Assignment:- 2

AI1110: Probability and Random Variables
Indian Institute of Technology, Hyderabad

CS22BTECH11001
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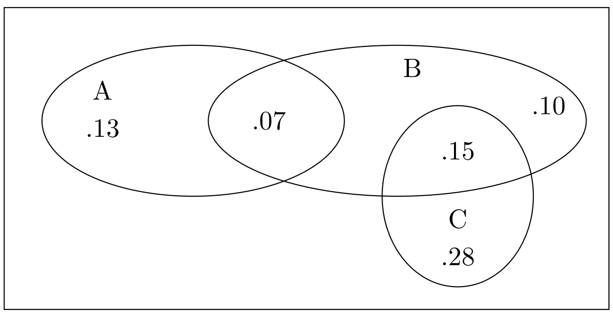
Exemplar 11.16.3.11 The accompanying Venn diagram shows three events, A, B, and C, and also the probabilities of the various intersections (for instance, $\Pr(AB) = .07$). Determine

(a) $\Pr(A)$
(b) $\Pr(BC')$
(c) $\Pr(A+B)$
(d) $\Pr(AB')$
(e) $\Pr(BC)$
(f) Probability of exactly one of the three occurs.



Solution.

(a) Clearly,

$$\Pr(A) = 0.13 + 0.07 \tag{1}$$
$$= 0.20 \tag{2}$$

(b) Clearly,

$$\Pr(B) = 0.10 + 0.07 + 0.15 \tag{3}$$
$$= 0.32 \tag{4}$$

Also,

$$A = A(B + B') = AB + AB' \tag{5}$$
$$[\because B + B' = 1] \tag{6}$$
$$\Pr(A) = \Pr(AB) + \Pr(AB') \tag{7}$$
$$[\because BB' = 0]$$

Using (7)

$$\Pr(BC') = \Pr(B) - \Pr(BC) \tag{8}$$
$$= 0.32 - 0.15 \tag{9}$$
$$= 0.17 \tag{10}$$

(c) From Axioms of Probability

$$\Pr(A+B) = \Pr(A) + \Pr(B) - \Pr(AB) \tag{11}$$
$$= 0.20 + 0.32 - 0.07 \tag{12}$$
$$= 0.45 \tag{13}$$

(d) Using (7)

$$\Pr(AB') = \Pr(A) - \Pr(AB) \tag{14}$$
$$= 0.20 - 0.07 \tag{15}$$
$$= 0.13 \tag{16}$$

(e) Clearly,

$$\Pr(BC) = 0.15 \tag{17}$$

(f) Let X be the event that exactly one of A, B or C occur.
Let Y be the event that at least one of A, B or C occur.
Using Boolean logic,

$$Y = A + B + C \tag{18}$$

Let Z be the event that at least two of A, B or C occur.
Writing down Z as at least one of AB, BC or AC occurring gives us.

$$Z = AB + BC + CA \tag{19}$$

We know that, all three events never occur simultaneously.
Therefore,
Z represents occurrence of exactly 2 of A,B and C.

X represents occurrence of exactly 1 of A,B and C.
Y can therefore be written as,

$$Y = X + Z \tag{20}$$
$$YZ' = (X + Z)Z' \tag{21}$$
$$YZ' = XZ' + ZZ' \tag{22}$$
$$YZ' = XZ' \tag{23}$$

Z' can be thought of as either none of the events occurring or exactly one of them occurring
XZ' would therefore be event that exactly one occurs.

$$XZ' = X \tag{24}$$
$$X = YZ' \tag{25}$$
$$= (A + B + C)(AB + BC + CA)' \tag{26}$$
$$= (A + B + C)(AB)'(BC)'(CA)' \tag{27}$$
$$= (A + B + C)(A' + B')(B' + C')(A' + C') \tag{28}$$
$$= (AA' + BA' + CA' + AB' + BB' + CB')(B'A' + C'A' + B'C' + C'C') \tag{29}$$
$$= (BA' + CA' + AB' + CB')(A'B' + A'C' + B'C') \tag{30}$$
$$\begin{aligned} &= (BA'A'B' + BA'A'C' + BA'B'C' \\ &+ CA'A'B' + BA'A'C' + CA'B'C' \\ &+ AB'A'B' + AB'A'C' + AB'B'C' \\ &+ CB'A'B' + CB'A'C' + CB'B'C') \\ &= (A'BC' + A'B'C + AB'C') \end{aligned} \tag{31}$$

Now, X has been represented as a union of 3 mutually exclusive events.
As any 2 of them has 0 intersection due of presence of complements.
Therefore, by Axioms of Probability

$$\Pr(X) = \Pr(AB'C') + \Pr(A'BC') + \Pr(A'B'C) \tag{32}$$

Clearly, from the figure

$$\Pr(X) = 0.13 + 0.10 + 0.28 \tag{33}$$
$$= 0.51 \tag{34}$$