the '`return`' statement is omitted, then the function will not return a value. In this case, the function is called by typing its name with whatever arguments it needs in parentheses.

Functions and good programming practice

When defining your own functions, the following are recognised as good programming practice:

- New functions should be defined at the top of the file, just under the import statements.
- A function should not use variables that are not passed into the function. That is, every piece of data your function needs to use in order to compute its output should be provided in the form of a value passed into the function.

A.9 Graphs

A.9.1 Plotting graphs

Drawing graphs is important in computer modelling. In the Python module `matplotlib.pyplot` (which must be imported), the `plot(...)` function *organises* a graph, and the `show(...)` function *displays* the graph.

To make the most basic graph, call the `plot(...)` function and then the `show(...)` function. To the `plot(...)` function you must pass two arrays of the same size: the first contains the x-coordinates of the points to be plotted; the second contains the corresponding y-coordinates. The `show(...)` function does not require anything to be passed.

You may pass more information to the `plot(...)` function if you would like to control more features of the plot, like the colour of the line or the way in which points are marked.

You may make more than one `plot(...)` call before calling `show(...)`. In this case your graphs will be plotted on the same set of axes.

The following example plots three graphs on the same set of axes. Make sure you remember to use the `show()` function at the end of your program; if you forget, the graph will not be displayed.

Program A.25: Using `plot(...)` and `show(...)`

```python
# This program demonstrates multiple plotting styles.
import numpy as np
import matplotlib.pyplot as plt

# Make some arrays of the same size
A = np.arange(0,10,1)
Exponential = A**2
Linear1 = 10 * A
Linear2 = 3 * A + 20

# Plot the exponential function using a solid line.
plt.plot(A, Exponential)
# Plot the first linear function using discrete points marked by 'x'.
plt.plot(A, Linear1, 'x')
# Plot the second linear function using a wide solid line.
plt.plot(A, Linear2, linewidth=3)
plt.show()
```

This program produces the plot of Figure A.2.

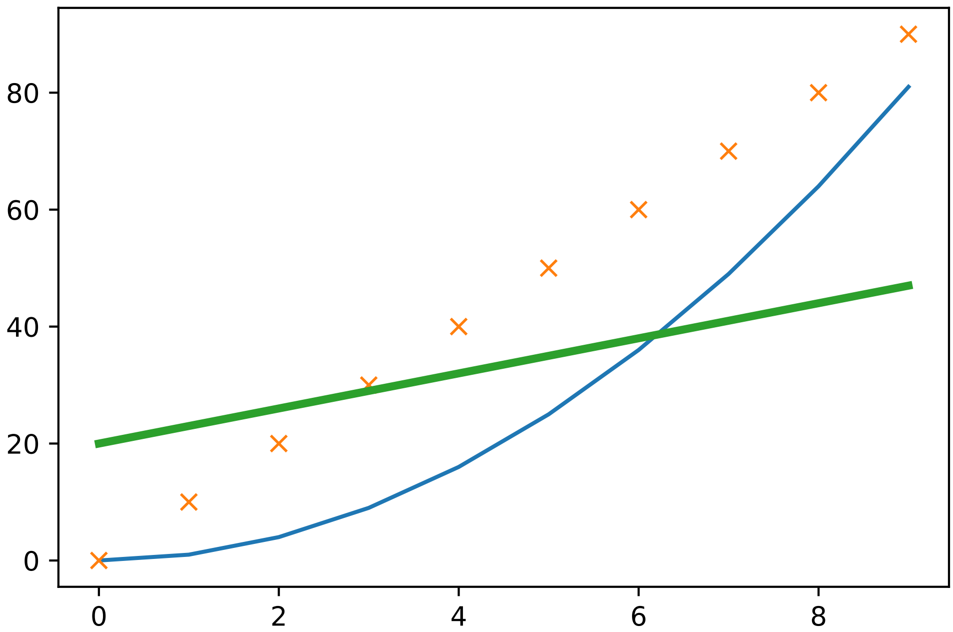


Figure A.2: Three graphs.

A.9.2 Graphing smooth functions

Computers cannot draw perfectly smooth curves: instead, they approximate smooth curves by drawing very short straight lines between points that are very close together. The more data points there are, the smoother the curve appears. The number of points needed to produce a smooth curve varies between different graphs. It is common to choose points whose x-coordinates are *equally spaced*, with a "small" spacing. Use the `arange(...)` function to do this.

Program A.26: Plotting graphs, 2

```
# This program shows how to use arange()
import numpy as np
import matplotlib.pyplot as plt

# Plot sin(x) with x-coordinates separated by 0.5.
X1 = np.arange(0.0, 4.1, 0.5)
plt.plot(X1, np.sin(X1), linewidth=3)

# Plot cos(x) with x-coordinates separated by 0.1.
X2 = np.arange(0.0, 4.1, 0.1)
plt.plot(X2, np.cos(X2), linewidth=3)
plt.show()
```

Figure A.3 shows the output from running the program.

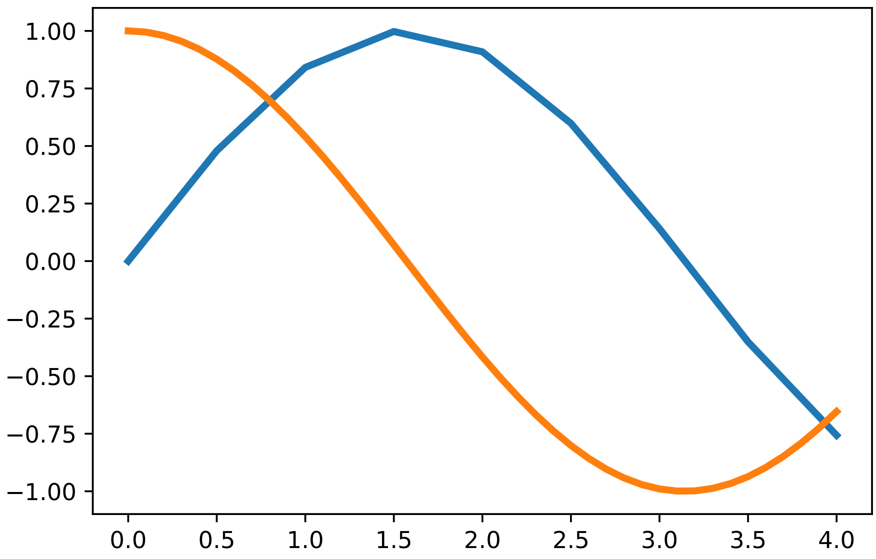


Figure A.3: Two graphs, with spacings of 0.5 and 0.1 between points.

A.9.3 Customising your graphs.

Python provides a number of commands to customise your graphs. The commands in the following program should be self-explanatory.

Program A.27: Some customised plots

```python
# This program shows how to customise graphs.
import numpy as np
import matplotlib.pyplot as plt

# Create equally spaced points and plot sin and cos.
X = np.arange(0.0, 4.1, 0.1)
plt.plot(X, np.sin(X), linewidth=3, label="sin(x)")
plt.plot(X, np.cos(X), "--", linewidth=3, label="cos(x)")

# Create title and label axes. Set axis limits.
plt.title("Graphs of sin(x) and cos(x)")
plt.xlabel("x")
plt.ylabel("y")
plt.xlim(0,4)
plt.ylim(-1,1)

# Draw a grid, create a legend and display the graph.
plt.grid(True)
plt.legend()
plt.savefig("graph.pdf")
plt.show()
```

Figure A.4 shows the output from running this program.

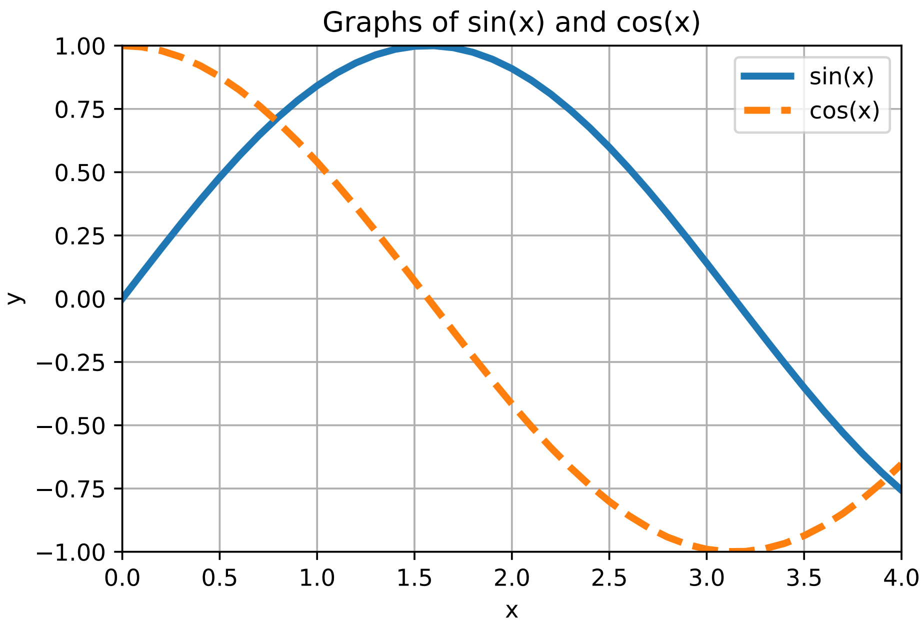


Figure A.4: Customising graphs.

A.10 Python summary

- If you are using mathematics functions or arrays then programs must commence with the line:

  ```
  import numpy as np
  ```

  Mathematical functions and arrays must have '`np.`' written before the name of the function.

- If you are using plotting commands then programs should also include the line:

  ```
  import matplotlib.pyplot as plt
  ```

  Plotting functions must have '`plt.`' written before them in your code.

- All lines commencing with '#' are comments.

  ```
  # Use lots of comments to explain what your program does.
  ```

- The `print(...)` command displays text on the screen. Text inside quotation marks is displayed. Text outside quotation marks is treated as calculations or variables, and the values are printed.

- Values can be assigned to variables using '='.

