5.1.1 Application: Sun Position Vector

We often want to find the position vector from the Earth to the Sun for uses such as determining solar-panel illumination and remote sensing. Very precise ephemerides of the Sun with respect to the Earth are available through the Jet Propulsion Laboratory (see Appendix D); however, it's often convenient to place a mathematical algorithm inside the particular program using a less precise technique. Some algorithms require us to consider which equator and equinox are needed (e.g. true-of-date, mean-of-date). Meeus (1991, 154–164) shows an analytical method which yields modest accuracy. He discusses how to select and calculate these vectors. We'll discuss a simpler technique from the *Astronomical Almanac* (1992, C24). This latter technique results in a J2000 (mean-equator, mean equinox) vector with an accuracy of 0.01°. It's valid from 1950 to 2050 because of the truncation of the expansions. Figure 5-1 shows the geometry necessary to visualize the problem.

The solution rests in part on expressions using the J2000 epoch. First, find the number of Julian centuries, T_{UT1}, from the epoch, using Eq. (3-40). Find the mean longitude of the Sun, $\lambda_{M_\odot}$, from (Seidelmann, 1992, 485).

$$\lambda_{M_\odot} = 280.460\,° + 36{,}000.770\ T_{UT1}$$

Find the mean anomaly (see Sec. 2.2) for the Sun, $M_\odot$, from Eq. (3-51). We show T_{TDB} to be precise, but it's acceptable to use T_{UT1} because this is only a low-precision formula.

$$M_\odot = 357.527\,723\,3° + 35{,}999.050\,34 T_{TDB}$$

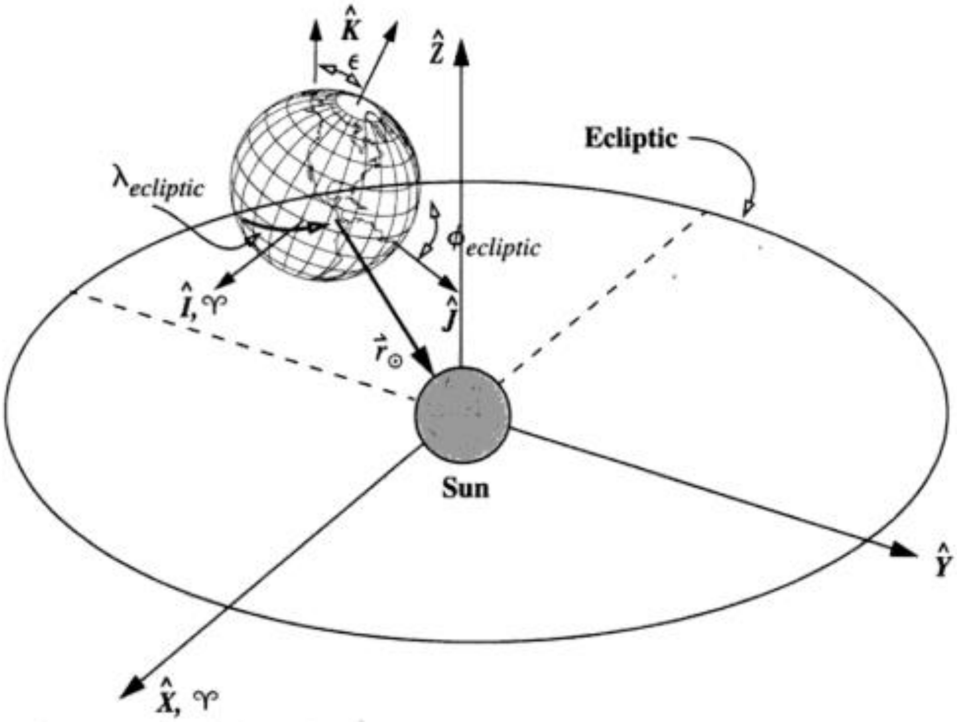

Figure 5-1. Geometry for the Sun Position Vector. The solution of the position vector to the Sun rests on determining the ecliptic longitude (ecliptic latitude is 0°) and the range. The *X* and *I* axes are parallel.

