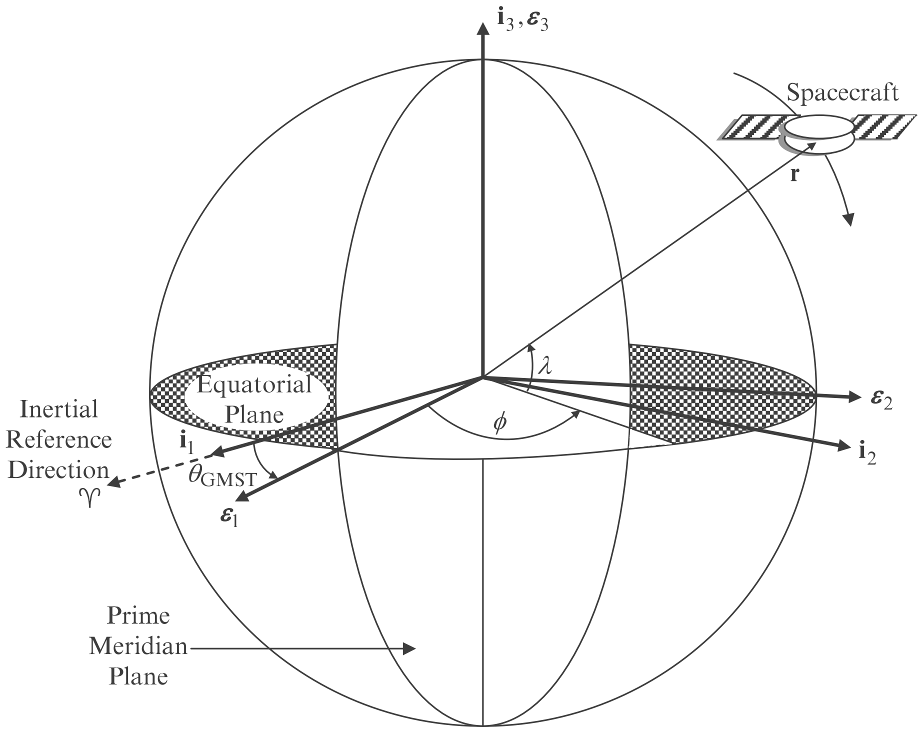

Fig. 2.3 Definitions of various reference frames

Determining the GMST angle requires the Julian date, JD. For a given year Y (between 1901 and 2099), month M, day D, hour h, minute m, and second s, the Julian date is calculated by [24]

$$JD(Y, M, D, h, m, s) = 1,721,013.5 + 367\,Y - \text{INT}\left\{\frac{7}{4}\left[Y + \text{INT}\left(\frac{M+9}{12}\right)\right]\right\} + \text{INT}\left(\frac{275\,M}{9}\right) + D + \frac{60\,h + m + s/60^*}{1440} \tag{2.68}$$

where INT denotes the integer part and 60* denotes using 61 s for days with a leap second. We need to compute T_0, the number of Julian centuries elapsed from the epoch J2000.0 to zero hours of the date in question:

$$T_0 = \frac{JD(Y, M, D, 0, 0, 0) - 2,451,545}{36,525} \tag{2.69}$$

The GCI coordinate system is fixed relative to the stars, not the Sun, so the GMST angle is the mean *sidereal* time at zero longitude. A *sidereal day* is the length of time that passes between successive crossings of a given projected meridian by a given fixed star in the sky. It is approximately 3 min and 56 s shorter than a *solar*

$1{,}000 \leq h < 1{,}500$ km

$$
\begin{aligned}
H &= (h - 1{,}000)/500 \\
F_{1,500} &= D_1 + D_2 \bar{F}_{10} + 1{,}500 D_3 + 1{,}500 D_4 \bar{F}_{10} \\
\frac{\partial F_{1,500}}{\partial h} &= D_3 + D_4 \bar{F}_{10} \\
F_{HA} &= 1 + \left(3F_{1,500} - 500 \frac{\partial F_{1,500}}{\partial h} - 3 \right) H^2 + \left(500 \frac{\partial F_{1,500}}{\partial h} - 2F_{1,500} + 2 \right) H^3
\end{aligned}
\tag{11.45}
$$

$1{,}500 \leq h$ km

$$
F_{HA} = D_1 + D_2 \bar{F}_{10} + D_3 h + D_4 h \bar{F}_{10} \tag{11.46}
$$

where the coefficients are given by

$$
\begin{aligned}
D_1 &= 2.2 \times 10^{-1}, \quad & D_2 &= -2.0 \times 10^{-3} \\
D_3 &= 1.15 \times 10^{-3}, \quad & D_4 &= -2.11 \times 10^{-6}
\end{aligned}
$$

Once the high altitude correction factor, and corrected density have been determined, the final atmospheric density is given by

$$
\rho = F_{HA} \rho_{\text{corr}} \tag{11.47}
$$

11.3 Sun Position, Radiation Pressure, and Eclipse Conditions

In order determine solar radiation pressure forces or torques acting on a spacecraft or to process Sun sensor data, we must first determine where the Sun is relative to the spacecraft and whether the spacecraft is shadowed by the Earth or the Moon. The position of the Sun with respect to the Earth can be determined as follows [17]. First, the mean longitude, $\phi_{\odot}$, and mean anomaly of the Sun, $M_{\odot}$, are determined in degrees as

$$
\phi_{\odot} = 280.460^{\circ} + 36{,}000.771 \; T_{UT1} \tag{11.48a}
$$

$$
M_{\odot} = 357.5277233^{\circ} + 35999.05034 \; T_{UT1} \tag{11.48b}
$$

where

$$
T_{UT1} = \frac{JD(Y, M, D, h, m, s) - 2{,}451{,}545}{36{,}525} \tag{11.49}
$$

with JD computed as in Sect. 2.6.3. Both $\phi_\odot$ and $M_\odot$ are reduced to the range 0° to 360° and the longitude of the ecliptic is determined in degrees as

$$\phi_{\text{ecliptic}} = \phi_\odot + 1.914666471^\circ \sin(M_\odot) + 0.019994643 \sin(2M_\odot) \tag{11.50}$$