- Use the operation `**` to find a power.

  ```
  print("x cubed =", x**3)
  ```

- Useful `numpy` functions include `sqrt(value)`, `sin(value)`, `exp(value)`, `log(value)` and `log10(value)`.

  ```
  print("The square root of x =", np.sqrt(x))
  ```

- The command `float(input(message))` reads a number from the keyboard.

```
num_fish = float(input("How many fish are there?"))
```

- The bodies of conditionals, loops and functions must be indented by four spaces.
- The conditional commands `if`, `else` and `elif` control which commands are run, when a particular condition is true or false.

```
if x==y:
    print("x equals y")
elif x<y:
    print("x is less than y")
else:
    print("x is greater than y")
```

- The `for` loop is used for repeating code a *fixed* number of times.

```
for i in range(5):
    print(i, "squared =", i**2)
```

- The `while` loop allows commands to be repeated until a condition is broken: Make sure that your loops eventually stop!

```
i = 0
while i<5:
    print(i, "squared =",i**2)
    i = i+1
```

- To create a new function, *define* it at the top of your file (just under the `import` statements), then *call* it wherever you need it.

```
def toRadians(deg):
    # A new function to convert degrees to radians.
    rad = deg * np.pi/180
    return rad

# Call the function:
print("Converted to radians, 90 degrees is", toRadians(90))
```

- Arrays allow multiple values to be stored in a "table", using a single variable name. The *index* of an element is its location in the array.

```
A = np.array([1, 10, 100, 1000])
print("The last entry is ", A[3])
B = np.log10(A)
C = np.zeros(int(10))
```

- Graphs are drawn using `plot()`, followed by `show()` (both imported from the `matplotlib.pyplot` module). Commands such as `title()`, `xlabel()` and `ylabel()` allow you to customise your graphs.

```
x = np.arange(0, 10.1, 0.1)
y = x**2
plt.plot(x, y)
plt.show()
```

Appendix B: Communication in Science

Image B.1: *Communication*, Joan M. Mas [38]

In SCIE1000 we will focus on four principles essential for effective communication of science which will be of lifelong use to you in this course, in your degree, and in your personal and professional life. You will apply one or more of these principles in a task each week in your SCIE1000 practicals. We have designed these tasks to build upon one another so that you develop good habits for communicating science.

Science by its nature aims to be:

- Precise
- Clear, and
- Concise

Consequently, **communication by scientists about science** aims to be:

- Precise
- Clear, and
- Concise

...and in that order. Communication that is vague is not scientific communication. Communication that is unclear or ambiguous is not scientific communication. And communication that is long-winded and rambles from one topic to another is not scientific communication. But do not sacrifice being **precise** and **clear** only to be concise! In other words, if you need to use more words, numbers or visuals to make something crystal **clear** to as many readers or listeners as possible, then do so.

Not all communication is **precise**, **clear** and **concise**, and nor should it be. There are situations and contexts where being precise, clear and concise is not the most effective way to communicate. For example, some communication aims to entertain, persuade, or even confuse. But in learning to become a scientist, it is important that you hone your skills at being precise, clear and concise. These characteristics are typical of the way scientists communicate with other scientists.

As developing scientists, we expect you to become **precise**, **clear** and **concise** in your writing and speaking about science. This is not a skill that comes naturally to most people, but your participation and efforts in this course and throughout your degree will have you communicating science precisely, clearly and concisely sooner than you think.

The four principles we focus on in SCIE1000 are:

1. Being clear
2. Knowing your purpose
3. Knowing your audience
4. Identifying key messages

Applying these four principles will get you well on the way to being precise, clear and concise when **you** communicate science. Here is why and how to do it...

B.1 Principle 1: Being clear

Why be clear?
Being clear is important because of the way the human brain works. The working memory part of the brain processes all incoming communication and turns it into understanding and long term memory. The working memory is only able to process a limited amount of information at any one time, and absorb only small amounts of new information at any one time. The brain then has to integrate the new information with its own mental models for it to be remembered in the long term.

Many concepts and ideas in science are new to people outside the specific field. So an effective way to communicate science is to combine information the person already knows with small chunks of information that is new to them. Doing so makes it easier for the brain to process, or understand, and is more likely to be remembered in the long term.

Useful tactics for being clear
Whether you use words, computer code, numbers, symbols and/or pictures, being clear in your communication is important. Some useful tactics to achieve clarity in your communication about science are:

1. **Identify what is FAMILIAR and NEW to your intended reader or listener.**
   People learn and remember information by adding and integrating it with what they already know. By identifying what is familiar and new, you can make decisions about how to present your information which will help another person learn and remember it (see below).

2. **Provide MORE explanation for new information than for the familiar.**
   The human brain requires more effort to understand and remember new information than the familiar. Providing extra explanation means less effort for the brain, and increased likelihood that it will be understood and remembered.

3. **Organise new information in SMALL CHUNKS AFTER familiar information.**
   The working memory of the brain can process only a limited amount of new information at any one time. By organising new information into small chunks you help ensure the working memory is able to process it. When you place new information after familiar you make it easier for the brain to combine the two and pass them together to long term memory.

4. **Present your work NEATLY and LOGICALLY.**
   Lay out your communication in a way that another person can easily see each step in your thinking; make it easy for them to follow the process you used to get from A to B.

5. **Provide DEFINITIONS and LABELS**
   Provide definitions and labels for all data, spreadsheet columns, variables, symbols, drawings, pictures, technical terms and jargon. It reduces effort required by the working memory in processing fiddly details and allows it to concentrate on the important stuff – remember there is only a limited amount the working memory can do at any one time.

6. **REVIEW your communication from a different perspective**
   Learning to see your work from the perspective of another person, including your future self, is a valuable skill. When you look back at your notes will you remember what it is you meant? Ask another person to look over your work and tell you what is unclear for them. It is not always possible to see your own work from another person's perspective because we all make assumptions about what other people know that may not be correct. In SCIE1000/1100 we encourage and expect you to ask other students for feedback on how to make your work clearer.

Examples of being clear

On the following pages you will find some examples of the tactics for being clear in a variety of contexts.

SENTENCES and **PARAGRAPHS** are simple tactics to help you be clear whether speaking or writing in words.

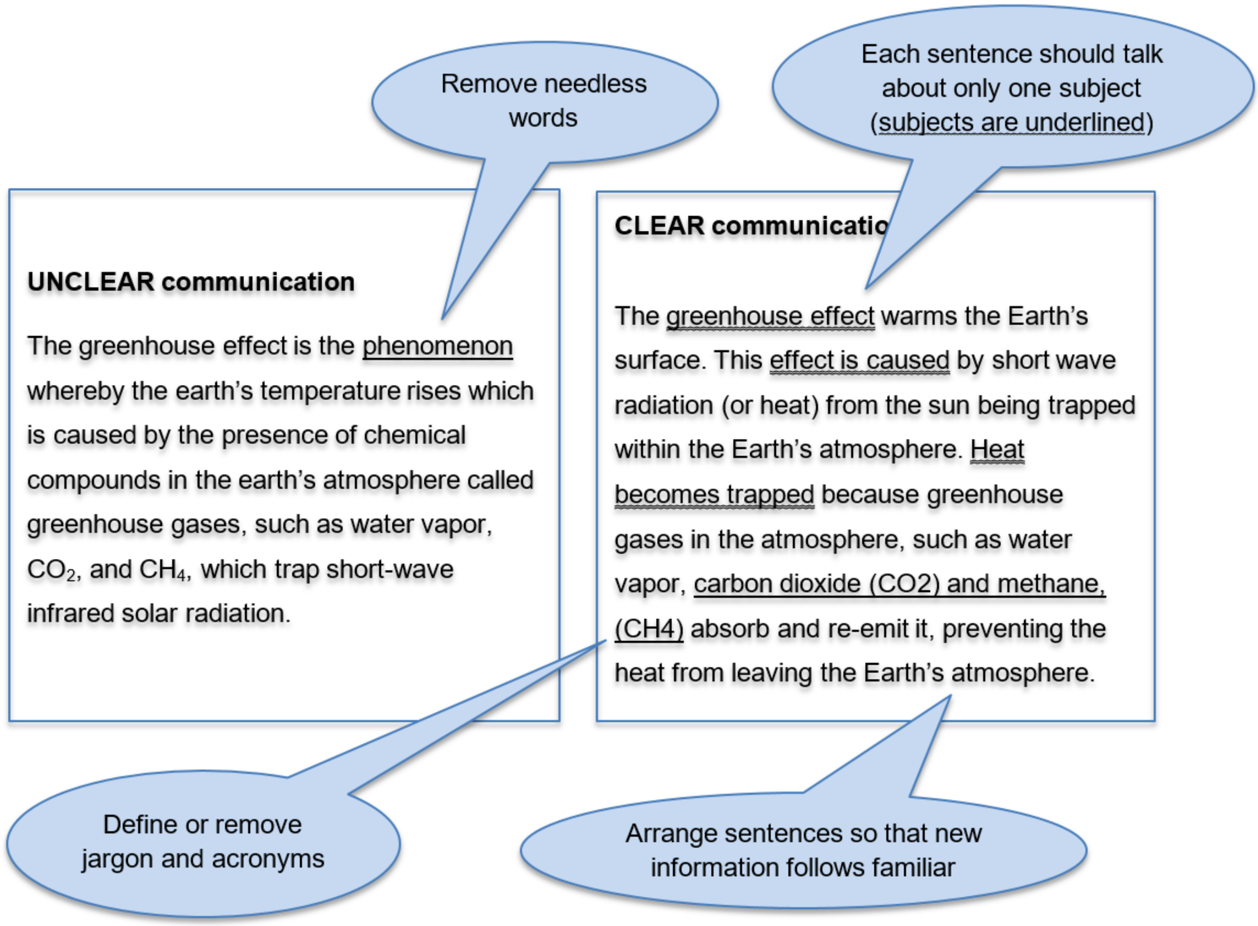


Figure B.1: Making text clear.

In the sample text in Figure B.1:

1. each sentence begins with FAMILIAR information with NEW information AFTER - each sentence (except the first) begins with information that was introduced in the previous sentence and ends with new information,
2. shorter sentences break new information into SMALL CHUNKS,
3. shorter sentences, and the patterns of new information after familiar information, help organise the information LOGICALLY so the reader can see each step of the writer's thinking.

