

Assignment:- 4

AI1110: Probability and Random Variables

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CS22BTECH11001

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12.13.6.4 Suppose that 90 % of people are right handed. What is the probability that at most 6 of a random sample of 10 people are right- handed.

Solution. Let X be a Binomial random Variable.

$$X = \text{Bin}(n, p) \quad (1)$$

$$= \text{Bin}(10, 0.9) \quad (2)$$

The mean μ of X ,

$$\mu = n * p \quad (3)$$

$$= 9 \quad (4)$$

The Variance σ^2 of X ,

$$\sigma^2 = n * p * (1 - p) \quad (5)$$

$$= 0.9 \quad (6)$$

Let,

$$Z = \frac{X - \mu}{\sigma} \quad (7)$$

Now, Z is a random variable with $\mu = 0$ and $\sigma^2 = 1$.

We can calculate the distribution of Z by assuming it be a set of discrete points on the Normal-Distribution.

Note:-The CDF of Z will converge to the normal distribution for large values of n .

The Normal-Distribution,

$$f(x) = \frac{1}{\sqrt{2\pi}} * e^{-\frac{x^2}{2}} \quad (8)$$

The CDF from the Normal-Distribution

$$F_Z(x) = \int_{-\infty}^{x+0.5} \frac{1}{\sqrt{2\pi}} * e^{-\frac{t^2}{2}} dt \quad (9)$$

Note:- The additional 0.5 correction term is present.

We want

$$X \leq 6 \quad (10)$$

$$\therefore Z \leq \frac{6 - \mu}{\sigma} \quad (11)$$

$$Z \leq -3.16 \quad (12)$$

$$F_Z(-3.16) = \int_{-\infty}^{-2.66} \frac{1}{\sqrt{2\pi}} * e^{-\frac{t^2}{2}} dt \quad (13)$$

On Computation,

$$F_Z(-3.16) = 0.0042 \quad (14)$$