Reduce both $\lambda_{M_\odot}$ and $M_\odot$ to the range of 0° to 360°. Approximate the ecliptic longitude ($\lambda_{ecliptic} \approx \nu_\odot$) by applying the equation of center [see Eq. (2-51)]:

$$\nu_\odot = M_\odot + 2e\,\mathrm{SIN}(M_\odot) + \frac{5e^2}{4}\mathrm{SIN}(2M_\odot) + \ldots$$

where we use the mean eccentricity of the Earth's orbit ($e_e = 0.016\,708\,617$) around the Sun. Don't confuse this term with the eccentricity used to determine site locations, $e_\oplus$. Meeus (1991, 151) shows a more accurate expression for the eccentricity of the Earth's orbit:

$$e_e = 0.016\,708\,617 - 0.000\,042\,037T_{TDB} - 0.000\,000\,123\,6T_{TDB}^2$$

This algorithm does *not* use this variable formula for eccentricity because the additional terms are so small. Also, because the Earth's orbit is approximately circular, we can assume the true anomaly is close enough to the longitude for the following expression to hold (remember to convert to units of degrees!). The ecliptic latitude of the Sun never exceeds 0.000 333° and is often set to 0.0°.

$$\lambda_{ecliptic} = \lambda_{M_\odot} + 1.914\,666\,471°\,\mathrm{SIN}(M_\odot) + 0.019\,994\,643\,\mathrm{SIN}(2M_\odot)$$
$$\phi_{ecliptic} = 0°$$

These expressions have several applications. The trigonometric terms in the ecliptic longitude expression come from the equation of time, introduced in Chap. 3 [Eq. (3-37)].

We can approximate the obliquity of the ecliptic using only the first two terms in Eq. (3-53) and letting $T_{TDB} \approx T_{UT1}$. Thus,

$$\epsilon = 23.439\,291° - 0.013\,004\,2T_{TDB}$$

Find the distance in AU from the Earth to the Sun using an expansion of elliptic motion (Taff, 1985, 60, or Brouwer and Clemence, 1961, 76):

$$\begin{aligned} r = a_e\Bigg[& 1 + \frac{e_e^2}{2} + \left\{-e_e + \frac{3e_e^3}{8} - \frac{5e_e^5}{192} + \frac{7e_e^7}{9216} + \ldots\right\}\mathrm{COS}(M) \\ & + \left\{-\frac{e_e^2}{2} + \frac{e_e^4}{3} - \frac{e_e^6}{16} + \ldots\right\}\mathrm{COS}(2M) + \left\{-\frac{3e_e^3}{8} + \frac{45e_e^5}{128} - \frac{567e_e^7}{5120} + \ldots\right\}\mathrm{COS}(3M) \\ & + \left\{-\frac{e_e^4}{3} + \frac{2e_e^6}{5} + \ldots\right\}\mathrm{COS}(4M) + \left\{-\frac{125e_e^5}{384} + \frac{4375e_e^7}{9216} + \ldots\right\}\mathrm{COS}(5M) \\ & + \left\{-\frac{27e_e^6}{80} + \ldots\right\}\mathrm{COS}(6M) + \left\{-\frac{16087e_e^7}{46{,}080} + \ldots\right\}\mathrm{COS}(7M)\Bigg] \end{aligned}$$

Find the position magnitude using values of the Earth from Table D-3 (a_e = 1.000 001 001 78 AU and e_e = 0.016 708 617):

$$r_{\odot} = 1.000\,140\,612 - 0.016\,708\,617\cos(M_{\odot}) - 0.000\,139\,589\cos(2M_{\odot})$$

If we use Eq. (4-15) and remember that the ecliptic latitude of the Sun is zero, the position vector in geocentric equatorial coordinates (J2000) is

$$\vec{r}_{\odot} = r_{\odot}\cos(\lambda_{ecliptic})\hat{I} + r_{\odot}\cos(\epsilon)\sin(\lambda_{ecliptic})\hat{J} + r_{\odot}\sin(\epsilon)\sin(\lambda_{ecliptic})\hat{K}$$

We find the right ascension and declination using Eq. (4-20) and other known values as necessary.