MATHEMATICS should be written with a mix of words, sentences and numbers so that mathematicians and non-math specialists alike can follow your logic. One reason for this is that most users of mathematical outcomes are not mathematicians.

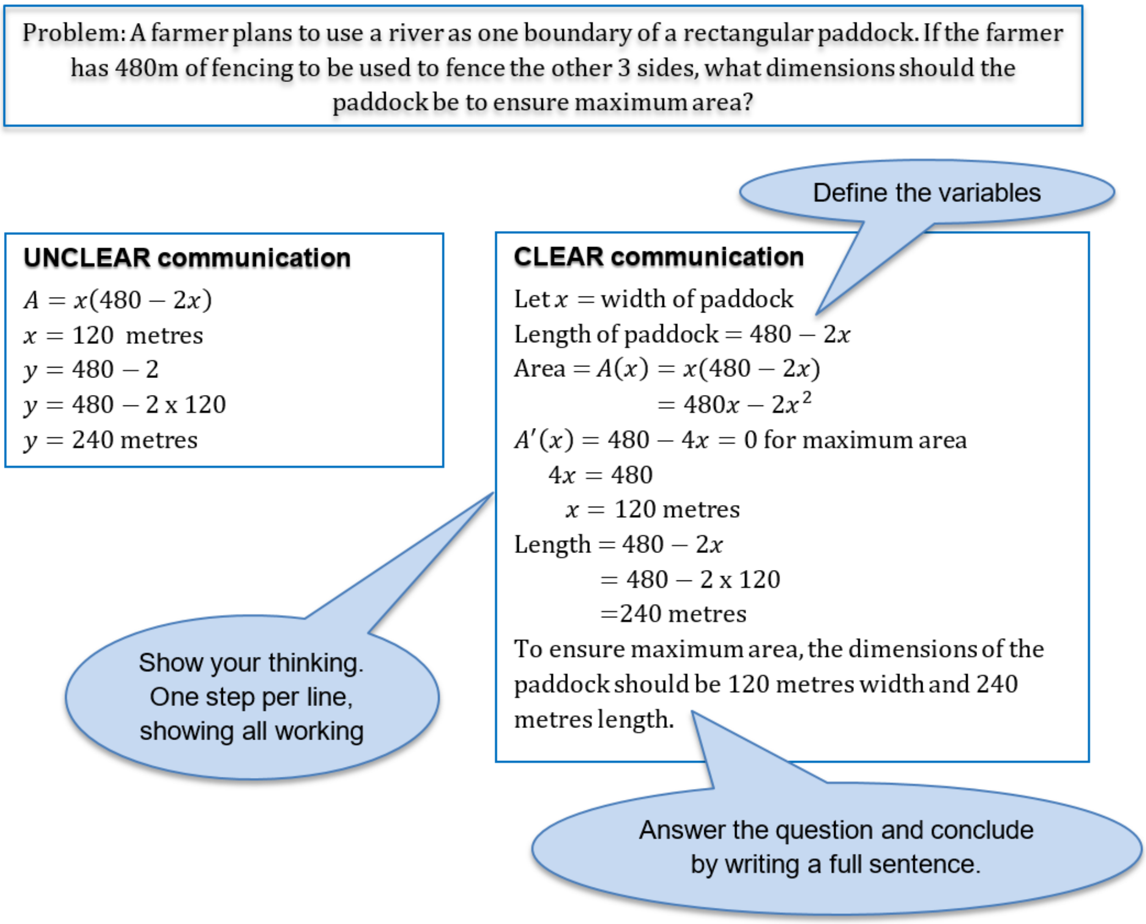


Figure B.2: Making mathematics clear.

In the example calculations shown in Figure B.2:

1. the variables and conclusion are the NEW information, the symbols are the FAMILIAR (e.g., $=$, $-$, $'$),
2. the DEFINITIONS of the variables and the full sentence explaining the conclusion provide MORE explanation about the new information,
3. by placing one step per line, the calculation is NEAT and LOGICAL.

COMPUTER CODE should be written for people, not computers. By being clear in your code, you make it easy for people to maintain, change and repair your code.

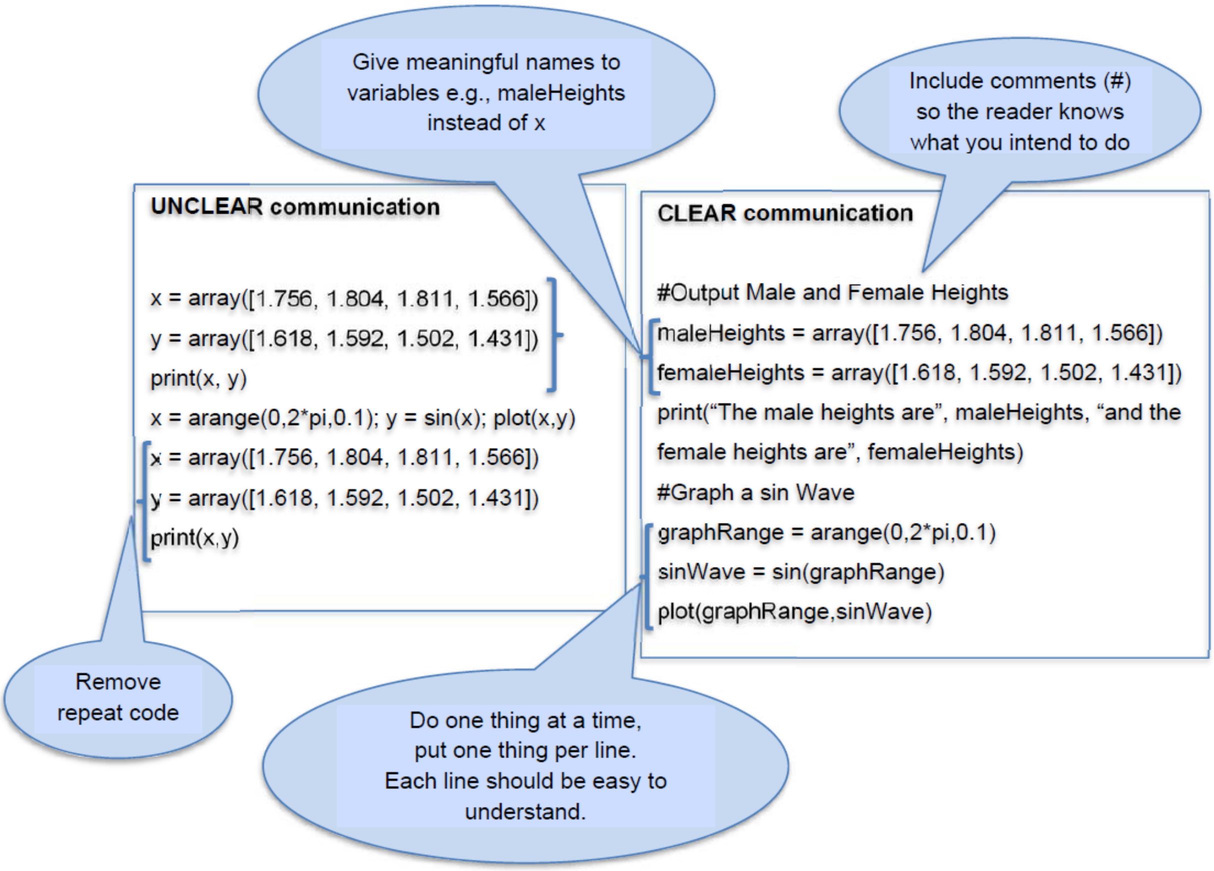


Figure B.3: Making computer code clear.

In the example code given in Figure B.3:

1. the variables are the NEW information while the functions are the FAMILIAR (plot, sin, etc),
2. the meaningful variable names provide MORE explanation about the new and act as LABELS,
3. by removing repeated code there is only a SMALL CHUNK of new information in this example,
4. by placing one step per line the code is NEAT and LOGICAL,
5. by adding comments that tell the reader the intentions of your code serves as a LABEL to enable the working memory to more easily process the code that follows.

B.2 Principle 2: Knowing your purpose

Why know your purpose?

Why am I communicating with you? That is a question you should always ask when you begin any form of communication. Knowing your specific purpose helps you to make decisions about how to tailor your communication to meet specific needs. If you don't know your purpose, your communication may not have the effect you want.

By knowing your purpose you can decide what information to include or exclude, what to emphasise or de-emphasise, what needs labelling or explaining, and how to structure and approach your communication for best effect.

How to know your purpose

Since communication usually occurs between two or more people, the best way to identify your purpose is to consider it from two perspectives: what is it that you want to do and what is it that you want your audience to do? By audience, we mean the person or people who will read, hear or see your communication.

It is quite normal for there to be more than one purpose for your communication. For example you might want to both inform and entertain your audience. You might also want your audience to understand and give you feedback. Recognising which purpose(s) are most important to you will help you prioritise decisions when creating your communication.

Some useful tactics to know your purpose include:

1. Be SPECIFIC. The more specific you are about your purpose, the easier it will be for you to make decisions about how to best communicate your information.

2. THINK and WRITE what you want to ACHIEVE. Each time you communicate, there is a reason you want other people to know about what you have to say, and the outcome you want to see happen. Try to articulate it and write it down in as much detail as possible. Time you spend doing this will save you time and make your job easier when you create

your communication.

3. THINK and WRITE how you want others to RESPOND. The outcome you hope to achieve will require the listener or reader to respond in some way. What do you want them to do? How do you want them to think, feel or act? What do you want them to comprehend? Write down your answers so you can refer back to them to help you make decisions when creating your communication.

4. STATE your purpose(s) near the start when you communicate. Doing so allows the reader or listener to be aware of their role, what will be useful to them, and think about how they want to respond.

5. ANALYSE the task. In the case of an assessment task, to identify the reason you are being asked to communicate, analyse what you are being asked to do and why. The 'why' is your purpose. A useful technique to analyse assessment questions is to underline words in the question you think are important.

Examples of purpose
Some examples of defining the purpose are given below.

