

# MXB105 Assessment Task 2

Caleb Yates

Noah Schmidt

Morgan Lawson

Justin Wade

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## Abstract

This report solves both Task 2's Problems.

## 1 Introduction

TODO write introduction if necessary

## 2 Problem 1

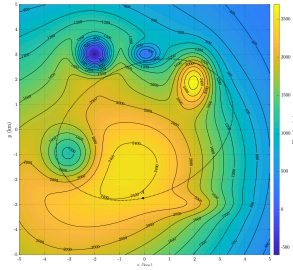


Figure 1: Problem 1 contour map

### 2.1 Hilltops

### 2.2 Saddle points

### 2.3 Freshwater or Saltwater Areas

### 2.4 Particularly Steep Areas

### 2.5 Relatively Flat Areas

### 2.6 Elevation Changes from Point A Clockwise

## 3 Problem 2

Problem 2 involved analysing the following function:

$$h(x, y) = -\exp\left(-\frac{5}{4}(x^2 + y^2)^2 + \frac{1}{4}x^3 + \frac{3}{2}y^3 + \frac{1}{5}x^2 + \frac{1}{5}y^2\right) \text{ for } -1.5 \leq x \leq 1.5 - 1.5 \leq y \leq 2$$

Immediately it became obvious to redefine this function in terms of just the exponent,  $g$ :

$$g(x, y) = -\frac{5}{4}(x^2 + y^2)^2 + \frac{1}{4}x^3 + \frac{3}{2}y^3 + \frac{1}{5}x^2 + \frac{1}{5}y^2$$

We can expand this into a quartic:

$$g(x, y) = -\frac{5}{4}(x^4 + 2x^2y^2 + y^4) + \frac{1}{4}x^3 + \frac{3}{2}y^3 + \frac{1}{5}x^2 + \frac{1}{5}y^2$$

$$g(x, y) = -\frac{5}{4}x^4 - \frac{5}{2}x^2y^2 - \frac{5}{4}y^4 + \frac{1}{4}x^3 + \frac{3}{2}y^3 + \frac{1}{5}x^2 + \frac{1}{5}y^2$$

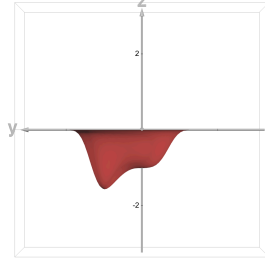


Figure 2: 3D visualization of  $g(x, y)$  looking down the  $+x$  axis

The way  $h(x, y)$  is defined in terms of  $g(x, y)$  implies that the stationary points / critical points of  $g(x, y)$  carry over to  $h(x, y)$ , as we will see later.

### 3.1 Critical Points

To analyze the critical points of  $h(x, y)$ , it's gradient function along  $\vec{v}$  was decomposed along the implicit basis vectors  $\hat{x}$ ,  $\hat{y}$  and  $\hat{z}$ :

$$\frac{\partial h}{\partial \vec{v}} = \nabla_{\vec{v}} \cdot h = \left\langle \frac{\partial h}{\partial x}, \frac{\partial h}{\partial y} \right\rangle \cdot \vec{v}$$

Similarly for  $g(x, y)$ :

$$\frac{\partial g}{\partial \vec{v}} = \nabla_{\vec{v}} \cdot g = \left\langle \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y} \right\rangle \cdot \vec{v}$$

Since the partial derivatives of  $g(x, y)$  were decidedly simpler to evaluate, they were evaluated first:

$$\frac{\partial g}{\partial x} = \frac{\partial}{\partial x} \left( -\frac{5}{4}x^4 - \frac{5}{2}x^2y^2 + \frac{1}{4}x^3 + \frac{1}{5}x^2 \right) = -5x^3 + \frac{3}{4}x^2 - 5y^2x + \frac{2}{5}x$$

$$\frac{\partial g}{\partial y} = \frac{\partial}{\partial y} \left( -\frac{5}{2}x^2y^2 - \frac{5}{4}y^4 + \frac{3}{2}y^3 + \frac{1}{5}y^2 \right) = -5y^3 + \frac{9}{2}y^2 - 5x^2y + \frac{2}{5}y$$

Solving for the critical points of  $g(x, y)$  will yield the same  $x$  and  $y$  coordinates of the critical points of  $h(x, y)$ , hence why  $g(x, y)$  is studied first. A 'critical point' is defined as a point where  $\frac{\partial g}{\partial \vec{v}} = 0$ , hence:

$$\text{let } \vec{v} = \langle v_x, v_y \rangle = \langle 0, 0 \rangle$$

$$0 = -5x^3 + \frac{3}{4}x^2 - 5y^2x + \frac{2}{5}x$$

$$0 = -5y^3 + \frac{9}{2}y^2 - 5x^2y + \frac{2}{5}y$$

TODO!

From 2 it can be seen that two critical points for  $h(x, y)$  exist:

1. A global minimum with  $y > 0$
2. A local minimum around  $y \approx 0$

### 3.2 Description of the topograph of the canyon

As shown in 2, the canyon consists of one major depression below  $z < 0$  with a secondary local depression around TODO.

### 3.3 Depth of a probe landing at $(0, -1)$

Since the height is given by  $h(x, y)$ , substituting the values of  $x = 0$  and  $y = -1$  yields the height at  $(0, -1)$ :

$$\begin{aligned} h(0, -1) &= -\exp\left(-\frac{5}{4}(0^2 + (-1)^2)^2 + \frac{1}{4}0^3 + \frac{3}{2}(-1)^3 + \frac{1}{5}0^2 + \frac{1}{5}(-1)^2\right) = -\exp\left(-\frac{5}{4} - \frac{3}{2} + \frac{1}{5}\right) \\ &= -e^{-\left(\frac{51}{20}\right)} \approx -0.07808167 \end{aligned}$$

Therefore the depth of the probe if it landed at  $(0, -1)$  would be  $78.08167 \approx 78$  meters.

### 3.4 Direction and slope of steepest descent at $(0, -1)$

### 3.5 Equation of motion around the canyon at constant height

### 3.6 Visualization of probe path at constant height of $z \approx 78$