Assignment:- 2

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11001
Aayush Adlakha
29 April, 2023

Exemplar 11.16.3.11 The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance, $\Pr(AB) = .07$). Determine

(a) $\Pr(A)$
(b) $\Pr(BC')$
(c) $\Pr(A+B)$
(d) $\Pr(AB')$
(e) $\Pr(BC)$
(f) Probability of exactly one of the three occurs.

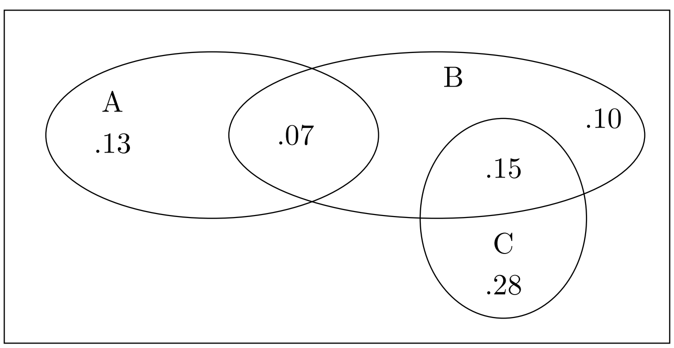

Fig. 0: Question Figure

Solution. Given that,

$$\Pr(AB) = 0.07 \tag{1}$$
$$\Pr(AB') = 0.13 \tag{2}$$
$$\Pr(BC) = 0.15 \tag{3}$$
$$\Pr(CB') = 0.28 \tag{4}$$
$$\Pr(AB'C') = 0.13 \tag{5}$$
$$\Pr(A'BC') = 0.10 \tag{6}$$
$$\Pr(A'B'C) = 0.28 \tag{7}$$

(a) From (1) and (2)

$$\Pr(A) = 0.13 + 0.07 \tag{8}$$
$$= 0.20 \tag{9}$$

(b) From (1), (3) and (6)

$$\Pr(B) = 0.10 + 0.07 + 0.15 \tag{10}$$
$$= 0.32 \tag{11}$$

Also,

$$A = A(B + B') = AB + AB' \tag{12}$$
$$[\because B + B' = 1] \tag{13}$$
$$\Pr(A) = \Pr(AB) + \Pr(AB') \tag{14}$$
$$[\because BB' = 0]$$

Using (14)

$$\Pr(BC') = \Pr(B) - \Pr(BC) \tag{15}$$
$$= 0.32 - 0.15 \tag{16}$$
$$= 0.17 \tag{17}$$

(c) From Axioms of Probability

$$\Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{18}$$
$$= 0.20 + 0.32 - 0.07 \tag{19}$$
$$= 0.45 \tag{20}$$

(d) Using (14)

$$\Pr(AB') = \Pr(A) - \Pr(AB) \tag{21}$$
$$= 0.20 - 0.07 \tag{22}$$
$$= 0.13 \tag{23}$$

(e) Clearly,

$$\Pr(BC) = 0.15 \tag{24}$$

(f) Let X be the event that exactly one of A, B or C occur.
Let Y be the event that at least one of A, B or C occur.
Using Boolean logic,

$$Y = A + B + C \tag{25}$$

Let Z be the event that at least two of A, B or C occur.

$$Z = AB + BC + CA \tag{26}$$

From (A.2.5)

$$X = AB'C' + A'B'C' + A'B'C \tag{27}$$

Now, X has been represented as a union of 3 mutually exclusive events.
As any 2 of them has 0 intersection due of presence of complements.
Therefore, by Axioms of Probability

$$\Pr(X) = \Pr(AB'C') + \Pr(A'BC') + \Pr(A'B'C) \tag{28}$$

From (5), (6) and (7)

$$\Pr(X) = 0.13 + 0.10 + 0.28 \tag{29}$$
$$= 0.51 \tag{30}$$