

Assignment:- 2

AI1110: Probability and Random Variables

Indian Institute of Technology, Hyderabad

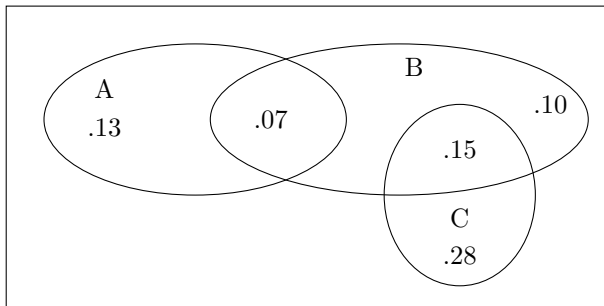
CS22BTECH11001

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Exexplar 11.16.3.11 The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance, $\Pr(AB) = .07$). Determine

- $\Pr(A)$
- $\Pr(BC')$
- $\Pr(A + B)$
- $\Pr(AB')$
- $\Pr(BC)$
- Probability of exactly one of the three occurs.



Solution.

(a) Clearly,

$$\Pr(A) = 0.13 + 0.07 \quad (1)$$

$$= 0.20 \quad (2)$$

(b) Clearly,

$$\Pr(B) = 0.10 + 0.07 + 0.15 \quad (3)$$

$$= 0.32 \quad (4)$$

Also,

$$B = B(C + C') \quad (5)$$

$$= BC + BC' \quad (6)$$

$$\Pr(BC') = \Pr(B) - \Pr(BC) \quad (7)$$

$$= 0.32 - 0.15 \quad (8)$$

$$= 0.17 \quad (9)$$

(c) From Axioms of Probability

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \quad (10)$$

$$= 0.20 + 0.32 - 0.07 \quad (11)$$

$$= 0.45 \quad (12)$$

(d)

$$\Pr(AB') = \Pr(A) - \Pr(AB) \quad (13)$$

$$= 0.20 - 0.07 \quad (14)$$

$$= 0.13 \quad (15)$$

(e) Clearly,

$$\Pr(BC) = 0.15 \quad (16)$$

(f) Let X be the event that exactly one of A, B and C occur.

Let Y be the event that atleast one of A, B and C occur.

Let Z be the event that exactly two of A, B and C occur.

By Principle of Inclusion and Exclusion

$$\Pr(Y) = \Pr(A) + \Pr(B) + \Pr(C) \quad (17)$$

$$- \Pr(AB) - \Pr(BC) - \Pr(AC)$$

$$[\because \Pr(ABC) = 0] \quad (18)$$

$$= 0.20 + 0.32 + 0.43 - 0.07 - 0.15 \quad (19)$$

$$= 0.73 \quad (20)$$

$$\Pr(Z) = \Pr(AB) + \Pr(BC) + \Pr(AC) \quad (21)$$

$$[\because \Pr(ABC) = 0] \quad (22)$$

$$= 0.07 + 0.15 \quad (23)$$

$$= 0.22 \quad (24)$$

Now,

$$\Pr(X) = \Pr(Y) - \Pr(Z) \quad (25)$$

$$= 0.73 - 0.22 \quad (26)$$

$$= 0.51 \quad (27)$$