SENTENCES AND PARAGRAPHS, either written or spoken, can incorporate clear purpose by:

1. either stating the purpose explicitly or inferring it. For example you can say 'In this essay I will...', or include a title such as 'A report on the evaluation of....', or you could pose a question, as seen in the example below, or choose a format or forum that has a commonly understood purpose, such as a court hearing or executive summary,

2. placing the purpose in the title or in the first few paragraphs, or both.

See Figure B.4 for an example.

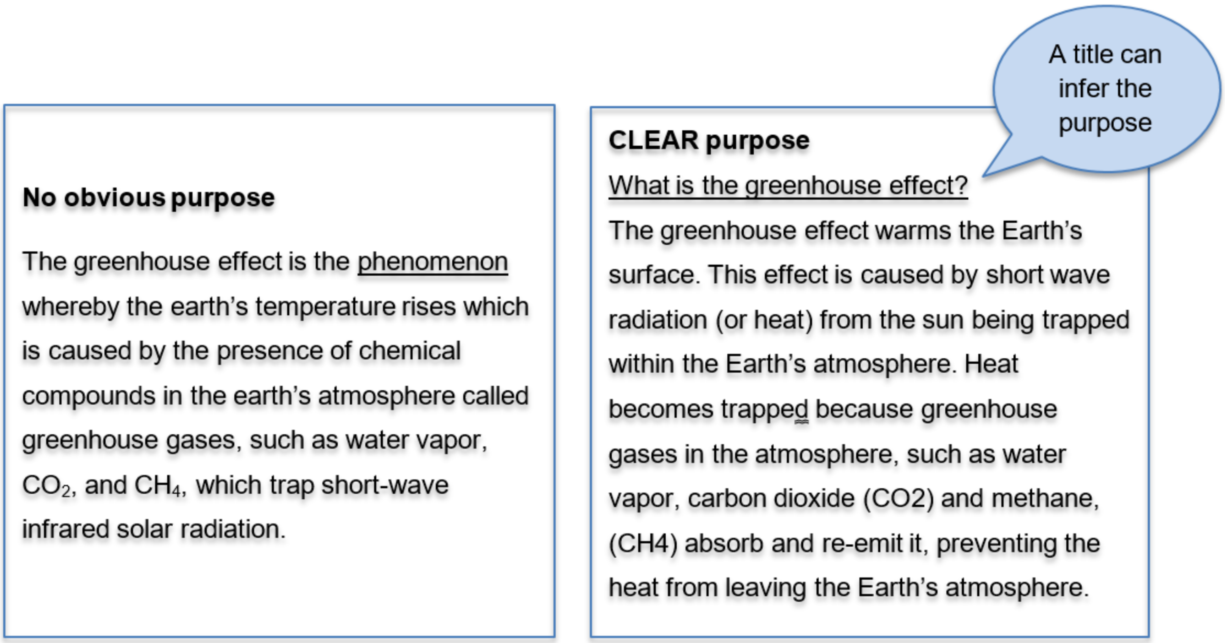


Figure B.4: Putting the purpose in text.

MATHEMATICS should be written with a clear purpose near the start. This may be as simple as including the question or rational for the calculations, or instead you may choose to state the purpose as shown in the example in Figure B.5.

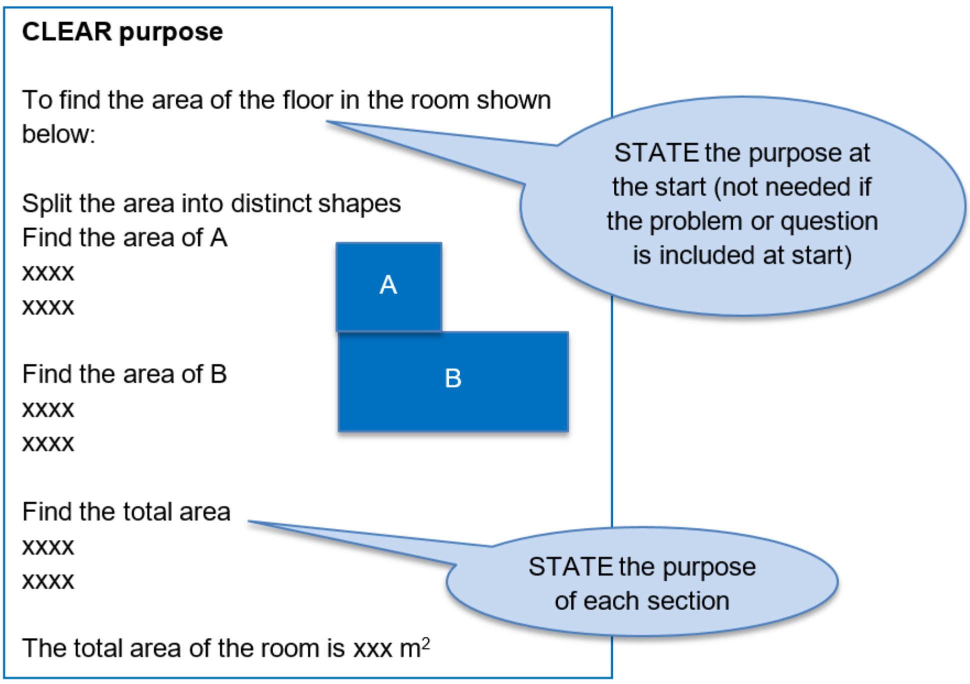


Figure B.5: Putting the purpose in mathematics.

COMPUTER CODE should be written for people, not computers. It is good practice to:

1. STATE the purpose for writing the code at the start of the code making it easier and faster for the reader (often the person who will maintain, repair or change the code) to follow your logic. In Python, purposes are denoted by ”’ at the start and end of a paragraph, collection of sentences or descriptive block.
2. STATE the purpose or intent at the start of each section of code as this prepares the reader's working memory for what to expect and makes easier the process of understanding the code that follows. In Python comments are denoted by a # at the start of the sentence.

See an example in Figure B.6.

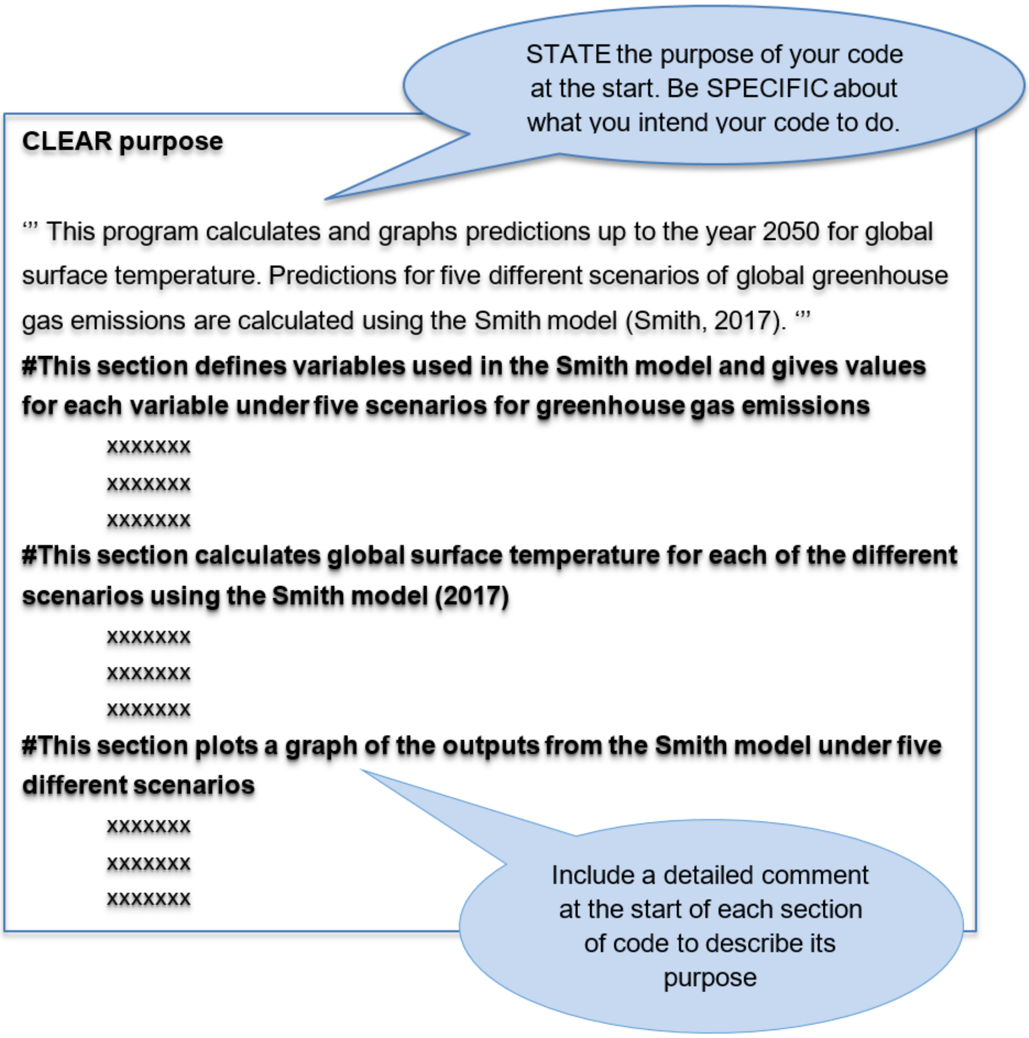


Figure B.6: Putting the purpose in computer code.

B.3 Principle 3: Knowing your audience

Why know your audience?
Communication usually involves more than one person. So if you are the communicator, the person to whom you are communicating is your audience. To communicate effectively, you need to adjust what you say or write to suit your audience. You are most likely already aware of this as you probably speak about what you did on your big night out in a different way when you speak with your best friend, your grandparents or your five year old cousin.

The more details you know about your audience the easier it is for you to make decisions about what information to include or exclude, what to emphasise or de-emphasise, what needs labelling or explaining, what types of words to use, and how to approach your communication for best effect.

