長庚大學期中、期末考試答案用紙

科目 機率

____學年度 第____學期 期中 考 資工 系 姓名 何亞甯 學號 B0729059

[1](1) $x=0$ $C^{10}_{0}\times(0.1)^{0}\times(0.9)^{10}=0.3487$, $x=1$ $C^{10}_{1}\times(0.1)^{1}\times(0.9)^{9}=0.3874$, $x=2$ $C^{10}_{2}\times(0.1)^{2}\times(0.9)^{8}=0.1937$, $x=3$ 0.0574 , $x=4$ 0.0111 ,

$x=5$ 0.0014 , $x=6$ 0.0001 , $x=7$ 8.7×10^{-6} , $x=8$ 3.6×10^{-7} , $x=9$ 9×10^{-9} , $x=10$ 10^{-10}

(2) $0.3487\times0+0.3874\times1+0.1937\times2+\cdots\cdots\doteq1$

(3) $(0-1)^{2}\times0.3487+(1-1)^{2}\times0.3874+(2-1)^{2}\times0.1937+\cdots\cdots\doteq0.9$

(4)

(5)

(6)

[2](1)

準差, $Std[X]$ =?
把球取出並登記
函數, $f_Y(y)$ =?
值+標準差, $E[Y$
球取出並登記顏
之總數定為隨機

千萬的國家在全
生的火災總件數
機率質量函數
平均值 $E[W]$ +
$- E[W]| \leq$
$> 120)$ =?
0" 這種情形
個假設，該接

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$b(x; n, p)$

[1] (4) [2]

$Y=0 \quad \frac{90}{100} \times \frac{89}{99} \times \cdots\cdots \times \frac{81}{91} = 0.3305$

$Y=1 \quad C^{10}_1 \times \frac{90 \times \cdots \times 82 \times 10}{100 \times \cdots \times 91} = 0.4080$

$Y=2 \quad C^{10}_2 \times \frac{90 \times \cdots \times 83 \times 10 \times 9}{100 \times \cdots \times 91} = 0.2015$

$Y=3 \quad C^{10}_3 \times \frac{90 \times \cdots \times 84 \times 10 \times \cdots \times 8}{100 \times \cdots \times 91} = 0.0518$

$Y=4 \quad C^{10}_4 \times \frac{90 \times \cdots \times 85 \times 10 \times \cdots \times 7}{100 \times \cdots \times 91} = 0.0076$

$Y=5 \quad C^{10}_5 \times \qquad = 0.0006$

$= 0.00003$

$Y=6 \qquad = 10^{-6}$

$Y=7 \qquad = 10^{-8}$

$Y=8 \qquad = 10^{-10}$

$Y=9 \qquad = 5.7769 \times 10^{-14}$

$Y=10$

(5) $0 \times 0.3305 + 1 \times 0.4080 + \cdots\cdots \doteq 1.0000 = E[Y]$

$(0-1)^2 \times 0.3305 + (1-1)^2 \times 0.4080 + \cdots\cdots \doteq 0.8180 = Std[Y]$

Sum: 1.818

(6) $f_Z(z) = b^*(z; 5, \frac{1}{10}) \cdot \binom{z-1}{4} \cdot \left(\frac{1}{10}\right)^5 \cdot \left(\frac{9}{10}\right)^{z-5}$

[2] (1)

100天 $\mu = 100 \frac{次火災}{100天}$

$f_W(W) = e^{-100} \times \frac{100^W}{W!}$

(2) $E[W] = \lambda t = 1$; $std(W) = \sqrt{\lambda t} = 1$ Sum = 2

(3) $|W - E[W]| \leq 2 \cdot std(W) \Rightarrow |W-1| \leq 2 \Rightarrow 1 \leq W \leq 3$

$P(\text{"}|W-E[W]| \leq 2 \cdot std(W)\text{"}) = P(1 \leq W \leq 3) = 0.6131$

(4) $P(W > 120) = 1 - P(W \leq 120) = 1 - \sum_{W=0}^{120} f_W(W) = 1 - 1 = 0$

(5) 由上題可知，機率接近0，因拒絕

C^{100}_0 0.0059 C^{100}_1 0.0312 C^{100}_2 0.0812 C^{100}_3 0.1396 C^{100}_4 0.1781 5 0.1800 6 0.1500 7 8 9

[3] (1) $P(X \geq 10) = 1 - P(0) - P(1) - P(2) \cdots - P(9) = 1 - b(0;100,0.05) - b(1;100,0.05) - \cdots - b(9;100,0.05) =$

$C^{100}_0 (0.05)^0 (0.95)^{100} + C^{100}_1 (0.05)^1 (0.95)^{99}$

(2) $P(X \geq 10 \mid p = 5\%) = P(X \geq 10)$

"$p=5\%$" ⇒ "$X \geq 10$" 機率很小

但 "$X \geq 10$" 確實發生

$P \Rightarrow q \quad \sim q \Rightarrow \sim P$

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[4] $P(X) = \binom{n}{x} p^x (1-p)^{n-x}$ for $x = 0, 1, 2, \cdots, n$

$\mu = np$, $\sigma^2 = np(1-p)$

$E(X)$ $\quad Var(X)$ $\qquad E(U_i) = p$ and $Var(U_i) = p(1-p)$

$X = U_1 + \cdots + U_i$ $\qquad p + \cdots + p$

$E(X) = E(U_1 + \cdots + U_n) = E(U_1) + \cdots + E(U_n) = np$

$Var(X) = Var(U_1 + \cdots + U_n) = Var(U_1) + \cdots + Var(U_n) = np(1-p)$

$p(1-p) + \cdots + p(1-p)$

$E(X) = \sum_x x p(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x} = \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x} = np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$

$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!((n-1)-(x-1))!} \times p^{x-1} (1-p)^{(n-1)-(x-1)}$ $\quad m = n-1$, $y = x-1$

$np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$

$= np \sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}$

$=$ 項分佈 $\sum_{y=0}^{m} \binom{m}{y} a^y b^{m-y} = (a+b)^m$ $\quad a = p$, $b = 1-p$

$E(X) = np(p + (1-p))^m = np(1)^m$

$= np$

$Var(X) = E[(X-\mu)^2] = \sum_x (x-\mu)^2 p(x)$

$E[(X-\mu)^2] = E(X^2) - [E(X)]^2$

$E(X^2) = \sum_{x=0}^{n} x^2 \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

$E[X(X-1)] = \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} = \sum_{x=2}^{n} \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x} = n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$

$E(X^2 - X)$

（請翻面繼續作答）

$= \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!((n-2)-(x-2))!} p^{x-2} (1-p)^{(n-2)-(x-2)} \times n(n-1)p^2$ $\quad m = n-2$, $y = x-2$

$= n(n-1)p^2 \times \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} = n(n-1)p^2 \times (p + (1-p))^m = n(n-1)$

由①知 $E(X^2) - E(X) = n(n-1)p^2$

np

$E(X^2) = n(n-1)p^2 + np$

$Var(X) = E(X^2) - [E(X)]^2 = n(n-1)p^2 + np - (np)^2$

$= np[(n-1)p + 1 - np]$

$= np(np - p + 1 - np)$

$= np(1-p)$

故得證 $b(x;n,p) \xrightarrow{n \to \infty} p(x;\mu)$

C^{100}_0 0.0059 C^{100}_1 0.0312 C^{100}_2 0.0812 C^{100}_3 0.1396 C^{100}_4 0.1781 5 0.1800 6 0.15

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