Assignment:- 3

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11001
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12.13.5.12 Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Solution. Let us define X to be the number of times six appears on the dice,

Such that,

$$\Pr(X = i | X = i) = \frac{5}{6} \tag{1}$$

$$\Pr(X = i + 1 | X = i) = \frac{1}{6} \tag{2}$$

Therefore, the process is a Markov process in which the ith state refers to six appearing i times.

Transition Probabilities

$$P_{i,i+1} = \frac{1}{6} \tag{3}$$

$$P_{i,i} = \frac{5}{6} \tag{4}$$

For 6 throws,

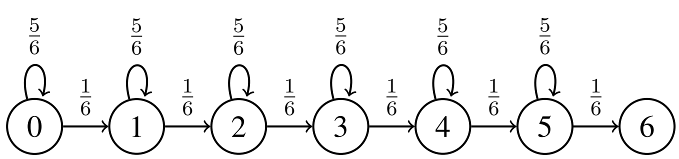

Fig. 0: Transition Graph

The Transition Matrix of this chain is given by,

$$A = \begin{bmatrix} \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{6} \end{bmatrix} \tag{5}$$

This matrix represents the probabilities after one throw,

To calculate probabilities after 6 throws, we need to raise this matrix to the power 6

[This is analogous to adjacency matrix in Graph Theory]

On computation,

$$A^6 = \begin{bmatrix} 0.335 & 0.402 & 0.201 & 0.054 & 0.008 & 0.001 & 0.0 \\ 0 & 0.335 & 0.402 & 0.201 & 0.054 & 0.008 & 0.001 \\ 0 & 0 & 0.335 & 0.402 & 0.201 & 0.054 & 0.008 \\ 0 & 0 & 0 & 0.335 & 0.402 & 0.201 & 0.054 \\ 0 & 0 & 0 & 0 & 0.335 & 0.402 & 0.201 \\ 0 & 0 & 0 & 0 & 0 & 0.335 & 0.402 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.335 \end{bmatrix} \tag{6}$$

Now, this is our transition matrix for 6 rolls of the dice.

Therefore, $P_{i,j}$ represents starting from ith state and ending up at jth state given 6 throws of dice.

For this question, we always start from $X = 0$ and are required to end up with at-most 2 sixes or $X \leq 2$.

$$P_{0,0} = 0.335 \tag{7}$$

$$P_{0,1} = 0.402 \tag{8}$$

$$P_{0,2} = 0.201 \tag{9}$$

From (7), (8) and (9)

$$\begin{aligned} \Pr(X \leq 2) &= P_{0,0} + P_{0,1} + P_{0,2} && (10) \\ &= 0.335 + 0.402 + 0.201 && (11) \\ &= 0.938 && (12) \end{aligned}$$