Tactics to know your audience
There are many sources of information you can use to get to know your audience. Whenever possible it is best to avoid making assumptions. If you have the opportunity, ask your audience questions directly. If that is not possible, do your research; ask people who know them and use the internet. Some useful tactics to get to know your audience include:

1. **Find out the DEMOGRAPHICS of your intended audience.**
   Demographics include factors such as age, gender, geographic location, ethnicity, language, the sector in which they work, etc.

2. **Determine what they ALREADY KNOW about the topic.**
   Having a good idea of what types of information are familiar and new to your audience will help you create clear communication. Their interests, education in science and age can provide clues to what aspects of science are likely to be familiar to them.

3. **Find out what they BELIEVE about the topic.**
   Beliefs have a strong influence on how people remember, interpret and use information. The cultural, religious and political background of your audience can provide clues.

4. **Determine their INTERESTS and PRIORITIES.**
   People are more likely to pay attention to, and remember, things that they find interesting, things that they need, or things which are important to them. Find out what it is that your audience values. Knowing the occupation, time availability, hobbies, studies, and/or company profile of your audience can provide clues.

5. **Ascertain what types of communication ENGAGE their attention.**
   Information is more likely to be processed by the working memory of the human brain if the person is paying attention, so finding out where the attention of your audience is focused is an advantage. For example, is your audience more likely to encounter and pay attention to a story, phone conversation, written report or video? Knowing this will help you decide what format to use in your communication.

Examples

Example Audience	Characteristics of Audience
A classmate who missed last week's lecture	*Demographic*: similar age & language to you *Knowledge*: similar scientific knowledge to you *Beliefs*: values science, values university study *Interests & priorities:* similar interests to yours (studying science), priority is to do well in studies for this class *Type of Comm*: In person conversation and/or shared study notes
A person who submits a question to the "Dr Science" Q&A Facebook page	*Demographic*: visit sites like https://sproutsocial.com/insights/new-social-media-demographics/ for detailed information *Knowledge*: you can gain clues about this from the question being asked *Beliefs*: values science, you can also gain clues from the question being asked *Interests & priorities:* interested in details of science, more clues can be gained from the question they ask *Type of Comm*: Facebook
The Australian association of farmers for sustainable futures need to provide a recommendation to its stakeholders about Y	*Demographic*: farmers, mostly male… look up members list for the association for more clues *Knowledge*: some scientific knowledge, plenty of on-ground observational knowledge of ecosystems, business knowledge *Beliefs*: value science, value practical solutions *Interests & priorities:* business, sustainability, land management *Type of Comm*: in person, social networks, local community groups

B.4 Principle 4: Identifying key messages

Why identify a key message?

Have you ever been to a lecture, or read an article and thought, wow that was fascinating! But when you try to summarise what it was about to another person, you can't quite remember? Chances are that the speaker or writer did not make their key message obvious, or maybe they didn't know what it was! Identifying key messages in communication from other people is a skill, but one that is made much easier when the person communicating has a clear and precise idea of the main message they want to convey.

Knowing your key message helps you make decisions about what information to include or exclude, what to emphasise or de-emphasise, what needs labelling or explaining, what types of words to use, and how to approach your communication for best effect.

Tactics to identify a key message

Your key message is the one thing you want your readers to know, consider, do or think about. It is your take home message; you will be satisfied if your audience remember nothing else. Ideally a key message should be easy to recall and repeat in conversation. Here are some tactics you can use to identify your key message:

1. **SUMMARISE in one sentence the specific purpose of communicating to your specific audience.**
   …you now have a rough draft for your key message.

2. **It is the IDEA not the words that is most important.**
   When developing your message, knowing what idea you want to get across is more important than worrying about the exact wording. Once you have crystalised the idea, you can find the best wording to make that idea stick, or elaborate that idea by using examples.

3. **REVISIT your key message as you create your piece of communication.**
   You will find that your message changes and improves as you clarify what it is you want to say. Often this only happens while you are creating the whole piece of communication. Revisiting your key message during the process also helps reminds you of your goals and assess if you are on the right track.

4. **TEST your message.**
   Have someone read/listen to your communication and tell you what they think the key message is, or what it is that they understood from your communication. If what they tell you is not the same as you intended, you need to revisit and

5. **CHANGE your message for different audiences and purposes.**
   What works in one context, may not work in another. The wording and sometimes the idea in your message may need to change when you communicate with different audiences or for different purposes.

6. **BEGIN and END with your key message.**
   You know you have a successful key message when your audience can recall and repeat what it is. To help this happen, place your message near the start when the attention of the audience is at its highest, and near then end because when it is repeated it is more likely to stay in people's memory.

Examples

SENTENCES and PARAGRAPHS Useful tactics for making your key message obvious in writing include those described above for being clear, but for different reasons as described below.

1. The paragraph in the clear example below BEGINS and ENDS with the same message, albeit stated in different words i.e., 'the greenhouse effect warms...' and 'prevents the heat from leaving...'

2. The first sentence (topic sentence) orients the reader's mind to what type of information will follow. The last sentence (concluding sentence) reiterates the key message so the reader can check they understood the content of the paragraph and to remind the reader of the key message.

3. Each sentence in the paragraph supports the key message stated in the first sentence. Doing so helps to reinforce the key message and makes it easier for the working memory part of the brain to process and move to long term memory.

MATHEMATICS Mathematical calculations and visual displays of data all have key messages.

1. Usually the key message is the outcome from the calculation or the conclusion from analysis of the data.

2. The key message should always be stated in words, either as a conclusion at the end of a calculation as shown in the example above for being clear, or as a caption or annotation on a table, figure or graph as shown below.

No obvious key message

The greenhouse effect is the <u>phenomenon</u> whereby the earth's temperature rises which is caused by the presence of chemical compounds in the earth's atmosphere called greenhouse gases, such as water vapor, CO_2, and CH_4, which trap short-wave infrared solar radiation.

The key message in this example is fuzzy: is it that temperatures rise? Or that chemical compounds are present? or that infra-red

CLEAR key message

<u>What is the greenhouse effect?</u>

The greenhouse effect warms the Earth's surface. This effect is caused by short wave radiation (or heat) from the sun being trapped within the Earth's atmosphere. Heat becomes trapped because greenhouse gases in the atmosphere, such as water vapor, carbon dioxide (CO2) and methane, (CH4) absorb and re-emit it, preventing the heat from leaving the Earth's atmosphere.

The paragraph BEGINS and ENDS with the key message i.e., that the greenhouse effect warms the Earth

3. The key message in mathematical work directs direct the reader to what you consider to be the most important message from the data or calculation.

Use the figure or table caption to tell the reader the key message

No obvious key message

Change in Median House Prices 2014-2017				
City	Jun-14	Jun-15	Jun-16	Jun-17
Sydney	$812,929	$1,000,616	$1,046,068	$1,178,417
Melbourne	$608,863	$668,030	$752,083	$865,712
Brisbane	$479,989	$490,855	$529,438	$546,043
Adelaide	$464,029	$479,285	$492,252	$524,968
Perth	$614,297	$605,089	$577,778	$555,788
Canberra	$583,473	$616,313	$664,133	$723,299
Darwin	$648,584	$325,972	$351,187	$404,522
Hobart	$326,460	$654,270	$594,144	$666,686
National	$628,561	$701,827	$743,264	$818,416

Remove needless columns and data. If appropriate and ethical, replace with summary data to illustrate your key message

CLEAR key message

Table 1: Median House Prices increased in most Australian Capital Cities between 2014 and 2017.

City	**Jun-14**	**Jun-17**	**% change over 3 years**
Sydney	$812,929	$1,178,417	45.0
Melbourne	$608,863	$865,712	42.2
Brisbane	$479,989	$546,043	13.8
Adelaide	$464,029	$524,968	13.1
Perth	$614,297	$555,788	-9.5
Canberra	$583,473	$723,299	24.0
Darwin	$648,584	$404,522	-37.6
Hobart	$326,460	$666,686	104.2
National	**$628,561**	**$818,416**	**30.2**

Use formatting to highlight the data that supports your key message

COMPUTER CODE Tactics for making your key message obvious to your reader when writing code is pretty much the same as identifying your purpose, the only difference being that the comments you add for every section of the code need to support the stated purpose. In the example shown above for purpose, the comment for each section refers to one or more parts of the description of purpose at the start of the code. This approach also helps make the code clear to the reader by laying out each step in your thinking.

Appendix C: Prerequisite maths review

C.1 Linear, quadratic, and power functions

- The basic mathematical tool used to describe quantitative relationships and patterns in models is the mathematical *function*.

Mathematical functions

A *function* is a rule that converts input value(s) to output value(s). There is exactly one output value for each input value. Often, the input values are called x or t (for time). On a graph, they are represented on the *horizontal* axis. If f is the name of a function, then $f(t)$ denotes the output that arises from applying f to the input value t. On a graph, these values are represented on the *vertical* axis.

- People study a range of functions, including: linear, quadratic, power, periodic, exponential, logarithmic, and combinations of these. These functions are interesting **precisely** because they model natural phenomena.

Linear functions

Linear functions have equations $y(x) = mx + c$, where m is the *slope* and c is the y-*intercept* of the line.

If (x_1, y_1) and (x_2, y_2) are two points on the line then

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

- Given a straight line, we can calculate the equation $y = mx + c$ that models the line by finding two points on the line and using them to calculate the slope m, and then using a point on the line to plug into the equation to find the value c.

- Note that sometimes it is easy to read the value of c straight from the graph. When doing this, be careful to read the labels of the axes very carefully.

Quadratics and modelling

Quadratic functions have a power of x (or t, or $\ldots$) equal to 2, with equations of the form $y(x) = ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$. The graphs of quadratics are parabolas.

- Quadratics are important in practical modelling, particularly when modelling over short time periods. They are the simplest functions with *local optimal* values, that is, local *maximum* or *minimum* values.
- Linear equations and quadratic equations are two special cases of a more general class of functions, called *power functions*.