The obliquity of the ecliptic is given by

$$\varepsilon = 23.439291^\circ - 0.0130042\, T_{UT1} \tag{11.51}$$

The unit vector in the direction from the Earth to the Sun is then

$$\mathbf{e}_{\oplus\odot} = \begin{bmatrix} \cos(\phi_{\text{ecliptic}}) \\ \cos(\varepsilon)\sin(\phi_{\text{ecliptic}}) \\ \sin(\varepsilon)\sin(\phi_{\text{ecliptic}}) \end{bmatrix} \tag{11.52}$$

We omit subscripts to indicate the coordinate frame because all vectors in this section are expressed in the GCI frame.

The distance, in AU, between the Earth and the Sun can be found by

$$r_{\oplus\odot} = 1.000140612 - 0.016708617\cos(M_\odot) - 0.000139589\cos(2M_\odot) \tag{11.53}$$

and the position vector from the Earth to the Sun is $\mathbf{r}_{\oplus\odot} = r_{\oplus\odot}\mathbf{e}_{\oplus\odot}$. After converting the spacecraft position vector $\mathbf{r}$ to AU, the position vector from the spacecraft to the Sun, expressed in the GCI frame is given

$$\mathbf{r}_{\text{sat}\odot} = \mathbf{r}_{\oplus\odot} - \mathbf{r} \tag{11.54}$$

The distance $r_{\text{sat}\odot}$, between the spacecraft and Sun (in AU) and the unit vector $\mathbf{e}_{\text{sat}\odot}$ are then given by

$$r_{\text{sat}\odot} = \|\mathbf{r}_{\text{sat}\odot}\| \tag{11.55a}$$

$$\mathbf{e}_{\text{sat}\odot} = \frac{\mathbf{r}_{\text{sat}\odot}}{r_{\text{sat}\odot}} \tag{11.55b}$$

The pressure of solar radiation at the position of the spacecraft is then

$$P_\odot = \frac{\mathscr{F}_\odot}{c\, r_{\text{sat}\odot}^2} \tag{11.56}$$

where $\mathscr{F}_\odot$, known as the *solar constant*, is the flux density of solar radiation at a distance of 1 AU from the Sun, and $c = 299{,}792{,}458$ m/s is the speed of light. The solar constant varies over the 11-year solar cycle from 1,361 W/m^2 at solar minimum to 1,363 W/m^2 at solar maximum and is subject to rapid fluctuations as large as 5 W/m^2 at times of high or low solar activity.[3] These fluctuations are very difficult to predict, although daily data does exist [5].

[3]These recent measurements [11] are about 5 W/m^2 lower than previous measurements [5].

Given the position vector of the spacecraft and the position vector of the Sun, it can now be determined whether or not the spacecraft is in the shadow of the Earth. There are two main approaches to shadowing. The first assumes that the shadow created by the Earth is a cylindrical projection of the Earth's diameter along the direction of the Sun to the Earth. In the cylindrical approximation, which is generally adequate for spacecraft in lower altitude orbits, the spacecraft is in the Earth's shadow if and only if

$$\mathbf{r} \cdot \mathbf{e}_{\oplus\odot} < -\sqrt{r^2 - R_\oplus^2} \tag{11.57}$$

where $\mathbf{r}$ is the spacecraft position vector and $R_\oplus$ is the equatorial radius of the Earth, which is approximated as a sphere. For a spacecraft in a geosynchronous orbit with radius 42,164 km, Eq. (11.57) gives an eclipse time of 69.6 min for the longest eclipse, which occurs at the equinoxes when the Sun is in the orbital plane of the spacecraft.

The second shadowing approach accounts for the finite diameters of both the Sun and the Earth. This more accurate approach has a conical shadow model with partial shadowing in a region called the penumbra. The more complex eclipse conditions for the conical shadow model, including illumination levels in the penumbra, can be found in Wertz [19]. His equations show that a spacecraft in a geostationary orbit spends 67.5 min in total shadow and 4.3 min in the penumbra during its longest eclipse.

11.4 Orbital Ephemerides of the Sun, Moon, and Planets

The best source for accurate orbital ephemerides of the Sun, Moon, and planets is DE405/LE405 computed by the Jet Propulsion Laboratory (JPL) of the California Institute of Technology [16]. These ephemerides are obtained by precise numerical integration of the equations of motion of the bodies. This computation has four main ingredients: formulating the equations of motion, determining the initial conditions, performing the integrations, and making the results available in a useful form. It is believed that the difficulty of accurately determining the initial conditions is the limiting factor in the accuracy of the solutions.

The equations of motion are described in detail in [16]. They include "(a) point-mass interactions among the Moon, planets, and Sun; (b) general relativity (isotropic, parameterized post-Newtonian); (c) Newtonian perturbations of selected asteroids; (d) action upon the figure of the Earth from the Moon and Sun; (e) action upon the figure of the Moon from the Earth and Sun; (f) physical libration of the Moon, modeled as a solid body with tidal and rotational distortion, including both elastic and dissipational effects; (g) the effect upon the Moon's motion caused by the tides raised upon the Earth by the Moon and Sun; and (h) the perturbations of 300 asteroids upon the motions of Mars, the Earth, and the Moon." The reference