Assignment:- 2

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11001
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29 April, 2023

Exemplar 11.16.3.11 The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance, $\Pr(AB) = .07$). Determine

(a) $\Pr(A)$
(b) $\Pr(BC')$
(c) $\Pr(A + B)$
(d) $\Pr(AB')$
(e) $\Pr(BC)$
(f) Probability of exactly one of the three occurs.

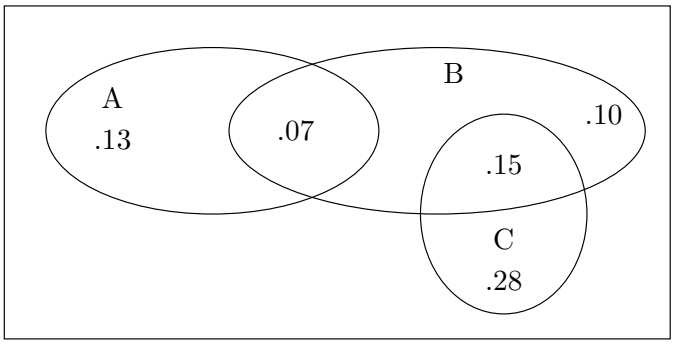

Fig. 0: Question Figure

Solution.

$$\Pr(AB) = 0.07 \tag{1}$$
$$\Pr(AB') = 0.13 \tag{2}$$
$$\Pr(A'B) = 0.25 \tag{3}$$
$$\Pr(BC) = 0.15 \tag{4}$$
$$\Pr(CB') = 0.28 \tag{5}$$
$$\Pr(AB'C') = 0.13 \tag{6}$$
$$\Pr(A'BC') = 0.10 \tag{7}$$
$$\Pr(A'B'C) = 0.28 \tag{8}$$

(a) From (1) and (2)

$$A = A(B + B') = AB + AB' \tag{9}$$
$$[\because B + B' = 1] \tag{10}$$
$$\Pr(A) = \Pr(AB) + \Pr(AB') \tag{11}$$
$$[\because BB' = 0]$$
$$\Pr(A) = 0.13 + 0.07 \tag{12}$$
$$= 0.20 \tag{13}$$

(b) From (1) and (3)

$$\Pr(B) = \Pr(A'B) + \Pr(AB) \tag{14}$$
$$\Pr(B) = 0.25 + 0.07 \tag{15}$$
$$= 0.32 \tag{16}$$

Using (11)

$$\Pr(BC') = \Pr(B) - \Pr(BC) \tag{17}$$
$$= 0.32 - 0.15 \tag{18}$$
$$= 0.17 \tag{19}$$

(c) From Axioms of Probability

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{20}$$
$$= 0.20 + 0.32 - 0.07 \tag{21}$$
$$= 0.45 \tag{22}$$

(d) Using (11)

$$\Pr(AB') = \Pr(A) - \Pr(AB) \tag{23}$$
$$= 0.20 - 0.07 \tag{24}$$
$$= 0.13 \tag{25}$$

(e) From (4)

$$\Pr(BC) = 0.15 \tag{26}$$

(f) Let X be the event that exactly one of A, B or C occur.

Let Y be the event that at least one of A, B or C occur.
Using Boolean logic,

$$Y = A + B + C \tag{27}$$

Let Z be the event that at least two of A, B or C occur.

$$Z = AB + BC + CA \tag{28}$$

From (A.2.5)

$$X = AB'C' + A'B'C' + A'B'C \tag{29}$$

Now, X has been represented as a union of 3 mutually exclusive events.
As any 2 of them has 0 intersection due of presence of complements.
Therefore, by Axioms of Probability

$$\Pr(X) = \Pr(AB'C') + \Pr(A'BC') + \Pr(A'B'C) \tag{30}$$

From (6), (7) and (8)

$$\Pr(X) = 0.13 + 0.10 + 0.28 \tag{31}$$

$$= 0.51 \tag{32}$$