General form of power functions

Power functions have equations

$$y(x) = Mx^p + c$$

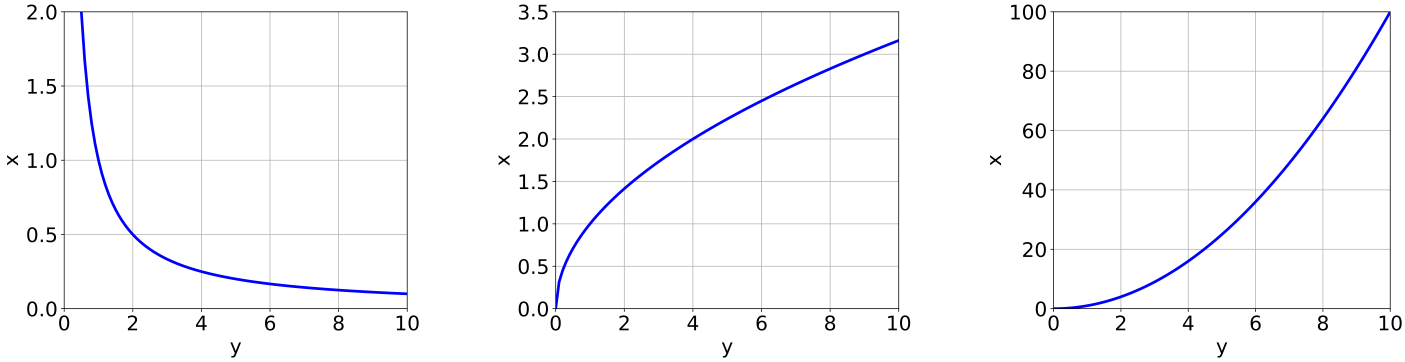
where M, p and c are constants. Changing the value of these constants generates graphs with different shapes, which makes power functions useful for modelling a range of phenomena. For example, changing the value of:

- the power p creates graphs that increase or decrease, at different rates;
- the constant M *scales* the vertical height of the graph at each point; and
- the constant c *shifts* the graph up or down.

Table C.1 illustrates how the value of the power p affects the general shape of the corresponding graph, **for positive values of C and x**. Figure C.1 illustrates this, showing some equations and their graphs.

Power, p	General shape of the graph
< 0	curve, **decreasing less rapidly** as x increases
0	horizontal line
> 0 and < 1	curve, **increasing less rapidly** as x increases
1	straight line
> 1	curve, **increasing more rapidly** as x increases

Table C.1: Different powers and the general shapes of the corresponding graphs.



$y = x^{-1}$ $\qquad$ $y = x^{1/2} = \sqrt{x}$ $\qquad$ $y = x^2$

Figure C.1: Graphs showing the shapes of some power functions.

C.2 Periodic functions

- To represent cyclic phenomena, or data that give rise to waves, the most common choices are the *trigonometric functions* sin and cos.
- These functions are defined in the context of geometry and angles. However, you **do not** need to think of them in a geometric context when modelling.
- In SCIE1000 we will always use the function sin (we could have used cos, noting that a cosine function can be considered as a shift of a sine function).

Sine functions

The function $\sin(t)$ has period 2π, amplitude 1, equilibrium (centre value) 0, and equals 0 when $t = 0$. Figure C.2 shows a simple sine wave $y(t) = \sin(t)$.

Sine functions *(continued)*

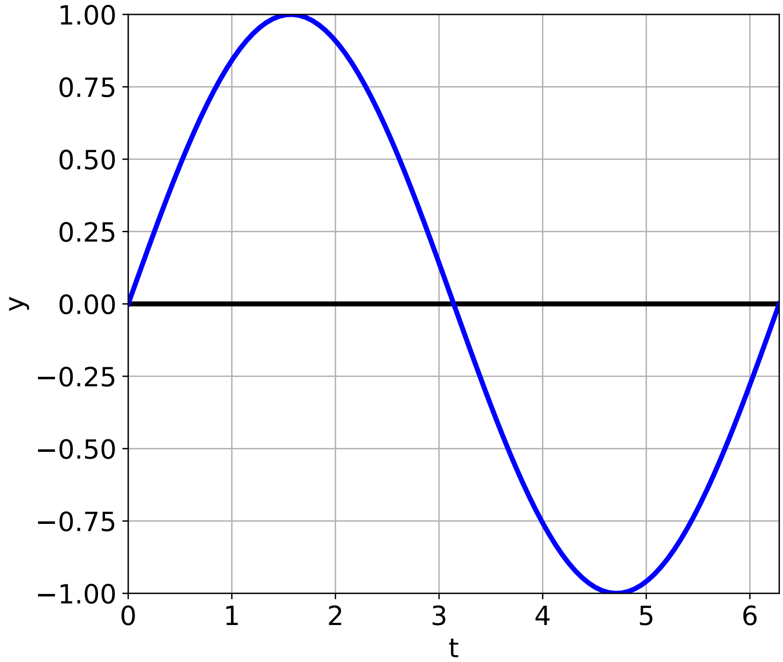


Figure C.2: Graph of $y(t) = \sin(t)$.

The general equation of a sine function is

$$y(t) = A \sin\left(\frac{2\pi}{P}(t - S)\right) + E.$$

The constants in this equation are

A: the amplitude of the sine wave

P: the period of the wave

S: the shift right of the wave

E: the equilibrium value

- Varying the values of the constants A, P, S and E within a general sin function alters the cyclic model, allowing us to model a range of cyclic phenomena.

C.3 Exponential and logarithmic functions

Exponential functions

Exponential functions have equations

$$f(x) = Ca^{kx},$$

where C, a and k are constants. The constant C is a scaling factor. The constant a is called the **base** of the exponential. The two most common values used for the base a are

- the number 10; and
- *Euler's number*, denoted e, where $e \approx 2.71828\ldots$.

The constant k is the **growth rate** or **decay rate**. If C is positive then:

- If k is *positive*, the function displays exponential *growth*.
- If k is *negative*, the function displays exponential *decay*.

Note that when $x = 0$ the function value equals C.

- Two useful rules for manipulating exponentials are:

$$a^x a^y = a^{x+y}$$

$$(a^x)^y = a^{xy}$$

Logarithmic functions

Suppose we have an exponential function of the form

$$y(x) = a^x.$$

The inverse function of the exponential is called the *logarithm* (or *log* for short), defined as

$$x(y) = \log_a(y).$$

- Note that there are a number of conventions when referring to the base. On your calculator you will notice a button labelled "ln" which means log to the base e (also called a natural logarithm), The button labelled "log" represents log to the base 10.
- However, mathematically, "log" (without a subscript to show the base) may sometimes be used to represent log to the base e, whereas in other cases "ln" will be used. Other bases are shown by writing the base as a subscript immediately after "log", eg $\log_2$. In this course we mostly work with log to the base e, and write this as ln, but always be careful to use the context to decide if this is true.
- Two useful rules for manipulating logarithmic functions (independent of base) are:

$$\log(x^n) = n\log(x)$$

$$\log(x \cdot y) = \log(x) + \log(y)$$

C.4 Rates of change

Average rate of change

Let (x_1, y_1) and (x_2, y_2) be two points. The **average rate of change of** y **with respect to** x between these points is the *slope of the straight line joining the points*. As we saw earlier, the slope equals the change in y values divided by the change in x values, so:

$$\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

(Note that Δ is the Greek capital letter "Delta", and usually means "the change in the value of".)

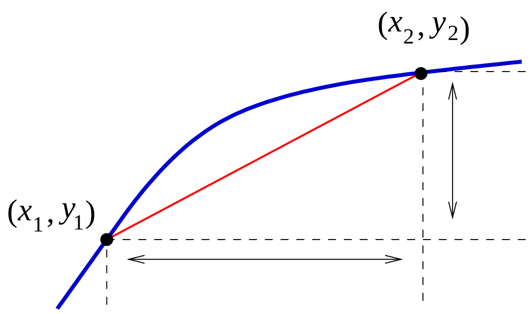


Figure C.3: Average rate of change.

- Rather than measure the average rate of change between two points, in many situations it is more useful to measure the *instantaneous* rate of change at a point. The mathematical term for an instantaneous rate of change is *derivative*. In SCIE1000, you will not be finding derivatives (in general), but will need to interpret and use them.

Derivatives

If $y = f(x)$ is a function, then the derivative y' is a new function that gives the instantaneous rate at which y is changing with respect to x.

The **value** of the **derivative** at any point describes the behaviour of the **function** at that point. At any point:

- if y' is **positive** then the function y is **increasing**;
- if y' is **negative** then the function y is **decreasing**; and
- if the function y has a **local maximum** (peak) or **local minimum** (trough) at a point, then y' **equals zero** at that point.

The *derivative of the derivative*, or *second derivative*, is denoted f''.

- A constant function has no slope and thus the derivative of a constant is 0. That is, if $y(x) = c$ for some constant c, then $y'(x) = 0$.
- The derivative of a linear function is equal to the slope of that function. That is, if $y(x) = mx + c$ for some constants m and c, then $y'(x) = m$.
- If $f(x)$ and $g(x)$ are functions whose derivatives are defined, then the derivative of $f(x) + g(x)$ is $f'(x) + g'(x)$ and the derivative of $f(x) - g(x)$ is $f'(x) - g'(x)$.

C.5 Area under the curve

- Given a graph, the *area under the curve* or *AUC* of that graph is the area bounded by that curve, the x-axis and two points on the x-axis.

AUC and Definite integrals

Given a function $f(x)$, the AUC from the point $x = a$ to the point $x = b$ is called the *definite integral of $f(x)$ from a to b*, written as

$$\int_a^b f(x)\,dx.$$

- There are two common ways of calculating AUCs.
- First, if the function $f(x)$ is known, then the *Fundamental Theorem of Calculus* gives an "easy" mathematical approach for finding the AUC between two points a and b on the x-axis:
  - Find an *antiderivative* or *integral* of $f(x)$, say $F(x)$.
  - Substitute the value b into $F(x)$.
  - Substitute the value a into $F(x)$.
  - Subtract the second value from the first one.
  - The answer gives the required AUC.
- More often, AUCs are used in practical applications in which the only available information is a collection of measured data values, and the function $f(x)$ is **not** known.
- In such cases, AUCs are estimated approximately, by summing the areas of geometric shapes of "narrow" width, such as rectangles (called *Riemann sums*), or *trapezoids* (called the *trapezoid rule*).

