Nombre: Lema del bombeo para lenguajes
Tema: libres de contexto

Día	Mes	Año	Folio
11	10	2023	

1.- Suponer que el lenguaje es $\exists LC$

2.- $\exists n \mid n \in \mathbb{N} \quad |\omega| \geq n \quad \forall \omega \in L$

3.- $\omega \mid \omega = xyzuv \mid x,y,z,u,v \in \Sigma$

$|yzu| \leq n \quad |yu| \geq 1$

4.- $\forall k \mid k \in \mathbb{N} \cup \{\emptyset\}$

$\omega' = x y^k z u^k v \quad \omega' \in L$

$L = \{a^n b^n c^n \mid n \in \mathbb{N}\}$

1.- Suponer que L sí es $\exists LL$

2.- $\exists n \mid n \in \mathbb{N} \quad \omega = a^n b^n c^n \quad |\omega| = 3n \geq n$

3.- $xyzuv = \omega$

Para a)

① $a^n b^n c^n = \omega$

$x = a^n$

$y = b^{n-1}$

$z = \lambda$

$u = \lambda$

$v = bc^n$

$n = n$

$|yzu| \leq n$

$|yu| \geq 1$

$k = 1$

$\omega = a^n b^{n-1} bc^n$

$k = 0$

$\omega' = a^n bc^n \quad x$

② $a^n b^n c^n = \omega$

$x = \lambda$

$y = \lambda$

$z = \lambda$

$u = a^{n-i} \quad i \geq 1 \quad i \leq n$

$v = a^i b^n c^n$

$k = 0$

$\omega' = \lambda \lambda^k \lambda a^{k(a-i)} a^i b^n c^n$

$i \leq n$ X

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③ $a^n b^n c^n = w$ $\quad |yzu| < n$ $\quad |yu| \geq 1$ $\quad k = 0$

$x = \lambda$

$y = \lambda$ $\quad j > 1$ $\quad w' = a^i a^{n-i-j} b^n c^n$

$z = a^i$ $\quad i + j < n$ $\quad a^{n-j} b^n c^n$ X

$u = a^j$

$v = a^{n-i-j} b^n c^n$

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④ $a^n b^n c^n = w$ $\quad k = 0$

$x = \lambda$ $\quad i + l \geq 1$ $\quad w' = \lambda a^{k \cdot i} a^j a^{l} a^{n-i-j-l} b^n c^n$

$y = a^i$ $\quad i + j + l < n$ $\quad w' = a^j a^{n-i-j-l} b^n c^n$

$z = a^j$ $\quad w' = a^{n-i-l} b^n c^n$ X

$u = a^l$ $\quad i + l = 0$

$v = a^{n-i-j-l} b^n c^n$ $\quad i + l \geq 1$

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⑤ $a^n b^n c^n = w$ $\quad k = 0$

$x = a^{n-i-j-l}$ $\quad w' = a^{n-i-j-l} a^{k \cdot i} a^j a^{k \cdot l} b^n c^n$

$y = a^i$ $\quad w' = a^{n-i-j-l} b^n c^n$

$z = a^j$ $\quad w' = a^{n-i-j} b^n c^n$ X

$u = a^l$ $\quad$ Para B)

$v = b^n c^n$

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⑥ $a^n b^n c^n = w$ $\quad k = 0$

$x = a^i$ $\quad w' = a^i a^{jk} a^l (b^n c^n)^k a^{n-i-j-l}$

$y = a^j$ $\quad w' = a^i a^l a^{n-i-j-l}$ X

$z = a^l$

$u = b^n c^n$

$v = a^{n-i-j-l}$

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$\omega = x y^k z\, v^k v$

(7) $a^{\wedge} b^{\wedge} c^{\wedge} = \omega$

$x = a^{\wedge}$
$y = b^{\wedge -1}$
$z = 1$
$v = b$
$v = b^{\wedge -i-j} c^{\wedge}$

$k = 0$

$\omega' = a^{\wedge} (b^{\wedge -1})^k (1) (b^k)\, b^{\wedge -i-j} c^{\wedge}$

$\omega' = a^{\wedge} (b^{\wedge -1})^0\, b^0\, b^{\wedge -i-j} c^{\wedge}$

$\omega' = a^{\wedge} b^{\wedge -i-j} c^{\wedge}$ X

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(8) $a^{\wedge} b^{\wedge} c^{\wedge} = \omega$

$x = a^{\wedge}$
$y = 1$
$z = b^i$
$v = b^j$
$v = b^{\wedge -i-j} c^{\wedge}$

$k = 0$

$\omega' = a^{\wedge} 1\, b^i b^{\wedge k}\, b^{\wedge -i-j} c^{\wedge}$

$\omega' = a^{\wedge} b^i b^{\wedge -i-j} c^{\wedge}$

$\omega' = a^{\wedge} b^{\wedge -j} c^{\wedge}$ X

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(9) $a^{\wedge} b^{\wedge} c^{\wedge} = \omega$

