Assignment:- 4

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11001
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12.13.6.4 Suppose that 90 % of people are right handed. What is the probability that at most 6 of a random sample of 10 people are right- handed.

Solution. Let X be a Binomial random Variable.

$$X = Bin(n, p) \tag{1}$$
$$= Bin(10, 0.9) \tag{2}$$

The mean μ of X,

$$\mu = n \times p \tag{3}$$
$$= 9 \tag{4}$$

The Variance σ^2 of X,

$$\sigma^2 = n \times p \times (1 - p) \tag{5}$$
$$= 0.9 \tag{6}$$

Let,

$$Z = \frac{X - \mu}{\sigma} \tag{7}$$

Now, Z is a random variable with $\mu = 0$ and $\sigma^2 = 1$.

We can calculate the distribution of Z by assuming it be a set of discrete points on the Normal-Distribution.

Note:-The CDF of Z will converge to the normal distribution for large values of n.

The Normal-Distribution,

$$f(x) = \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} \tag{8}$$

The CDF from the Normal-Distribution

$$F_Z(x) = \int_{-\infty}^{x+0.5} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{t^2}{2}}\, dt \tag{9}$$

Note:- The additional 0.5 correction term is present. We want

$$X \leq 6 \tag{10}$$
$$\therefore Z \leq \frac{6 - \mu}{\sigma} \tag{11}$$
$$Z \leq -3.16 \tag{12}$$

$$F_Z(-3.16) = \int_{-\infty}^{-2.66} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{t^2}{2}}\, dt \tag{13}$$

On Computation,

$$F_Z(-3.16) = 0.0042 \tag{14}$$