Appendix D: The use of units in Science

D.1 SI Units and prefixes

Australia adopted the **International System of units**, or **SI units**, in 1960 and this is the system still in use today. There are seven **SI base units**. The kinds of things they measure, their standard names and their symbols are shown in Table D.1.

Base quantity	SI base unit	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Table D.1: The seven SI base units.

Multiple	Prefix	Symbol	Multiple	Prefix	Symbol
10^1	deca	da	10^{-1}	deci	d
10^2	hecto	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atto	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y
10^{27}	ronna	R	10^{-27}	ronto	r
10^{30}	quetta	Q	10^{-30}	quecto	q

Table D.2: The 20 SI prefixes.

Some lengths, like the charge radius of a proton, are much less than a metre; others, like the distance from the Earth to the centre of the Milky Way, are much more than a metre. To save writing numbers with many digits before or after the decimal place, we use one of the SI prefixes, as shown in Table D.2, in front of a unit of measurement to indicate a multiple of the unit.

D.2 Derived units

Many natural and scientific quantities require more complex units than SI base units. These **can always be defined** in terms of the seven base units, and are called **SI derived units.**

Example *D.2.1*

Some frequently-used SI derived units have been given special names and symbols. Table D.3 shows some well-known examples.

Quantity	Name	Symbol	In SI base units	In other SI Units
frequency	hertz	Hz	s^{-1}	-
force	newton	N	$m \cdot kg \cdot s^{-2}$	-
pressure, stress	pascal	Pa	$m^{-1} \cdot kg \cdot s^{-2}$	$N \cdot m^{-2}$
energy, work, quantity of heat	joule	J	$m^2 \cdot kg \cdot s^{-2}$	$N \cdot m$
power, radiant flux	watt	W	$m^2 \cdot kg \cdot s^{-3}$	$J \cdot s^{-1}$
electric potential difference, electromotive force	volt	V	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$	$W \cdot A^{-1}$
volume	litre	L or l	10^{-3} m^3	-
time	day	d	$86,400$ s	24 h
time	hour	h	$3,600$ s	60 min
time	minute	min	60 s	-

Table D.3: Some well-known derived units and their SI base units.

D.3 Algebra for quantities and units

If quantities have the same units then they can be added or subtracted and the result has the same units.

If quantities are multiplied or divided then the corresponding units are multiplied or divided (or cancelled) using the familiar algebra rules.

We often write a dot, or leave a space, between units when they are to be multiplied, and use exponent notation to indicate "powers" of units. Negative exponents indicate quotients. So, for example, we write m^3 instead of $\mathrm{m}\cdot\mathrm{m}\cdot\mathrm{m}$, and $\mathrm{m}\cdot\mathrm{s}^{-1}$ or $\mathrm{m\ s}^{-1}$ instead of $\frac{\mathrm{m}}{\mathrm{s}}$.

D.4 Unit conversions

The algebra rules for quantities and units provide a neat way to convert between different units for the same base quantity. For example, an atmosphere, abbreviated to atm, is a unit for pressure. By definition, 1 atm is equal to 101.325 kPa. It follows that $\frac{1\ \mathrm{atm}}{101.325\ \mathrm{kPa}}$ is like 1, and we may introduce it as a multiplicative factor whenever we like. For example, we can convert 233.05 kPa to the units of atm as follows:

$$P = 233.05\mathrm{kPa} \times \frac{1\mathrm{atm}}{101.325\mathrm{kPa}} \approx 2.30\mathrm{atm}$$

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These notes contain a large number of original photographs. The photographer or photograph owner is identified, as follows:

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Bibliography

[1] Australian Institute of Health and Welfare (AIHW) and Australasian Association of Cancer Registries (AACR), *Cancer in Australia 2001*. AIHW cat. no. CAN 23. Canberra: AIHW (Cancer Series no. 28, 2004).

[2] Australian Medical Association, accessed 27 December 2010, `<ama.com.au/node/2931>`

[3] Benowitz *et al.*, *Nicotine intake and dose response when smoking reduced-nicotine content cigarettes*, Clinical Pharmacology and Therapeutics **80** (2006) 703–714.

[4] Bonani *et al.*, *AMS 14C age determination of tissue, bone and grass samples from the Ötzal Ice Man*, Radiocarbon **36:2** (1994) 247-250.

[5] Caswell, *Matrix population models: Construction, analysis and interpretation*, 2nd Edition, 2001. Sinauer Associates, Sunderland, Massachusetts. ISBN 0-87893-096-5.

[6] Centers for Disease Control and Prevention, accessed 27 December 2010, `www.cdc.gov`

[7] Crouse *et al.*, *A Stage-Based Population Model for Loggerhead Sea Turtles and Implications for Conservation*, Ecological Society of America **68** (1987) 1–13.

[8] Crowder *et al.*, *Predicting the impact of turtle excluder devices on loggerhead sea turtle populations*, Ecological Applications **4(3)** (1994) 437–445.

[9] DaliPaintings.com, accessed February 7, 2023,

`<https://www.dalipaintings.com/persistence-of-memory.jsp>`

[10] DaliPaintings.com, accessed February 7, 2023,

`<https://www.dalipaintings.com/three-sphinxes-of-bikini.jsp>`

[11] Damon *et al.*, *Radiocarbon Dating of the Shroud of Turin*, Nature **337: 6208** (1989) 611–615.

[12] Dingle *et al.*, *What Is Migration?*, BioScience **57(2)** (2007) 113–121.

[13] Drugs.com, accessed 11 September 2010, `<www.drugs.com>`

[14] Egevang *et al.*, *Tracking of Arctic terns* Sterna paradisaea *reveals longest animal migration*, PNAS **107** (2010) 2078–2081.

[15] Engber, *Vodka + movies = pseudoscience*, Chronicle of Higher Education, October 29, 2004.

[16] Engber, “Dodgy Boffins”, accessed 17th August 2012,

`<https://slate.com/technology/2012/05/science-in-the-telegraph-and-the-daily-mail-whats-wrong-with-british-journalism.html>`

[17] Duijm *et al.*, *Sensitivity, specificity and predictive values of breast imaging in the detection of cancer*, British Journal of Cancer **76(3)** (1997) 377–381.

[18] Explorers Web 2004, accessed 6 September 2010,

`<www.explorersweb.com/adventureweather/charts>`

[19] Framingham Heart Study 2010, accessed 31 August 2010,
<www.framinghamheartstudy.org/participants/index.html>

[20] Framingham Heart Study 2010, accessed 31 August 2010,
<www.framinghamheartstudy.org/about/index.html>

[21] Framingham Heart Study 2010, accessed 31 August 2010,
<www.framinghamheartstudy.org/about/milestones.html>

[22] Galileo, *Dialogue Concerning the Two Chief World Systems*, trans. Stillman Drake, Berkeley: Univ. California Press (1953) 207.

[23] Gigerenzer *et al.*, *Helping Doctors and Patients Make Sense of Health Statistics*, Psych. Science in the Public Interest **8 (2)** (2007) 53–96.

[24] Goldacre, "Bad Science", accessed 19th September 2012, <www.badscience.net>

[25] Goldman *et al.*, *A re-examination of the age and growth of sand tiger sharks, Carcharias taurus, in the western North Atlantic: the importance of ageing protocols and use of multiple back-calculation techniques*, Environ Biol Fish **77:3-4** (2006) 241–252.

[26] Grau *et al.*, *Breast cancer in Rubens paintings*, Breast Cancer Research and Treatment **68** (2001), 89–93.

[27] Greenbaum *et al.*, *Impaired β-cell function, incretin effect, and glucagon suppression in patients with type 1 diabetes who have normal fasting glucose*, Diabetes **51** (2002) 943 – 950.

[28] Haddaway-Riccio (Secretary), *Full Report of the 2020 Update Stock Assessment of the Eastern Oyster, Crassostrea virginica, in the Maryland waters of Chesapeake Bay*, accessed January 2022,
<dnr.maryland.gov/fisheries/Pages/Oyster_Stock_Assess/oysters.aspx>.

[29] Haub, "How many people have ever lived on Earth?", accessed 5th July 2013, <www.prb.org>

[30] Hoegh-Guldberg *et al.*, *Coral Reefs Under Rapid Climate Change and Ocean Acidification*, Science **318:5857** (2007) 1737 – 1742.

[31] International Organization for Standardization, ISO 2533:1975.

[32] John Simon Guggenheim Memorial Foundation web-site, accessed February 6, 2023,
<https://www.gf.org/fellows/margaret-morrison/>

[33] Kasperski and Wieland, *When is it Optimal to Delay Harvesting? The Role of Ecological Services in the Northern Chesapeake Bay Oyster Fishery*, Marine Resource Economics, **24**, 4, (2009), 361-385.

[34] Keeling *et al.*, *Exchanges of atmospheric CO_2 and ${}^{13}CO_2$ with the terrestrial biosphere and oceans from 1978 to 2000*, I. Global aspects, SIO Reference Series, No. 01-06, Scripps Institution of Oceanography, San Diego, (2001), 88 pages.

[35] Lundquist *et al.*, *Cyanide concentrations in blood after cigarette smoking, as determined by a sensitive fluorimetric method*, Clinical Chemistry, **33:7** (1987), 1228 – 1230.

[36] MacLulich, *Fluctuations in the numbers of the varying hare (Lepus americanus)*, University of Toronto Studies, Biological series **43** (1937).

[37] Maher, *Growth and development of the Yellow-bellied Sunbird* Nectarinia jugularis *in North Queensland*, Emu **91** (1991) 58–61.

[38] Mas, *Communication*, <https://www.flickr.com/photos/dailypic/1459055735>, CC BY-NC 2.0.

[39] Maskel. *et al.*, *Evidence based survey of the distribution volume of ethanol: Comparison of empirically determined values with anthropometric measures*, Forensic Science International **294** (2019) 124 – 131.

[40] Moolenaar *et al.*, *Absorption rate and bioavailability of paracetamol from rectal aqueous suspensions*, Pharmacy World and Science, **1:1** (1979), 201–206.

[41] Morgan, Mary S., and Morrison, Margaret (eds.) (1999), *Models as Mediators: Perspectives on Natural and Social Science*, Cambridge: Cambridge University Press.