$x = a^{\wedge}$
$y = b^{\wedge -1}$
$z = 1$
$v = b$
$v = c^{\wedge}$

$k = 0$ Para B)

$\omega' = a^{\wedge} (b^{\wedge -1})^k (1) (b^k) (c^{\wedge})$

$\omega' = a^{\wedge} c^{\wedge}$ X

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(10) $a^{\wedge} b^{\wedge} c^{\wedge} = \omega$

$x = a^{\wedge}$
$y = 1$
$z = 1$
$v = b^{\wedge -i-j}$
$v = b^{i+j} c^{\wedge}$

$k = 0$

$\omega' = a^{\wedge} (1)^k (1) (b^{\wedge -i-j})^k (b^{i+j} c^{\wedge})$

$\omega' = a^{\wedge} b^{i+j} c^{\wedge}$ X

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(11) $a^{\wedge} b^{\wedge} c^{\wedge} = w$

$x = a^{\wedge}$

$y = \lambda$

$z = \lambda$

$u = b^{\wedge} - i - j$

$v = b^{\wedge} c^{\wedge}$

$k = 0$

$w' = a^{\wedge}(b^{\wedge} - i - j)^{k} b^{\wedge} c^{\wedge} x$

$w' = a^{\wedge} b^{\wedge} c^{\wedge}$

No cumple

$|y \times z| < \wedge \; |y \cdot u| > L$

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(12) $a^{\wedge} b^{\wedge} c^{\wedge} = w$

$x = a^{\wedge} - i - j - k$

$y = a^{i}$

$z = a^{j}$

$u = a^{k}$

$v = b^{\wedge} c^{\wedge}$

$k = 0$

$w' = a^{\wedge - i - j - k} a^{i} a^{j} a^{k} b^{\wedge} c^{\wedge}$

$w' = a^{\wedge - i - j - k + i + j + k} b^{\wedge} c^{\wedge}$ *

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(13) $a^{\wedge} b^{\wedge} c^{\wedge}$

$x = a^{\wedge}$

$y = b^{\wedge - i - j - k}$

$z = b^{i}$

$u = b^{j}$

$v = b^{k} c^{\wedge}$

$k = 0$

$w' = a^{\wedge} b^{\wedge - i - j - k + i + j + k} b^{i} b^{j} b^{k} b^{\wedge} c^{\wedge}$

$w' = a^{\wedge} b^{i} b^{\wedge} c^{\wedge}$

$w' = a^{\wedge} b^{\wedge + i} c^{\wedge}$ X

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(14) $a^{\wedge} b^{\wedge} c^{\wedge}$

$x = a^{\wedge} b^{\wedge}$

$y = c^{\wedge + 1}$

$z = c^{i}$

$u = \lambda$

$v = \lambda$

Para c)

$k = 0$

$w' = a^{\wedge} b^{\wedge} c^{\wedge - 1 + k} c^{i} (\lambda)(\lambda)$

$w' = a^{\wedge} b^{\wedge} c^{i}$ X

Nombre:

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(15) $a^{\wedge} b^{\wedge} c^{\wedge} = \omega$

$x = a^{\wedge} b^{\wedge}$

$y = c^{i}$

$z = c^{j}$

$u = c^{\wedge - i - j}$

$v = \lambda$

$k = 0$

$\omega' = a^{\wedge} b^{\wedge} c^{(\wedge - i - j)k} (\lambda)$

$\omega' = a^{\wedge} b^{\wedge} c^{j}$ X

(16) $a^{\wedge} b^{\wedge} c^{\wedge} = \omega$

$x = a^{\wedge} b^{\wedge}$

$y = c^{\wedge - i}$

$z = c$

$u = \lambda$

$v = \lambda$

$k = 0$

$\omega' = a^{\wedge} b^{\wedge} c^{(\wedge - i)k} (\lambda)(\lambda)$

$\omega' = a^{\wedge} b^{\wedge} c$ X

(17) $a^{\wedge} b^{\wedge} c^{\wedge} = \omega$

$x = a^{\wedge} b^{\wedge}$

$y = c^{i}$

$z = c^{j}$

$u = c^{l}$

$v = c^{\wedge - l - j - l}$

$k = 0$

$\omega' = a^{\wedge} b^{\wedge} c^{ik} c^{jk} c^{lk} c^{\wedge - i - j - l}$

$\omega' = a^{\wedge} b^{\wedge} c^{j} c^{\wedge - i + j - l}$

$\omega' = a^{\wedge} b^{\wedge} c^{\wedge - i - l}$