MXB105 Assessment Task 2

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Abstract

This report solves both Task 2's Problems.

1 Introduction

TODO write introduction if necessary

2 Problem 1

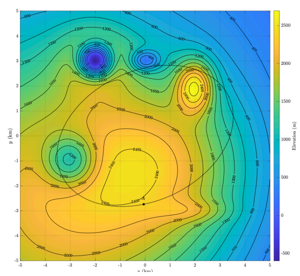

Figure 1: Problem 1 contour map

2.1 Hilltops

2.2

3 Problem 2

Problem 2 involved analysing the following function:

$$h\left(x, y\right) = -\exp\left(-\frac{5}{4}\left(x^2 + y^2\right)^2 + \frac{1}{4}x^3 + \frac{3}{2}y^3 + \frac{1}{5}x^2 + \frac{1}{5}y^2\right)$$

Immediately it became obvious to redefine this function in terms of just the exponent, g:

$$g\left(x, y\right) = -\frac{5}{4}\left(x^2 + y^2\right)^2 + \frac{1}{4}x^3 + \frac{3}{2}y^3 + \frac{1}{5}x^2 + \frac{1}{5}y^2$$

We can expand this into a quartic:

$$g\left(x, y\right) = -\frac{5}{4}\left(x^4 + 2x^2y^2 + y^4\right) + \frac{1}{4}x^3 + \frac{3}{2}y^3 + \frac{1}{5}x^2 + \frac{1}{5}y^2$$

$$g\left(x,\ y\right) = -\frac{5}{4}x^4 - \frac{5}{2}x^2y^2 - \frac{5}{4}y^4 + \frac{1}{4}x^3 + \frac{3}{2}y^3 + \frac{1}{5}x^2 + \frac{1}{5}y^2$$

The way $h(x, y)$ is defined in terms of $g(x, y)$ implies that the stationary points / critical points of $g(x, y)$ carry over to $h(x, y)$, as we will see later.

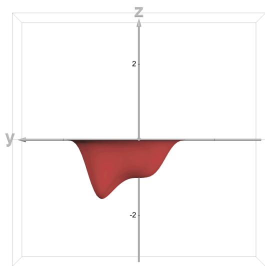

Figure 2: 3D visualization of $g(x, y)$ looking down the +x axis

3.1 Critical Points

To analyze the critical points of $h(x, y)$, it's gradient function along $\vec{v}$ was decomposed along the implicit basis vectors $\mathbf{\hat{x}}$, $\mathbf{\hat{y}}$ and $\mathbf{\hat{z}}$:

$$\frac{\partial h}{\partial \vec{v}} = \nabla_{\vec{v}} \cdot h = \langle \frac{\partial h}{\partial x}, \frac{\partial h}{\partial y} \rangle \cdot \vec{v}$$

Similarly for $g(x, y)$:

$$\frac{\partial g}{\partial \vec{v}} = \nabla_{\vec{v}} \cdot g = \langle \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y} \rangle \cdot \vec{v}$$

Since the partial derivatives of $g(x, y)$ were decidedly simpler to evaluate, they were evaluated first:

$$\frac{\partial g}{\partial x} = \frac{\partial}{\partial x}(-\frac{5}{4}x^4 - \frac{5}{2}x^2y^2 + \frac{1}{4}x^3 + \frac{1}{5}x^2) = -5x^3 + \frac{3}{4}x^2 - 5y^2x + \frac{2}{5}x$$
$$\frac{\partial g}{\partial y} = \frac{\partial}{\partial y}(-\frac{5}{2}x^2y^2 - \frac{5}{4}y^4 + \frac{3}{2}y^3 + \frac{1}{5}y^2) = -5y^3 + \frac{9}{2}y^2 - 5x^2y + \frac{2}{5}y$$

Solving for the critical points of $g(x, y)$ will yield the same x and y coordinates of the critical points of $h(x, y)$, hence why $g(x, y)$ is studied first. A 'critical point' is defined as a point where $\frac{\partial g}{\partial \vec{v}} = 0$, hence:

$$\text{let } \vec{v} = \langle v_x, v_y \rangle = \langle 0, 0 \rangle$$
$$0 = -5x^3 + \frac{3}{4}x^2 - 5y^2x + \frac{2}{5}x$$
$$0 = -5y^3 + \frac{9}{2}y^2 - 5x^2y + \frac{2}{5}y$$

TODO!

From 2 it can be seen that two critical points for $h(x, y)$ exist:

1. A global minimum with $y > 0$
2. A local minimum around $y \approx 0$

3.2 Description of the topograph of the canyon

As shown in 2, the canyon consists of one major depression below $z < 0$ with a secondary local depression around TODO.

3.3 Depth of a probe landing at $(0,-1)$

Since the height is given by $h(x,y)$, substituting the values of $x=0$ and $y=-1$ yields the height at $(0,-1)$:

$$h\left(0,-1\right) = -\exp\left(-\frac{5}{4}\left(0^2+(-1)^2\right)^2+\frac{1}{4}0^3+\frac{3}{2}(-1)^3+\frac{1}{5}0^2+\frac{1}{5}(-1)^2\right) = -\exp\left(-\frac{5}{4}-\frac{3}{2}+\frac{1}{5}\right)$$

$$= -e^{-}\left(\frac{51}{20}\right) \approx -0.07808167$$

Therefore the depth of the probe if it landed at $(0,-1)$ would be $78.08167 \approx 78$ meters.

3.4 Direction and slope of steepest descent at $(0,-1)$

3.5 Equation of motion around the canyon at constant height

3.6 Visualization of probe path at constant height of $z \approx 78$