

# Assignment:- 2

## AI1110: Probability and Random Variables

### Indian Institute of Technology, Hyderabad

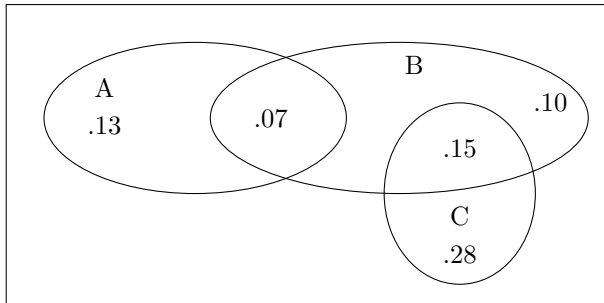
CS22BTECH11001

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**Exemplar 11.16.3.11** The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance,  $\Pr(AB) = .07$ ). Determine

- $\Pr(A)$
- $\Pr(BC')$
- $\Pr(A + B)$
- $\Pr(AB')$
- $\Pr(BC)$
- Probability of exactly one of the three occurs.



**Solution.**

- (a) Clearly,

$$\Pr(A) = 0.13 + 0.07 \quad (1)$$

$$= 0.20 \quad (2)$$

- (b) Clearly,

$$\Pr(B) = 0.10 + 0.07 + 0.15 \quad (3)$$

$$= 0.32 \quad (4)$$

Also,

$$A = A(B + B') = AB + AB' \quad (5)$$

$$[\because B + B' = 1] \quad (6)$$

$$\Pr(A) = \Pr(AB) + \Pr(AB') \quad (7)$$

$$[\because BB' = 0]$$

Using (7)

$$\Pr(BC') = \Pr(B) - \Pr(BC) \quad (8)$$

$$= 0.32 - 0.15 \quad (9)$$

$$= 0.17 \quad (10)$$

- (c) From Axioms of Probability

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \quad (11)$$

$$= 0.20 + 0.32 - 0.07 \quad (12)$$

$$= 0.45 \quad (13)$$

- (d) Using (7)

$$\Pr(AB') = \Pr(A) - \Pr(AB) \quad (14)$$

$$= 0.20 - 0.07 \quad (15)$$

$$= 0.13 \quad (16)$$

- (e) Clearly,

$$\Pr(BC) = 0.15 \quad (17)$$

- (f) Let X be the event that exactly one of A, B or C occur.

Let Y be the event that at least one of A, B or C occur.

Using Boolean logic,

$$Y = A + B + C \quad (18)$$

Let Z be the event that at least two of A, B or C occur.

Writing down Z as at least one of AB, BC or AC occurring gives us.

$$Z = AB + BC + CA \quad (19)$$

We know that, all three events never occur simultaneously.

Therefore,

Z represents occurrence of exactly 2 of A, B and C.

X represents occurrence of exactly 1 of A,B and C.

X can be thought of as occurrence of at least 1 but not 2 events

$$X = YZ' \quad (20)$$

$$X = (A + B + C)(AB + BC + CA)' \quad (21)$$

From (A.2.5)

$$X = AB'C' + A'B'C' + A'B'C \quad (22)$$

Now, X has been represented as a union of 3 mutually exclusive events.

As any 2 of them has 0 intersection due of presence of complements.

Therefore, by Axioms of Probability

$$\Pr(X) = \Pr(AB'C') + \Pr(A'B'C') + \Pr(A'B'C) \quad (23)$$

Clearly, from the figure

$$\Pr(X) = 0.13 + 0.10 + 0.28 \quad (24)$$

$$= 0.51 \quad (25)$$