Assignment:- 2

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11001
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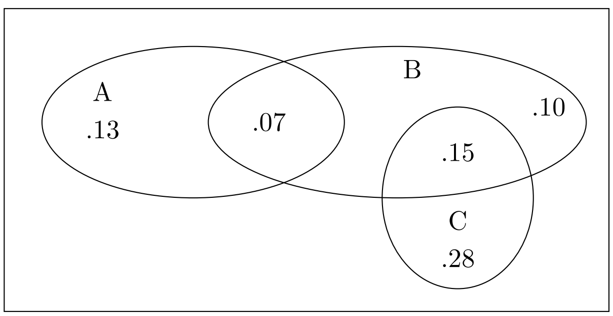
Exemplar 11.16.3.11 The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance, $\Pr(AB) = .07$). Determine

(a) $\Pr(A)$
(b) $\Pr(BC')$
(c) $\Pr(A+B)$
(d) $\Pr(AB')$
(e) $\Pr(BC)$
(f) Probability of exactly one of the three occurs.



Solution.

(a) Clearly,

$$\Pr(A) = 0.13 + 0.07 \tag{1}$$
$$= 0.20 \tag{2}$$

(b) Clearly,

$$\Pr(B) = 0.10 + 0.07 + 0.15 \tag{3}$$
$$= 0.32 \tag{4}$$

Also,

$$A = A(B + B') = AB + AB' \tag{5}$$
$$[\because B + B' = 1] \tag{6}$$
$$\Pr(A) = \Pr(AB) + \Pr(AB') \tag{7}$$
$$[\because BB' = 0]$$

Using (7)

$$\Pr(BC') = \Pr(B) - \Pr(BC) \tag{8}$$
$$= 0.32 - 0.15 \tag{9}$$
$$= 0.17 \tag{10}$$

(c) From Axioms of Probability

$$\Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{11}$$
$$= 0.20 + 0.32 - 0.07 \tag{12}$$
$$= 0.45 \tag{13}$$

(d) Using (7)

$$\Pr(AB') = \Pr(A) - \Pr(AB) \tag{14}$$
$$= 0.20 - 0.07 \tag{15}$$
$$= 0.13 \tag{16}$$

(e) Clearly,

$$\Pr(BC) = 0.15 \tag{17}$$

(f) Let X be the event that exactly one of A, B or C occur.
Let Y be the event that at least one of A, B or C occur.
Let Z be the event that at least two of A, B or C occur.
Using (11)

$$\Pr((A+B)+C) = \Pr(A+B) + \Pr(C) - \Pr(C(A+B)) \tag{18}$$
$$\Pr(Y) = \Pr(A+B) + \Pr(C) - \Pr(AC+BC) \tag{19}$$
$$\tag{20}$$

Applying (11) again,

$$\Pr(Y) = \Pr(A) + \Pr(B) + \Pr(C) - \Pr(AB) - \Pr(BC) - \Pr(AC) + \Pr(ACBC) \quad (21)$$

$$\Pr(Y) = \Pr(A) + \Pr(B) + \Pr(C) - \Pr(AB) - \Pr(BC) - \Pr(AC) + \Pr(ABC) \quad (22)$$

$$[\because CC = C] \quad (23)$$

$$\Pr(Y) = \Pr(A) + \Pr(B) + \Pr(C) - \Pr(AB) - \Pr(BC) - \Pr(AC) \quad (24)$$

$$[\because ABC = 0] \quad (25)$$

$$= 0.20 + 0.32 + 0.43 - 0.07 - 0.15 \quad (26)$$

$$= 0.73 \quad (27)$$

Applying (22)

$$\Pr(Z) = \Pr(AB) + \Pr(BC) + \Pr(AC) - \Pr(ABBC) - \Pr(BCAC) - \Pr(ACAB) + \Pr(ABBCAC) \quad (28)$$

$$\Pr(Z) = \Pr(AB) + \Pr(BC) + \Pr(AC) - 2\Pr(ABC) \quad (29)$$

$$= \Pr(AB) + \Pr(BC) + \Pr(AC) \quad (30)$$

$$[\because ABC = 0] \quad (31)$$

$$= 0.07 + 0.15 \quad (32)$$

$$= 0.22 \quad (33)$$

We know that, all three events never occur simultaneously.
Therefore,
Z represents occurrence of exactly 2 of A,B and C.
X represents occurrence of exactly 1 of A,B and C.
Y can therefore be written as,

$$\Pr(Y) = \Pr(X) + \Pr(Z) \quad (34)$$

Upon rearranging

$$\Pr(X) = \Pr(Y) - \Pr(Z) \quad (35)$$

$$= 0.73 - 0.22 \quad (36)$$

$$= 0.51 \quad (37)$$