[42] NASA Ozone Watch, accessed 15 January 2022,
<https://ozonewatch.gsfc.nasa.gov/meteorology/annual_data.html>

[43] National Breast and Ovarian Cancer Centre, *Breast cancer risk factors: a review of the evidence*, National Breast and Ovarian Cancer Centre, Surry Hills, NSW, 2009.

[44] National Health Service, accessed 19 September 2012,
<https://www.nhs.uk/%2FNews%2FPages%2FNewsIndex.aspx>

[45] National Heart, Lung and Blood Institute 2008, accessed 31 August 2010,
<www.nhlbi.nih.gov/health/dci/Diseases/Hbc/HBC_WhatIs.html>

[46] Newman *et al.*, What would have happened to the ozone layer if chlorofluorocarbons (CFCs) had not been regulated?, *Atmos. Chem. Phys.*, 9 (2009) 2113–2128.

[47] Niyomnaitham *et al.*, *Bioequivalence study of 50 mg sertraline tablets in healthy Thai volunteers*, J Med Assoc Thailand **92:9** (2009), 1229–33.

[48] Pan American Health Organization, *EPI Newsletter Volume XX, Number 4*, August 1998.

[49] Ponce-Reyes *et al.*, *Vulnerability of cloud forest reserves in Mexico to climate change*, Nature Climate Change **2** (2012), 448–52.

[50] Popper, *The Logic of Scientific Discovery*, London: Hutchinson (1934).

[51] Posey and Mozayani, *The estimation of BAC, Widmark revisited*, Forensic Science, Medicine and Pathology, **3** (2007) 33–39.

[52] Pyle *et al.* (2005). Ozone and Climate: A Review of Interconnections. In Metz *et al.* (Eds), *Safeguarding the Ozone Layer and the Global Climate System* (p. 83 - 132). Cambridge University Press.

[53] Rawlins *et al.*, *Pharmacokinetics of paracetamol (Acetaminophen) after intravenous and oral administration*, European Journal of Clinical Pharmacology **11** (1977), 283 – 286.

[54] Rodenhouse *et al.*, *Potential effects of climate change on birds of the northeast*, Mitigation and Adaptation Strategies for Global Change, **13** (2008) 517–540.

[55] Ronfeld *et al.*, *Pharmacokinetics of sertraline and its N-demethyl metabolite in elderly and young male and female volunteers*, Clinical Pharmacokinetics, **32** Suppl 1 (1997) 22–30.

[56] Salm *et al.*, *Blood flow in coronary artery bypass vein grafts: volume versus velocity at cardiovascular MR imaging*, Radiology, **232** 915–920.

[57] Schwartz *et al.*, *The role of numeracy in understanding the benefit of screening mammography*, Annals of Internal Medicine, **127** (1997) 966–972.

[58] Scripps Institution of Oceanography 2010, La Jolla, CA, accessed 24 September 2013,

`<scrippsco2.ucsd.edu/data/atmospheric_co2.html>`

[59] Seidl *et al.*, *The calculation of blood ethanol concentrations in males and females*, International Journal of Legal Medicine, **114** (2000) 71–77.

[60] Sheen and Hwang, *Mathematical modelling the cross-contamination of E. coli* O157:H7 *on the surface of ready-to-eat meat product while slicing*, Food Microbiology **27** (2010) 37–43.

[61] Steadman, Norms of apparent temperature in Australia, *Aust. Met. Mag.*, **Vol 43**, 1994, 1–16.

[62] Stenseth *et al.*, *Population regulation in snowshoe hare and Canadian lynx*, PNAS **94** (1997), 5147–5152.

[63] Stephenson *et al.*, *Mars Climate Orbiter Mishap Investigation Board, Phase I Report*,

`<https://llis.nasa.gov/llis_lib/pdf/1009464main1_0641-mr.pdf>`

[64] United Nations, Department of Economic and Social Affairs, Population Division, *World Population Prospects: The 2010 Revision*, CD-ROM Edition (2011).

[65] Wikimedia Commons contributors, "File:Arctic tern.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

`<https://commons.wikimedia.org/w/index.php?`
`title=File:Arctic_tern.jpg&oldid=639977830>`

[66] Wikimedia Commons contributors, "File:Åsgårdsreien.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

`<https://commons.wikimedia.org/w/index.php?`
`title=File:%C3%85sg%C3%A5rdsreien.jpg&oldid=707402192>`

[67] Wikimedia Commons contributors, "File:Bicknells Thrush From The Crossley ID Guide Eastern Birds.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

`<https://commons.wikimedia.org/w/index.php?`
`title=File:Bicknells_Thrush_From_The_Crossley_ID_Guide_Eastern_Birds.jpg`
`&oldid=696545696>`

[68] Wikimedia Commons contributors, "File:Canadian lynx by Keith Williams.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

`<https://commons.wikimedia.org/w/index.php?`
`title=File:Canadian_lynx_by_Keith_Williams.jpg&oldid=644840697>`

[69] Wikimedia Commons contributors, "File:Caretta caretta 060417w2.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

`<https://commons.wikimedia.org/w/index.php?`
`title=File:Caretta_caretta_060417w2.jpg&oldid=691327877>`

[70] Wikimedia Commons contributors, "File:Da Vinci Vitruve Luc Viatour.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,
<https://commons.wikimedia.org/w/index.php?title=File:Da_Vinci_Vitruve_Luc_Viatour.jpg&oldid=690516986>

[71] Wikimedia Commons contributors, "File:Ecoli colonies.png," Wikimedia Commons, the free media repository, accessed February 6, 2023,
<https://commons.wikimedia.org/w/index.php?title=File:Ecoli_colonies.png&oldid=452411550>

[72] Wikimedia Commons contributors, "File:Edmonton Leporidae (winter 6).jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,
<https://commons.wikimedia.org/w/index.php?title=File:Edmonton_Leporidae_(winter_6).jpg&oldid=598892785>

[73] Wikimedia Commons contributors, "File:Emilie Chatelet portrait by Latour.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,
<https://commons.wikimedia.org/w/index.php?title=File:Emilie_Chatelet_portrait_by_Latour.jpg&oldid=642023067>

[74] Wikimedia Commons contributors, "File:Helen E Longino.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,
<https://commons.wikimedia.org/w/index.php?title=File:Helen_E_Longino.jpg&oldid=631082689>

[75] Wikimedia Commons contributors, "File:Hieronymus Bosch - The Garden of Earthly Delights - Hell.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,
<https://commons.wikimedia.org/w/index.php?title=File:Hieronymus_Bosch__The_Garden_of_Earthly_Delights_-_Hell.jpg&oldid=697051758>

[76] Wikimedia Commons contributors, "File:Justus Sustermans - Portrait of Galileo Galilei, 1636.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,
<https://commons.wikimedia.org/w/index.php?title=File:Justus_Sustermans_-_Portrait_of_Galileo_Galilei,_-1636.jpg&oldid=726672335>

[77] Wikimedia Commons contributors, "File:Global Warming Predictions.png," Wikimedia Commons, the free media repository, accessed November 1, 2023,
<https://commons.wikimedia.org/w/index.php?title=File:Global_Warming_Predictions.png>

[78] Wikimedia Commons contributors, "File:Karl Popper.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,
<https://commons.wikimedia.org/w/index.php?title=File:Karl_Popper.jpg&oldid=719172722>

[79] Wikimedia Commons contributors, "File:Lascaux painting.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,
<https://commons.wikimedia.org/w/index.php?title=File:Lascaux_painting.jpg&oldid=708513921>

[80] Wikimedia Commons contributors, “File:Los borrachos o el triunfo de Baco 1629 Velázquez.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

<https://commons.wikimedia.org/w/index.php?title=File:Los_borrachos_o_el_triunfo_de_Baco_1629_Vel%C3%A1zquez.jpg&oldid=692865202>

[81] Wikimedia Commons contributors, "File:Mary S. Morgan at HSS 2009.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

<https://commons.wikimedia.org/w/index.php?title=File:Mary_S._Morgan_at_HSS_2009.jpg&oldid=454534706>

[82] Wikimedia Commons contributors, "File:Painting of David Hume.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

<https://commons.wikimedia.org/w/index.php?title=File:Painting_of_David_Hume.jpg&oldid=704209033>

[83] Wikimedia Commons contributors, "File:Skull with a Burning Cigarette.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

<https://commons.wikimedia.org/w/index.php?title=File:Skull_with_a_Burning_Cigarette.jpg&oldid=617140355>

[84] Wikimedia Commons contributors, "File:Spanish flu death chart.png," Wikimedia Commons, the free media repository, accessed February 6, 2023,

<https://commons.wikimedia.org/w/index.php?title=File:Spanish_flu_death_chart.png&oldid=718745141>

[85] Wikimedia Commons contributors, "File:"The School of Athens" by Raffaello Sanzio da Urbino.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

<https://commons.wikimedia.org/w/index.php?title=File:%22The_School_of_Athens%22_by_Raffaello_Sanzio_da_Urbino.jpg&oldid=728266201>

[86] Wikimedia Commons contributors, "File:Tsunami by hokusai 19th century.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

<https://commons.wikimedia.org/w/index.php?title=File:Tsunami_by_hokusai_19th_century.jpg&oldid=711119869>

[87] Wikimedia Commons contributors, "File:Van Gogh - Starry Night - Google Art Project.jpg," Wikimedia Commons, the free media repository, accessed February 6, 2023,

<https://commons.wikimedia.org/w/index.php?title=File:Van_Gogh_-_Starry_Night_-_Google_Art_Project.jpg&oldid=720670399>

[88] Wilkinson *et al.*, *Pharmacokinetics of ethanol after oral administration in the fasting state*, Journal of Pharmacokinetics and Biopharmaceutics, **5:3** (1977) 207–224.

[89] Wolever *et al.*, *The glycemic index: methodology and clinical implications*, American Journal of Clinical Nutrition, **54** (1991) 846–854.

[90] World Health Organisation, *Gender*, accessed 5 November 2020,

<https://www.who.int/health-topics/gender>

[91] World Health Organisation, *Global burden of disease*, accessed 10 October 2012, <https://www.who.int/healthinfo/global_burden_disease/en>

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