ALGORITHM 29: SUN $(JD_{UT1} \Rightarrow \vec{r}_{\odot})$

$$T_{UT1} = \frac{JD_{UT1} - 2{,}451{,}545.0}{36{,}525}$$

$$\lambda_{M_{\odot}} = 280.460^{\circ} + 36{,}000.770\, T_{UT1}$$

LET $T_{TDB} \cong T_{UT1}$

$$M_{\odot} = 357.527\,723\,3^{\circ} + 35{,}999.050\,34 T_{TDB}$$

$$\lambda_{ecliptic} = \lambda_{M_{\odot}} + 1.914\,666\,471^{\circ}\sin(M_{\odot}) + 0.019\,994\,643\sin(2M_{\odot})$$

$$r_{\odot} = 1.000\,140\,612 - 0.016\,708\,617\cos(M_{\odot}) - 0.000\,139\,589\cos(2M_{\odot})$$

$$\epsilon = 23.439\,291^{\circ} - 0.013\,004\,2 T_{TDB}$$

$$\vec{r}_{\odot} = \begin{bmatrix} r_{\odot}\cos(\lambda_{ecliptic}) \\ r_{\odot}\cos(\epsilon)\sin(\lambda_{ecliptic}) \\ r_{\odot}\sin(\epsilon)\sin(\lambda_{ecliptic}) \end{bmatrix} \text{AU}$$

▼ **Example 5-1: Finding the Sun Position Vector.**

GIVEN: April 2, 1994, 00:00 UTC

FIND: Geocentric position vector of the Sun

Begin by finding the Julian date using Algorithm 14, JD_{UTC} = 2,449,444.5. Letting $JD_{UTC} \approx JD_{UT1}$, the Julian centuries are

$$T_{UT1} = \frac{JD_{UT1} - 2{,}451{,}545.0}{36{,}525} = -0.057\ 508\ 555$$

Let $T_{TDB} \cong T_{UT1}$. The mean longitude of the Sun and other parameters are

$$\lambda_{M_\odot} = 280.460\ ° + 36{,}000.770\ T_{TDB} = 10.107\ 710°$$

$$M_\odot = 357.527\ 723\ 3° + 35{,}999.050\ 34 T_{TDB} = 87.274\ 329°$$

$$\lambda_{ecliptic} = \lambda_{M_\odot} + 1.914\ 666\ 471° \sin(M_\odot) + 0.019\ 994\ 643 \sin(2M_\odot) = 12.022\ 110°$$

$$\phi_{ecliptic} = 0°$$

As a check, the *Astronomical Almanac* lists ecliptic latitude as 0.12" or 0.000 033 3° and longitude as 12°01'20.39" or 12.022 330 56°. The obliquity of the ecliptic is

$$\epsilon = 23.439\ 291° - 0.013\ 004\ 2 T_{TDB} = 23.440\ 038\ 8°$$

The magnitude of the distance to the Sun is

$$r_\odot = 1.000\ 140\ 612 - 0.016\ 708\ 617 \cos(M_\odot) - 0.000\ 139\ 589 \cos(2M_\odot) = 0.999\ 485\ 0 \text{ AU}$$

The vector referred to the epoch of date is

$$\vec{r}_\odot = \begin{bmatrix} r_\odot \cos(\lambda_{ecliptic}) \\ r_\odot \cos(\epsilon) \sin(\lambda_{ecliptic}) \\ r_\odot \sin(\epsilon) \sin(\lambda_{ecliptic}) \end{bmatrix} = \begin{bmatrix} 0.977\ 563\ 6 \\ 0.191\ 002\ 0 \\ 0.082\ 812\ 5 \end{bmatrix} \text{AU} = \begin{bmatrix} 146{,}241{,}432 \\ 28{,}573{,}499 \\ 12{,}388{,}571 \end{bmatrix} \text{km}$$

D. A. Vallado, *Fundamentals of Astrodynamics and Applications*, El Segundo, CA: Microcosm Press and Kluwer Academic Publishers, 2001, pp. 265-268